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Nonlinear damping to increase energy transfer in tuning fork gyroscope

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Abstract: In this work, the dynamic behavior of a gyroscope of the tuning fork type for energy recovery will be studied. The gyroscope consists of two vertical posts embedded on a mass of suspension. The suspended mass is subject to vertical excitation force, and it can be shown that the system is a parametrically excited system. When the beams oscillate in phase, the parametric excitation can lead to vibration in the horizontal movement of the suspension mass. The problem is particularly interesting for capture or energy recovery. Due to the interaction between the degrees of freedom of the system, energy is transferred from the vertical toward the horizontal direction. Parametric studies will be performed to analyse the dynamic behavior of the system with respect to energy recovery performance. The inclusion of nonlinear damping on the energy transfer between the degrees of freedom will be analysed, including in phase and out of phase movement of the vertical beams, under harmonic excitation. The existence of periodic, quasiperiodic and chaotic regimes, and its influence on the energy transfer between the vertical and horizontal directions will be studied.

Keywords: Tuning fork gyroscope, Parametric excitation, Nonlinear damping

NOMENCLATURE

m_1 and m_2: mass of each pendulum
l_1 and l_2: length of massles rods
k_1 and k_2: linear torsion stiffness
h_1 and h_2: nonlinear torsional stiffness
c_1 and c_2: linear torsion damping
c_{n1} and c_{n2}: nonlinear torsion damping
M: suspended mass
k_x: equivalent horizontal stiffness
k_y: equivalent vertical stiffness
c_4: equivalent horizontal damping
c_5: equivalent vertical damping
θ_1 and θ_2: rotation of each pendulum
x and y: horizontal and vertical motion of suspended mass
q: electrical charge
R: electrical resistance
L: electrical inductance
C_0: linear electrical capacitance
α_3: nonlinear electrical capacitance
K: coupling factor
B: magnetic field
l_c: coil length
e: input voltage
e_0: input amplitude
Ω: input frequency
t: time

INTRODUCTION

The concept of energy harvesting is based on the capture of energy present in the environment and conversion to electricity. Applications of the energy harvesting principle extend to various fields. The energy from the environment can be transformed using various types of transducers, such as chemical, pyroelectric, piezoelectric, electrostatic and electromagnetic. Electromechanical transducers are well explored in engineering, particularly piezoelectric or electromagnetic transducers. The latter uses a coil with permanent magnet, converting mechanical movement into electromagnetic energy (Preumont, 2006).

Energy harvesting devices can replace external sources of energy in embedded systems, providing electricity recovered from mechanical vibrations in the environment. For example, energy harvesters can be used to supply power to wireless sensor systems for structural monitoring, allowing great flexibility and positioning sensors in places which are difficult to access (Beeby *et al.*, 2007).

The maximum possible harvested energy is subject to restrictions on size, weight and cost of the system. It is intuitive that a device which has resonant behavior will provide the best performance, and the potentially beneficial effects of adding stiffness and nonlinear damping are also considered (Langley, 2014). There are several configurations of energy harvesters. For example, Beeby *et al.* (2007) and Kulkarni *et al.* (2008) analysed electromagnetic microgenerators, optimized for energy harvesting in low-vibration environments. The fact that the system has a reduced size provides a better use of the large coil winding density and the magnetic properties of the material. Therefore, the results show that it is possible to obtain usable energy from micro generators, which proves to be a promising future for the wireless sensors, systems in which there is the need for device independence or the reuse of dissipated energy.

Considering the applicability of these devices for large scale systems, Munteanu *et al.* (2013) explored the use of electromechanical devices such as vibration absorbers in small buildings. Brasil *et al.* (2006) studied energy recovery through a electromechanical system for wind turbine structures. Zhu *et al.* (2012) analysed the application of electromagnetic damping systems of linear motion in civil structures in order to reduce vibrations and recover the vibrational energy. Besides acting as a damping device, these system harvests energy and can be used in wireless sensors for data transmission and information about the structure, for example. In the study, four different types of electric circuits connected to the electromagnetic absorber are studied, both theoretically and experimentally. The performance of the electromagnetic damper and energy recovery are highly affected by the type of electrical circuit connected to the shock absorber. The authors state that the use of these types of dampers in civil structures is promising.

