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Investigating Interface Modes on Periodic Acoustic Waveguides and Elastic Rods Using Spectral Elements

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Abstract: Phononic crystals and acoustic metamaterials are periodic structures that have drawn much attention of the engineering and physics communities due to their numerous applications in passive vibration and noise control. In this context, some geometric-phase concepts originally developed in electronics have inspired a number of applications in photonics and phononics. These concepts allow the manipulation of the topological behavior, such as edge modes or interface modes, as a result of the non-trivial topology of the band structure of the material. For acoustic systems, it has been recently shown that interface modes appear at the boundary separating two phononic crystals having different band-gap topological characteristics, which can be explained in terms of the geometric phase called Zak phase. In this paper, one-dimensional spectral elements are used to reproduce and investigate the interface modes present on these acoustic systems, as well as to show they can also be found in similar periodic systems such as elastic rods. Wave confinement at an interface can have useful applications in both acoustic systems (e.g. local sound enhancement) and elastic structures (e.g. energy harvesting).

Keywords: Phononic Crystals, Interface Modes, Acoustic Waveguides, Elastic Rods, Spectral Elements

NOMENCLATURE (OPTIONAL)

Latin symbols

r: tube radius
d: tube length
L: unit cell length
E: Elasticity modulus
c: speed of sound
f: frequency
k: wavenumber

Greek symbols

ρ: mass density
η: loss factor

Subscripts

a: relative to tube a
b: relative to tube b
A: relative to air
s: relative to steel
p: relative to polyacetal
i: relative to the interface mode

INTRODUCTION

The study of phononic crystals and acoustic metamaterials for wave control applications is an emerging topic in engineering and physics. A phononic crystal (PC) is a structure with geometric and/or material periodicity, while an acoustic metamaterial is usually characterized by the presence of substructures that work like resonators tuned to a narrow frequency band. In both cases their wave propagation characteristics can be derived from the analysis of a single unit cell. This is done through the dispersion diagram, which can be computed by different methods and essentially relates the angular frequency to the wavenumber of the travelling waves. Wave control can therefore be achieved by manipulating the pass and stop bands on these diagrams, *i.e.* frequency ranges in which the waves are propagating or evanescent, respectively. This allows a number of applications, such as vibration isolation, vibroacoustic barriers, wave tunneling, and cloaking. A comprehensive review on the subject is presented in Hussein *et al.*, 2014

Many applications of photonic and phononic crystals, as well as mechanical metamaterials, were inspired by concepts that were originally developed in electronic band theory. In particular, the geometric-phase concept has been explored in order to produce topological behavior that emerges from the non-trivial topology of some material band structures. One example is the creation of one-dimensional elastic edge waves, which are envisioned for the design of loss-free, one-way acoustic waveguides. In this case, non-trivial band gaps are caused by breaking the time-reversal symmetry, in analogy to the quantum anomalous Hall effect counterpart in electrocnics (Haldane, 1988). The time-reversal symmetry breaking has been achieved on solid lattice structures by inducing Coriolis forces via rotation (Wang *et al.*, 2015a) or by gyroscopic intertial effects (Wang *et al.*, 2015b and Nash *et al.*, 2015).

Another recent development of non-trivial topological behavior is the formation of interface modes on periodic acoustic systems. Xiao *et al.*, 2015, have shown that its possible to realize the band inversion concept on acoustic PC's, which also has an analogy in electronic systems (Pankratov *et al.*, 1987). In this case, the geometric phase involved is called Zak phase, which is a special type of Berry phase for one-dimensional (1D) periodic systems (Zak, 1989). The Zak phase of a particular band is related to the symmetry properties of the band-edge states, and the topological characteristics of a band gap are determined by the summation of the Zak phases of all the bands below the gap. Broadly speaking, interface modes are formed on the boundary separating two phononic crystals having different bandgap topological characteristics.

In this paper the interface modes present on these periodic acoustic systems are explored. It is shown that the band inversion concept can be observed in the dispersion diagrams of these PC's by varying one of the system parameters, resulting in the gap closing and reopening process. Therefore, it is not strictly necessary to calculate or measure the Zak phase in order to design simple periodic systems with interface modes. It is also shown that the same concepts can be applied to periodic elastic rods, where interface modes can also be created.

