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PERFORMANCE ANALYSIS OF BROADBAND AND MULTIDIRECTIONAL PIEZOELECTRIC VIBRATION-BASED ENERGY HARVESTERS

Virgilio Junior Caetano, vj.caetano@mecanica.coppe.ufrj.br[1]
Marcelo Amorim Savi, savi@mecanica.coppe.ufrj.br[1]

[1]Department of Mechanical Engineering – Center for Nonlinear Mechanics, Universidade Federal do Rio de Janeiro, 21.941.972, Rio de Janeiro, Brazil

Abstract: *Vibration-based energy harvesting systems have been investigated as an alternative to power sources like batteries that have limited lifetime, need periodic recharge, and eventually replacement. In this regard, ambient vibration excitations may feature in a multidirectional and broadband frequency spectrum. Usually, piezoelectric energy harvesting devices presented in the literature are still associated with unidirectional behavior, being suitable for operating under transverse vibration direction, which cannot explore all energy potential available in the environment. This work presents a comparative analysis between two multimodal piezoelectric mechanical energy harvesting systems to operate under multidirectional and broadband excitation sources: 8-irregular-slice pizza-shaped (8-ISPS-PM) and star-shaped (SSH) configurations. Multiple inertial pendular-like masses are used to harvest energy from three-direction excitation sources with a broadband spectrum. The devices are modeled based on the finite element method using the ANSYS software. Modal, harmonic, and transient simulations are carried out in order to design the system to operate in the required frequency range and to investigate the system performance. Results for in-plane and out-of-plane excitations show that the proposed devices have potential to operate in a wideband frequency spectrum, serving as an alternative to conventional linear cantilever-type energy harvester with a single vibration mode.*
Keywords: *Energy Harvesting, Multimodal systems, Multidirectional harvester*

1. INTRODUCTION

The continuous evolution of electronic devices has been observed in recent decades toward the development of smaller, more robust and low-power consuming electronic systems. The energy harvesting concept has been considered as a promising solution to power portable autonomous electronic devices due to its low maintenance cost and sustainability. Especially nowadays with the movement toward the *Internet of Things* (IoT), with smart devices, sensors and wireless microelectronic, energy harvesting devices has been studied as an alternative to conventional power sources like batteries because of their limited lifetime and need for periodic recharging or replacement.

Energy harvesting is the process of generating electricity from harness energy available in the surrounding environment that, otherwise, would be wasted. Among several smart materials used as transducers mechanism, piezoelectric energy harvesters (PEH) have received most attention due to its simplicity, compatibility and high-power density. A typical linear energy harvester usually presents some limitations such as narrow operational bandwidth and unidirectional behavior, regardless of the structure and transduction materials (Erturk and Inman, 2008a; Erturk, A and Inman, 2008b; Erturk and Inman, 2008c; b; Erturk, Renno and Inman, 2009). This limitation encouraged a variety of strategies to broaden the operational frequency range of linear energy harvesters. Multimodal energy harvesters constitute an interesting alternative to increase the operational frequency range by establishing multiple resonant peaks, but also could be designed to operate under multidirectional excitations.

Multimodal energy harvesters constitute an interesting approach where different oscillators can be coupled together. Several works (Wang and Liao, 2017; Zhao *et al*., 2018; Suresh *et al*., 2019) have been developed devices that can efficiently generate more energy. Energy harvester devices with two (Jang *et al*., 2011; Kim *et al*., 2011) and three-DOFs (Upadrashta and Yang, 2018; Li *et al*., 2019a) have been designed to be very close, expanding the operational frequency range. Li *et al*. (2019a) and Upadrashta and Yang (2018) proposed multi-branch device configurations to generate electrical energy from low frequency, amplitude, and wide bandwidth vibrations, achieving an average power around 60 μW. Sandwich structure harvesters with different core materials are evaluated showing an increase of 22% on the output voltage and 26% reduction on the operating frequency compared with the conventional device without core material (Li *et al*., 2019b). Caetano and Savi (2022) proposed and analyzed multimodal pizza-shaped piezoelectric energy harvesters. The authors proposed a finite element-based analysis to designs, optimized and investigate multimodal systems to operate under different types of ambient vibration sources with wideband spectrum.

