

Operational Modal Parameters Identification Using the ARMAV Model

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Abstract: Applied system identification is an important issue in the science and engineering. Experimental modal analysis is used to describe the dynamical behavior of structures, in general, for a given set of input and output data. This article deals with multidimensional modal parameters identification valid for output-only data. This approach is interesting when the input is not known or difficult to be measured. A linear, time-invariant and finite dimensional mechanical system is considered, which is described mathematically by an autoregressive-moving-average-vector (ARMAV) model, excited by unknown operating forces assumed to be a white Gaussian process – a persistent excitation. The focus of the study is on, both, theoretical and practical aspects of the ARMAV model. Specifically, it is discussed the need of using an output-vector as reference for output-only parameters identification scheme. The model order is identified by inspection of more significant singular values of a block Hankel matrix derived directly from the formulation of the model. The AR parameters matrices of the ARMAV model, contained in a companion matrix, are determined via least-squares technique. Natural frequencies, damping factors and modal shapes are identified by means of eigenvalues and eigenvectors of that companion matrix. Examples using computational simulation are presented.

Keywords: operational modal analysis, ARMAV model, least squares approach

NOMENCLATURE

Latin symbols

$\mathbf{e}(k)$: error vector
 $h_{ij}(k)$: impulse response
 $\mathbf{H}^{(1)}, \mathbf{H}^{(2)}$: block Hankel matrices
 m : number of outputs
 n : minimal order of the system
 p : order of the AR part of model
 q : order of the MA part of model
 $\mathbf{R}_i, \mathbf{R}_i^r$: covariance and referenced-covariance matrices
 $\mathbf{y}(k)$: output vector
 z : discrete-time pole

Greek symbols

α_i : AR matrix parameters
 β_i : AR matrix parameters
 Δt : time sampling interval
 λ : continuous- time pole
 ζ : viscous damping factor
 ϕ : mode-shape

Subscripts

T : vector transposition
 $*$: complex conjugation

INTRODUCTION

Mathematical modeling is an analytical approach used to describe the dynamic behavior of a natural phenomenon based on physical laws. System identification is an experimental approach, where parametric models are fitted from measured data (Söderström and Stoica, 1987). Both approaches are important in system analysis, design and control problems.

Modal parameters can be used in analysis, updating modal via finite elements, damage detection and control. Modal parameters identification techniques, in the time domain, are classically based on the information contained in the impulse response functions (IRF) or in the input-output relationship (Maia and Silva, 1997). In general terms, a modal identification test is conducted under certain laboratory conditions, where the structure is fixed to a test bench and hammers or actuators are used to produce controlled types of input forces, which are required to match a linear time-invariant mathematical model, covering a certain frequency range of interest. However, in many applications, the real

operating conditions may differ significantly from those applied during the modal tests, where the input forces are not known, or just impossible to be measured. Parameters identification based on the knowledge of output-only responses, without using excitation information, is known as operational modal analysis (OMA) (Lardies, 2010). The subject is of actual scientific and industrial interest in mechanical and civil engineering opening a way for damage detection and structural health analysis (Mevel et al., 1999, Lardies, 2010, Lardies and Ta, 2010, Vu et al., 2013).

OMA presents several practical engineering applications. Zaghbani and Songmene (2009) have proposed a methodology based on OMA to compare the modal parameters of machine tools, demonstrating how OMA can be industrially exploited. Rainieri and Fabbrocino (2010) present a literature review on automated operational modal-based damage detection for civil engineering structures. Vu et al. (2013) proposed a method for the automatic identification procedure for discriminate physical modes from spurious ones is developed using a multivariate autoregressive (AR) model and which parameters are estimated via least-squares (LS) method. Lardies and Ta (2011) have used OMA to assess the structural health and damage detection of stay cables in cable stayed bridges. Ramos et al. (2010) performed structural identification of monuments in Portugal by OMA to obtain the damage assessment by means of vibration signatures.

According to Peeters and Roeck (1999), there are many methods used to performing the OMA parameters identification. Formally, for a completely unknown input, it can be assumed that the system is excited by a white Gaussian process known as a persistent excitation. A multivariate linear time-invariant autoregressive-moving-average-vector (ARMAV) model can be used to fit the data, adopting a least squares, maximum likelihood or prediction method as optimization criterion to calculate the model's parameters (Soderstrom and Stoica, 1987 and Lardies, 2010). Maximum likelihood optimization procedure leads to a highly non-linear minimization problem in order to calculate the parameters of the model. The solution of such a problem has a very high computational cost, especially for the multivariable parameters case.

