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BICYCLE WHEEL DESIGN: THE IMPORTANCE OF LATERAL EFFORTS IN DESIGN PROJECTS

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Resumo: *The bicycle is one of the most efficient machines created by man as it transforms almost all the energy generated by them into propulsion. This is mainly due to the wheels. In literature, the analysis of stress in bicycle wheels is almost unanimous considering only radial efforts, that is, disregarding the effects of lateral efforts caused by the way of riding and accelerations outside the saddle produced by professional athletes. This work proposes a design project of a high-performance mountain bike wheel and shows what has been most used in the literature in calculating efforts and stiffness analyzes. The 29-inch mountain bike wheel created through a computer-aided design is analyzed using the finite element method. Still, it shows how the maximum stresses generated in the wheel due to lateral efforts generated by cyclists in accelerations out of the saddle are up to 38.2% higher than when compared to the tension calculated in a highly sudden braking with the cyclist's weight being doubled and the wheel torque being approximately 6 times higher. The location of the maximum stress confirms the main place of failure of the wheels.*

Keywords: *Bicycle wheel design, Mountain Bike, Lateral efforts, finite element method*

1. INTRODUCTION

The bicycle can be one of the most popular and used vehicles in the world and is capable of converting much of the energy generated by man into propulsion. Today there are several models on the market adapted for each type of application. If the sport is mountain biking, there are wheels more adapted to this type of terrain with greater robustness. If the sport is road cycling, aerodynamics certainly becomes a predominant factor.

The bicycle spoked wheel project has been isolated by the literature and few works have been found in recent years, although technological and industrial development has certainly not stopped, but it is hardly publicized. The last relevant works on the subject were: Brandt (1988), Salamon and Oldham (1991), Brandt (1993), Hartz (2002) and Mariappan, *et al*. (2003) and were subsidized by finite element techniques.

Some researchers have done laboratory studies (experimental on real models) to understand the behavior of the wheel on some issues, such as: Gavin (1996), Burgoyne and Dilmaghanian (1993) and Price & Akers (1985). Others researchers sought analytical solutions for the bicycle's spoked wheel, in order to predict behavior and improve it. Such as: Pippard, *et al*. (1952); Goldenberg (1980); Salomon and Oldham (1991); Burgoyne and Dilmaghanian (1993); Gavin (1996) and Wilson and Papadopoulos (2004). The solutions of Pippard, *et al*. (1952) are still used today to compare the results of numerical solutions using the finite element method, but he considers the wheel as if it were in a single plane, that of the rim.

All of these researchers concentrated on analyzing the wheel in two dimensions, where the main efforts are on the radial plane (the rim plane), and this approximation causes the differences between experimental and analytical data to reach differences of 800%, according to Keller (2013). The high complexity of the wheel structure in all directions has shown that numerical methods have been more consistent with reality. Even so, Keller (2013) did the only work found in the bicycle spoked wheel project considering lateral efforts using the finite element method.

Motivated by the lateral fragility evidenced by the appearance of larger wheels for mountain bikes, the author researched more about lateral stiffness trying to find the best bicycle spokes mount in order to optimize radial, torsional and lateral stiffness at the same time. The author also compared them with identical and real models in the laboratory, finding the lags of the theoretical results as in the previous cases and approving the finite element method as closest to the laboratory results.

This work deals with the pre-design of a 29-inch mountain bike type wheel and will perform the calculus of the main efforts on a bicycle wheel considering all directions: radial, lateral and torsional. A discussion will be made of what is found in the literature about the wheel design and its rigidity in all directions. A wheel model will be proposed and the maximum von Mises stresses and strains will be calculated by the finite element method (FEM).

The proposed wheel will face two scenarios of great tensions found in competitions: cyclists accelerating the bikes off the saddle (sprint) and abrupt braking. This is the first work that will analyze the effect of off-saddle accelerations by professional cyclists on wheel stress, confronting the old research that considered efforts only on the radial plane and that even, according to Brand (1993), that the efforts generated by cyclists would be insignificant in the analysis of wheel stresses. The bicycle is one of the few devices that allows the energy generating force of the mechanism to tilt the wheel to discharge part of the cyclist's weight against the pedals and the same in order to obtain more power.