Another studied application of energy harvesting systems involves regenerative automotive and rail suspension systems. In a typical vehicle suspension system, viscous dampers dissipate energy as heat, ie, energy is lost to the environment. The implementation of an energy harvesting system allows part of this energy to be reused. These devices have advantages compared to conventional viscous damper, such as ease of application of an active control and parameter setting, absorbing a greater range of frequencies, and the possibility of system energy harvesting (Amati *et al.*, 2011; Tonoli *et al.*, 2013). An electromagnetic damper model was demonstrated by Kawamoto *et al.* (2007). The use of active control for the suspension system proved that considerable reduction of vibrations with a positive energy balance is possible.

Some authors studied the ideal conditions for maximum energy recovery. Stephen (2006), for example, studied the energy recovery arising from vibration in the environment, reminding that the amount of power supplied to the electrical part of the system cannot be greater than the power dissipated by the mechanical part. Moreover, a system with high damping will harvest more energy. Therefore, for a maximum energy flow in the system, the optimal conditions are required on two factors, the environment (frequency and amplitude) and the device size.

Recently there has been much interest in the concept of low power microelectromechanical systems (MEMS) that are capable of harvesting energy from its operating environment. As the energy to be harvested is low, a smart sensor for measuring vibration causes the device to harvest and store energy during a period of time before performing a read, and then makes a new harvesting period. Such devices can be used in hostile or difficult to access environments, and require little or no maintenance. The displacement amplitude of a mass is limited in any practical device, and this, coupled with the magnitude and frequency of the excitation, defines the maximum power that can be harvested from the environment (Stephen, 2006).

Renno *et al.* (2009) considered the mechanical and electrical subsystems for optimization of the energy harvesting system. Regarding the mechanical damping ratio, it is shown that certain damping ratios result in two values (resonance and antiresonance) where harvesting is maximum. Furthermore, materials with larger electromechanical coupling values do not always lead to a greater recovery of energy. In relation to the electrical system, the use of an ideal inductor in the circuit may greatly increase the energy recovery. Using the inductor is not required to meet the natural resonance or antiresonance frequency of the material for maximum energy recovery.

A resonant linear generator has been one of the most common type of generator used for energy harvesting. However, the performance of the generator is limited to a narrow frequency band, when the device is tuned so that its natural frequency coincides with the frequency of excitation. If poorly tuned, the performance of the resonance generator falls rapidly. Analyses based on the principle of energy conservation show an efficiency of any nonlinear device for a linear device for such application. Several works explore nonlinear characteristics of electromagnetic energy recovery systems to increase energy efficiency. For example, the presence of geometrical and ferroresonance nonlinearities can improve the system performance (Tékam *et al.*, 2014). An electromagnetic system considering a nonlinear relationship between the magnetic field and the coil position showed that the energy transfer is greatly impact on transitions between regular and chaotic oscillations (Siewe & Buckjohn, 2014).

Two nonlinear mechanisms were considered by Ramlan *et al.* (2010). The first system has a bi-stable nonlinear mechanism of the *snap-through* type with one degree of freedom (Fig. 1 -a), which has the effect of increasing the mass displacement time response, resulting in higher velocity for a given input excitation, thereby increasing the amount of harvested energy. The other nonlinear system studied has a hardening spring which has the effect of shifting the resonance frequency.

With respect to nonlinear damping, Ghandchi Tehrani & Elliot (2014) used cubic nonlinear damping in a mechanical system for energy harvesting (Fig. 1 -b) in order to investigate a method to increase the dynamic range energy harvesting operation. It was found that a nonlinear harvester provides more power at resonance as compared with a linear harvester.

Vibratory gyroscopes of the tuning fork type are used to measure angular velocity. They are non-rotational devices and based on the Coriolis acceleration effect. They consist of two extensions, which vibrate in opposite directions to measure angular velocity. The performance of micro tuning fork gyroscopes is limited due to its small size and imperfections in manufacturing. Small imperfections can cause significant error in the operation and accuracy of such devices. The excitation amplitude or increased speed can improve its performance. However, this depends on the power supplied by the external source.