To this end, the Spectral Element Method (SEM) (Doyle, 1997) is used to compute the dispersion relations and the forced response of elastic rods and acoustic PC's. The dynamic stiffness matrices produced by SEM are exact within the scope of the theory used to obtain the equations of motion, and thus the method is a suitable tool for fast and accurate computations of the forced response and dispersion relations of structures with simple geometry and kinematic behavior, such as rods and beams. An one-dimensional spectral element for cylindrical ducts can be derived in analogy to the 1D spectral element for elementary rods, assuming the presence of plane waves only, as their dynamics are both governed by the same 1D wave-equation.

INTERFACE MODES ON PERIODIC ACOUSTIC SYSTEMS

To demonstrate the presence of interface modes on acoustic systems, simple one-dimensional PC's are used here. Figure 1a shows the geometry of the unit cell that is used for both acoustic and elastic rod systems. Each unit cell is composed of a narrower tube (Tube B) of length d_b and radius r_b located between two wider tubes (Tube A) of length $\frac{1}{2}d_a$ and radius r_a. For acoustic systems, these tubes are hollow cylindrical ducts and are filled with air (mass density ρ_A=1.3 kg m^{-3} and speed of sound c_A = 343 m s^{-1}). For both acoustic and elastic rod systems a loss factor $\eta = 0.001$ is used on the wavenumber primarily to avoid numerical instabilities ($k = k(1+i\eta)$).

For the examples in this section, the tubes radii are set to $r_a = 2.4$cm and $r_b = 1.5$cm, and the total length of the unit cell is fixed at $L = 8.5$cm. Figure 1b shows the band structure, calculated via SEM, for an acoustic PC with $d_a = 2.25$cm and $d_b = 6.25$cm. In all dispersion diagrams shown in this article the real part of the wavenumber is plotted on the positive axis while the imaginary part is plotted on the negative axis.

The band inversion for this PC can be observed on the second band gap of the band structure as one of the system parameters is varied, defined as $\Delta d = \frac{d_a - d_b}{2}$. The topological transition point for this system occurs when $d_a = d_b$ and therefore $\Delta d = 0$. Figure 1c shows the gap closing and reopening process, indicating that the band inversion occurs when Δd shifts from negative to positive values. It should be noted that the band structure for two PC's with the same absolute value for Δd is exactly the same, the only difference being the symmetry properties of the band-edge states. In other words, the topological characteristics of the band gap for two PC's having the same $|\Delta d|$ with opposite signs are different, and their Zak phases, if computed, would also be different.

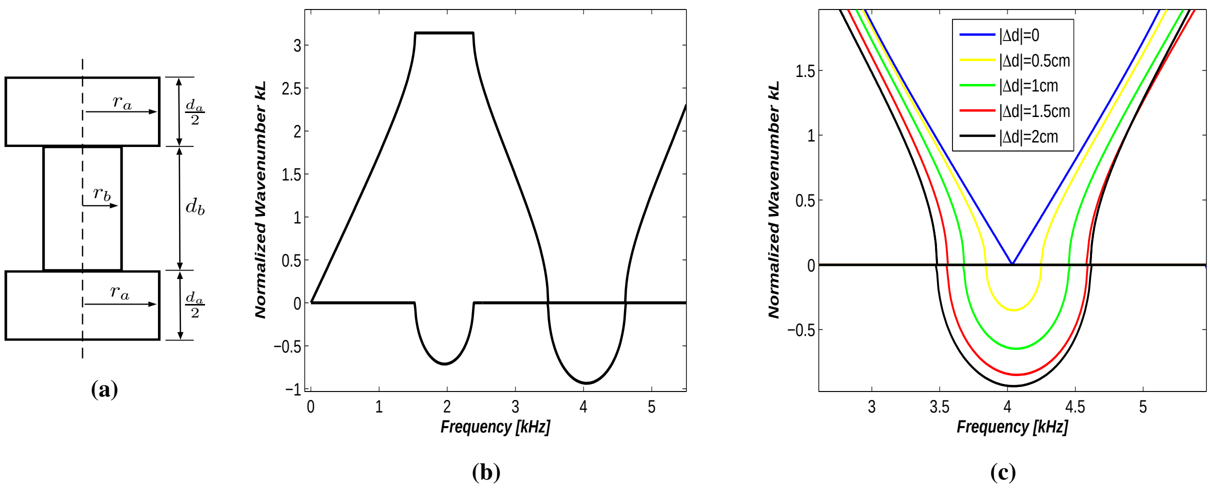
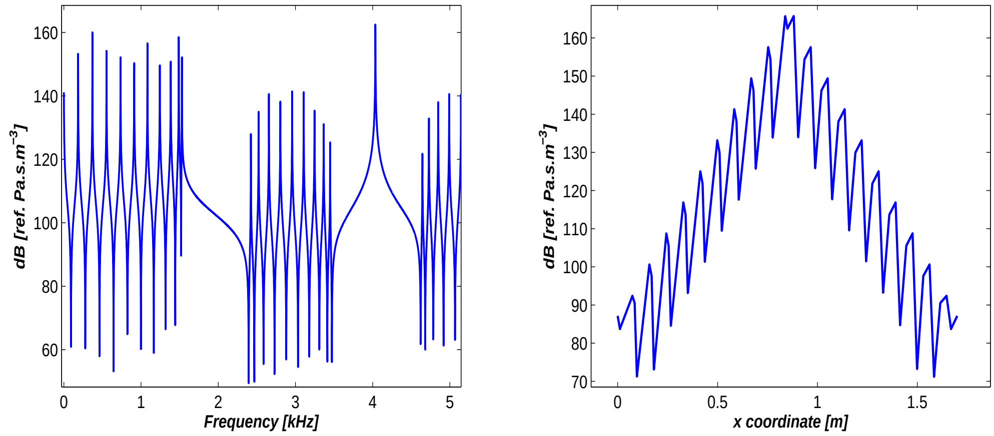


