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Stochastic analysis of vortex induced vibrations using a reduced model

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Abstract: Vortex-Induced vibrations plays an important role in many fields and is crucial in the design of offshore engineering systems and has received special attention lately due to its relevance for design of mooring and risers' system as well to mono-column platforms for hydrocarbon exploration in deep waters. The use of phenomenological reduced models to analyses this phenomena can be a very useful tool to complement the results achieved via CFD (Computational Fluid Dynamics). The phenomenon plays an important role in many fields and is crucial in the design of offshore engineering systems, where the accurate prediction of structural instability is extremely important due that the vortex shedding behind bluff bodies may lead to degradation of structural performance or even structural failure. This work presents a stochastic analysis of this phenomena, taking into account uncertain input parameters in a phenomenological model taken into account results from Navier-Stokes equations model using a CFD solver as reference. This analysis can help to understand the behavior of the structure to critical situations and the effects of varying parameters in the response variables. The statistics moments are approximated by the non-intrusive sparse grid stochastic adaptive collocation method. This method has emerged in recent years as an attractive technique, due to that allows obtaining approximations of the interpolating function in the stochastic space. The method approximates the solution in the stochastic space using Lagrange polynomial interpolation, providing a simple way to approach the statistical moments of the system outputs. To improve the method when there are steep gradients or finite discontinuities in the stochastic space an adaptive technique was adopted to perform the analysis. Numerical simulations are performed to demonstrate the appropriateness of method applied for characterization of the statistical moments of the critical points as well as for the determination of the probability of occurrence of undesired phenomenon.

Keywords: Flow-induced Vibrations, Uncertainty Quantification, Adaptive Stochastic Collocation Method

1 INTRODUCTION

The complexity involved in engineering systems has been, frequently, tackled with the use of sophisticated computational models. That, from the decision makers standpoint, requires the use of robust and reliable numerical simulators. Often, the reliability of those simulations is disrupted by the inexorable presence of uncertainty in the model data, such as inexact knowledge of system forcing, initial and boundary conditions, physical properties of the medium, as well as parameters in constitutive equations. These situations underscore the need for efficient uncertainty quantification methods for the establishment of confidence intervals in computed predictions, the assessment of the suitability of model formulations, and/or the support of decision-making analysis. The traditional statistical tool for uncertainty quantification within the realm of Engineering is the Monte Carlo method, see [2]. This method requires, first, the generation of an ensemble of random realizations associated to the uncertain data, and then it employs deterministic solvers repetitively to obtain the ensemble of results. The ensemble results should be processed to estimate the mean and standard deviation of the final results. The implementation the Monte Carlo is straightforward, but its convergence rate is very slow (proportional to the inverse of the square root of the realization number) and often infeasible due the large CPU time needed to run the model in question. Other technique that has been applied recently is the so called Stochastic Galerkin Method (SG), which employs Polynomial Chaos expansions to represent the solution and inputs to stochastic differential equations, see [1]. The Galerkin projection minimizes the error of the truncated expansion and the resulting set of coupled equations is solved to obtain the expansion coefficients. SG methods are highly suited to dealing with ordinary and partial differential equations, even in the case of nonlinear dependence on the random data. The main drawback with SG relies on its need of solving a system of coupled equations that requires efficient and robust solvers and, most importantly, the modification of existing deterministic code. This last issue entails difficulties on using commercial or already in use codes. A non-intrusive method, referred to as Stochastic Collocation (SC), see [9], arises towards addressing this point. SC methods are built on the combination of interpolation methods and deterministic solvers, likely Monte Carlo. A deterministic problem is solved in each point of an abstract random space. Similarly to SG methods, SC methods achieve fast convergence when the solution possesses sufficient smoothness in random space. Thus when there are steeps gradients or finite discontinuities in the stochastic space, these methods converge very slowly or even fail to converge. In this work, we present an adaptive sparse grid collocation strategy with the aim of obtaining greater accuracy in nonlinear systems analysis.