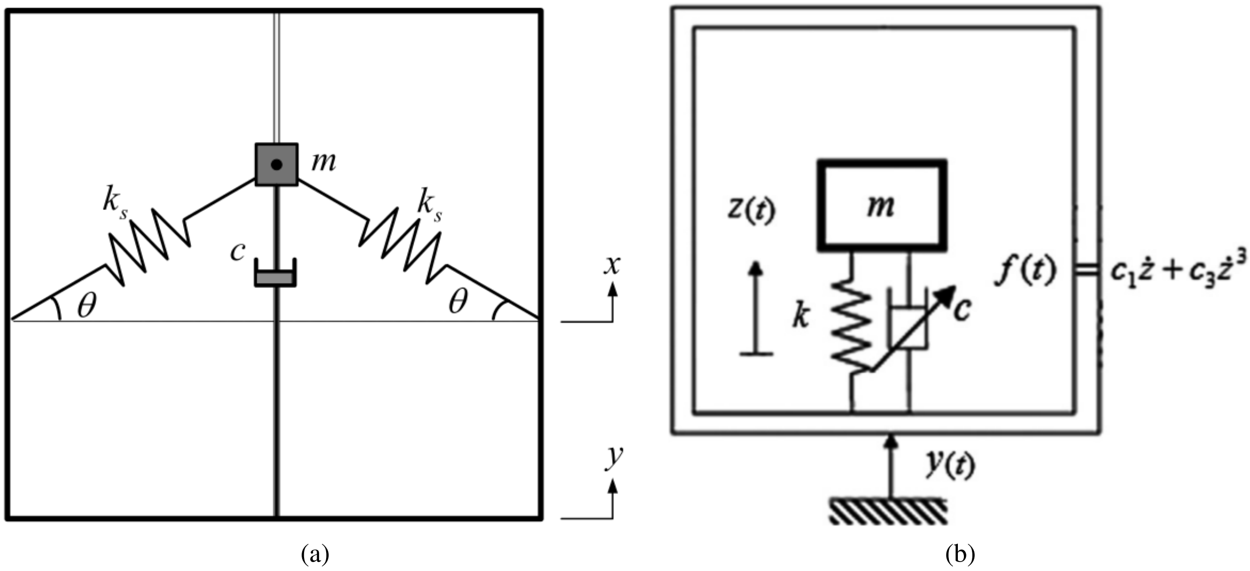

Figure 1 – System with nonlinear stiffness (a) and system with nonlinear damping (b) to increase energy harvesting. Sourcess: Ramlan *et al.* (2010); Ghandchi Tehrani & Elliot (2014)

It has been seen that parametric excitation can be used to increase the amplitude of the vibrations of the two extensions. For example, Ghandchi Tehrani & Kalkowski (2015) experimentally investigated the instability caused by parametric excitation in a slender beam subject to base excitation. When the base excitation frequency is twice the natural frequency of the beam, large vibration amplitudes can occur in the direction perpendicular to the movement of the base. Lee *et al.* (2008) showed the response of a tuning fork subjected to parametric excitation experimentally. It was shown that the parametric excitation can be exploited to increase the sensitivity of the gyro pitch tuning. Vibration of parametrically excited systems with ideal and non-ideal energy sources have been investigated, such as a system consisting of a nonlinear mechanical oscillator with a damped pendulum. The dynamic interaction between pendular systems and base excitation has also been considered. Xu *et al.* (2007) used numerical and analytical techniques to explore the dynamics of parametric pendulum harmonically excited when it displays rotational and oscillatory orbits.

Ghandchi Tehrani *et al.* (2016) analysed the dynamics of a model of a gyroscope-type tuning fork. The system is subject to base excitation through an electromechanical shaker, which interacts with the dynamic structure. There is coupling between the degrees of freedom by parametric excitation, inertial and non-geometric linearity. The dynamic equations were demonstrated using Lagrange's method. Parametric study investigated the effect of initial conditions, the amplitude and frequency of arousal. It was seen that the system may have periodic, quasiperiodic and chaotic behaviour depending on the parameters related to base excitation. It was shown that it is possible to have coexistence of periodic and chaotic attractors, and windows for a wide range of frequencies in which there is high frequency response. Due to the interaction between the degrees of freedom of the system, energy is transferred from the vertical direction to the horizontal direction.

In this work, the dynamic behavior of a gyroscope of the tuning fork type for energy recovery will be studied. The gyroscope consists of two vertical posts embedded on a mass of suspension. The suspended mass is subject to vertical excitation force, and it can be shown that the system is a parametrically excited system. When the beams oscillate in phase, the parametric excitation can lead to vibration in the horizontal movement of the suspension mass. The problem is particularly interesting for capture or energy recovery. Due to the interaction between the degrees of freedom of the system, energy is transferred from the vertical toward the horizontal direction. Parametric studies will be performed to analyse the dynamic behavior of the system with respect to energy recovery performance. The inclusion of nonlinear damping on the energy transfer between the degrees of freedom will be analysed, including in phase and out of phase movement of the vertical beams, under harmonic excitation. The existence of periodic, quasiperiodic and chaotic regimes, and its influence on the energy transfer between the vertical and horizontal directions will be studied.