Figure 1: 1D Phononic Crystal System. a- Phononic Crystal's Geometry, **b**- Band Structure for PC with $d_a = 6.25$cm and $d_b = 2.25$cm, **c** Band gap closing and reopening process.

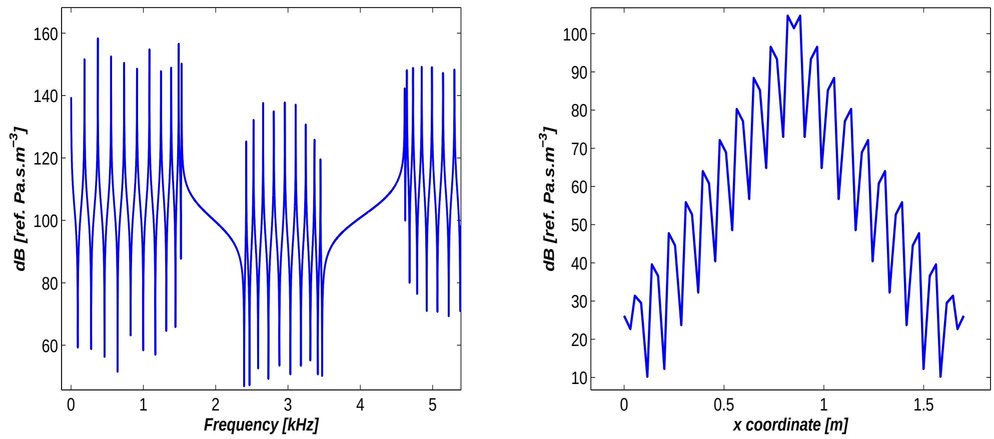
With the topological transition point for the system defined, interface modes can be observed by calculating the forced response of different PC couplings. In this work, four specific PC's are used to investigate these interface modes: PC_1 ($d_a = 2.25$cm, $d_b = 6.25$cm and $\Delta d = -2$cm), PC_2 ($d_a = 6.25$cm, $d_b = 2.25$cm and $\Delta d = 2$cm), PC_3 ($d_a = 3.75$cm, $d_b = 4.75$cm and $\Delta d = -0.5$cm) and PC_4 ($d_a = 4.75$cm, $d_b = 3.75$cm and $\Delta d = 0.5$cm). Each structure analysed consists of ten unit cells of one PC on the left side and ten unit cells of another PC on the right side, with the interface separating them at the center of the structure. All forced responses are computed via SEM, with closed-closed or forced-closed boundary conditions when the excitation is at the interface or at the left end, respectively. The spectral element used to model the acoustic PC was derived in analogy to the 1D spectral element for elementary rods (Doyle, 1997), as they are both governed by the same 1D wave-equation.