Therefore, this work will be divided as follows: section 2 brings the main concepts about the spoked wheel. Section 3 presents the methodology of the work: the proposal of a wheel with greater resistance and lateral stiffness, the efforts estimated on this wheel in two highly demanding scenarios and the modeling by FEM. Section 4 contains the results and discussions of these simulations and section 5 concludes this work.

2. BACKGROUNDS

In general, the current wheel is composed of the following components: rim, spoke, hub and nipple. The assembly of the current wheel has an efficient arrangement with regard to the stress relief arrangement. In general, the assembly takes place by positioning the spokes to the hub and then, through the rotation of the nipples, there is the tensioning of the spokes at the union between the spoke and the rim, forcing the rim in the direction of the hub.

The quality of this traction directly interferes with the rigidity of the wheel in all directions and guarantees the union of the elements. A low traction does not generate enough friction in the nipples to guarantee the spoke fixation to the wheel, which can severely compromise the structural integrity of the wheel. On the other hand, it can leave it with low rigidity, compromising the transmission efficiency of the energy produced by the cyclist (Brandt, 1993).

The spoke is the element that suffers the greatest and most varied stresses in the bicycle spoke wheel and the predominant is that of traction, due to the pre-tension inserted in the assembly of the wheel. If the spoke enters liquid compression, the wheel structure is compromised (Mariappan, *et al*. 2003). It was the element whose evolution led to a great reduction in mass and has the greatest influences on the structure of the wheel, it is the main element that its shape and adjustment leads to the definition of all types of wheel stiffness.

Radial stiffness is the resistance of the wheel to undergo deformation when subjected to stress on the rim plane. The first and most important effort suffered by the wheel is considered radial, the cyclist's weight. According to Brandt (1993), the weight of the cyclist can be distributed on both wheels with the percentages of approximately 60% for the rear wheel and 40% for the front wheel. In all the solutions found in the literature, the cyclist's weight is considered purely radial in the design of the wheel. In a dynamic loading situation, when the wheel is impacted by holes or large obstacles, the catastrophic impact force on the wheels can reach up to five times the weight of the cyclist. But when there is sudden braking and tolerable impacts, the force is up to 500N. There is also the compression due to the tire air, which pushes the rim against the spokes and it can be reached for a typical pressure of 0.86 Pa at approximately 300N (BRANDT, 1993).

It can be said that the pre-tensioning of the spokes is also an effort that is in the plane of the rim and in the radial direction. This is a function of the number of spokes in the wheel, because the smaller the number of spokes, the greater the pre-tension. However, according to Brandt (1993), this value should not exceed about 1000N of traction. The intensity of the pre-tension has the intensity calculated by the Equation (1).

$$T_{pt} = \frac{A_r\ \sigma_{esc}}{8} \tag{1}$$

being T_{pt} the pre-tension [N], A_r the spoke cross section area [mm²] and σ_{esc} the yield stress of spoke material [MPa].

Although the lateral efforts do not seem to be as intense or numerous as the radial ones, to the point that they are even neglected in analytical solutions and in the literature itself, they exist and will be estimated in this work. It is common to see wheels fail when subjected to acceleration by the cyclist outside the saddle. (The cyclist when tilting, turns part of the weight into lateral effort) or turns with high speed. Thus, in Section 3, lateral efforts will be modeled for analysis in the simulation, based on the inclination and weight of the athlete in a standing acceleration.

The torque generated by the cyclist when pedaling and that generated by the actuation of disc brakes is a dynamic load and they do not increase the stresses in the spokes by more than 5%, being similar to the effect of tire pressure: Minimum. However, if the wheel has a small flange diameter, it can have considerable effects on the life of the spokes (BRANDT, 1993).

3. NUMERICAL WHEEL DESIGN AND MODELLING

Whether by tilting the bike at accelerations, lateral impacts or entering corners with the bike not perpendicular to the ground (causing a lateral effort that, depending on the speed and radius of the curve, can be large), lateral efforts exist and will be addressed in this work. In this sense, this section aims to obtain these types of efforts and to propose a wheel model that distributes all tensions more evenly.