The focus of present paper is on, both, theoretical and practical aspects of the time-domain ARMAV model. Despite OMA parameters identification now to be quite advanced field, some problems remains to be studied. Specifically, the principal objective here is to discuss the need of using an output-vector as reference for output-only parameters identification scheme. Practical aspect consists in the computational implementation of an ARMAV model identification algorithm.

In the present technique, the OMA parameters are identified from eigendecomposition of a companion matrix that contains the AR coefficients of the ARMAV model obtained via least squares optimization of a block Hankel matrix formed by correlation matrices between output measured data. The problem with the least squares approach is the adopting an initial overparametrization of the model order resulting in a number of spurious numerical modes that must be separated from the true modes of the system. The correct order of the ARMAV model is identified via inspection of the more significant singular value of that block Hankel matrix above mentioned, using singular value decomposition (SVD).

The performance of the presented technique is demonstrated using data generated by means of computational simulation. Impulse responses (which have a type of own-reference given by their impulsive force) and input-output data without using excitation information are considered.

The paper is organized as follows: Next sections present the multivariate the ARMAV model. The algorithm is then introduced. An application based on simulation using data from mechanical system is discussed. Finally, it is brought the main conclusions of the work.

THE ARMAV MODEL

The autoregressive-moving-average-vector (ARMAV) model is largely used in multivariate system identification (Lardies, 2010). ARMAV model can represent a multivariate time series from a linear time-invariant dynamical system by means of a multivariate difference equation as,

$$\mathbf{y}(k+p) - \sum_{i=1}^p \boldsymbol{\alpha}_i \mathbf{y}(k+p-i) = \sum_{i=1}^q \boldsymbol{\beta}_i \mathbf{e}(k+q-i) \quad (1)$$

where $\mathbf{y}(k) = \{y_1(k) \ \cdots \ y_m(k)\}^T$ are the $m \times 1$ vectors representing the measurements of m outputs variables of the system at discrete time $k \Delta t$, with the superscript “ T ” denoting vector transposition. The vector $\mathbf{e}(k) = \{e_1(k) \ \cdots \ e_m(k)\}^T$ is a non-observable stochastic $m \times 1$ vector process of with zero mean and nonsingular $m \times m$ covariance matrix Σ , representing the extraneous noise contained in the measurements. The limits p and q represent, respectively, the orders of the autoregressive (AR) and moving-average (MA) matrix parameters. The generic scalar elements $\boldsymbol{\alpha}_i$'s and $\boldsymbol{\beta}_i$'s are, respectively, the p AR and the q MA parameter matrices of dimension $m \times m$.

Multivariable ARMAV model described by Eq. (1) can be converted to following first order difference equations as,

$$\mathbf{Y}(k+1) = \boldsymbol{\alpha} \mathbf{Y}(k) + \boldsymbol{\beta} \mathbf{E}(k) \quad (2)$$

where

$$\mathbf{Y}(k) = \{\mathbf{y}^T(k) \ \mathbf{y}^T(k+1) \ \cdots \ \mathbf{y}^T(k+p-1)\}^T \quad (3)$$

and

$$\mathbf{E}(k) = \left\{ \mathbf{e}^T(k) \quad \mathbf{e}^T(k+1) \quad \cdots \quad \mathbf{e}^T(k+q-1) \right\}^T \quad (4)$$

More specifically, Eq. (2) can be written as,

$$\begin{Bmatrix} \mathbf{y}(k+1) \\ \mathbf{y}(k+2) \\ \vdots \\ \mathbf{y}(k+p) \end{Bmatrix} = \begin{bmatrix} \mathbf{0} & \mathbf{I} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{I} & \cdots & \mathbf{0} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{I} \\ \boldsymbol{\alpha}_p & \boldsymbol{\alpha}_{p-1} & \boldsymbol{\alpha}_{p-2} & \cdots & \boldsymbol{\alpha}_1 \end{bmatrix} \begin{Bmatrix} \mathbf{y}(k) \\ \mathbf{y}(k+1) \\ \vdots \\ \mathbf{y}(k+p-1) \end{Bmatrix} + \begin{bmatrix} \mathbf{0} & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} \\ \boldsymbol{\beta}_q & \boldsymbol{\beta}_{q-1} & \boldsymbol{\beta}_{q-2} & \cdots & \boldsymbol{\beta}_1 \end{bmatrix} \begin{Bmatrix} \mathbf{e}(k) \\ \mathbf{e}(k+1) \\ \vdots \\ \mathbf{e}(k+q-1) \end{Bmatrix} \quad (5)$$