The interface mode appears when the coupled PC's have different band gap topological characteristics. Figure 2a shows the forced response at the interface for a PC_1-PC_2 coupling excited directly at the interface, where the presence of the interface mode is observed at frequency $f_i = 4035$Hz. For this frequency, the spacial distribution of the pressure field is concentrated at the interface, which can be observed on Fig. 2b. If a PC_1-PC_1 coupling is used instead (same band gap topological characteristics), the interface mode is not present, which is shown on Fig. 3. In such case, the spacial distribution of the pressure field for frequency f_i is concentrated at the interface but only because the frequency is inside the band gap, and therefore the waves are evanescent. The amplitudes of the pressure field, however, are much lower then when compared with the PC_1-PC_2 coupling.



(a) (b)

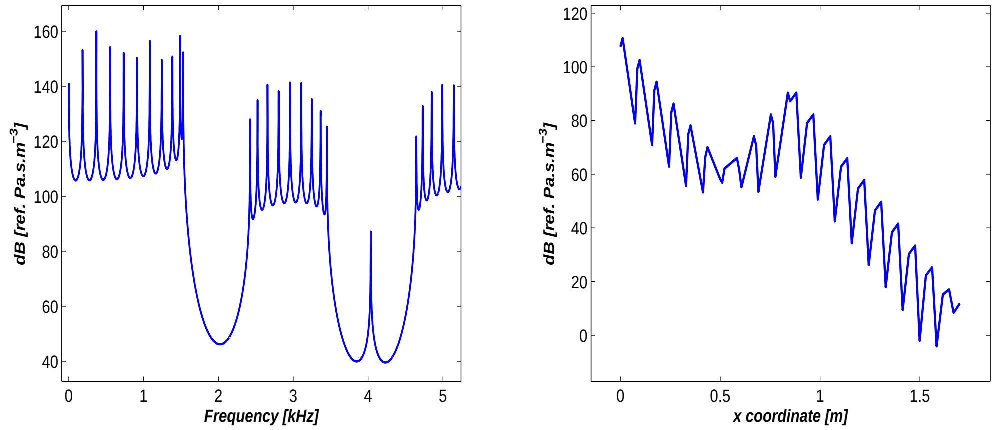
Figure 2: PC_1-PC_2 excited at interface. a- FRF at the interface, **b**- Spacial distribution of Pressure at $f_i = 4035Hz$.



(a) (b)

Figure 3: PC_1-PC_1 excited at interface. a- FRF at the interface, **b**- Spacial distribution of Pressure at $f_i = 4035Hz$.

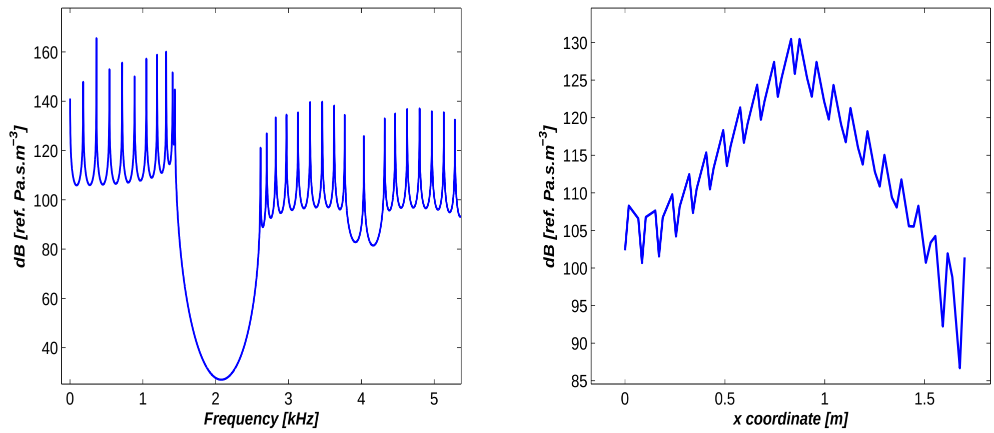
If the same PC_1-PC_2 coupling is excited at the left end instead of directly at the interface, the interface mode is still present, which is shown on Fig. 4. However, the magnitude of the interface peak is much smaller because the frequency f_i lies inside a band gap and, therefore, the waves will undergo significant attenuation before reaching the interface.



