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Effect of Mistuning on the Harvested Energy from a Nonlinear System with Weak Coupling
Nonlinear Dynamics:

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Abstract: In this paper, the potential benefit of mistuning on the harvested energy from a nonlinear system is investigated. Previously, it has been demonstrated that the mistuning of the linear periodic weakly coupled system can increase the harvested power significantly. In this paper, the work is extended to a nonlinear system with weak linear coupling as shown in Figure 1. The nonlinearities take into account the Duffing nonlinear spring due to an electromagnetic set-up and the cubic electrical damping, which is used in the electrical circuit as nonlinear resistive load to harvest energy. For simulation, a two degree-of-freedom (dof) spring-mass-damper system is considered. The mistuning is achieved by a slight variation of one of the masses. An electromagnetic energy harvester is used to harvest energy from the motion of the mistuned mass. The response and the harvested power of the mistuned system is obtained using harmonic balance method as a function of the mistuning parameter. The effects of the mistuning, damping and excitation level on the harvested power are investigated. The performance (harvested power) of the linear and nonlinear electrical damper is also compared for different values of mistuning and excitation levels. The system can easily be generalized to n DOF to represent a quasi-periodic structure.

Keywords: energy harvesting, mistuning, nonlinear system, weakly coupled system.

NOMENCLATURE

Latin symbols

A : coefficient
B : coefficient
C : coefficient
D : coefficient
X : amplitude of displacement
P : harvested power
c : damping
k : stiffness
m : mass
n : number of degrees-of-freedom
p : number of mistuning
t : time
x : displacement

Greek symbols

Ω : excitation frequency
δ : mistuning
μ : nonlinear stiffness

Subscripts

e : electrical
e N: nonlinear electrical
m : mechanical
c : coupling
g: base excitation
j: degree of freedom

INTRODUCTION

Periodic structures can be found in several engineering systems such as aircraft rotor and turbine blades (Yoo et al. 2003). Periodic structures are usually manufactured identically, however there always exists some slight differences among the components due to manufacturing tolerances, which refer to mistuning. The mistuning can cause significant increase in the vibration response of a mistuned component. Therefore, the mechanical energy stored in the mistuned component is larger than of a perfectly tuned component. This is called the vibration localization of a mistuned periodic structure. The vibration localization due to mistuning can be exploited for vibration energy harvesting.

Energy harvesting has received considerable attention in the past (Williams and Yates (1996), Stephen (2006)). Number of vibration energy harvesting devices have been developed using electromagnetic, electrostatic and piezoelectric technologies. To increase the range of excitation frequency over which the vibration energy harvester operates, various nonlinear arrangements have been suggested, particularly using semi-active control (Di Monaco et al (2013)), nonlinear damping (Ghandchi Tehrani and Elliott 2014) to extend the dynamic performance range, parametric damping (Scapolan et al, (2016)) to achieve parametric resonance for harvesting and nonlinear springs (Mann and Sims (2008) and Ramlan et al (2010)) to increase the frequency bandwidth.

Previously, Malaji and Ali (2014) investigated the effect of stiffness mistuning on a 2- dof pendulum system. The mistuning is achieved by a small variation of the length of the pendulum. They demonstrated that the slight mistuning can increase the maximum displacement and hence the harvested power significantly. Multiple harvesters are also used with intentional mistuning to obtain the wider bandwidth (Malaji et al 2016). Each harvester is mistuned to a slightly different resonance frequency to extend the bandwidth.

In this paper, we extend the work by Malaji and Ali (2014) to nonlinear weakly coupled system. The nonlinearities take into account the Duffing nonlinear spring due to an electromagnetic set-up and the cubic electrical damping, which is used in the electrical circuit as nonlinear resistive load to harvest energy. The displacement and the harvested power is obtained for the system using harmonic balance method. A parametric study is carried out and the effect of the mistuning on the harvested power is investigated.

THEORY ON HARVESTED ENERGY FROM A NONLINEAR SYSTEM

A general weakly coupled nonlinear system is subjected to a harmonic base excitation, having the excitation frequency of Ω and the acceleration amplitude of X_g, as shown in Fig. 1. The mistuned mass is also shunted to an electrical circuit as shown in Fig.2 to harvest energy from the motion of the mass.