Although these devices are proven to extend the operational bandwidth, they still suffer from unidirectional sensitivity, usually been suitable for operating under transverse vibration direction. Little scientific effort (Hung *et al*., 2015; Su and Zu, 2013; Yu *et al*., 2015; Fattahi and Mirdamadi, 2019, 2020) have been carried out on PEH subjected to multidirectional and broad bandwidth excitations, such as ocean wave, wind and human motion. Yu *et al*. (2015) investigated the mechanism of vibro-impact between a spiral cylindrical spring system and multi-piezoelectric beams to harness energy from three-directional vibration excitations. The bandwidth gain assumes that the system operates associated with high energy orbits that would require a controller to maintain the harvester operation efficiency. Hung *et al*. (2015) that employed an inertial pendular mass to convert three-axis vibration energy into electricity using four piezoelectric beams. Although the harvester provides multidirectional sensitivity, being interesting for extracting energy from different direction excitations, the system presents narrow frequency bandwidth characterized by a single mode and losing performance for wideband excitations. Caetano and Savi (2022) proposed a multimodal star-shaped energy harvester device (MSS) that employs inertial pendular masses to extract energy from broadband excitations in three-axis directions.

This work presents a comparative analysis between two multidirectional and multimodal piezoelectric mechanical energy harvesting systems: 8-irregular-slice pizza-shaped (8-ISPS-PM) and star-shaped (SSH) configurations, that are based on previously proposed configurations (Caetano and Savi, 2021, 2022). The devices are modeled using ANSYS finite-element software. Modal and harmonic analyses are carried out to determine the system response in the steady-state regime. Two performance metrics, the frequency bandwidth (δ) and area under the power density spectrum curves (A), are utilized to establish suitable performance conditions for both devices under multidirectional ambient excitation.

2. HARVESTERS DESIGN CONCEPT AND MODELING

The two multimodal piezoelectric mechanical energy harvesting systems investigated are showed in Fig. 1. These devices are composed of three main structures: substrate, piezoelectric patch and proof mass. The 8-ISPS-PM configuration consists of an eight-slice pizza-shaped design that utilizes eight piezoelectric patches to convert the bending strain of the structure into electricity. The star-shaped harvester (SSH) is composed of three-leg structure having six piezoelectric patches and three inertial pendular masses, strategically positioned in the substructure.

Applying an excitation to the harvesters in the *x*-, *y*- or *z*-axis direction, the pendular masses experience inertial forces and moments due to Newton's law of inertia. Consequently, the pendular masses rotate, deforming the structure and producing voltage outputs because of the piezoelectric effect. Both harvester configurations are composed of an aluminum substrate, piezoelectric macro fiber composite material (MFC) and inertial pendular masses are made of steel alloy. The MFC material operates under d_{31} piezoelectric mode, being used to convert the ambient vibration into electric potential. Material properties of the harvester are presented in Tab. 1. Additionally, the ambient vibration is transferred to the system through a base support located at the center of the substrate as shown in Fig. 1.