where the AR parameters are contained in the following $mp \times mp$ companion matrix as,

$$\boldsymbol{\alpha} = \begin{bmatrix} \mathbf{0} & \mathbf{I} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{I} & \cdots & \mathbf{0} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{I} \\ \boldsymbol{\alpha}_p & \boldsymbol{\alpha}_{p-1} & \boldsymbol{\alpha}_{p-2} & \cdots & \boldsymbol{\alpha}_1 \end{bmatrix} \quad (6)$$

and the MA parameters are contained in the $mp \times mq$ matrix as,

$$\boldsymbol{\beta} = \begin{bmatrix} \mathbf{0} & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \boldsymbol{\beta}_q & \boldsymbol{\beta}_{q-1} & \boldsymbol{\beta}_{q-2} & \cdots & \boldsymbol{\beta}_1 \end{bmatrix} \quad (7)$$

Let's define the following vectors,

$$\mathbf{Y}^{fut}(k) = \left\{ \mathbf{y}^T(k) \quad \mathbf{y}^T(k+1) \quad \cdots \quad \mathbf{y}^T(k+p-1) \right\}^T \quad (8)$$

$$\mathbf{Y}^{pas}(k) = \left\{ \mathbf{y}^T(k) \quad \mathbf{y}^T(k-1) \quad \cdots \quad \mathbf{y}^T(k-s+1) \right\}^T \quad (9)$$

$$\mathbf{E}^{fut}(k) = \left\{ \mathbf{e}^T(k) \quad \mathbf{e}^T(k+1) \quad \cdots \quad \mathbf{e}^T(k+p-1) \right\}^T \quad (10)$$

where \mathbf{Y}^{fut} and \mathbf{Y}^{pas} are, respectively, $mp \times 1$ and $ms \times 1$ vectors and \mathbf{E}^{fut} is a $mp \times 1$ vector, with the superscripts *fut* and *pas* denoting, respectively, future and past data.

Now, for a quantity of measured data of N_p points, post-multiplying Eq. (2) by $\mathbf{Y}^{pasT}(k-1)$ and taking the expectation values results,

$$E[\mathbf{Y}^{fut}(k+1) \mathbf{Y}^{pasT}(k-1)] = \boldsymbol{\alpha} E[\mathbf{Y}^{fut}(k) \mathbf{Y}^{pasT}(k-1)] + \boldsymbol{\beta} E[\mathbf{E}^{fut}(k) \mathbf{Y}^{pasT}(k-1)] \quad (11)$$

where E means expectation operation.

Assuming that the process \mathbf{E}^{fut} and \mathbf{Y}^{pas} are uncorrelated, i.e., $E[\mathbf{E}^{fut}(k) \mathbf{Y}^{pasT}(k-1)] = \mathbf{0}$, Eq. (11) reduces to,

$$E[\mathbf{Y}^{fut}(k+1) \mathbf{Y}^{pasT}(k-1)] = \boldsymbol{\alpha} E[\mathbf{Y}^{fut}(k) \mathbf{Y}^{pasT}(k-1)] \quad (12)$$

Let's define,

$$\mathbf{H}^{(1)} = E[\mathbf{Y}^{fut}(k) \mathbf{Y}^{pasT}(k-1)] = \begin{bmatrix} \mathbf{R}_1 & \mathbf{R}_2 & \cdots & \mathbf{R}_s \\ \mathbf{R}_2 & \mathbf{R}_3 & \cdots & \mathbf{R}_{s+1} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{R}_p & \mathbf{R}_{p+1} & \cdots & \mathbf{R}_{p+s-1} \end{bmatrix} \quad (13)$$

and

$$\mathbf{H}^{(2)} = E[\mathbf{Y}^{fut}(k+1) \mathbf{Y}^{pasT}(k-1)] = \begin{bmatrix} \mathbf{R}_2 & \mathbf{R}_3 & \cdots & \mathbf{R}_{s+1} \\ \mathbf{R}_3 & \mathbf{R}_4 & \cdots & \mathbf{R}_{s+2} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{R}_{p+1} & \mathbf{R}_{p+2} & \cdots & \mathbf{R}_{p+s} \end{bmatrix} \quad (14)$$

as block Hankel matrices of dimension $mp \times mr$ formed by $m \times m$ covariance matrices defined as,

$$\mathbf{R}_i = E[\mathbf{y}(k) \mathbf{y}^T(k-i)] \quad (15)$$

where i means the correspondent lag of \mathbf{R}_i , for a quantity of lags used to build the matrices $\mathbf{H}^{(1)}$ and $\mathbf{H}^{(2)}$ equal to $N_{lags} = p + s$.