The general equations of motion for the general nDOF can be written as,

$$m\ddot{x}_i + c_m\dot{x}_i + (k + 2k_c)x_i - k_c x_{i-1} - k_c x_{i+1} = -m\ddot{x}_g, \quad \text{for } i = 1:n \tag{1}$$

where, m, c_m, and k are the mass, mechanical damping and stiffness, respectively. It is assumed that the damping is proportional to the mass. For weak coupling, we have $k_c << k$.

If p number of mistuning and also harvesters are considered for the periodic system, then the equations of the mistuned DOF becomes,

$$\delta_j m\ddot{x}_j + \delta_j c_m\dot{x}_j + (k + 2k_c)x_j - k_c x_{j-1} - k_c x_{j+1} + \mu x_j^3 + \delta_j c_{ej}\dot{x}_j + \delta_j c_{eNj}\dot{x}_j^{\,3} = -\delta_j m\ddot{x}_g, \quad \text{for } j = 1:p \tag{2}$$

where, δ_j is the mistuning parameter, which represents the slight change in the *j*th-mass . The electrical damping, which is used to harvest energy consists of linear, c_e and nonlinear parts c_{eN}, which relates the current to the relative velocity of the mistuned mass. For example, for a linear circuit shown in Fig 2. , the linear electrical damping can be obtained from the electromechanical coupling α, the resistive load of R_L, and the internal resistance of R_I, $c_e = \dfrac{\alpha^2}{R_L + R_I}$.

The nonlinear stiffness μ is due to the electromagnetic set-up and c_{eN} is introduced in the electrical circuit to harvest energy. Ghandchi Tehrani and Elliott (2014) demonstrated the benefit of using cubic nonlinear damper for extending the dynamic range.

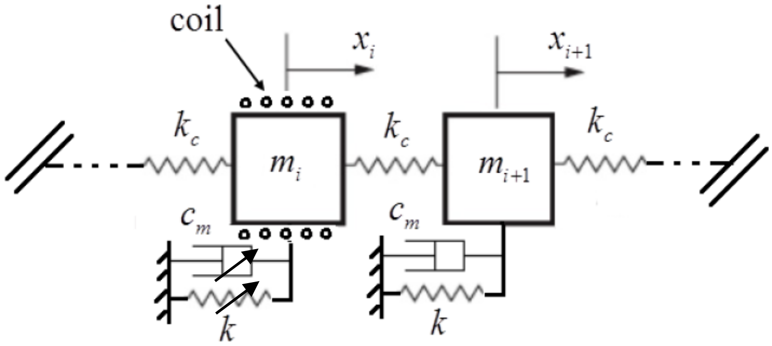


Figure 1 - A general weakly coupled periodic system with n dof and damping proportional to mass.

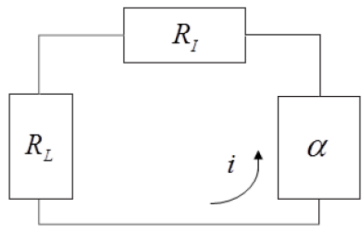


Figure 2 - Electrical harvester circuit shunted to the mistuned mass with electromechanical coupling of α , linear and nonlinear resistive load of R_L , the internal resistance of R_I , and the current of $i(t)$ through the coil.

The harvested energy can then be calculated from the electrical damping and the velocity as,

$$P_j(\omega) = \delta_j c_{e_j} \Omega^2 X_j^2 + \frac{3}{4}\delta_j c_{eN_j} \Omega^4 X_j^4 \quad \text{for} \quad j = 1:p \tag{3}$$

The total maximum power harvested is the sum of the individual maximum power $P_{\max} = \sum_{j=1}^{p} P_{\max\ j}$.

For nonlinear damper, harmonic balance method (HBM) is a good approximation as the backbone curve is a straight line and there is no jump or bifurcation. Assuming that the response is at the fundamental frequency, then we have,

$$x_1 = A\cos\Omega t + B\sin\,\Omega t\ ,\ x_2 = C\cos\Omega t + D\sin\,\Omega t \tag{4,5}$$

Substituting the two responses and their derivatives into the dynamic equations and partitioning the cosine and sine coefficients results in,

$$\begin{bmatrix} k+k_c-m\Omega^2 & c_m\Omega & -k_c & 0 \\ -c_m\Omega & k+k_c-m\Omega^2 & 0 & -k_c \\ -k_c & 0 & k+k_c-\delta m\Omega^2 & (c_e+c_m)\delta\Omega+\frac{3}{4}\delta c_{eN}\Omega^3(C^2+D^2) \\ 0 & -k_c & -\delta(c_e+c_m)\Omega-\frac{3}{4}\delta c_{eN}\Omega^3(C^2+D^2) & k+k_c-\delta m\Omega^2 \end{bmatrix} \begin{pmatrix} A \\ B \\ C \\ D \end{pmatrix} = \begin{pmatrix} X_g\Omega^2 \\ 0 \\ \delta X_g\Omega^2 \\ 0 \end{pmatrix} \tag{6}$$

The amplitude of the two responses are $X_1 = \sqrt{A^2 + B^2}$ and $X_2 = \sqrt{C^2 + D^2}$.