The pre-project refers to the rear wheel, which undergoes the greatest efforts, as it carries most of the cyclist's weight and transfers torque. Two scenarios will be created for structural validation: scenario 1, where the cyclist accelerates off the bench and scenario 2, where there is intense braking and impact**.**

Some information about the bicycle-spoked wheel are inaccessible, making computational tools take on crucial importance in the development of this pre-project. In this way, several dimensions, stresses, deformations and properties of the pre-project were obtained with the SolidWorks ™ and ANSYS® programs.

3.1 Proposed Wheel Geometry

This wheel has 20 spokes and the types of spokes are radial and without elbow, in order to make the distribution of tensions more homogeneous and to reduce it, since there is no curved head at its end. The spoke is of the crossed type and the angles are based on the result of the optimization proposed by Keller (2013) aiming to maximize the rigidity of the wheel considering the directions of all efforts: *i*) Angle of spoke for the 77 ° LHS, *ii*) spoke angle for the RHS of 81° and *iii*) Equal number of spokes in the LHS and RHS. The spokes are 285 mm long on both sides of the wheel spokes. Figure 1 shows the proposed wheel.

Figure 1. **Proposed 29 Inch Mountain Bike Wheel**

3.2 Estimated efforts in the proposed bicycle spoked wheel Model

The first effort suffered by the wheel comes from the assembly, the pre-tension. According to Equation (1), seen in the previous section and having the stainless steel yield stress equal to 275 *MPa* (BEER, 1995) and the diameter of the proposed spoke of 3 mm, results:

$$T_{PT} = \frac{A_r\ \sigma_{esc}}{8} = \frac{\pi \times 1.5^2 \times 275}{8} \cong \mathbf{240N}$$

In the rear wheel there is a difference in the reinforcement angle between the LHS and the RHS, which is guaranteed by the uneven tensioning of the spokes, with the RHS spokes being more stressed. As the values of the reinforcement angle for the RHS and LHS are, respectively, 6.86 ° and 3.65 °, a proportion was made to distribute the pre-tension on both sides. The most tensioned side, represented by the red line, will have a pre-tension value of 240N. In the LHS, a trigonometric relationship will be made that cancels the horizontal components, in order to balance the wheel. See Figure 2.

From the trigonometric relations of the right triangle and equaling x (Equation 2) to y (Equation 3), the expression that provides a spoked wheel balanced in terms of pre-tension is given by Equation (4):

$$sen\beta = \frac{x}{T_{LHS}} \tag{2}$$

$$sen\beta = \frac{x}{T_{LHS}} \tag{3}$$

$$T_{LHS} = \frac{sen \propto\ T_{PT}}{sen\beta} \tag{4}$$

being T_{LHS} the claim of spokes at LHS, $\propto$ the arm angle on the RHS and β on the LHS and T_{PT} is the pre-tension calculated by Equation (1) and is the same for the spokes in the RHS.

$$\therefore T_{LHS} = \frac{sen\, 3.65 \ \times 240}{sen\, 6.85} = 128.10\, N$$

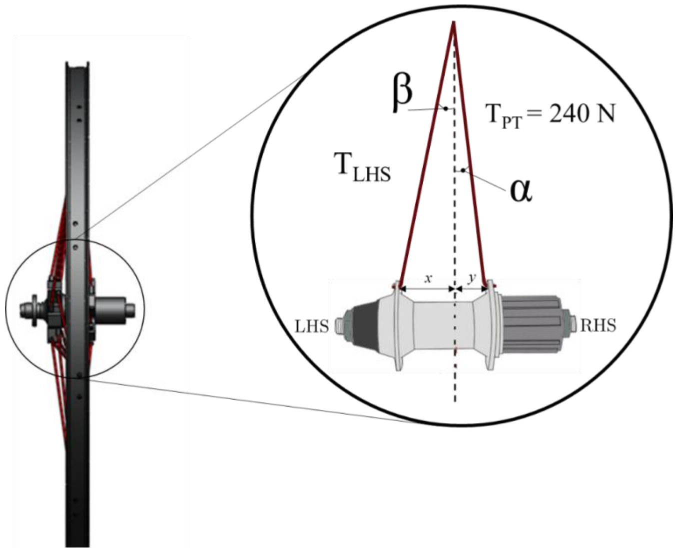


Figure 2. **Pre-tension no RHS e LHS**

These values were checked in the numerical simulation so that there is no liquid compression in the spokes, which would result in the catastrophic failure of the wheel. The divisor number was obtained through several numerical simulations of the proposed model for different pre-tension values.