Eq. (12) can be rewritten as,

$$\mathbf{H}^{(2)} = \boldsymbol{\alpha} \mathbf{H}^{(1)} \quad (16)$$

and assuming that the matrix $\mathbf{H}^{(1)}$ to be nonsingular, it follows that the companion matrix $\boldsymbol{\alpha}$ can be calculated by solving the overdetermined system of linear equation as,

$$\boldsymbol{\alpha} = \mathbf{H}^{(2)} \mathbf{H}^{(1)T} (\mathbf{H}^{(1)} \mathbf{H}^{(1)T})^{-1} = \mathbf{H}^{(2)} \mathbf{H}^{(1)+} \quad (17)$$

where $\mathbf{H}^{(1)+} = \mathbf{H}^{(1)T} (\mathbf{H}^{(1)} \mathbf{H}^{(1)T})^{-1}$ denotes the Moore-Penrose pseudo inverse of $\mathbf{H}^{(1)}$.

REFERENCE-VECTORS FOR OMA IDENTIFICATION SQUEME

Theoretically, the impulsive or white Gaussian forces, used as excitation for, respectively, IRF's or input-output modal tests, have constant spectrum that represents the denominator of the frequency response function. These signals work as a type of reference in the modal parameters identification, for a certain frequency range of interest. In OMA parameters identification, where the input forces are not known, is also important to define some coordinates of reference to calculate the modal parameters of the system. The output vector of dimension $m \times 1$ is defined as,

$$\mathbf{y}(k) = \begin{Bmatrix} \mathbf{y}_r(k) \\ \mathbf{y}_{nr}(k) \end{Bmatrix} \quad (18)$$

where $\mathbf{y}_r(k)$ is the reference-output vector of dimension $r \times 1$. The vector $\mathbf{y}_{nr}(k)$ of dimension $(m-r) \times 1$ represents the part of non-referenced of the output vector $\mathbf{y}(k)$. The relation between $\mathbf{y}_r(k)$ and $\mathbf{y}_{nr}(k)$ is given by,

$$\mathbf{y}_r(k) = \mathbf{L} \mathbf{y}(k) \quad (19)$$

with $\mathbf{L} = [\mathbf{I}_r \quad \mathbf{0}]$ of dimension $r \times m$.

In OMA parameters identification, the non-referenced covariance matrices defined by Eq. (15) must be substituted by referenced-covariance matrices between the complete output vector $\mathbf{y}(k)$ and the reference-output-vector $\mathbf{y}_r(k)$ defined as,

$$\mathbf{R}_i^r = E[\mathbf{y}_r(k) \mathbf{y}_r^T(k-i)] = \mathbf{R}_i \mathbf{L}^T = E[\mathbf{y}(k) \mathbf{y}^T(k-i)] \mathbf{L}^T \quad (20)$$

For the example, in the case of only one reference as the j th variable $y_j(k)$, the referenced-covariance matrix \mathbf{R}_i^r becomes,

$$\mathbf{R}_i^r = \mathbf{R}_i \mathbf{L}^T = \begin{bmatrix} \mathbf{R}_i(1,1) & \cdots & \mathbf{R}_i(1,j-1) & \mathbf{R}_i(1,j) & \cdots & \mathbf{R}_i(1,m) \\ \mathbf{R}_i(2,1) & \cdots & \mathbf{R}_i(2,j-1) & \mathbf{R}_i(2,j) & \cdots & \mathbf{R}_i(2,m) \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ \mathbf{R}_i(m,1) & \cdots & \mathbf{R}_i(m,j-1) & \mathbf{R}_i(m,j) & \cdots & \mathbf{R}_i(m,m) \end{bmatrix} \begin{Bmatrix} 0 \\ \vdots \\ 0 \\ 1 \\ \vdots \\ 0 \end{Bmatrix} = \begin{Bmatrix} \mathbf{R}_i(1,j) \\ \mathbf{R}_i(2,j) \\ \vdots \\ \mathbf{R}_i(m,j) \end{Bmatrix} \quad (21)$$

The above equation shows how referenced covariance matrix \mathbf{R}_i^r can be obtained from non-reference covariance matrix \mathbf{R}_i .