NUMERICAL SIMULATION

For numerical simulation, a two DOF system with mistuning on the second mass is considered. The damping is proportional to the mass. The following system parameters are chosen:

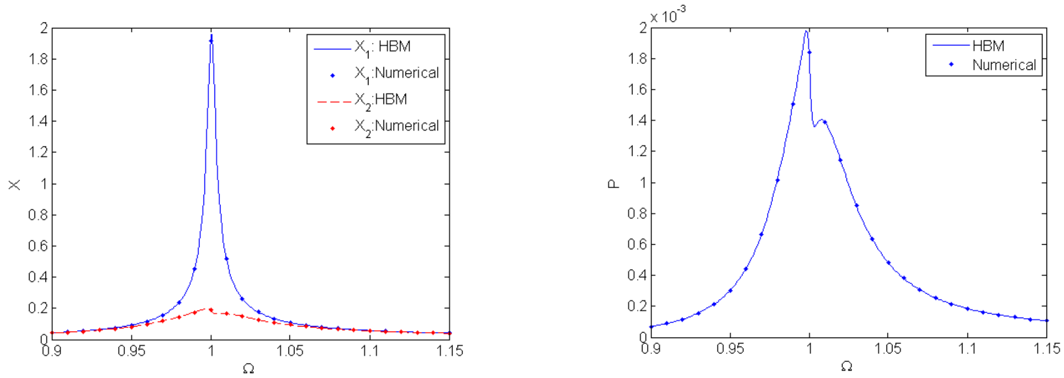
$$m = 1kg, k = 1N/m, c_m = 0.005Ns/m, k_c = 0.001N/m, X_g = 0.01m \tag{7}$$

The response and the harvested power are obtained for different parameters. In this example, to compare the linear with nonlinear dampers, the maximum displacement of the harvester is considered to be the same at resonance for both devices. Therefore, optimum load condition is not considered. First, harmonic balance method is used and the harvested power is obtained for linear and nonlinear electrical loads. Then, the nonlinear cubic stiffness with linear electrical load is considered. Finally, a combined nonlinear electrical load and nonlinear stiffness is considered and the harvested power is obtained for different mistuning and amplitude levels.

Case 1. Linear stiffness and nonlinear electrical damping

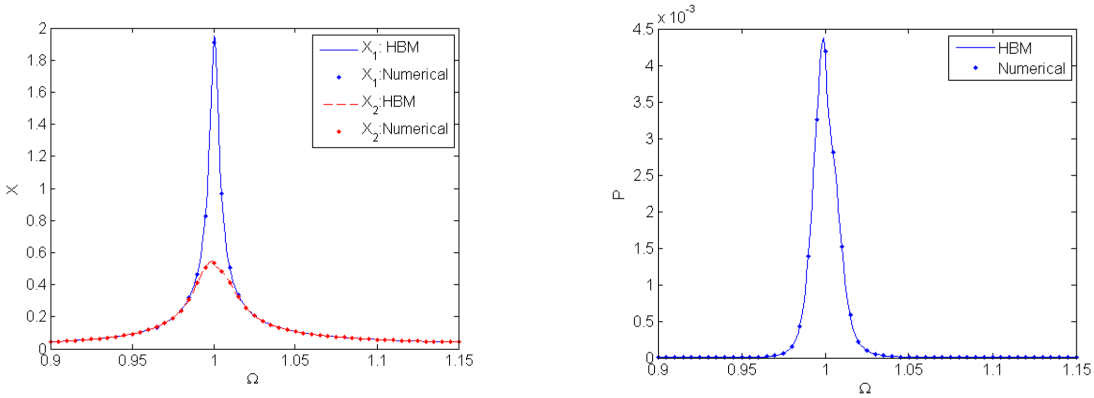
The amplitude of the response and the harvested power for the linear electrical damping with, $c_e = 0.05\, Ns/m$ and for purely cubic electrical damping, with $c_{eN} = 0.0667\, Ns^3/m^3$ are shown in Figs. 3 and 4 respectively, when there is no mistuning. The value of the nonlinear damper is chosen so that the equivalent linear damper, $c_{eqe} = \frac{3}{4} c_{eN} X_2^2 \Omega^2$ at resonance $\Omega = 1$ and at the response amplitude of one $X_2 = 1$ is equal to the linear damper. The nonlinear damper has higher amplitude of response and therefore the harvested power is also higher than the linear damper, when comparing Fig. 3(b) with 4(b).