TUNING FORK GYROSCOPE

Ghandchi Tehrani *et al.* (2016) analysed the dynamic response of a tuning fork gyroscope as shown in Fig. 2. It consists of two pendulums attached to a suspension mass. Each pendulum has a lumped mass, m_1 and m_2, which are connected to massless rods, with lengths l_1 and l_2. The stiffness of the pendulums includes both linear and nonlinear springs. The linear springs are k_1 and k_2. The nonlinear torsional springs are h_1 and h_2. Linear damping coefficients c_1 and c_2 are considered for the pendulums. The mass M is suspended via horizontal and vertical springs, k_3 and k_4, respectively, to an electrodynamic shaker. The total mass of the system is $M_t = M + m_1 + m_2$. The equivalent springs for horizontal and vertical directions are k_x and k_y. Two linear dampers c_4 and c_5 are considered for horizontal and vertical directions. There are five degrees of freedom, two rotations of the pendulums (θ_1 and θ_2), horizontal and vertical motion of the suspended mass (x and y) and the electrical charge (q) of the shaker. The shaker generates a base displacement along the vertical direction and has an electrical circuit with resistance R, inductance L and capacitance C. The coupling through the electromagnetic force can be realised as $F_{em} = K\dot{q}$, with $K = Bl_c$ being the coupling factor, B the magnetic field and l_c the length of the conductor. A sinusoidal voltage input $e(t) = e_0\cos(\Omega t)$ with an amplitude e_0 and excitation frequency Ω drives the shaker. The voltage across the capacitor is a nonlinear function of charge q, with a linear capacitor C_0 and a cubic capacitor α_3.

The problem is complex, as there is coupling between the degrees of freedom by parametric excitation, inertia and geometrical nonlinearities. The dynamical equations were obtained using the Lagrange method, and are given in nondimensional form by Eq. (1), with nondimensional parameters given by Eq. (2). A prime ($'$) denotes derivative with respect to τ. Note that this representation of the system considers linear damping. The inclusion of nonlinear damping terms will be described in a following section.

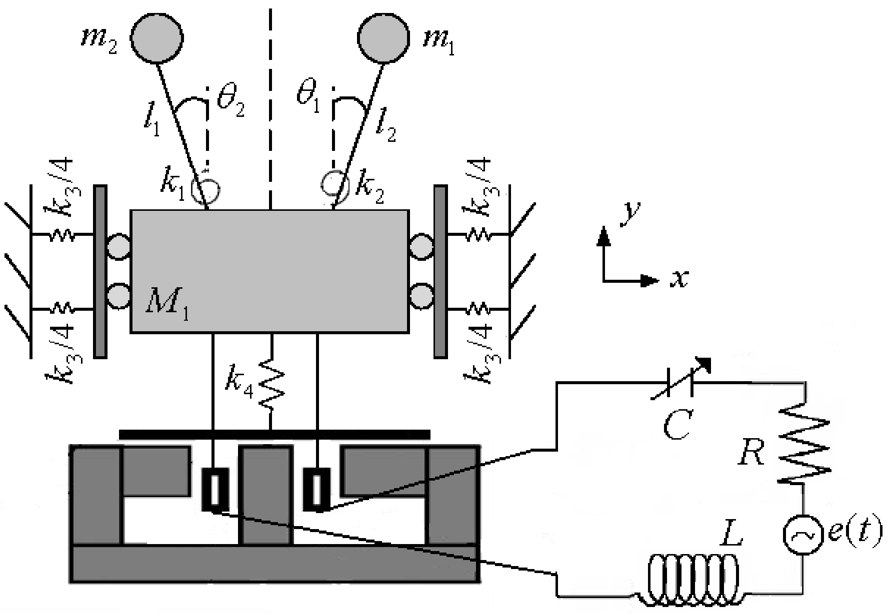

Figure 2 – Schematic drawing of tuning fork energy harvester as shown by Ghandchi Tehrani *et al.* (2016).