Specifically in the present work, is examined a interesting situation involving fluid-structure interaction in a model used in preliminary approach of Engineering projects. The analysis done takes into account uncertainties in the input variables of the prediction model The problem is then formulated through the probabilistic approach where the uncertainties are characterized by a probability density function. Particular emphasis is placed on investigating uncertainty propagation in the nonlinear response of fluid-structure interaction, see [10]. Therefore, the Stochastic Collocation method is used to propagate uncertainties through the model. Stochastic modeling seems to offer an appropriate framework to for example, tackle the external forces and uncertainties in the input data, like, for instance, damping and boundary conditions. Here, the fluid-structure interaction is modeled in a simple way focusing the assessment of an SC method as an effective tool for uncertainty quantification. Results are presented as probability density functions for the amplitude of vibration and free stream velocity in the range of synchronization.

2 STOCHASTIC COLLOCATION METHOD

The main idea of this method is approximate the multidimensional stochastic space building a interpolation function on a set of collocation points $\{\mathbf{Y}_i\}_{i=1}^M$ in the stochastic space $\Gamma \subset \mathbb{R}^M$. The method, similarly to Monte Carlo methods, requires only the solution of a set of decoupled equations, allowing the model to be treated as a black box and solved it with existing deterministic solvers.

The multidimensional interpolation can be built through either full-tensor product of 1D interpolation rule or by the so called sparse grid interpolation based on the Smolyak algorithm. This algorithm provides a way to construct interpolations functions based on minimal number of points in multidimensional space, extending in a easy way the univariate interpolation to the multivariate case.

Hence, considering a smooth function $f : [-1,1]^N \to \mathbb{R}$, for the $1D$ case $(N = 1)$, f can be approximated by the following interpolation formula $\mathscr{U}^i(f)(y)$,

$$\mathscr{U}^i(f)(y) = \sum_{j=1}^{m_i} f(\mathbf{Y}_j^i)a_j^i, \tag{1}$$

in the set of support nodes,

$$X^i = \mathbf{Y}_j^i | \mathbf{Y}_j^i \in [0,1] for j = 1, \ldots, m_i, \tag{2}$$

where, $i \in \mathbb{N}$, $a_i(\mathbf{Y}_j^i) \in C[0,1]$ are the interpolation basis functions and m_i is the number of elements of the set X^i. The multivariate case, i.e. $(N > 1)$, the tensor product formula is:

$$(\mathscr{U}^{i_1} \otimes \ldots \otimes \mathscr{U}^{i_N})(f) = \sum_{j_1=1}^{m_1} \cdots \sum_{j_N=1}^{m_N} f(Y_{j_1}^{i_1} \ldots Y_{j_N}^{i_N}).(a_{j_1}^{i_1} \otimes \cdots \otimes a_{j_N}^{i_N}), \tag{3}$$

which serve as building blocks for the Smolyak algorithm. So, The Smolyak algorithm build the interpolant $\mathscr{A}_{q,N}(f)$ using products of $1D$ functions as given in [7].

$$\mathscr{A}_{q,N}(f) = \sum_{q-N+1 \leq |i| \leq q} (-1)^{q-|i|} \binom{N-1}{q-|i|} (\mathscr{U}^{i_1} \otimes \ldots \otimes \mathscr{U}^{i_N}), \tag{4}$$

with $q \geq N$, $\mathscr{A}_{N-1,N} = 0$ and where the multi-index $i = (i_1, \ldots, i_N) \in \mathbb{N}^N$ and $|i| = i_1 + \cdots + i_N$. Here $i_k, k = 1, \ldots, N$, is the level of interpolation along the $k-th$ direction. The Smolyak algorithm builds the interpolation function by adding a combination of $1D$ functions of order i_k with the constraint that the sum total $(|i| = i_1 + \ldots + i_N)$ across all dimensions is between $q - N + 1$ and q.

$$\mathscr{A}_{q,N}(f) = \sum_{|i| \leq q} (\triangle^{i_1} \otimes \ldots \otimes \triangle^{i_N}) = \mathscr{A}_{q-1,N}(f) + \sum_{|i|=q} (\triangle^{i_1} \otimes \ldots \otimes \triangle^{i_N}). \tag{5}$$