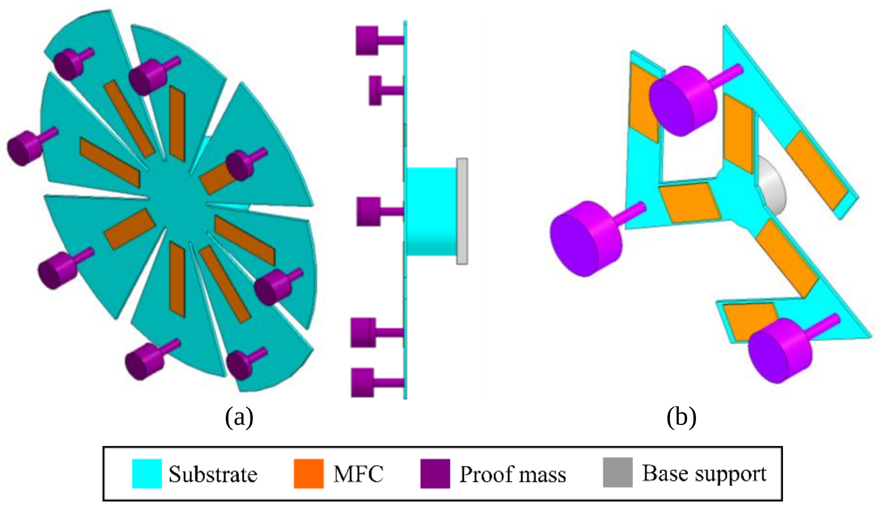


Figure 1. **(a) 8-ISPS-PM and (b) SSH devices.**

Table 1. **Material properties of energy harvesters.**

Parameter	Substrate	SSH	8-ISPS-PM
Material	Aluminum	MFC2814	MFC2807
Density (kg/m^3)	2700	5440	5440
Elastic modulus (GPa)	69	30.336	30.336
Poisson ratio	0.33	0.3	0.3
Length (mm) × width (mm) × thickness (mm)	–	28 × 14 × 0.3	28 × 07 × 0.3
Piezoelectric constant (C/m^2)	–	−5.16	−5.16
Relative permittivity	–	1900	1900
Permittivity constant (pF/m)	–	8.854	8.854
Modal damping ratio	0.02	–	–

A piezoelectric analysis involves bidirectional coupling between structural and electrical fields which are represented, respectively, by a displacement vector, $\{u\}$, and electrical potential, φ. Thus, in a tridimensional analysis, the system is described by four partial differential equations with four unknown variables, three mechanical displacements u_i, and one electric potential φ.

The linear piezoelastic equations, including the mechanical and electrical equations of motion are given by

$$\frac{\partial T_{ij}}{\partial x_j} + f_i = \rho \ddot{u}_i \tag{1}$$

$$\frac{\partial D_j}{\partial x_j} - q = 0 \tag{2}$$

where ρ is the material density and q is the electrical charge density. The kinematic relation, Maxwell law and constitutive equations for a linear piezoelectric material can be defined as follows

$$S_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) \tag{3}$$

$$E_i = -\frac{\partial \phi}{\partial x_i} \tag{4}$$

$$\begin{aligned} T_{ij} &= c_{ijkl}^{E} S_{kl} - e_{kij} E_k \\ D_i &= e_{ikl} S_{kl} + \varepsilon_{ik}^{S} E_k \end{aligned} \tag{5}$$

where T_{ij} and S_{ij} are, respectively, the stress and strain tensor components; E_k and D_i are, respectively, the electric field and electrical displacement components. Noting that the indexes $i, j, k = 1,2,3$.

After the application of the variational principle and finite element discretization (Allik, and Hughes, 1970), interpolation functions are used to express the continuous displacement and potential in terms of nodal values, resulting in the electromechanical equation of motion for a single element:

$$\begin{bmatrix} [m] & [0] \\ [0] & [0] \end{bmatrix} \begin{Bmatrix} \{\ddot{u}\} \\ \{\ddot{\varphi}\} \end{Bmatrix} + \begin{bmatrix} [c^s] & [0] \\ [0] & -[c^d] \end{bmatrix} \begin{Bmatrix} \{\dot{u}\} \\ \{\dot{\varphi}\} \end{Bmatrix} + \begin{bmatrix} [k] & [k^c] \\ [k^c]^t & -[k^d] \end{bmatrix} \begin{Bmatrix} \{u\} \\ \{\varphi\} \end{Bmatrix} = \begin{Bmatrix} \{f\} \\ \{q\} \end{Bmatrix} \tag{6}$$