The results from HBM, marked with solid lines are in agreement with the results from the numerical time integration as shown with dotted line. For further parametric study, the HBM method is used.



(a) (b)

Figure 3 - Linear electrical damping $c_e = 0.05\, Ns/m$ with $\delta = 1$ (a) amplitude of the responses for the two masses, (b) harvested power, solid line (HBM) and dotted line (numerical integration)

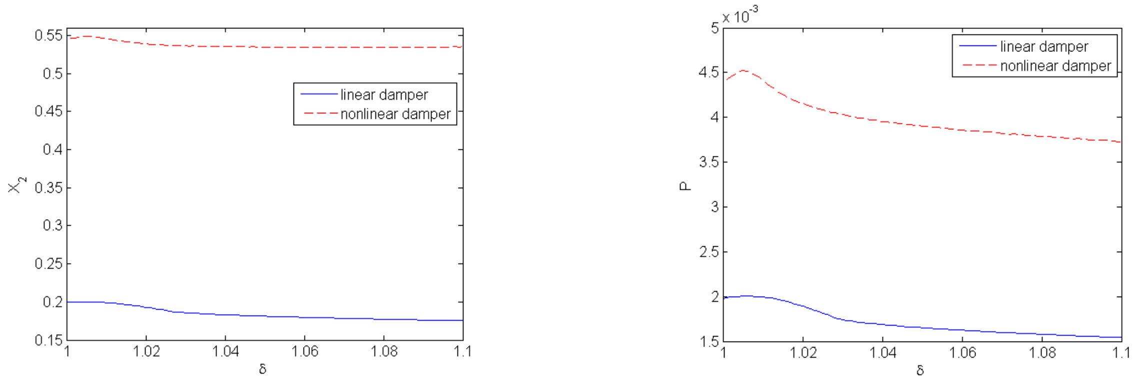


(a) (b)

Figure 4 - Nonlinear electrical damping $c_{eN} = 0.0667\, Ns^3/m^3$ with $\delta = 1$ (a) amplitude of the responses for the two masses, (b) harvested power, solid line (HBM) and dotted line (numerical integration)

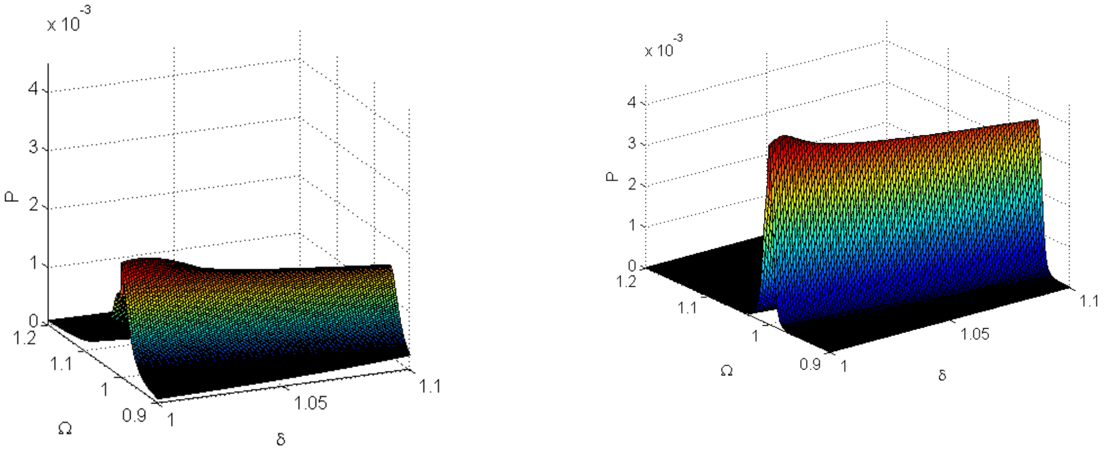
Effect of mistuning

The maximum amplitude of response of the second mass is plotted as a function of the mistuning parameter as shown in Fig.5 (a) for the linear damper marked with blue solid line and nonlinear electrical damper marked with red dashed line. The maximum harvested power is also compared between the linear damper and the nonlinear damper in Fig. 5(b). At a particular mistuning value of $\delta = 1.005$, the harvested power is maximum for the nonlinear damper. The harvested power is also plotted as a function of the excitation frequency and the mistuning parameter in Fig. 6.