According to Brandt (1993), 60% of the cyclist's weight is on the rear wheel. According to NG (2012), the average world weight of a man is 196lbf (872N), so the radial force due to the weight for a rear wheel is defined in Equation (5):

$$P_{rw} \ = \ 0.6\, P_{cy} \tag{5}$$

being P_{wr} the weight of the cyclist on the rear wheel and Pcy the cyclist's weight.

$$\therefore P_{rt} = \ 0.6 \times 872 = 523.20\, N$$

According to Brandt (1993), for a typical tire pressure, the radial force is approximately 300N and in a sudden braking or impacts, a tolerable force of up to 500N can arise.

Regarding the Torque, it can exist in two ways: acceleration or braking. A professional cyclist can generate a power of 1300 W at 116 rpm. This power is generated on the pedal, coupled to the front crown, which on a typical competition bike has 42 teeth. Transmitted to the rear wheel, which in an end-of-race sprint must be in the last gear set, with 11 teeth and knowing that the power is conserved, we have from Equations (6) and (7) the Equation (8):

$$P_p = P_p\, \omega = T_p \times 2 \times \pi \times \frac{f}{60} \tag{6}$$

$$\frac{N_p}{N_c} = \frac{T_p}{T_c} \tag{7}$$

$$\therefore T_c = T_r = \frac{T_p\, N_c}{N_p} = \frac{P_p\, 60 \ \ N_c}{2\, \pi\, f\, N_p} \tag{8}$$

being P_p the power of the cyclist on the pedal, N_c the number of teeth in the gear set, N_p the number of teeth in the crown and f the frequency in RPM.

$$\therefore \ \ T_r = \frac{P_p\, 60 \ \ N_c}{2\, \pi\, f\, N_p} = \frac{1300 \times 60 \times 11}{2 \times \pi \times 116 \times 42} = 28.03\, Nm$$

The force of the hydraulic disc brake is a static load, but with varying intensity depending on the climate, speed and weight of the cyclist, etc. However, for safe braking, it is recommended that between the tire and the ground, there is a force that is half the cyclist's weight (ALLEN, 2013). So we have Equation (9):

$$T_r = -\, \frac{P_{cy}}{2} \times \frac{2.54 \times D_r}{2 \times 100} = 0.00635\, P_{ci}\, D_r \tag{9}$$

being P_{cy} the cyclist´s weight and D_r the wheel diameter.

Therefore, we have through Equation (9) and for a cyclist weighing 872N on a 29-inch wheel bike:

$$\therefore\ T_r\ = 0.00635 \times 872 \times 29 = -160.58\ Nm$$

Now a new concept will be addressed: the determination of the lateral force generated by the acceleration of a cyclist off the saddle. No work was found in the literature that approached this way. A lateral effort value will be estimated for the situation in which the cyclist tilts the bicycle and unloads all its weight on a single pedal. Figure 3 shows the right triangle used to deduce the equation for determining lateral strength, represented in Equation 10 and still provides a typical slope value for a professional athlete.