$$
\begin{aligned}
\theta_1'' + \delta_1\theta_1 + \lambda_1\theta_1^3 + \mu_1\theta_1' - \eta_1\left(X''\cos\theta_1 + \left(Y'' + g_1\right)\sin\theta_1\right) &= 0 \\
\theta_2'' + \delta_2\theta_2 + \lambda_2\theta_2^3 + \mu_2\theta_2' - \eta_2\left(X''\cos\theta_2 + \left(Y'' + g_1\right)\sin\theta_2\right) &= 0 \\
X'' + \mu_4 X' + \omega_x^2 X - \sum_{i=1}^{2} F_i\left(\theta_i'' cos\theta_i - \theta_i'^2\sin\theta_i\right) &= 0 \\
Y'' + \mu_5 Y' + \omega_y^2 Y - \sum_{i=1}^{2} F_i\left(\theta_i'' sin\theta_i - \theta_i'^2\cos\theta_i\right) - \gamma_i Q' &= 0 \\
Q'' + \mu_3 Q' + Q + \lambda_3 Q^3 + \gamma_2 Y' &= E_0\cos\left(\omega\tau\right)
\end{aligned}
\tag{1}
$$

$$
\begin{array}{llllll}
X = \dfrac{x}{l_c} & Y = \dfrac{y}{l_c} & Q = \dfrac{q}{q_0} & \delta_1 = \dfrac{k_1}{m_1 l_1^2\omega_e^2} & \delta_2 = \dfrac{k_2}{m_2 l_2^2\omega_e^2} & g_1 = \dfrac{g}{\omega_e^2 l_c} \\
\omega_x = \sqrt{\dfrac{k_x}{M_t\omega_e^2}} & \omega_y = \sqrt{\dfrac{k_y}{M_t\omega_e^2}} & \zeta_1 = \dfrac{l_1}{l_c} & \zeta_2 = \dfrac{l_2}{l_c} & \gamma_1 = \dfrac{Kq_0}{M_t\omega_e l_c} & \gamma_2 = \dfrac{Kl_c}{M_t\omega_e q_0} \\
F_1 = \dfrac{m_1 l_1}{M_t l_c} & F_2 = \dfrac{m_2 l_2}{M_t l_c} & \lambda_1 = \dfrac{h_1}{m_1 l_1^2\omega_2^2} & \lambda_2 = \dfrac{h_2}{m_2 l_2^2\omega_2^2} & \lambda_3 = \dfrac{\alpha_3 q_0^2}{L\omega_e^2} & \\
\mu_1 = \dfrac{c_1}{m_1\omega_e} & \mu_2 = \dfrac{c_2}{m_2\omega_e} & \mu_3 = \dfrac{R}{L\omega_e} & \mu_4 = \dfrac{c_4}{M_t\omega_e} & \mu_5 = \dfrac{c_5}{M_t\omega_e} & \\
\omega_e = \dfrac{1}{\sqrt{LC_0}} & \omega = \dfrac{\Omega}{\omega_e} & E_0 = \dfrac{e_0}{L\omega_e^2 q_0} & \tau = \omega_e t & &
\end{array}
\tag{2}
$$

LINEAR DAMPING

The system as described by Eq. (1) has linear damping terms in the angular (θ_1 and θ_2) and translational (x and y) displacements. Bifurcation diagrams as a function of the excitation frequency are shown in Fig. 3 for both forward (black) and backward (red) sweep. The response of the system presents periodic, quasi-periodic and chaotic behaviour, and presents co-existence of multiple attractors. It is also possible to show that there are responses with high periodicity in very small windows among the chaotic regions.

