THE INFLUENCE OF THE VISCOUS PHENOMENON ON THE MECHANICAL PERFORMANCE OF A KNEE LIGAMENT

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Abstract. *The proper mechanical description of the mechanical performance of knee ligaments depends on the right modeling of its constitutive relationships. This research aims to analyze the viscoelastic behavior of porcine anterior cruciate ligament, through the realization of relaxations tests, for 3, 4, 5 and 6% of imposed strains. To accomplish this objective, a numeric model, based in Schapery's equations, was implemented using both, C# programming language and MATLAB software. The numerical implementation of Schapery's model, through the obtainment of the constants for Schapery's equations, successfully reproduced the results of stress evolution of experimental relaxation tests in porcine anterior cruciate ligament. The numerical data fits with the experimental ones with a maximum error of approximately 3%.*

Keywords*: knee ligaments, viscoelasticity, Schapery model.*

1. INTRODUCTION

The description of the mechanical behavior of the knee ligaments, considering the effect of viscous behavior can be very useful to aid to model the knee performance. Thus, researchers have been attempting to macroscopically analyze the knee ligaments/tendons through the utilization of different viscoelastic mechanical models. Bernardes *et al.* (2005) sought to determine the biomechanical parameters for modeling the human knee joint through the imposition of extensive exercises, together with images obtained by videofluoroscope, with good results.

Viscoelasticity can be understood as the property of materials that present viscous and elastic behavior at the same time. Haj-Ali and Muliana (2003) applied non-linear viscoelastic models, with a tridimensional micromechanical approach, to describe the behavior of some polymers. This concept can also be used to describe mechanical behavior of the knee ligaments. It has been verified that the viscoelastic models must be non-linear to minimally meet the real ligament behavior. Provenzano *et al.* (2002) analyzed whether both, non-linear Schapery's model and the Modified Superposition Method, could adequately model the stress behavior of ligaments subjected to relaxation. It was concluded that both models achieve this objective. Blandford, (2017) also used both models, the non-linear Schapery's model and the Modified Superposition Method, to model the behavior of medial collateral ligament (MCL) subjected to relaxation. It produced satisfactory results for both models. Ramo *et al.* (2018) developed a strain-dependent numerical integration approach for a non-linear model, that enables to calculate the current stress using the previous value of stress. Both fitting efficiency and computational tractability were improved. This methodology was validated by comparing the numerical results with experimental relaxation tests, with six strain levels, and with dynamic tests, with three frequencies. For this approach, they obtained satisfactory results with errors smaller than 3%.

The aim of this paper is to explain the numerical implementation of the Schapery's non-linear viscoelasticity model. To compare the results of the numerical model and experimental data obtained from stress-relaxation curves at 3, 4, 5 and 6% imposed strain of porcine anterior cruciate ligament (ACL). For this research, the C# programming language was used to develop all the numerical routines.

2. SCHAPERY'S NON-LINEAR VISCOELASTIC MODEL

The non-linear Schapery's model (1969, 1997, 2000) uses the Boltzmann superposition method, with thermodynamics concepts, to determine a non-linear strain-stress relation. Although, it is possible to consider the influence of temperature, in this paper it will not be used, because the relaxation tests were executed in a steady laboratory temperature.

The main limitation of Schapery's model is the dependence on the relaxation function value in the equilibrium condition (when the time tends to infinity). This is considered a limitation since the accuracy of the numerical model depends on finding the equilibrium state in the experimental data. This may require periods in the range of hours, as observed by several research. However, Blandford (2017) presented a way to avoid that problem, using curve fitting.

2.1. Mathematical description

Schapery (1969) established a non-linear stress-strain relation for the creep case. The relaxation case, an analogous equation can, also, be written as will be presented next. Moreover, it was assumed that the strain is applied as fast as a step and the loading ramp time is not considered, as in Schapery (1969), Provenzano *et al*. (2002), Duenwald *et al*. (2009) and Blandford (2017).