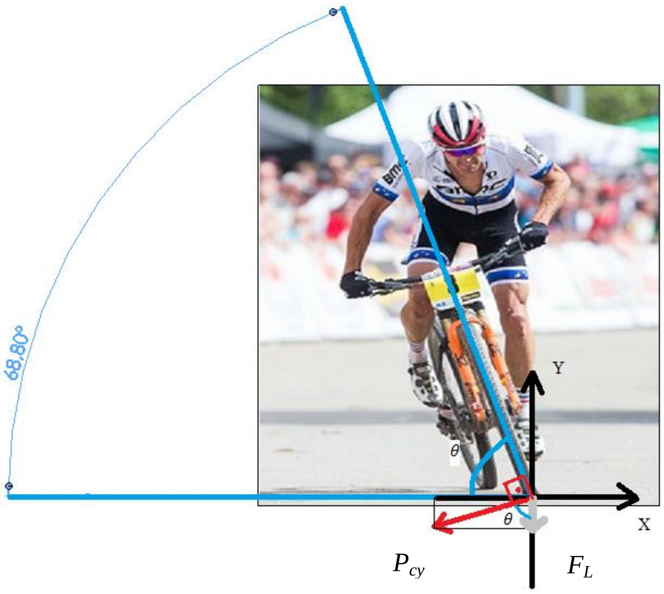


Figure 3. **Cyclist in a sprint**

$$F_L\ =\ G\ P_{cy}\ cos\ \theta \tag{10}$$

being FL the lateral effort on the wheel, PCY the cyclist´s weight, G the weight distribution between the front and rear wheels (for Brandt it is 0.6 at the rear and 0.4 at the front) and θ the angle between the rim-containing plane and the ground plane.

$$\therefore F_L = 0.6 \times 872 \times cos\ 68{,}8 = 189.20\ N$$

All the efforts calculated above can be summarized and distributed through the two scenarios as shown in Table 1.

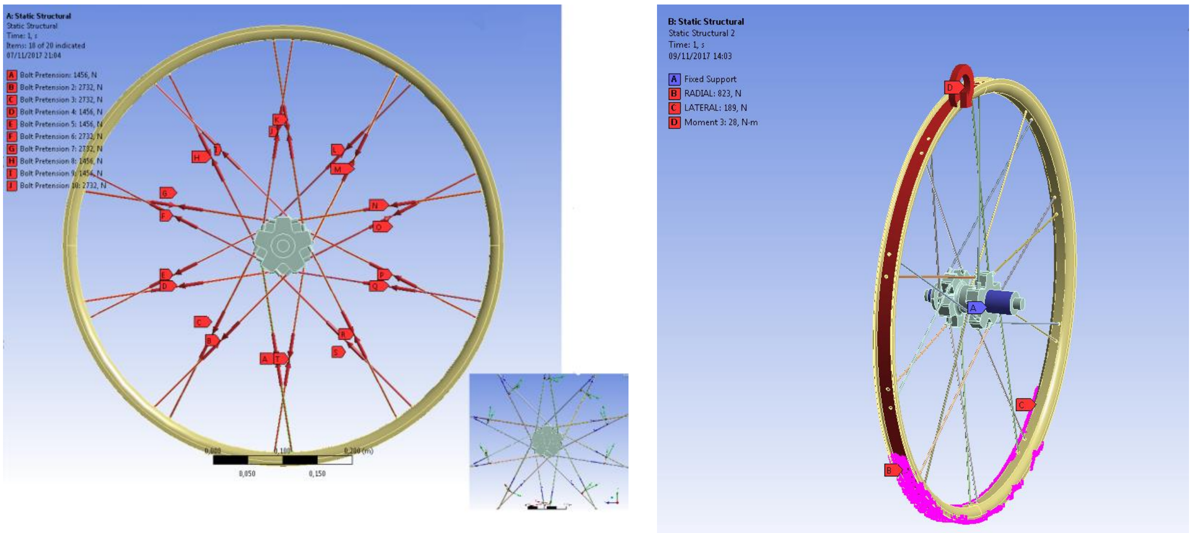
Table 1. **Summary of estimated efforts for the two study scenarios**

Scenario	Description	Radial Forces (N)	Torque (Nm)	Lateral Forces (N)	Pre-tension
1	Acceleration Torque	-	28.03	-	
	Inclination of the cyclist	-	-	189.20	
	cyclist weight	523.20	-	-	
	tire pressure	300	-	-	
SUM		**823.20**	**28.03**	**189.20**	**240 (RHS) 130 (LHS)**
2	braking torque	-	-160.58	-	
	Radial braking force	500.00	-	-	
	cyclist weight	523.20	-	-	
	tire pressure	300.00	-	-	
SUM		**1323.20**	**-160.58**		

3.3 Finite Element Modeling

Having the proposed wheel modeled CAD, its geometry is imported into ANSYS® to calculate the stresses and deformations in the face of the estimated efforts. The materials used in the simulation were Stainless Steel 302 (spokes) and Aluminum 6061 T6 (other components), whose yield limits are, respectively: 275MPa and 290MPa. It as defined that every contact, between Spoke / Rim and Hub / Spoke would be of the Bonded type, or fixed (bonded). In addition, that every encounter between spokes, compared to the reality where, after pre-tensioning the spokes are very close, the type of contact was rough.