where $[m]$, $[k]$ and $[c^s]$ are the inertial, stiffness and structural damping matrices, respectively; $[k^d]$ and $[c^d]$ are the dielectric permittivity and dielectric damping matrices, respectively; $[k^c]$ is the piezoelectric coupling matrix where the subscript $(\ )^t$ indicates that the matrix is transpose; $\{f\}$ and $\{q\}$ are, respectively, the vectors of force and electric charge. Therefore, by performing a simple nodal addition of elemental contributions, the equation of motion for a linear structural system with piezoelectric coupling can be rewritten in a compact form (Allik, and Hughes, 1970).

$$[M]\{\ddot{\tilde{u}}\} + [C]\{\dot{\tilde{u}}\} + [K]\{\tilde{u}\} = \{F\} \tag{7}$$

where $[M]$, $[C]$ and $[K]$ are the global inertial, damping and stiffness matrices, respectively; $\{F\}$ is the global load vector; $\{\tilde{u}\}$ is the global nodal vector composed of the nodal displacement vector, $\{u\}$, and the nodal electrical potential vector, $\{\varphi\}$.

The energy harvesters are composed of three structures: substrate, piezoelectric patch and proof mass. ANSYS Workbench is employed for numerical simulations and three-dimensional structural elements are employed. Specifically, SOLID186 is employed to model the substrate that is a higher-order 20-nodes element that exhibits quadratic displacement behavior, having three translational degrees-of-freedom per node in the *x*-, *y*- and *z*-directions. Element SOLID226 is employed to model the piezoelectric patches, being similar to the previous one but presenting an additional degree-of-freedom of electric potential for each node. The piezoelectric material uses the d_{31} sensing mode to convert bending strain into electric potential. The harvester configurations have the proof masses assumed to be rigid bodies and modeled as point mass elements (MASS21). The electronic circuit is represented by an electric load resistance using element CIRCU94. Three types of analyses, including modal, harmonic and transient, are employed to either analyze and investigate energy harvester devices.

The performance of the energy harvesters is evaluated considering the frequency bandwidth (δ) and the area under the power density curve (A). The power density is defined as the generated output power over unit of piezoelectric volume. The frequency bandwidth (δ) is estimated by applying the following relation, $P(\omega_1) = P(\omega_2) = 0.02P_{max}$, where $\delta = \omega_2 - \omega_1$ and P_{max} is the peak output power taken from the frequency response curve. The performance metric A provides a measure of the generated power density within the operational frequency range defined in Eq. (8). Figure 2 provides a schematic representation of a frequency response curve utilized to determine the δ and A performance metrics.

$$A = \int_{\omega_1}^{\omega_2} PD(\omega) d\omega \tag{8}$$

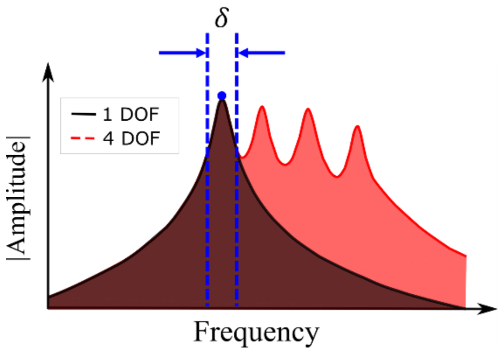


Figure 2. **Schematic representation of frequency response curve**.