(a) (b)

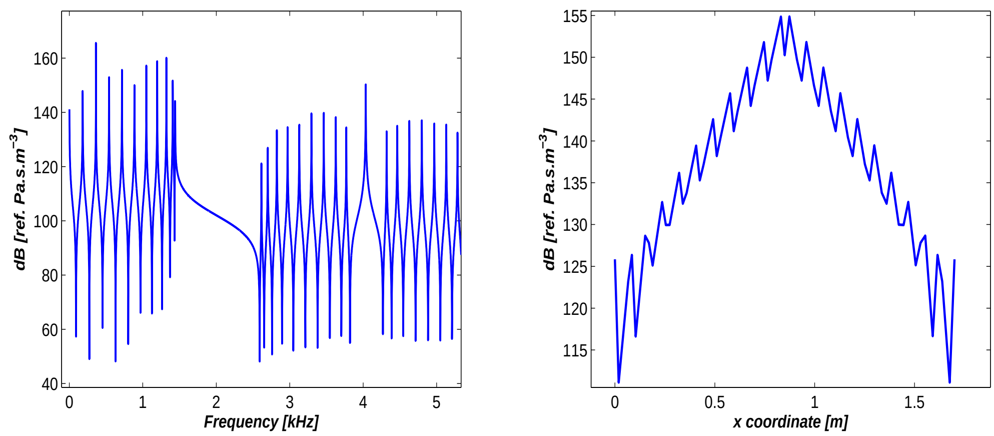
Figure 4: PC$_1$-PC$_2$ excited at left end. a- FRF at the interface, **b-** Spacial distribution of Pressure at $f_i = 4035Hz$.

For a higher interface peak, when the structure is excited away from the interface, a PC$_3$-PC$_4$ coupling can be used. In this case, the band gap is smaller and, therefore, the waves will suffer less attenuation before reaching the interface. The forced response for this coupling when excited at the left end is shown on Fig. 5, which clearly presents a higher interface peak when compared to the PC$_1$-PC$_2$ coupling. However, for the same reasons, this coupling has a lower peak when excited directly at the interface, which is shown on Fig. 6.



(a) (b)

Figure 5: PC$_3$-PC$_4$ excited at left end. a- FRF at the interface, **b-** Spacial distribution of Pressure at $f_i = 4035Hz$.



(a) (b)

Figure 6: PC$_3$-PC$_4$ excited at interface. a- FRF at the interface, **b-** Spacial distribution of Pressure at $f_i = 4035Hz$.

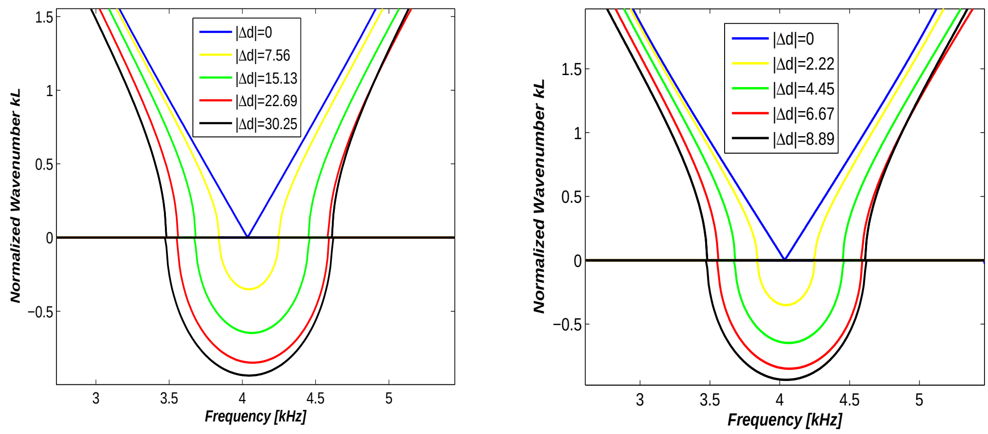
INTERFACE MODES ON PERIODIC ELASTIC RODS

The same concepts shown for acoustic systems can be extended to similar systems of elastic rods, where interface modes can also be created. To that end, PC's with the same geometry described in the previous section (shown on Fig. 1a) are used, but the tubes are now elastic rods instead of hollow ducts.