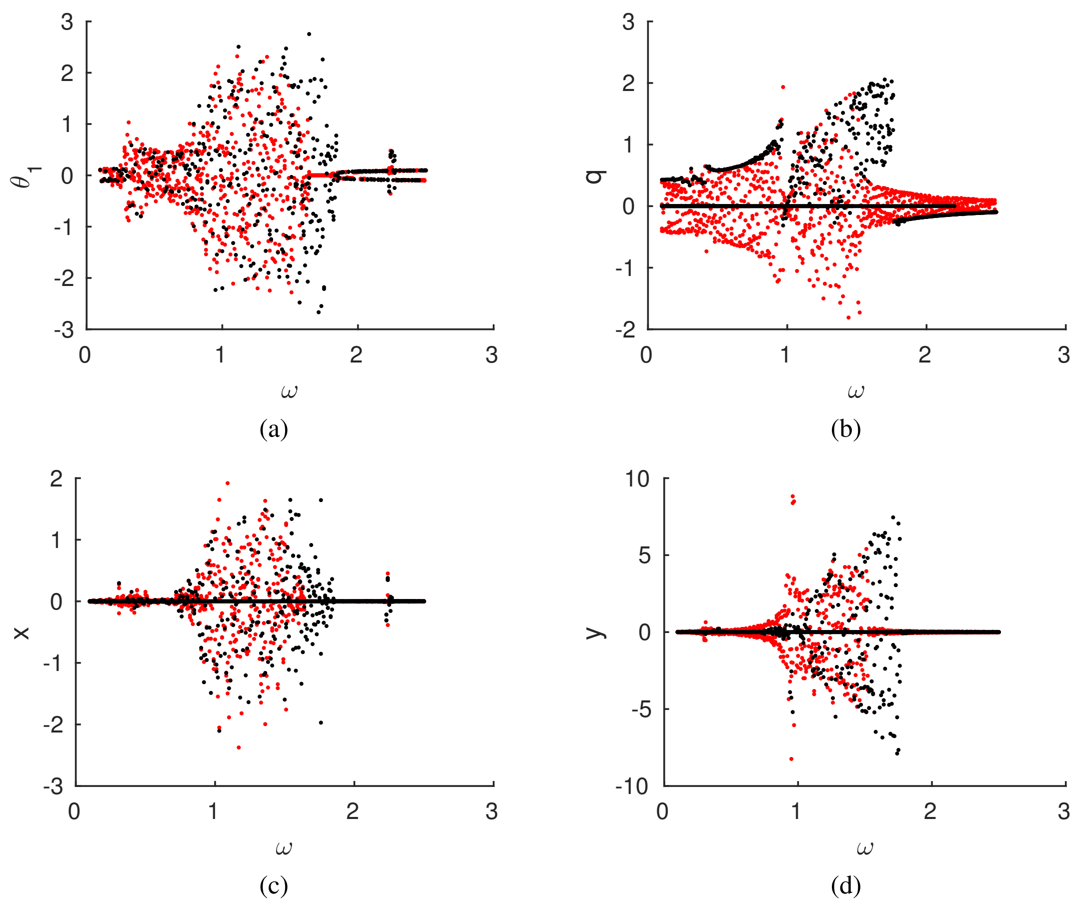

Figure 3 – Bifurcation diagrams with forward (black) and backward (red) sweep: (a) θ_1**, (b)** q**, (c)** x**, (d)** y**.**

NONLINEAR DAMPING

In this section, nonlinear damping terms are considered to investigate their influence on the response of the system. The nonlinear damping terms for each rotation and translation damper are considered as:

$$F_{c1} = \mu_1 \dot{\theta}_1 + \beta \mu_1 \dot{\theta}_1^3 \quad (3)$$

$$F_{c2} = \mu_2 \dot{\theta}_2 + \beta \mu_2 \dot{\theta}_2^3 \quad (4)$$

$$F_{c4} = \mu_4 \dot{X} + \beta \mu_4 \dot{X}^3 \quad (5)$$

$$F_{c5} = \mu_5 \dot{Y} + \beta \mu_5 \dot{Y}^3 \quad (6)$$

Bifurcation diagrams were built with the nonlinear terms included, and are shown in Fig. 4 to 6, with respective values for β of 0.1, 0.5 and 1. It is possible to observe that as the value of β is increased, amplitudes are altered and bifurcation points are shifted. Specifically, with $\beta = 0.1$, amplitudes in x direction are lower, although the large chaotic attractor exists for a larger range of higher frequencies. These changes are reflected on the angular amplitudes (θ_1 and θ_2), electrical charge (q) and horizontal movement (y), although the bifurcation structure is not altered. As β is further increased to 1, θ_1, θ_2 and q do not show significant change in behaviour, although displacements in x and y are considerably reduced. As the range of frequencies for existance of the chaotic attractor is enlarged further, it is possible to harvest more energy at higher frequencies, as it has been shown that trajectories in the chaotic has higher energy content.