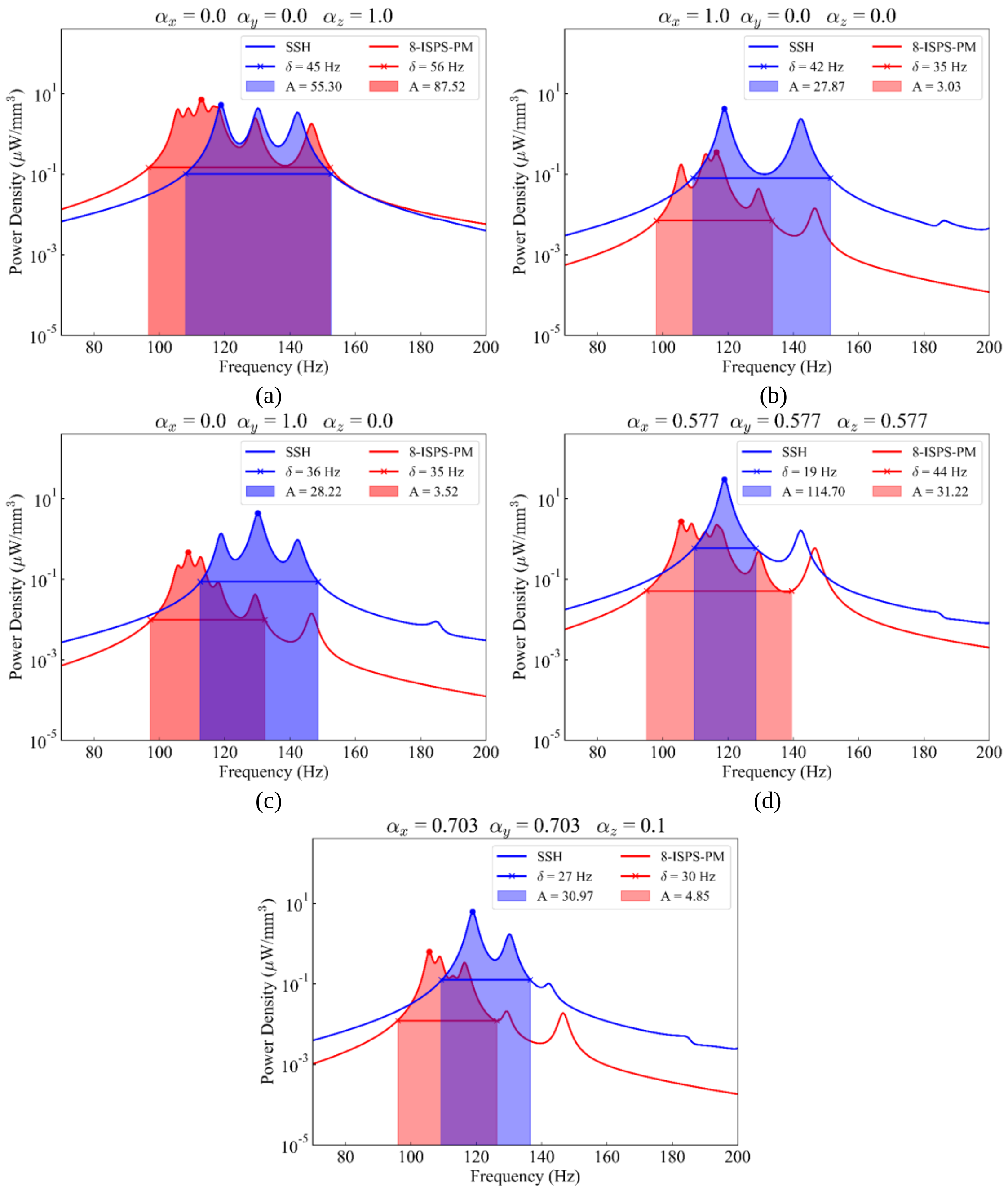
3. RESULTS ANS DISCUSSION

Multidirectional energy harvester designs employing inertial pendular masses to both tuning the resonant frequencies and extract energy from three-directional excitation sources showed to be an interesting feature to explore all potential energy available from the environment (Caetano and Savi, 2022). In this section, a comparative analysis is performed to establish suitable performance conditions for both multidirectional energy harvesting devices: 8-ISPS-PM and star-shaped (SSH) devices.