MODAL PARAMETERS IDENTIFICATION

The input-output relationship, based on Equation (1), can be written as,

$$\mathbf{y}(k+p) - \sum_{i=1}^p \mathbf{a}_i \mathbf{y}(k+p-i) = \mathbf{u}(k) \quad (22)$$

where $\mathbf{u}(k)$ denotes the $m \times 1$ input-vector related to the external forces applied to the system.

In order to obtain a scheme to estimate the modal parameters of the mechanical system, the z -transform is applied to both sides of Eq. (22) giving the following equation,

$$\left[z^p \mathbf{I} - z^{p-1} \mathbf{a}_1 - z^{p-2} \mathbf{a}_2 - \cdots - \mathbf{a}_p \right] \mathbf{Y}(z) = \mathbf{U}(z) \quad (23)$$

or,

$$\left[z^p \mathbf{I} - z^{p-1} \mathbf{a}_1 - z^{p-2} \mathbf{a}_2 - \cdots - z \mathbf{a}_{p-1} - \mathbf{a}_p \right] \mathbf{H}(z) = \mathbf{I} \quad (24)$$

where $\mathbf{Y}(z)$ and $\mathbf{U}(z)$ are, respectively, the z -transform of $\mathbf{y}(k)$ e $\mathbf{u}(k)$ and $\mathbf{H}(z)$ is the $m \times m$ transfer function between $\mathbf{Y}(z)$ and $\mathbf{U}(z)$ in the z -domain.

Eq. (24) can be re-written in terms of a companion matrix \mathbf{a} as,

$$\left\{ \begin{bmatrix} z\mathbf{I} & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & z\mathbf{I} & \cdots & \mathbf{0} & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & z\mathbf{I} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} & z\mathbf{I} \end{bmatrix} - \begin{bmatrix} \mathbf{0} & \mathbf{I} & \cdots & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{I} \\ \mathbf{a}_p & \mathbf{a}_{p-1} & \cdots & \mathbf{a}_2 & \mathbf{a}_1 \end{bmatrix} \right\} \begin{Bmatrix} \mathbf{I} \\ z\mathbf{I} \\ \vdots \\ z^{p-2}\mathbf{I} \\ z^{p-1}\mathbf{I} \end{Bmatrix} \mathbf{H}(z) = \begin{Bmatrix} \mathbf{0} \\ \mathbf{0} \\ \vdots \\ \mathbf{0} \\ \mathbf{I} \end{Bmatrix} \quad (25)$$

The above equation can be written in a more compactly form as,

$$\left[z\mathbf{I} - \mathbf{a} \right] \tilde{\mathbf{I}}_z \mathbf{H}(z) = \tilde{\mathbf{B}}(z) \quad (26)$$

where

$$\tilde{\mathbf{I}}_z = \begin{Bmatrix} \mathbf{I} \\ z\mathbf{I} \\ \vdots \\ z^{p-2}\mathbf{I} \\ z^{p-1}\mathbf{I} \end{Bmatrix} \quad \text{and} \quad \tilde{\mathbf{B}}(z) = \begin{Bmatrix} \mathbf{0} \\ \mathbf{0} \\ \vdots \\ \mathbf{0} \\ \mathbf{I} \end{Bmatrix} \quad (27)$$

are $pm \times m$ matrices.

The eigenvalue problem of companion matrix \mathbf{a} can be written, from Eq. (26), as,

$$[z_j \mathbf{I} - \mathbf{a}] \tilde{\phi}_j = \mathbf{0} \quad (28)$$

which leads to the calculation of a quantity of mp z -poles z_j 's, where $mp - n$ of them are computational poles and may be separated from the identification process.

The minimal order of the model n can be identified by inspection of the more significant singular of matrix $\mathbf{H}^{(1)}$.

In general, mechanical systems are modeled in continuous-time in nature using for example the Newton's second law. The relation between the continuous-time poles λ_j 's and the discrete-time poles z_j 's is given by, (Maia e Silva, 1997),

$$\lambda_j = \frac{\ln(z_j)}{\Delta t} \quad \text{with} \quad j = 1:n \quad (29)$$

where Δt is the sampling time interval.

The natural frequencies ω_j and modal damping ξ_j , for the case of underdamped vibratory systems, are estimated from λ_j , respectively, according to,

$$\omega_j = |\lambda_j| \quad \text{and} \quad \xi_j = \frac{\text{Re}(\lambda_j)}{\omega_j} \quad (30)$$

where symbol $||$ denotes absolute value.