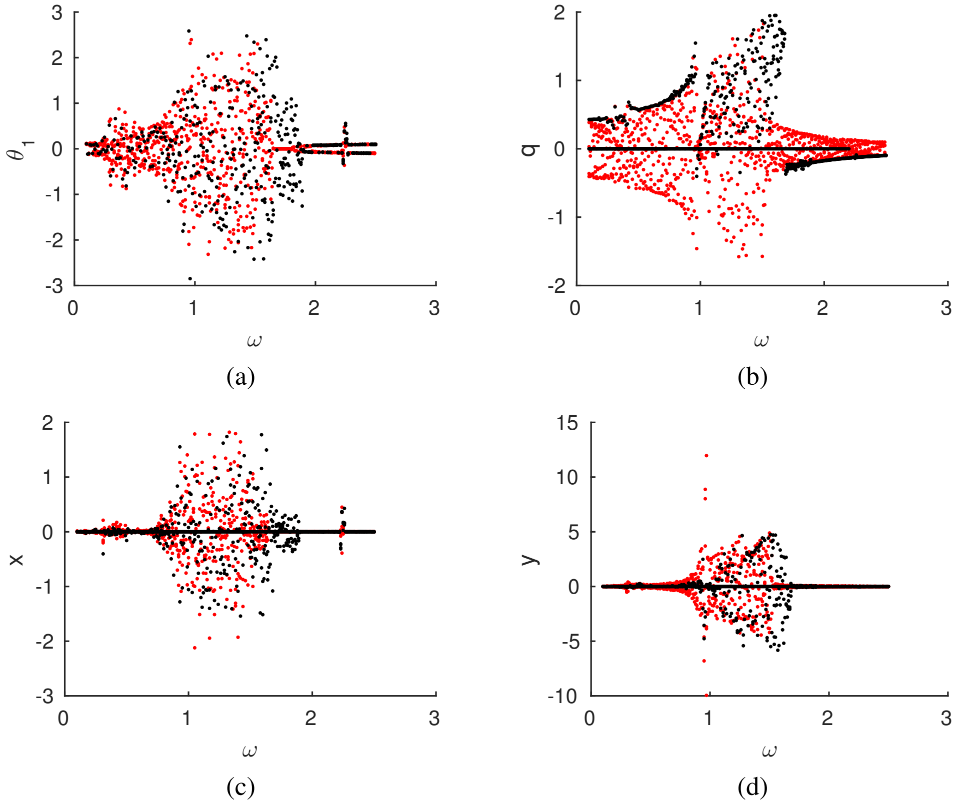

Figure 4 – Bifurcation diagrams for $\beta = 0.1$ with forward (black) and backward (red) sweep: (a) θ_1, (b) q, (c) x, (d) y.

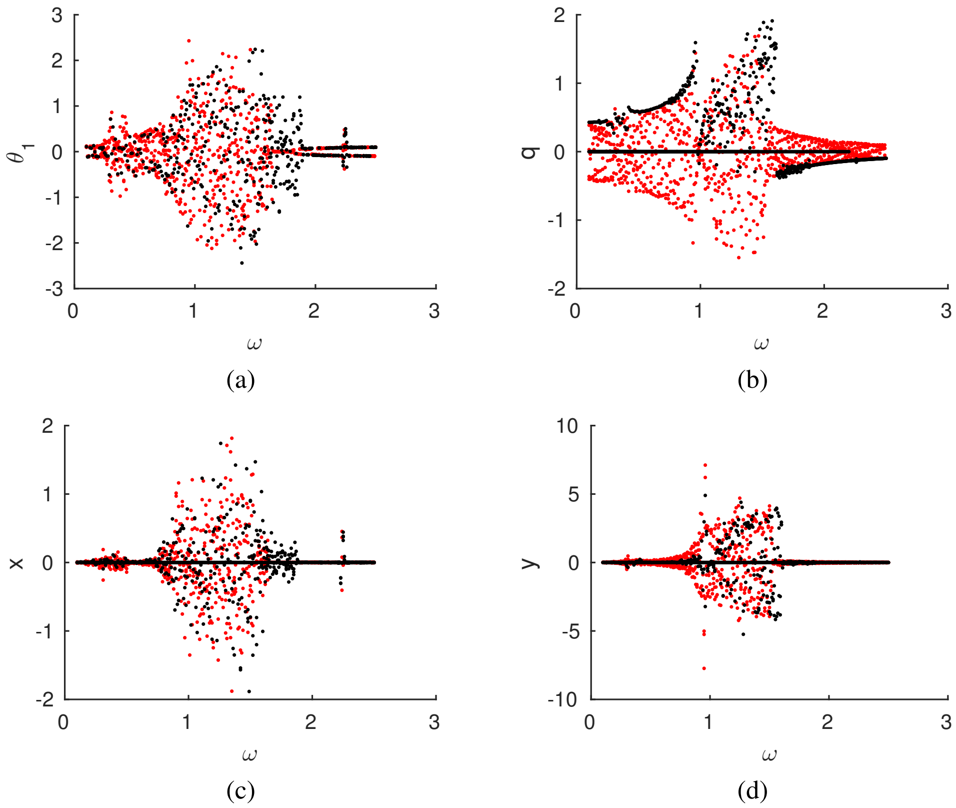

Figure 5 – Bifurcation diagrams for $\beta = 0.5$ with forward (black) and backward (red) sweep: (a) θ_1, (b) q, (c) x, (d) y.