To compute the interpolant $\mathscr{A}_{q,N}(f)$ is necessary to compute the function at the nodes covered by the sparse grid $\mathscr{H}_{q,N}$,

$$\mathscr{H}_{q,N}(f) = \bigcup_{q-N+1 \leq |i| \leq q} (X^{i_1} \times \cdots \times X^{i_N}). \tag{6}$$

The construction of the algorithm allows to utilizing all the previous results generated to improve the interpolation. By choosing the appropriate points for interpolating the $1D$ function, it is possible ensure that the sets of points are nested $X^i \subset X^{i+1}$. Where to extend the interpolation from level $i-1$ to i, one only has to evaluate the function at grid points that are unique to X^i. Hence, to go from an order $q-1$ to q in N dimensions, one only needs to evaluate the function at the differential nodes,

$$\triangle\mathscr{H}_{q,N}(f) = \bigcup_{|i|=q} (X^{i_1} \otimes \cdots \otimes X^{i_N}). \tag{7}$$

Finally after the choice of collocation points and the nodal basis functions, any function $u \in \Gamma$ can be approximated by,

$$u(x,\mathbf{Y}) = \sum_{|i|\leq q}\sum_{j\in B_i} w_j^i(x)a_j^i(\mathbf{Y}). \tag{8}$$

This equation is a simple weighted sum of the value of the basis functions for all collocations points in the sparse grid, being an approximation to the solution of the equations of the system. From this equation, it is possible calculate easily the useful statistics of the solution for example, the mean of the random solution can be evaluated as follow:

$$\mathbb{E}[u(x)] = \sum_{|i|\leq q}\sum_{j\in B_i} w_j^i(x).\int_\Gamma a_j^i(\mathbf{Y})d\mathbf{Y}, \tag{9}$$

where denoting $\int_\Gamma a_j^i(\mathbf{Y})d\mathbf{Y} = I_j^i$ we can write,

$$\mathbb{E}[u(x)] = \sum_{|i|\leq q}\sum_{j\in B_i} w_j^i(x).I_j^i, \tag{10}$$

the mean is an arithmetic sum of the product of the hierarchical surpluses and the integral weight at each interpolation point. To obtain the variance of the random solution we can be calculate first,

$$u^2(x,\mathbf{Y}) = \sum_{|i|\leq q}\sum_{j\in B_i} v_j^i(x)a_j^i(\mathbf{Y}), \tag{11}$$

and then,

$$\mathbb{V}ar[u(x)] = \mathbb{E}[u^2(x)] - (\mathbb{E}[u(x)])^2 = \sum_{|i|\leq q}\sum_{j\in B_i} v_j^i(x).I_j^i - (\sum_{|i|\leq q}\sum_{j\in B_i} w_j^i(x)I_j^i)^2. \tag{12}$$

The method allows us to obtain an approximation of the solution dependent random variables and also easily extract the mean and variance analytically as well its probability density function (PDF) by simple sampling of this function, leaving only the interpolation error [9].

3 ADAPTIVE SPARSE GRID COLLOCATION METHOD

When the the smoothness condition in the stochastic space is not fulfilled it is possible to use adaptive strategies to improve de interpolation function in the stochastic space. The basic idea here is to use hierarchical surpluses $w_j^i(x)$ as an error indicator to detect the smoothness of the solution and refine the grid around the discontinuity region and less points in the region of smooth variation. This method proposed in [7], automatically detect the discontinuity region in the stochastic space and refine the collocation points in this region. Then, considering the interpolation level of a grid point Y as the depth of the tree $D(Y)$. After denote the father of a grid point as $F(Y)$, where the father of the root 0.5 is itself. Thus, the conventional sparse grid in the N-dimensional random space Equation 6 can be reconsidered as,

$$\mathscr{H}_{q,N}(f) = \{\mathbf{Y} = \{Y_1\dots Y_N\}|\sum_{i=1}^{N} D(Y_i) \leq q\}. \tag{13}$$

Where we call their sons of a grid point $\mathbf{Y} = (Y_1\dots Y_N)$ by:

$$Sons(\mathbf{Y}) = \mathbf{S} = (S_1,S_2,\dots,S_N)|(F(S_1),S_2,\dots,S_N) = \mathbf{Y}, \tag{14}$$

or

$$(S_1,F(S_2),\dots,S_N) = \mathbf{Y},\dots,or(S_1,S_2,\dots,F(S_N)) = \mathbf{Y}. \tag{15}$$