Both multidirectional energy harvesters are investigated under harmonic excitation, considering that the vibration source can be from distinct directions. Harmonic analyses are carried out for a frequency range between 70 and 200 Hz with amplitude acceleration of $a_0 = 1.0\ g$ ($\approx$ 9.81 m/s^2). A base excitation of $u_0 = a_0/\Omega^2$ displacement amplitude is applied at nodes of the base support considering $a_0 = 1.0$ g ($\approx$ 9.81 m/s^2). Different displacement amplitudes are utilized in each direction following the relations: $u_x = \alpha_x\, u_0, u_y = \alpha_y\, u_0$ and $u_z = \alpha_z\, u_0$. This is done to provide the same level of inputted energy to the harvester device for excitation in different directions by respecting the following relation: $u_0 = \sqrt{u_x^2 + u_y^2 + u_z^2}$. Table 2 summarizes five vibration conditions (I–V) including unidirectional and multidirectional excitations. Figure 3 presents a comparison of power density (PD) spectrum curves for each vibration condition.

Table 2. **Values of amplitude parameters considered for the ambient excitation.**

	Excitation	Parameters		
I	unidirectional: *z*-axis Direction	$\alpha_x = 0.0$	$\alpha_y = 0.0$	$\alpha_z = 1.0$
II	unidirectional: *x*-axis Direction	$\alpha_x = 1.0$	$\alpha_y = 0.0$	$\alpha_z = 0.0$
III	unidirectional: *y*-axis Direction	$\alpha_x = 0.0$	$\alpha_y = 1.0$	$\alpha_z = 0.0$
IV	multidirectional: *xyz*-axis direction	$\alpha_x = 1/\sqrt{3}$	$\alpha_y = 1/\sqrt{3}$	$\alpha_z = 1/\sqrt{3}$
V	multidirectional: *xyz*-axis direction	$\alpha_x = 0.703$	$\alpha_y = 0.703$	$\alpha_z = 0.1$



(e)

Figure 3. **(a) Frequency response curves of power density under 1 g amplitude excitation in different directions; performance comparison for frequency bandwidth (δ) and area under PD spectrum (A).**

The ambient excitation condition has an evident effect in the number of resonant peaks, amount of energy generated and bandwidth for both harvesters. Based on the frequency response curves, a performance analysis is presented in Fig. 4, providing a quantitative comparison regarding the frequency bandwidth (δ) and area under PD curves (A) metrics. Results show that the 8-ISPS-PM harvester presents better performance for out-of-plane (z-axis direction) excitation (I) compared with the SSH harvester, providing gain of 37% in PD and 20% in bandwidth since eight resonant peaks are designed to be within the desired frequency range while the SSH device has three resonant peaks. Under unidirectional in-plane excitation in the x-axis (II) and y-axis (III) directions, the 8-ISPS-PM configuration presents poor performance as can be observed for narrow bandwidths and more than 700% less PD compared with the SSH device. Moreover, for a multidirectional excitation with the same level of energy in the three-axis directions (IV), besides having narrow bandwidth (58%), the SSH device provides better performance (267%) than the 8-ISPS-PM device regarding parameter A as shown in Fig. 4. For condition V, where the excitation energy in mainly in the x-axis and y-axis directions, the SSH device outperforms the 8-ISPS-PM presenting higher values for metric A which correspond to an improvement of 538%, however, narrow bandwidth (10%) is obtained.

The 8-ISPS-PM device has a better performance only under unidirectional out-of-plane excitation as observed in Fig. 3.a and Fig. 4. In this scenario, the pizza-slices from the 8-ISPS-PM undergoes bending motion, being more efficient in terms of generating energy per unit of piezoelectric volume as can be observed Tab. 3. Besides, the SSH device undergoes bending motion, the strain distribution is concentrated near the center of structure, as consequence, only the piezoelectric patches P4–P6 are essentially contributing to generated output power and therefore, the 8-ISPS-PM has better PD performance under unidirectional out-of-plane excitation condition.