Finally, the mp eigenvectors $\tilde{\phi}_j$ of the companion matrix \mathbf{a} , from Eq. (28), can be used to estimate the mode-shapes ϕ_j of the mechanical system using the following relation, (Maia and Silva, 1997),

$$\tilde{\phi}_j = \mathbf{I}_z \phi_j \quad (31)$$

where ϕ_j is identified as,

$$\phi_j = (\mathbf{I}_z^T \mathbf{I}_z)^{-1} \mathbf{I}_z^T \tilde{\phi}_j \quad (32)$$

where $\tilde{\phi}_j$ is a $pm \times 1$ vector and ϕ_j is a $m \times 1$ mode-shape vector.

THE ARMAV ALGORITHM

The ARMAV algorithm for OMA parameters identification consists in compliance of the following steps:

- 1) Calculating the matrices $\mathbf{H}^{(1)}$ and $\mathbf{H}^{(2)}$, as Eqs. (13) and (14), for a total of a number of lags equal to $N_{lags} = p + s$, using referenced-covariance-matrices \mathbf{R}_i^r obtained by Eq. (20) for a quantity of N_p measured data,
- 2) Calculating of the companion matrix \mathbf{a} that contains the matrices parameters of the ARMAV model by Eq. (17),
- 3) Calculating a quantity of mp eigenvalues z_i 's and the associated mp eigenvectors $\tilde{\phi}_i$'s of the companion matrix \mathbf{a} , with $i = 1 \dots, mp$. The poles mp λ_i 's of the mechanical system described in continuous time are calculated according to Eq. (29). For underdamped systems, the natural frequencies ω_j and damping factors ξ_j are calculated by Eq. (30),
- 4) The minimum order of the system n can be obtained by means of inspection of the number of repeated poles identified by the former step or by the number of significant singular values of Hankel block matrix $\mathbf{H}^{(1)}$.

EXAMPLES OF APPLICATION

In order to show the capabilities of the present OMA parameters identification technique using the ARMAV model, a SIMO numerical experiment is conducted. The collection of impulse responses and only-output data are obtained by numerical simulation from a five degrees of freedom mass-spring oscillator, as shown in Fig. (1), using the following parameters: $m_1 = \dots = m_7 = 0.5$ kg and $k_1 = \dots = k_7 = 2000$ N/m. The mass \mathbf{M} and stiffness \mathbf{K} matrices take the

following form,

$$\mathbf{M} = \begin{bmatrix} m_1 & 0 & 0 & \dots & 0 \\ 0 & m_2 & 0 & \dots & 0 \\ 0 & 0 & m_3 & \dots & \vdots \\ \vdots & \vdots & \vdots & \ddots & 0 \\ 0 & 0 & 0 & 0 & m_5 \end{bmatrix} \quad (33)$$

and

$$\mathbf{K} = \begin{bmatrix} k_1 + k_2 & -k_2 & 0 & \dots & 0 \\ -k_2 & k_1 + k_2 & -k_3 & \dots & \vdots \\ 0 & -k_3 & k_3 + k_4 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & -k_5 \\ 0 & 0 & 0 & -k_5 & k_5 + k_6 \end{bmatrix} \quad (34)$$

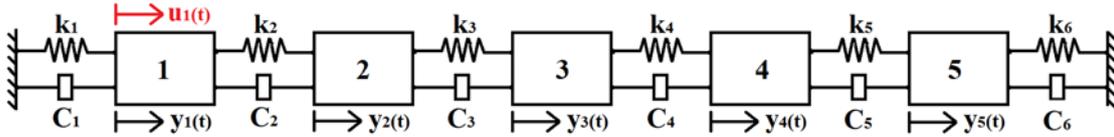


Figure 1- Five Degrees of Freedom Oscillator System

OMA Parameter Identification Using IRF's

In this test the impulse response $h_{ij}(k) = h_{ij}(k\Delta t)$ is the response in the block i at time $k\Delta t$, due to a unit impulse applied to the block j at time 0. The mathematical expression for the IRF's can be easily derived from the parameters of the system as,

$$h_{ij}(k) = \sum_{l=1}^n \left[r_{ij(l)} e^{\lambda_l k\Delta t} + r_{ij(l)}^* e^{\lambda_l^* k\Delta t} \right] \quad (35)$$

where the terms $r_{ij(l)} = \phi_{il} \phi_{jl}$ are the residues associated to eigenvalues λ_l (or z_l), ϕ_{ij} is the i -th element of the mode shape vector ϕ_j and the term “*” denotes complex conjugation.