From this definition, is noted that in general for each grid point here there are two sons in each dimension, therefore, for a grid point in a N-dimensional stochastic space, there are 2N *Sons*. Therefore, by adding the neighbor points, we actually add the support nodes from the next interpolation level, so that the magnitude of the hierarchical surplus satisfies $|w_j^i \geq \varepsilon|$. If this criterion is satisfied, one only add the $2N$ neighbor points of the current point to the sparse grid. It is noted that the definition of level of the Smolyak interpolation for the ASGC method is the same as that of the conventional sparse grid even if not all point are included. A more detailed explanation of the method and algorithm can be found in, [7].

4 NUMERICAL EXAMPLE

In the following section we present an interesting situation involving a structure coupled with flow induced vibrations model used in preliminary analysis of Engineering projects. The choice was guided by the challenge that potentially can bring to the SGC even as the adaptive form ASGC, as robust tool applied to Uncertainty Quantification.

4.1 Model of structure coupled with flow induced vibrations

As is well known, Vortex-Induced Vibration are motions induced on bodies interacting with an external fluid flow. It is caused by the vortex shedding behind bluff bodies and may lead to degradation of structural performance or even structural failure. In offshore structures such as pipes, risers and mooring lines, it is a particularly important. Exist several ways to predict the dynamic response of structures undergoing large amplitude vibrations induced by the surrounding flow. One of the most effective prediction method consists of solving the coupled fluid-structure system modeling the flow through Navier-Stokes equations. The structure, due to its rigidity, can be often characterized by a simple oscillator with one or two degrees of freedom. The problem of this approach, when applied to real problems is that sometime lead in to highly cost models for UQ analysis or in preliminary phase os design. An alternative, is to use a phenomenological model based on wake oscillators [4] that replace the vortex shedding mechanisms of the flow by simple models. In this example we use a similar model as presented in [6] and [8] that captures important features of the VIV dynamics after a calibration of its parameters, taking into consideration the available experimental data. Due this reason, the calibration is an activity that leads with parameters subject to uncertainties that can be modeled. With this purpose a CFD code, COMSOL Multiphysics was used to obtain references solutions to analyse the parameters of our phenomenological model with the following equations model

$$\begin{aligned}(m_s+m_f)\ddot{x}+(c_s+c_f)\dot{x}+kx &= F_x(CFD)\\ (m_s+m_f)\ddot{y}+(c_s+c_f)\dot{y}+ky &= F_y(CFD)\end{aligned}$$

where m_s and m_s are the structural and added mass ande c_s,c_f the structural and added damping coefficients. Figure 1 show a sample COMSOL reference solution, considering an elastically supported rigid circular spring-mounted cylinder

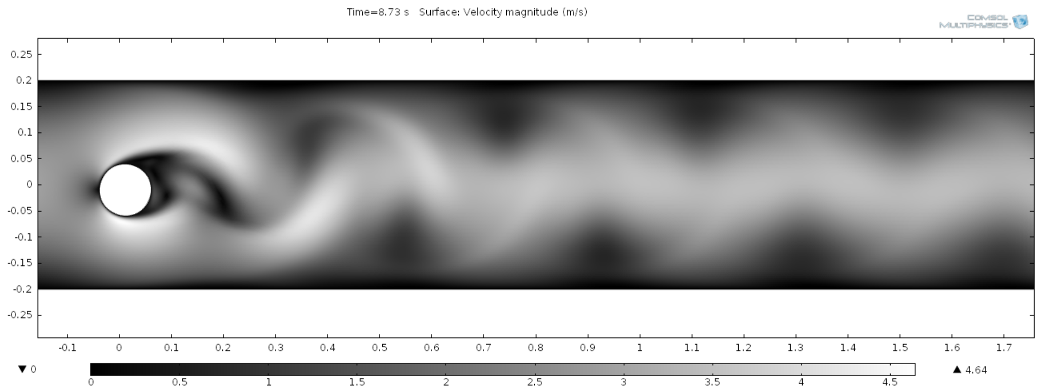

Figure 1 – Sample of statistics for reduced velocity $U_r = 12$

with two degree of freedom shown schematically in Fig.2. In other hand, as was stated herein, it is possible to replace the CFD code to calculate the hydrodynamic loads through a phenomenological model based on oscillator. This new system , after calibrated can be able to produce the same effect on the structure like a surrogate model with less computational cost solved by Runge-Kutta. Following are shown the system of equations of the surrogate model:

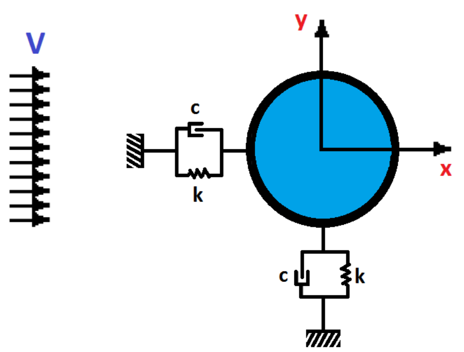

Figure 2 – Wake oscillator model for Vortex-induced vibration

$$\begin{aligned}(m_s+m_f)\ddot{x}+(c_s+c_f)\dot{x}+kx &= F_x\\ (m_s+m_f)\ddot{y}+(c_s+c_f)\dot{y}+ky &= F_y\end{aligned}$$

$$\begin{aligned}\ddot{q}_x+2\varepsilon_x\Omega_f(q_x^2-1)\dot{q}_x+4\Omega_f^2 q &= \frac{A_x}{D}\ddot{x}\\ \ddot{q}_y+\varepsilon_y\Omega_f(q_y^2-1)\dot{q}_y+\Omega_f q &= \frac{A_y}{D}\ddot{y}\end{aligned}$$

$$F_x = \tfrac{1}{2}\rho DL[C_D(Ux-\dot{x})-C_L(Uy-\dot{y})]\sqrt{(Ux-\dot{x})^2+(Uy-\dot{y})^2}$$
$$F_y = \tfrac{1}{2}\rho DL[C_D(Uy-\dot{y})+C_L(Ux-\dot{x})]\sqrt{(Ux-\dot{x})^2+(Uy-\dot{y})^2}$$

where

$$C_D = C_o(1+Kq_y{}^2)+C_{io}\frac{q_x}{2} \qquad c_s = 2m\Omega_s\xi \qquad \Omega_s = \sqrt{\frac{k}{m}}$$

$$C_L = C_{Lo}\frac{q_y}{2} \qquad V = \frac{D\Omega_s U_r}{2\pi} \qquad c_f = \gamma\Omega_f\rho LD^2 \qquad \Omega_f = 2\pi S$$

Where, the dimensions of the numerical model are $L = 1$m and $D = 0.1$ m. The aspect ratio of this system is therefore L/D = 0.4. In the structural oscillator, the structural mass in air is $m = 45.71$ kg; the structural damping coefficient is a given parameter estimated $\xi = 4.4$ and the stiffness constant is $k = 5.$ kN/m. The vortex shedding lift coefficient $C_{Lo} = 0.3$ is taken as in [3] and the reference drag is taken as $C_o = 0.7$, which was obtained from the experiments reported in [8]. The Strouhal number is equal to 0.2 and $C_{io} = 0.1$. The parameter $\gamma = 2.14$ and A_x and A_y correspond respectively to cross-flow and in-line amplification factors and ε_x and ε_y to the cross-flow and in-line damping respectively with K parameter that couples cross-flow and in-line motions. Being the numerical values the followings $A_x = 6$, $A_y = 12$, $\varepsilon_x = 0.3$, $\varepsilon_y = 0.15$ and $K = 0.05$ like in [8].