However, the speed of sound in solids is in general much greater than the speed of sound in the air. Therefore, if the same frequency for the interface mode is sought ($f_i = 4035$Hz), the dimensions of the unit cells need to be much larger. To demonstrate that, two materials are used: steel ($E_s = 210$Gpa, $\rho_s = 7800$kg m^{-3} and $c_s = \sqrt{\frac{E_s}{\rho_s}} = 5188.7$ m s^{-1}) and polyacetal ($E_p = 3.3$Gpa, $\rho_p = 1418$kg m^{-3} and $c_p = \sqrt{\frac{E_p}{\rho_p}} = 1525.5$ m s^{-1}).

The speed of sound in steel is 15.13 times greater than the speed of sound in the air. Therefore, all the PC's dimensions are multiplied by this factor in order to maintain the bandgaps and the interface modes at the same frequencies as in the acoustic systems. The same is done for polyacetal, but by a factor of 4.45. The tubes radii are multiplied by this factor only to keep the same geometric proportions, as they don't influence the location of the interface mode when the ratio $\frac{r_a}{r_b}$ is kept constant.

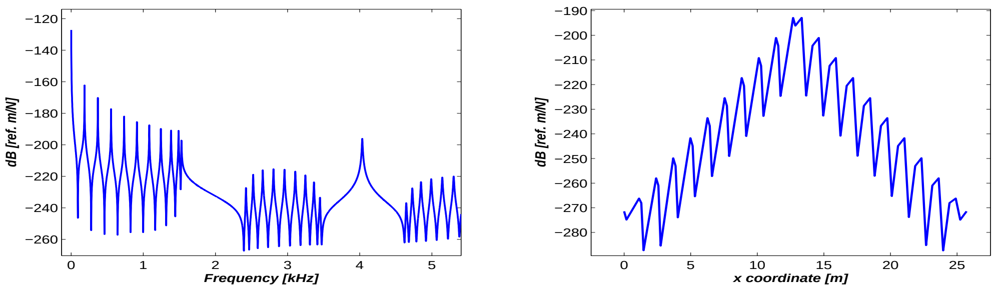
By doing that, the PC's dimensions for steel becomes $L = 1.29$m, $r_a = 36.31$cm, $r_b = 22.69$cm while for polyacetal they are set to $L = 37.8$cm, $r_a = 10.67$cm and $r_b = 6.67$cm. The band gap closing and reopening process on the second band gap of both materials band structure is very similar to the one that occurs for acoustic systems, and is shown on Fig. 7.



(a) **(b)**

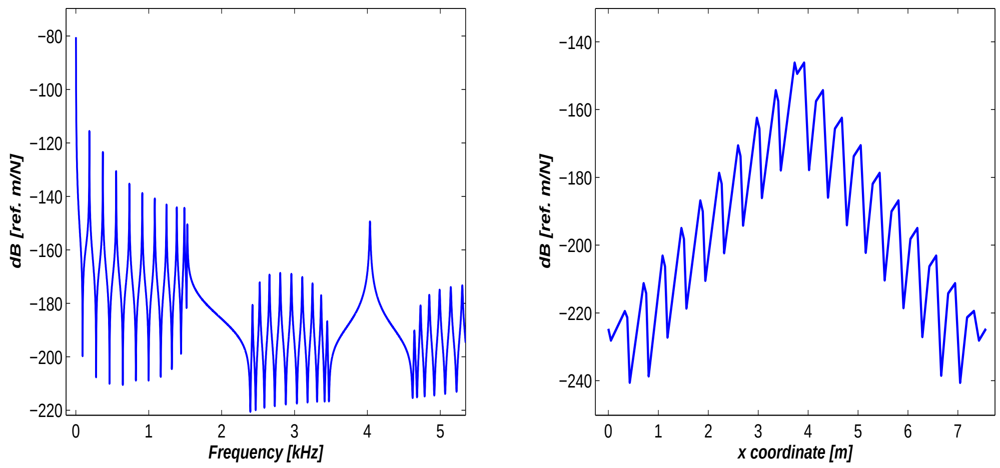
Figure 7: Band gap closing and reopening for elastic rods. a- PC's made of steel, **b**- PC's made of polyacetal. Δd dimensions in cm.