The SSH harvester showed superiority under unidirectional in-plane excitation conditions compared with the 8-ISPS-PM device as shown in Fig. 3b-c and Fig. 4. For in-plane *x*-axis direction excitation, the slices (7) and (8) undergoes torsion motion while the remaining slices undergoes bending with strain distribution concentrated near the center. Nevertheless, for this scenario, the SSH device provides more evenly strain distribution over the whole area of piezoelectric patches P4–P6, as consequence, better PD performance is obtained. Similar behavior is observed under in-plane *y*-axis direction excitation where torsion is observed in the slices (5)–(6) and bending for the remaining slices of 8-ISPS-PM device. The better PD performance of SSH device compared with the 8-ISPS-PM can be understood from the strain distribution presented in Tab. 3.

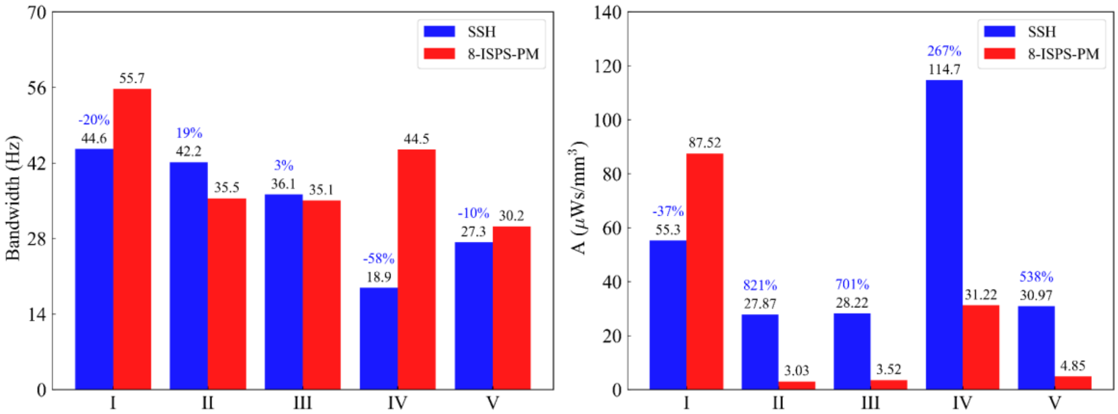
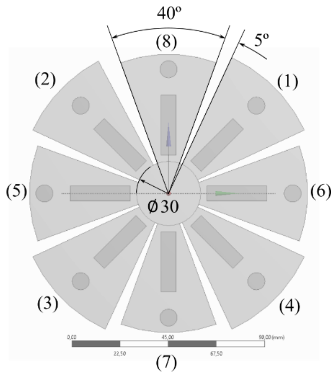
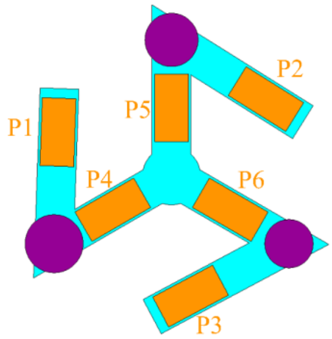
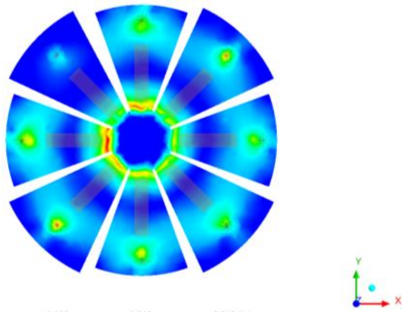
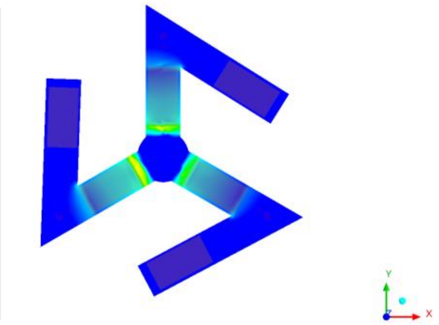
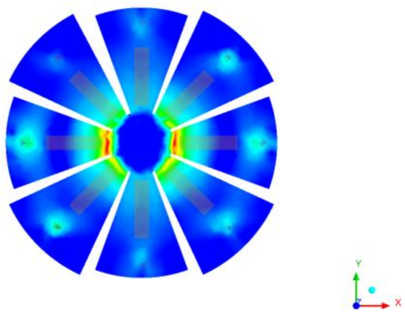
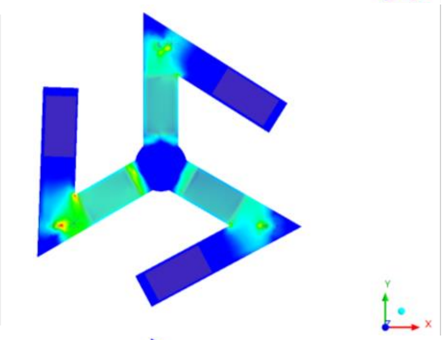
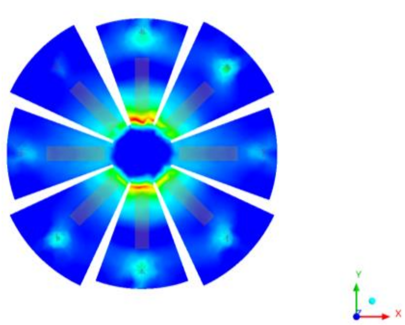
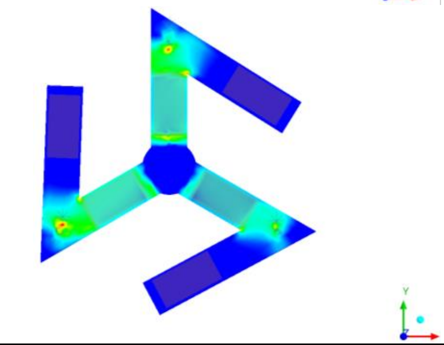