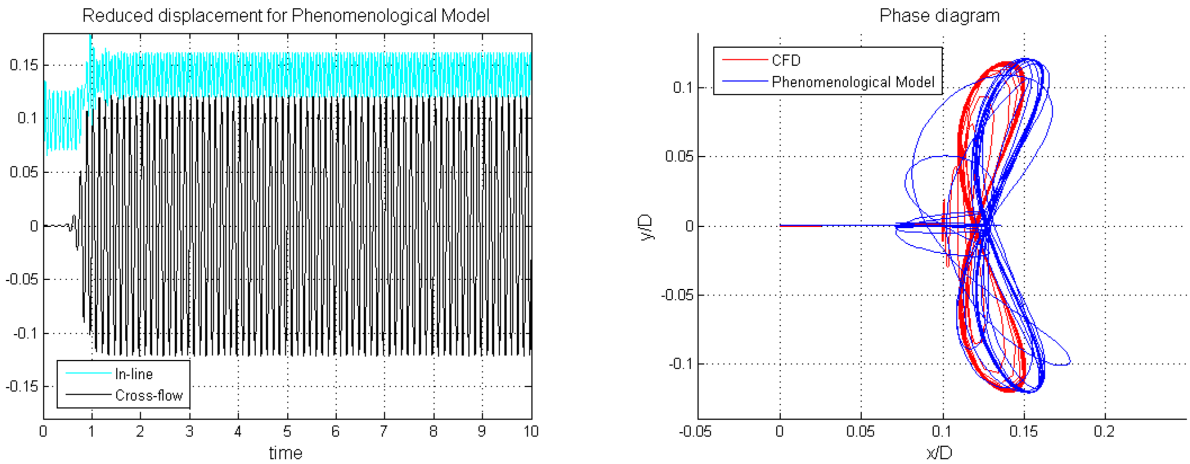

Figure 3 – Reduced in-line and cross flow displacement vs Phase diagram for $U_r = 2.58$

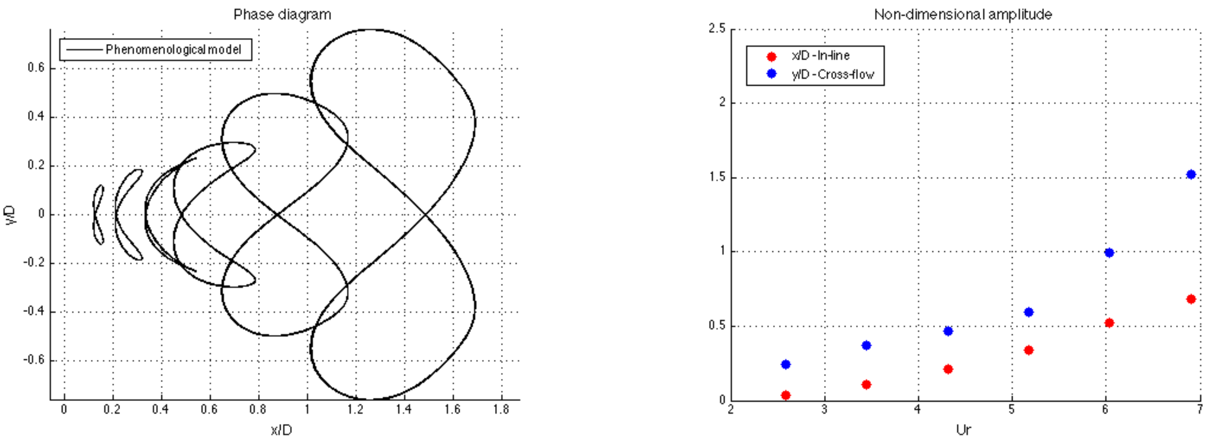

Figure 4 – Phase diagram vs amplitude for increasing reduced velocity

Being y the dimensionless cross-wise displacement of the structure and x the stream displacement, q_x and q_y are the dimensionless wake variables associated with the fluctuating lift and drag coefficient of the structure. V is the free stream velocity of the uniform flow, Ω_s and Ω_f are the vortex shedding angular frequency and the structure angular frequency, respectively, U_r is the reduced velocity, C_L and C_D is the lift and drag coefficients of the structure, m is the structural mass, including the fluid-added mass, and ρ is the fluid density. The response of the model to stochastic variation in the empirical parameter will be analysed in this work.