2.1.1. Stress-strain relation

Schapery (1969, 1997, 2000) stated the Eq. (1), in function of strain ε and time t, to describe the stress-strain relation for the general case of relaxation.

$$\sigma(\varepsilon,t) = h_e(\varepsilon)\cdot G_e\cdot\varepsilon(t) + h_1(\varepsilon)\cdot\int_0^t \Delta G\big(\rho(t)-\rho(\tau)\big)\cdot\frac{d[h_2(\varepsilon)\cdot\varepsilon(\tau)]}{d\tau}d\tau\,,\ with\ \rho(t)=\int_0^t\frac{dx}{a_\varepsilon[\varepsilon(x)]}\ and\ \rho(\tau)=\int_0^\tau\frac{dx}{a_\varepsilon[\varepsilon(x)]} \quad (1)$$

Where ΔG and G_e represent, respectively, the transient relaxation function and the relaxation function at equilibrium. The variables h_e, h_1, h_2 and a_ε are material constants and depends on strain. The first three variables depend on Helmholtz free energy and the last variable is related to the effect of temperature and depends on the entropy and the free energy of the system. The variables $\rho(t)$ and $\rho(\tau)$ represent the reduced time function and depends on a_ε. Note in Eq. (1) that the interval of integration ranges from 0 to time t and the variable of integration is τ.

The Eq. (1) can be rewritten according to Schapery (1969), Provenzano *et al*. (2002) and Duenwald *et al*. (2009), where $h_1(\varepsilon)$ and $a_\varepsilon(\varepsilon)$ can be set equal to 1, because tissue tests present low stresses. Also, for relaxation tests, the temperature variation is not considered. The Eq. (2) development is available in Silveira (2021).

$$\sigma(\varepsilon,t) = h_e(\varepsilon)\cdot G_e\cdot\varepsilon(t) + \int_0^t \Delta G(t-\tau)\cdot\frac{d[h_2(\varepsilon)\cdot\varepsilon(\tau)]}{d\tau}d\tau \quad (2)$$

The Eq. (2) can be rewritten according to Provenzano (2002) and Blandford (2017), for strains being considered constant during the test, $\varepsilon(t)=\varepsilon_i$.

$$\sigma(\varepsilon_i,t) = h_e(\varepsilon_i)\cdot G_e\cdot\varepsilon_i + h_2(\varepsilon_i)\cdot\varepsilon_i\cdot\Delta G(t) \quad (3)$$

Therefore, it was possible to obtain a simpler equation for relaxation with constant imposed strain that improves the curve fitting and the numerical implementation.

2.1.2. Transient relaxation function

Duenwald *et al*. (2009), Provenzano (2002) and Blandford (2017) proposed the utilization of a power law equation to describe the transient relaxation function (ΔG). For soft tissues, as the knee ligaments, time and stress during relaxation are supposed to be logarithmically correlated. In the Eq. (4), the parameter C represents the material's stiffness to relaxation at the initial time and n represents the relaxation rate, which is the slope on a log-log plot of stress versus time.

$$\Delta G(t) = C\cdot t^{-n} \quad (4)$$

Applying Eq. (4) in Eq. (3), it is possible to obtain the Eq. (5), which has been widely used in several studies. It was used in this research as a basis for obtaining the Schapery's constants by curve fitting.

$$\sigma(\varepsilon_i,t) = h_e(\varepsilon_i)\cdot G_e\cdot\varepsilon_i + h_2(\varepsilon_i)\cdot\varepsilon_i\cdot C\cdot t^{-n} \quad (5)$$

3. NUMERICAL IMPLEMENTATION

All codes developed in this research was based in Oriented Object Programming, with the utilization of classes and interfaces, and the SOLID principles, as in Thelma (2020). In Fig. 1, it is shown a flowchart illustrating the principal steps of the model numerical implementation.