The data of a SIMO, 1-input and 5-outputs, test is then numerically simulated with the unit impulse force acting in block 1. In the present test, it is adopted a number of 2000 data samples for each term $h_{ij}(k)$ for a quantity of 500 lags to build the covariance matrix \mathbf{R}_i^1 and the order of AR part of the model $p = 4$, resulting in a pair of matrices $\mathbf{H}^{(1)}$ and $\mathbf{H}^{(2)}$ both of dimension 20×496 . The time sampling interval Δt used is 0.025 second.

The order of the system is identified to be equal to 10 by inspection of most significant singular values of matrix $\mathbf{H}^{(1)}$ is shown in Fig. (2). Based on this criterion, it is adopted $p = 2$ in the identification process resulting the five modes and modal identified parameters present in the Tab. 1. Tab.1 shows the exact and identified modal.

Fig. 3 shows the five identified mode shapes associated to five natural frequencies as compared to exact modes derived from numerical simulation.

Theoretically, it is important to note that the modal parameters identification using IRF's data using the present method does not require the use of reference-vectors in the identification scheme, as discussed in previous sections. This type of data has their own references due to the impulsive forces as integrant part of calculation of IRF's.

Table 1 – Exact and identified modal parameters

Mode Number	Exact Natural Frequency (Hz)	Identified Natural Frequency (Hz)	Error (%)
1	5.2105	5.2105	0
2	10.0658	10.0658	0
3	14.2353	14.2353	0
4	17.4346	17.4346	0
5	19.4457	19.4459	0.001

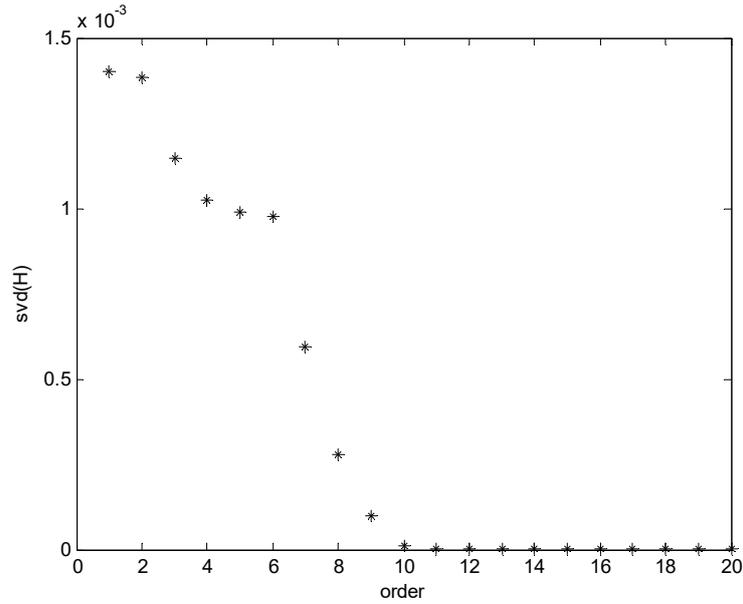


Figure 2 – Singular Values of Matrix $H^{(1)}$

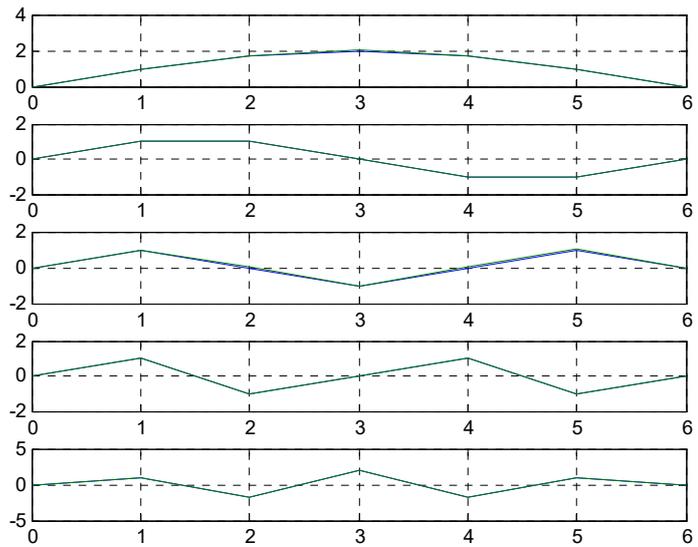


Figure 3 – Exact Mode Shapes (Green) and Identified Mode Shapes (Blue)