The dependency of the empirical coupling parameter A on the frequency ratio δ, including a jump, might entail abrupt changes on the structure response. Whenever the shedding frequency approaches the natural frequency of the structure, the coupled system enters in the lock-in regime. In this case, the amplitude of the structural response during achieves a maximum and, thus, this is considered a critical mode of vibration. A schema of the lock-in is depicted in Fig. 5, where the maximum amplitude in the steady state regime is plotted as a function of the reduced velocity U_r in the range

$[0,8]$. To avoid problems, computational models are built aiming at identifying the narrow band of the reduced velocity corresponding to lock-in. Hence, in this example we investigate the impact of uncertainties on the predictions of quantities of interest. The choice of those two parameters relies on a previous sensitivity analysis about the model's parameters. Moreover, both parameters are to be determined through experiments and then subject to unavoidable variability. In order to illustrate impact of uncertainties propagation on the computed results, we assume the following probabilistic models.

4.2 Stochastic analysis model

Assuming the following probabilistic models for the parameters:

$$k = \bar{k} + \delta_k \phi_1 \tag{16}$$
$$c = \bar{c} + \delta_c \phi_2 \tag{17}$$

being k the structural stiffness constant and c the structural damping coefficient. Mean and variability are given by $(\bar{k}, \delta_k) = (5000, 0.05)$ and $(\bar{c}, \delta_c) = (100, 0.05)$ respectively. The random variables ϕ_i with support [-1,1] are assumed independent and uniformly distributed. The analysis performs a parameter sweep over reduced velocity U_r in the range $[2-8]$

As we can see in Fig. 5, the lock-in response shows a significant sensitivity with respect to those parameters, which motivates the following analysis, which is carried out using [5], as its predictions are closer to the available experimental data. The model parameters adopted are, C_{L_0} and c_f, i.e., the lift coefficient of reference and a coefficient that controls the amplitude of the coupling parameter A.

In this case this case the ASGC method was used in order to avoid problems with the presence of discontinuities or sharp gradients in the solution with respect to the random space. With this aim, we adopted here the adaptive sparse grid collocation (ASGC) introduced in [7]. This method introduces a local refinement departing from the hierarchical sparse grid collocation discussed before. It is also to be remarked that the discontinuity on the uncertain parameter A might contribute to the existence of sharp gradients on the response. We performed a parameter sweep over U_r in the range $[2-8]$ with constant step 0.1. This yields 60 stochastic problems to be solved using both Monte Carlo and ASGC methods.

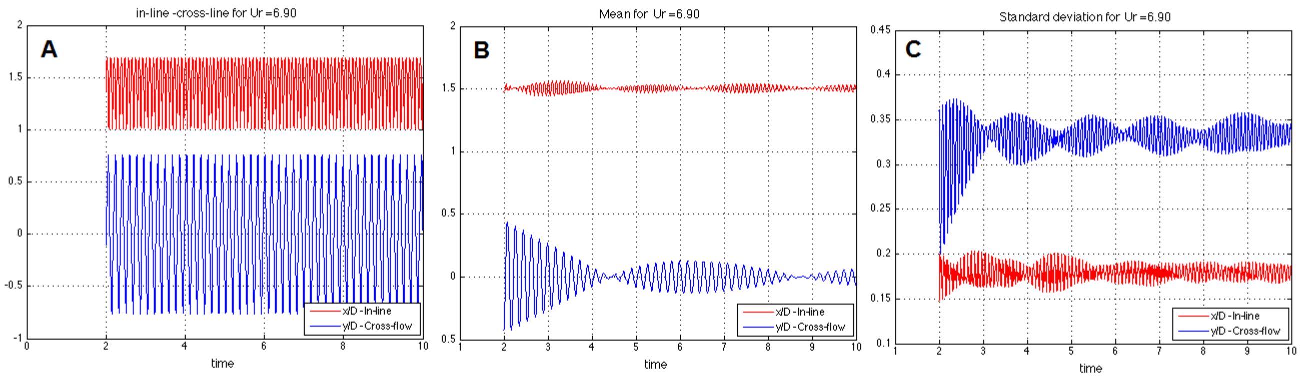

Figure 5 – Sample of statistics for reduced velocity $U_r = 6.9$

The analysis, summarized in Fig. 5 was performed using the uncertain parameters. In A we show the deterministic output for the nominal values of parameters. while in B and C we show the evolution in time of statistical moments computed with the ASCM for a $Ur = 6.9$. The Figures 5 and 6 show the evolution of the mean and standard deviation of the structure response for increasing velocity.