Once the physical parameters of the model are determined, the mesh can be generated. According to Keller (2013), defining a 7.5mm mesh size in the entire geometry of the wheel generates credible results and with rapid simulation. The next step is to insert the efforts estimated to then solve the simulation. However, due to the complexity of the efforts, which involve Radial, Lateral, Torsional and Pre-tension forces, it was necessary to create a system composition (Component Systems). First, the pre-tension effect is simulated and then the other efforts are incorporated in a second simulation. See Figure 4.



a) Pre-tension simulation b) Efforts Insertion

Figure 4. **Double Simulation in Design wheel**

Pre-tension values in the RHS were estimated and simulated until they converged to the maximum allowable stress in the spoke. In the LHS, the force was calculated in front of Equation 1. After this step, a static balance of the wheel was observed in the application of the pre-tensions. The second simulations included the insertion of efforts related to Scenarios 1 and 2, as shown in Table 1. The wheel was fixed by the hub. The torque is also applied with reference to a surface, the outer rim of the rim, where there would be contact with the tire. After defining the efforts and the simulations, ANSYS® is asked to calculate the pre-project analysis of the maximum, minimum and local values of the equivalent Von Mises stress.

4. RESULTS AND DISCUSSION

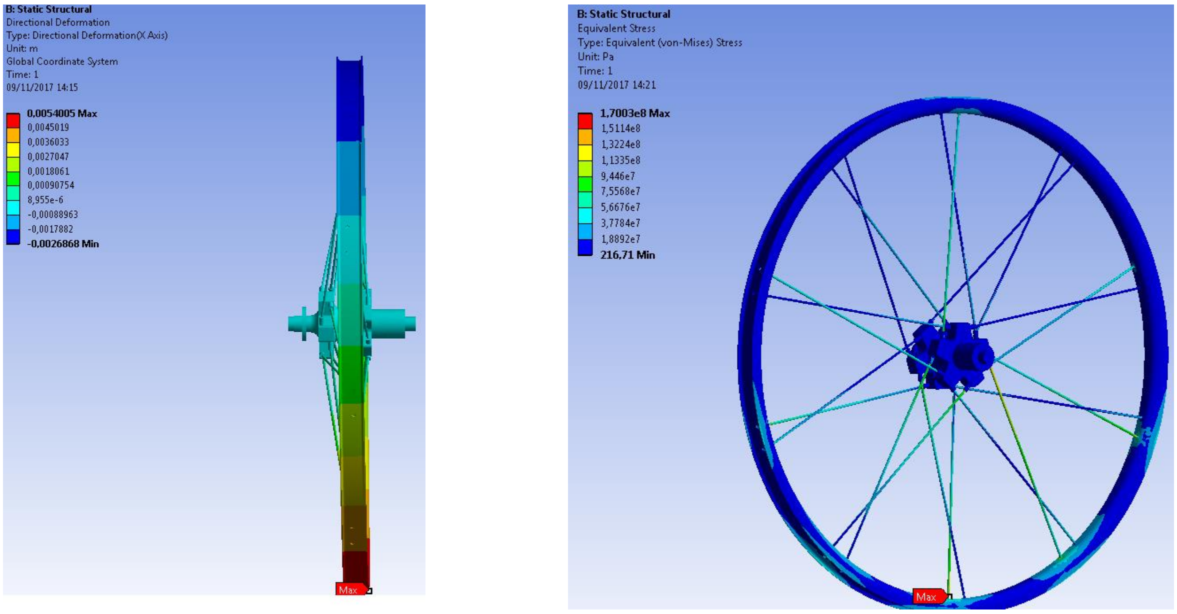
The first simulation, related exclusively to the pre-tension of the spokes, had a semi-interactive system to determine the pre-tension of the pre-projected wheel. This was necessary due to the great difference in the results of the literature with this model. No other had the same cube geometry, with an almost perpendicular exit angle and radial radios. In addition, it did not have a 29 inch wheel diameter, which greatly changes the stress field.