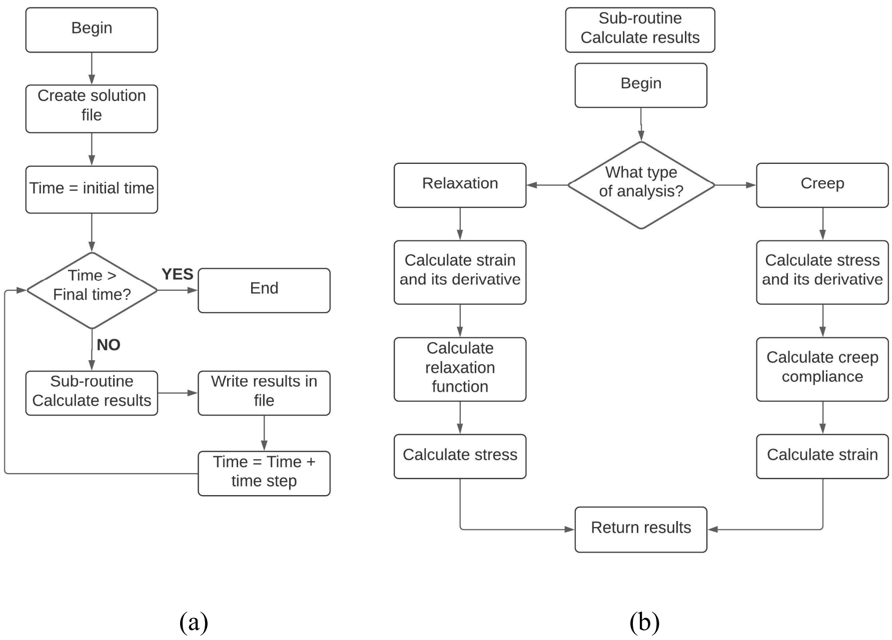

Figure 1. Flowchart for: (a) the main operation and (b) the sub-routine "Calculate Results"

The numerical implementation was made in two steps. At first step, it was developed the main class which validate the input data and orchestrates the code execution. At the second step, it was developed the class that contains all equations for Schapery's model and one method that returns the model's results for a specific time, represented in Fig. (1.b) as the "Sub-routine Calculate results". It is noteworthy that the code can perform calculations for both creep and relaxation, since the logics and equations for both analyses were implemented.

4. RESULTS

The Schapery's model was successfully applied to experimental stress-relaxation data obtained from a porcine ACL. The constants for the model were successfully obtained by curve fitting using the MATLAB software. The parameter R-Squared for the imposed strain levels 3%, 4%, 5% and 6% was, respectively, 1, 0.9998, 0.9999 and 1. The utilized approach was implemented according to Provenzano *et al*. (2002), which the curve with 3% of strain was used as base for obtaining the constants, and Blandford (2017), which G_e was obtained by curve fitting. The value of each determined constant is present in the Tab. (1).

Table 1. Constants for Schapery's model.

Constants	Value	Constants	Value
$h_e(3\%)$	1	$h_2(3\%)$	1
$h_e(4\%)$	0.8472	$h_2(4\%)$	0.6713
$h_e(5\%)$	0.8432	$h_2(5\%)$	0.6466
$h_e(6\%)$	0.852	$h_2(6\%)$	0.6411
C	-1.308	G_e	7.308
n	-0.1478		

It is noteworthy that h_2 decreased by approximately 36% from 3% to 6% strain. According to Provenzano *et al.* (2002), it indicates a reduction in relaxation rate with increasing of the imposed strain. Moreover, h_2 also decreased by approximately 16% from 3% to 4% strain. For other strain levels, the greatest variation was approximately of 1%.

The numerical results obtained using the Eq. (5) coincide with the experimental ones. In Fig. (2), it is shown the comparison between these values for each imposed strain level, 3%, 4%, 5% and 6%, where the hollow square and filled circular points represent, respectively, the experimental and numerical data.