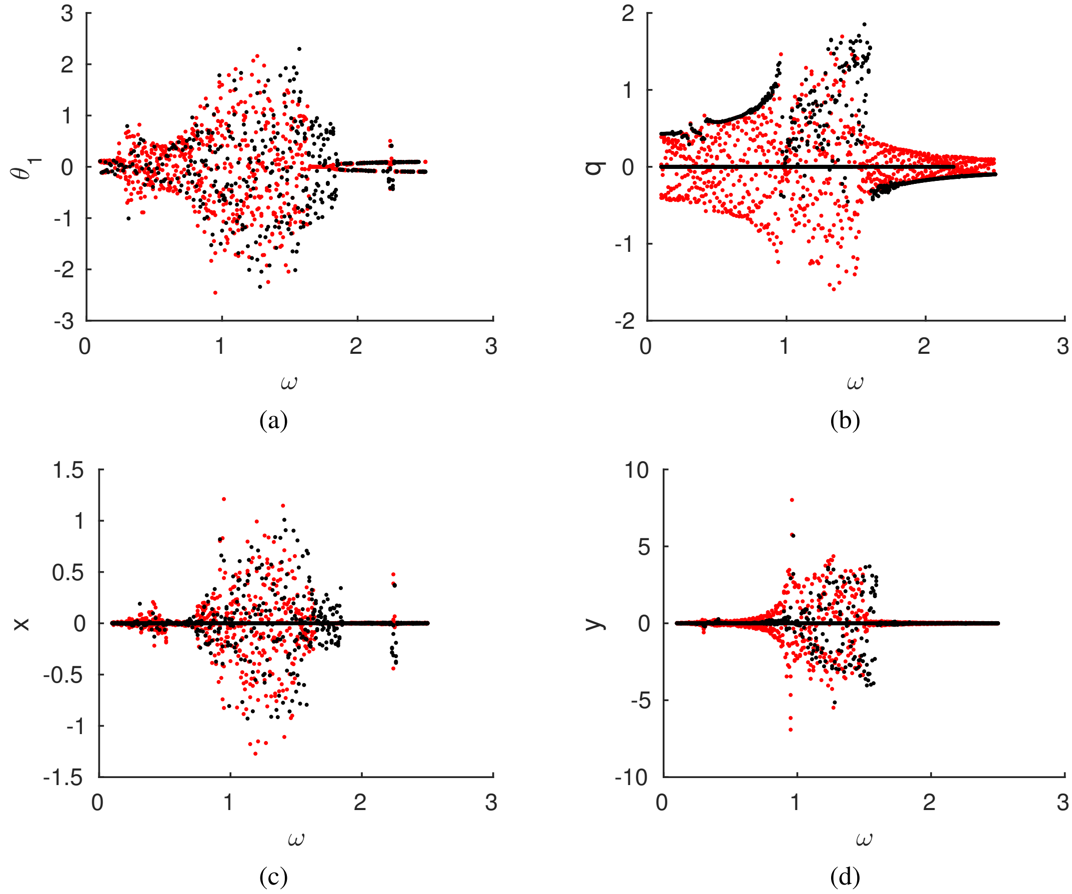

Figure 6 – Bifurcation diagrams for $\beta = 1$ with forward (black) and backward (red) sweep: (a) θ_1, (b) q, (c) x, (d) y.

CONCLUSIONS

In this paper, a numerical study is carried out on a macro-scale tuning gyroscope. The gyroscope consists of two inverted pendulums on a suspension mass. The suspension mass is subjected to base excitation generated by an electromagnetic shaker. It is demonstrated that the system can have periodic, quasi-periodic and chaotic behaviour depending on the parameters using bifurcation diagrams. Co-existence of periodic and chaotic attractors is possible, and there are narrow windows for a broad range of frequencies with high-periodic response. Due to the interaction between the system degrees of freedom, the energy is transferred from vertical to horizontal direction. The inclusion of nonlinear damping terms has resulted in increased range of frequencies in which chaotic attractors exists, which results in larger energy being harvested in higher frequencies.

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