At the end of the process, it was determined that the ideal pre-tension for this model is 240N for the RHS and 130N for the LHS. Intensity differences explained in Equation 1. These values resulted from the search for a quarter of the flow stress of the material in the spoke, exclusively pre-tensioned and ensuring that there is no liquid compression in the spokes. The permitted stress was approximately 68 MPa, since the yield stress of the spoke material, Stainless Steel 302, is 275MPa.

A reasonable static balance of the wheel was obtained, where the maximum total deformation of the model was approximately 0.12 mm, on the rim itself. While in the Lateral direction, it was 0.06 mm and did not occur in the rim, but in the spoke itself, close to the cube.

After this simulation, another one is added considering Scenario 1 or 2. Scenario 1 is a cyclist accelerating and 2 is a cyclist under intense braking. According to the literature and the old wheel models, smaller in size to 29 inches, a higher maximum tension is expected for Scenario 2.

In the acceleration scenario, the wheel suffered a total deformation of 5.46 mm, with 5.4 mm in the lateral direction, at the base of the rim (in contact with the ground). The highest equivalent stresses (Von Mises) were at the base of the rim, close to the ground. Both the rim and the spoke. The maximum equivalent tension was 170 MPa, in the spoke, almost in the rim. It can be seen in the Figure 5.



a) Deformation b) Equivalent Stress

Figure 5. **Full-scale visual results for the acceleration scenario**

It can be seen above that the RHS spokes close to the ground and a pair of the LHS, also closer to the ground, were the most critical for the analysis of the equivalent voltage. In other words, of the compositions of all efforts, the geometry of the beam proved to be the most fragile. In the cube, the highest equivalent stress observed was 23 MPa. Place where there was the lowest tension of the entire wheel. In the rim, the highest tension found was 98 MPa.

Knowing the maximum yield stresses of these materials and dividing them by the stresses found, we have what can be called a safety factor (SF). Therefore, we have for the rim: 2.95; Spoke: 1.7 and hub: 12.6. It is possible to observe an over-dimensioning of the cube, where there is room for removal of mass. For the kidney and spoke, the values found are considered consistent with a good project, mainly because there is no fatigue study.

With the concern of not letting the rim into liquid compression, an analysis of Maximum Stress was made and analyzed spoke by spoke. The lowest value of tension found was in the rim, close to the spoke, but not in it. The lowest traction value found in the spokes was approximately 0.6 MPa, showing that there was no liquid compression in the spokes.

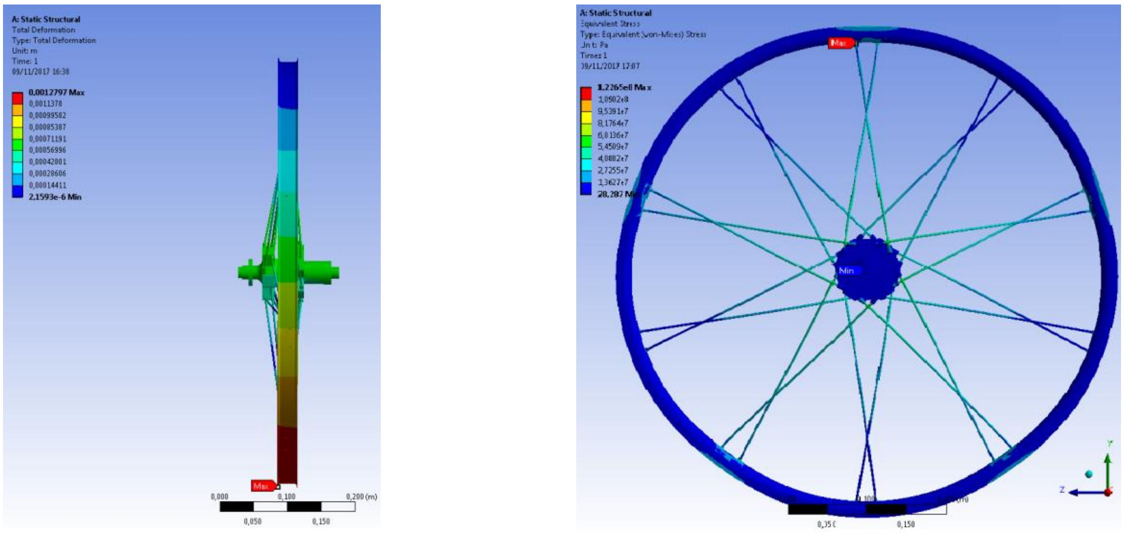
Scenario 2 represents the situation that would be possibly the most critical: when there is sudden braking. The torque generated by the brake is almost five times greater than that generated by a professional athlete (according to the calculations in Table 1) and the radial force is twice as large on the wheel. Despite having the impression that the rear wheel suffers load relief during braking, due to the cyclist's inertia, in some situations it can be even more critical. When applying the brakes in a rough terrain, the cyclist's inertia on the rear wheel can generate a destructive compressive impact system on the rear wheel.