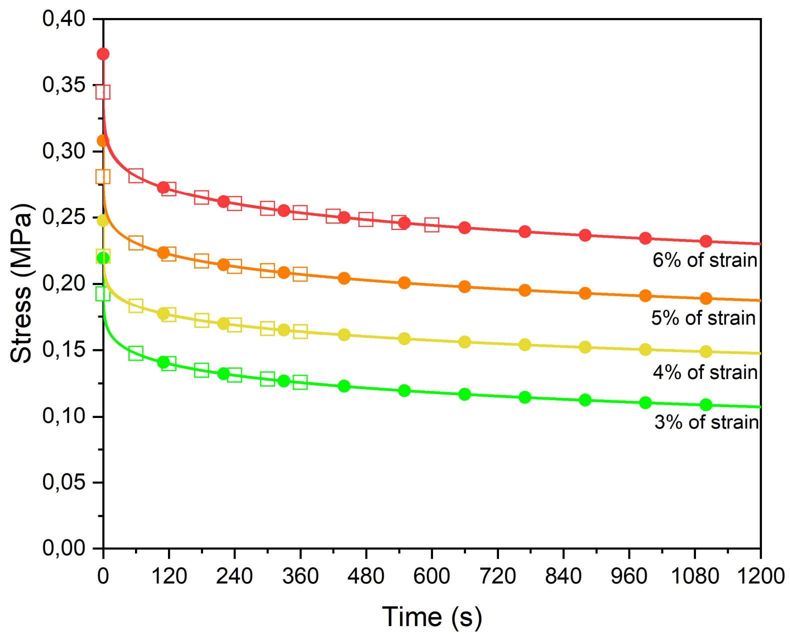

Figure 2. Relaxation tests - Stress vs time for porcine ACL

As shown in Fig. (2), the stress decreased over the time, as expected for relaxation tests. In Tab. (2), it is presented the initial stress and final stress for experimental and numerical data, for each imposed strain level. It is, also, presented how much the stresses decreased from the begin to the end of the relaxation tests.

Table 2. Stress analysis for each imposed strain, for experimental and numerical results.

Data	Strain	Initial stress (MPa)	Final stress (MPa)	Decrease
Experimental	3%	0.1924	0.1241	35.5%
Experimental	4%	0.2209	0.1623	26.5%
Experimental	5%	0.2808	0.2055	26.8%
Experimental	6%	0.3445	0.2441	29.2%
Numerical	3%	0.2192	0.1073	51.0%
Numerical	4%	0.2477	0.1475	40.4%
Numerical	5%	0.3081	0.1875	39.1%
Numerical	6%	0.3736	0.2301	38.4%

As shown in Tab. (2), although the initial stress for experimental and numerical data did not coincide, the comparation between experimental and numerical data during the development of the relaxation tests, for the 3%, 4%, 5% and 6% of imposed strains, presented diminished errors, respectively, around 2%, 3%, 1% and 1%.

5. CONCLUSIONS

The proposed numerical model performed quite well, although the initial stress diverged considerably from the experimental results. Also, it was not possible to reach to a real asymptotic value. This occurred because, as previously mentioned, for a greater precision of the Schapery's model, a prolonged test time should be necessary.

Moreover, the numerical model predicted a reduction in stress rate with increasing of the imposed strain, the same behavior found for h_2 values. Alternatively, when the numerical data is used only to predict the final stress for experimental data, the stress decreased approximately 44.2% for 3% of strain and approximately 33.2% for the others strain levels. It indicates that for the first relaxation at 3% of strain, the ligament shows a greater decrease of stiffness and for the next strain levels this decrease was reduced.

With the numerical and experimental data, it was possible to analyze the influence of the viscous phenomenon on the mechanical performance of the knee ligament, in particular, to a porcine ACL.

This text covers the numerical and experimental part of an extensive research about the porcine ACL's mechanical characterization, with focus in their viscoelastic behavior. With the development of this research, the porcine ligaments experimental/numerical results can be utilized to increase the knowledge about effect of viscous behavior on the mechanical behavior of knee ligaments.

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8. RESPONSIBILITY NOTICE

The authors are the only responsible for the printed material included in this paper.