As was the case for acoustic systems, interface modes will appear on systems of elastic rods when the PC's coupled have different band gap topological characteristics. Four different PC's are used in this section: PC$_5$ (steel, $d_a = 94.55$cm, $d_b = 34.04$cm and $\Delta d = -30.25$cm), PC$_6$ (steel, $d_a = 34.04$cm, $d_b = 94.55$cm and $\Delta d = 30.25$cm), PC$_7$ (polyacetal, $d_a = 27.8$cm, $d_b = 10.01$cm and $\Delta d = -8.9$cm) and PC$_8$ (polyacetal, $d_a = 10.01$cm, $d_b = 27.8$cm and $\Delta d = 8.89$cm). All forced responses are again computed via SEM, with free-free or forced-free boundary conditions when the applied force is at the interface or at the left end, respectively. Figures 8 and 9 shows the forced responses for the PC$_5$-PC$_6$ and PC$_7$-PC$_8$ couplings, respectively. As expected, the interface modes for both materials occur at the same frequency as in the acoustic systems ($f_i = 4035Hz$).



(a) **(b)**

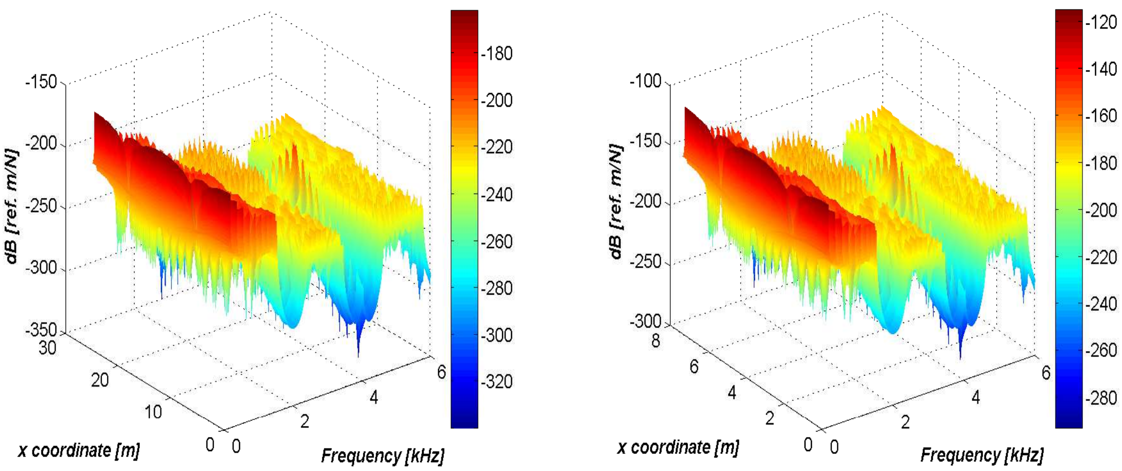
Figure 8: PC$_5$-PC$_6$ excited at interface. a- FRF at the interface, **b**- Spacial distribution of displacement amplitude at $f_i = 4035Hz$.



(a) **(b)**

Figure 9: PC_7-PC_8 excited at interface. a- FRF at the interface, **b-** Spacial distribution of displacement amplitudes at $f_i = 4035Hz$.

These interface modes can be better visualized in a surface containing the spacial distribution of displacements amplitudes for a wide frequency range (100Hz-6kHz). This is shown for both PC_5-PC_6 and PC_7-PC_8 couplings on Fig. 10, highlighting that interface modes may be seen as structural resonance peaks that are not only concentrated in frequency but also in space.



(a) **(b)**

Figure 10: Surfaces of spacial distribution of displacement amplitudes a- PC_5-PC_6 coupling, **b-** PC_7-PC_8 coupling.

CONCLUSION

Interface modes that appear in the interface of two phononic crystals are characterized by a large local dynamic response and fast spatial attenuation. This feature may be useful in applications such as energy harvesting. The topic involves geometrical phase concepts that are new to the structural dynamics community . Using finite elements, Xiao et al. (2015) showed that interface modes can be generated in periodic acoustic waveguides. In this paper, one-dimensional spectral elements were used to investigate the interface modes present on acoustic systems and to show they can also be created on elastic structures such as periodic systems of elastic rods. However, as expected, for elastic rods larger dimensions are required for generating interface modes at lower frequencies. This motivates our current research efforts in analyzing more complex geometries and material distributions aiming at producing bandgaps at lower frequencies with smaller dimensions and, therefore, obtaining interface modes at lower frequencies. The spectral element method is shown to be a suitable tool to analyse and optimize geometry and materials of the periodic cells with low computational cost before performing experiments or more detailed dynamic analyses.

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