Figure 4. **Comparative analysis of frequency bandwidth and performance metric *A* for energy harvesters under excitation in different directions**

Table 3. **Relative strain distribution for the 8-ISPS and SSH devices under excitation in distinct directions.**

Excitation	8-ISPS-PM	SSH
		
Out-of-plane *z*-axis direction		
In-plane *x*-axis direction		
In-plane *y*-axis direction		

4. CONCLUSIONS

This work presented a comparative analysis between to two different piezoelectric mechanical energy harvesting devices designed to operate in a wideband frequency spectrum and from multidirectional vibration sources. The energy harvesters utilize pendular masses to take advantage of Newton's law of inertia to extract ambient energy from multi directions. The system is modeled and investigated using the finite element method. Modal and harmonic simulations are conducted, respectively, to obtain the system response in the steady-state regime. Results for in-plane and out-of-plane vibration show the performance of both in terms of power density, broadband frequency spectrum and multidirectional excitation capability. The SSH harvester showed superiority under multidirectional excitation conditions compared with the 8-ISPS-PM device, converting the bending strain into electricity more efficiently from three-axis directions excitations and therefore, being able to explore all potential energy available from the ambient excitation. Nevertheless, in the SSH configuration, the piezoelectric patches P1–P3 are not being used efficiently.

5. ACKNOWLEDGMENTS

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7. DECLARATION OF CONFLICTING INTERESTS

The authors declared no potential conflict of interest with respect to the research, authorship, and/or publication of this article.