OMA Parameter Identification Using Output-Only Data

A SIMO 1-input and 5-outputs test is shown. Adopting as input $u_1(k)$, a mean zero white Gaussian noise signal with amplitude equal to 10 N, that acts on the blocks 1, the responses $y(k)$'s are obtained by evaluating the following sum of convolution,

$$y_i(k) = \sum_{s=0}^{N_p-1} h_{i1}(s) u_1(k-s) \quad i = 1, \dots, 5 \tag{36}$$

In the present test, it is adopted a number of 2000 data samples for each term $y_i(k)$ for a quantity of 500 lags to build the covariance matrix \mathbf{R}_i^1 and the order of AR parameters of the model $p = 4$, resulting in a pair of matrices $\mathbf{H}^{(1)}$ and $\mathbf{H}^{(2)}$ both of dimension 20×496 . The time sampling interval Δt used is 0.025 second.

The order of the system is identified to be equal to 10 by inspection of most significant singular values of matrix $\mathbf{H}^{(1)}$ shown in Fig. (4). Based on this criterion, the order of AR parameters is changed to $p = 2$ resulting in a total of five identified modes. The exact and identified parameters present in the Tab. 2.

Fig. 5 shows the five identified mode shapes associated to five first natural frequencies as compared to exact modes derived from numerical simulation.

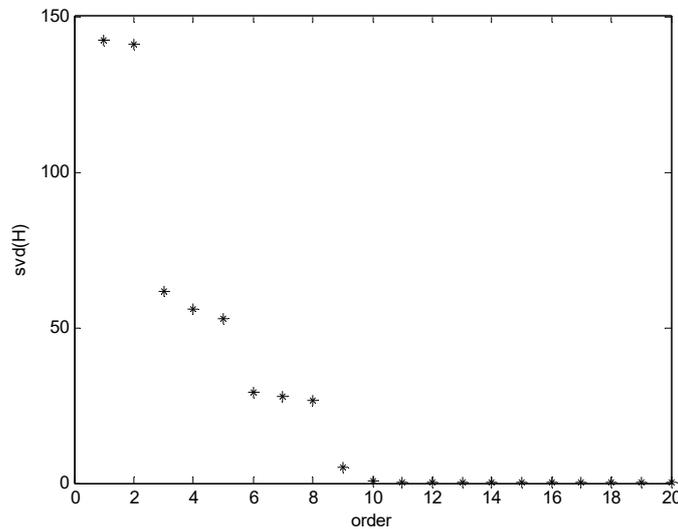


Figure 4 – Singular Values of Matrix \mathbf{H}^1

Table 2 – Exact and identified modal parameters

Mode Number	Exact Natural Frequency (Hz)	Identified Natural Frequency (Hz)	Error (%)
1	5.2105	5.2049	0.1074
2	10.0658	10.0713	0.0546
3	14.2353	14.2413	0.0415
4	17.4346	17.4381	0.0207
5	19.4457	19.4479	0.0113

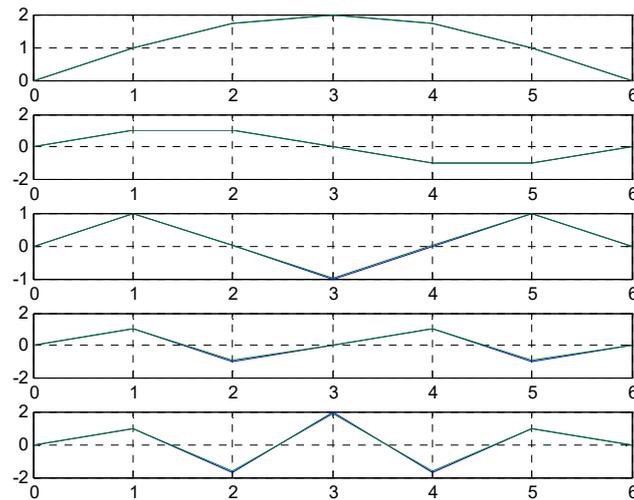


Figure 5 – Exact Mode Shapes (Green) and Identified Mode Shapes (Blue)

CONCLUSION

OMA is a very attractive field in mechanical and civil engineering and several techniques has been proposed in the literature. The present OMA parameter identification, method based on ARMAV model, is very simple, has robust numerical properties and relatively low computational cost, using only linear algebra manipulations. The tests based on numerical simulated data show that the presented method can be regarded as a way to perform the modal identification – natural frequencies, damping factors and associated mode shapes. The need of an output-vector as reference in the output-only parameters identification is highlighted. The present paper encourages for future implementation of the present algorithm using a more precise (accuracy) optimization technique for the parameters identification using, for example, maximum likelihood technique.

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