In this situation, the wheel underwent a maximum total deformation of 1.3 mm, at the base of the rim in contact with the ground, where the amplified radial load was applied. The lateral deformation was 0.6 mm, the radial deformation was 0.15 mm and in the tangent direction to the rim (torsional), it was 0.14 mm.

The highest equivalent tension (Von Mises) recorded in this one in the wheel in this one was 123Mpa, in the spoke, close to the rim. The curious thing is that it was in the part diametrically opposite to the ground. This can be attributed to two truths seen in the literature: The torque causes the addition of pre-tension in the spokes that have the opposite direction of rotation. In the same way as it was explained in acceleration, but now in braking. As the intensity of this torque was higher, it was more significant. The highest equivalent stresses found in the Cube and Rim were 17 MPa and 57 MPa, respectively. The greatest tension in the rim is close to the spoke, revealing that the wheel is rigid enough to distribute the tension throughout its structure and not just locally. It can be seen in the Figure 6.

Due to the greater radial load, it is possible to observe the equivalent stress relief in the spokes close to the ground, mainly on the RHS side, which has a higher pre-tension value. In the upper part of the wheel it was practically the opposite, because due to the greater rigidity of the wheel, this load was not absorbed locally. Then there was compression in the spokes below the hub and increased traction (pre-tension) in the spokes above the hub, especially at the most extreme, where it was the highest equivalent tension. The minimum tension was again on the hub. Thus, with the yield stress of the spoke and rim material known, the SF for rim is $SF_R = 5$, spoke $SF_S = 2.2$ and hub $SF_H = 17$.

As the radial load is greater, more care should be taken to check for liquid compression in the spokes through the main maximum stress. It was observed that the lowest stress value found in the spokes was 0.46 MPa. Again, there was no liquid compression on the spokes. Table 2 summarizes the main results found.



a) Deformation b) Equivalent Stress

Figure 6. **Full-scale visual results for the sudden braking**

Table 2. **Comparison between the two proposed scenarios**

Item	Wheel Acceleration	Wheel braking
SF of the spoke	1.70	2.20
SF of the rim	2.95	5.00
SF of the hub	12.60	17.00
Maximum equivalent stress (*MPa*)	170.00	123.00

5. CONCLUSION

Due to the high complexity of analytical solutions, the study of the spoked wheel in the literature lacks three-dimensional studies and analysis, where lateral efforts are erroneously neglected. This weakness is evident with the development of larger rims, used mainly on the mountain bike: the 29-inch wheels. This work discusses the ways to increase the rigidity and resistance of these types of wheels and proposes a new model, where it is tested through two demanding situations in the use of them: accelerations by professional athletes outside the saddle and in sudden braking.

Contrary to what the literature says about cyclists pedaling efforts are negligible in the analysis of wheel stress, this study shows that this type of effort must be calculated and shows how. In the comparative study, it was demonstrated that the maximum equivalent stresses generated in the bicycle wheel when a cyclist accelerates while standing off the saddle in the plane is 38.2% higher than if the same cyclist is on rough terrain with maximum braking, where the torque on the wheel it is 5.71 times higher and the radial stress are 60% higher. The inclination of the bicycle by the cyclist during acceleration and the unloading of part of his weight laterally on the wheel is potentially destructive and must be considered in projects.

In addition, the main site of wheel failure in the field is in the spokes at the end, close to the rim. This study shows and coincides with this fact since in the two situations of analysis, the greatest tensions were in these places. Although in the case of acceleration, it was close to the ground and in sudden braking, in the highest spoke.

6. ACKNOWLEDGEMENTS

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