A more refined analysis, involving reliability of the structure, which often is linked with events of low probability, is enabled by obtaining the statistics tails contained in a PDF that can be approximated by simple sampling of the interpolation function calculated through the ASGC method.

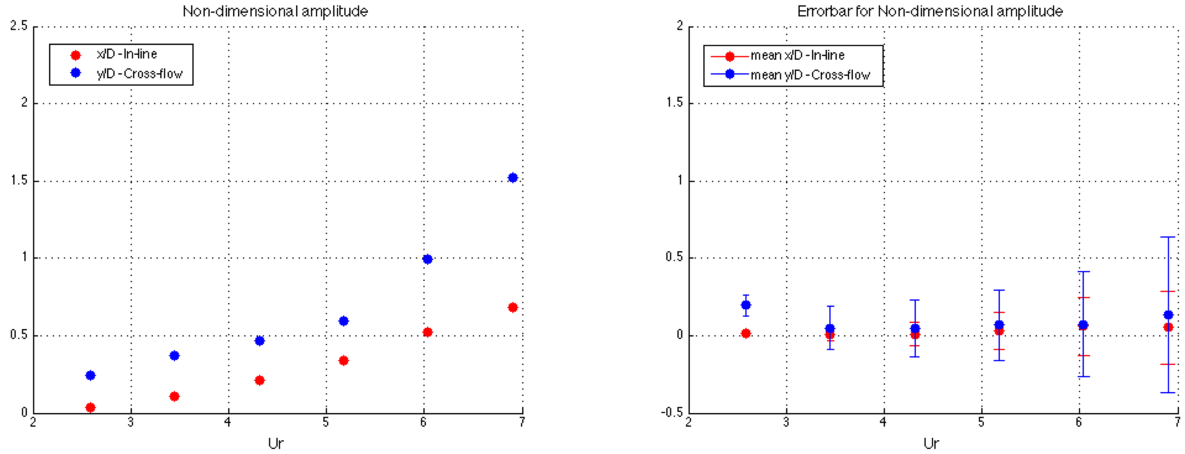

Figure 6 – Amplitude vs errorbar of mean amplitude for increasing reduced velocity

5 CONCLUSIONS

In this work, we have explored the capabilities of Adaptive Sparse Grid Collocation Method was used to estimate the statistical moments. The aim of this work was try to capture the response sensitivity to the change of parameters in the surrogate model to compare with the CFD model. The results obtained are preliminary and need to be analyzed deeper. The adaptive sparse grid collocation method was used to estimate the statistical moments refining the grid locally when necessary, identifying non smooth regions.

The example presented deals with the vibrational response structures excited by vortex detachments in the surrounding flow. We adopted a popular engineering model to predict this dynamics, with special attention to the lock-in phenomena. Two of the parameters used by this modeling were considered uncertain, inasmuch as they often are obtained by means of model calibration based on more complex models.

An Adaptive Sparse Grid Collocation Method was used to estimate the statistical moments. Like the Monte Carlo method, the ASGC method leads to the solution of uncoupled deterministic problems and, as such, it is simple to implement and parallelize. This non-intrusive method, allow convert any deterministic code into a code that solves the corresponding stochastic problem. Compared with the Monte Carlo Simulation method, the ASGC method presents a significative reduction in the number of experiments required to achieve the same level of accuracy. The results obtained, show that it is possible refine the grid locally identifying automatically non smooth regions in the stochastic space achieving the same accuracy and reducing significatively the cost by the use of less collocations points in smooth regions of the stochastic space. Due to that the majority of engineering problems varying rapidly in only some dimensions, remaining much smoother in other dimensions and in general it have more stochastic dimensions. Future work of this research will include analysis of sensitivity for others parameters as well the use of experimental data to validate and calibrate the models aiming to solve practical problems in Engineering.

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