(a) (b)

Figure 5- (a) Maximum amplitude of the response of the second mass as a function of the mistuning parameter (b) maximum harvested power as a function of the mistuning parameter, linear damper (solid line), nonlinear damper (dashed line)

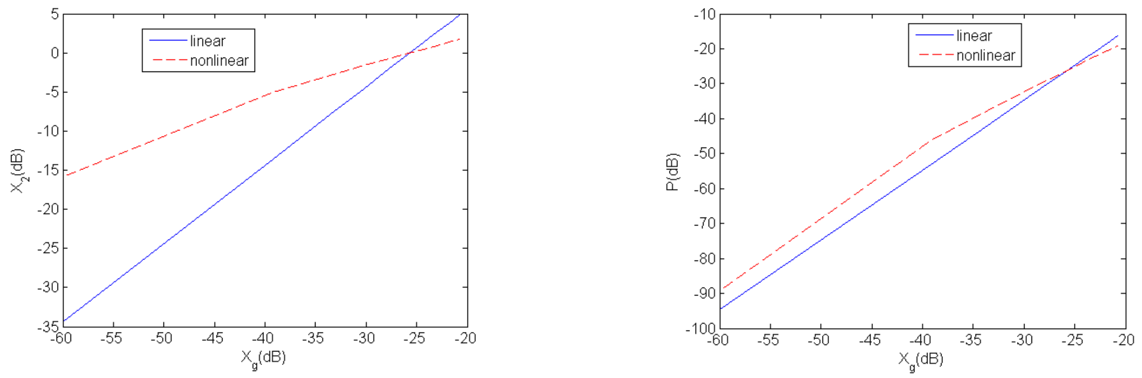


(a) (b)

Figure 6- Harvested power (a) linear electrical damping (b) nonlinear electrical damping for $X_g = 0.01$

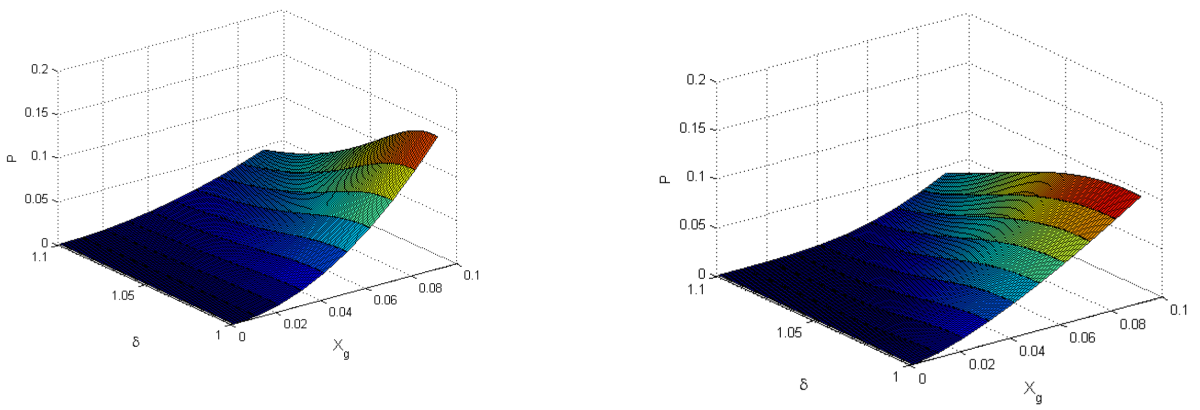
Effect of excitation amplitude at resonance

The excitation amplitude, X_g is varied between 0.001 and 0.1 and the response of the second mass and the harvested power are plotted for the linear and nonlinear damper when there is no mistuning as shown in Fig.7. The nonlinear damper outperforms the linear damper below the maximum excitation level. Figures 8 and 9 show the effect of mistuning on the harvested power at resonance for the linear and nonlinear damper when the excitation level varies. From Fig. 9, it can be seen that at resonance, the mistuning reduces the power significantly at lower excitation levels particularly for the nonlinear damper. Not much benefit is gained from mistuning in this case, which is a trade-off between the robustness and performance.



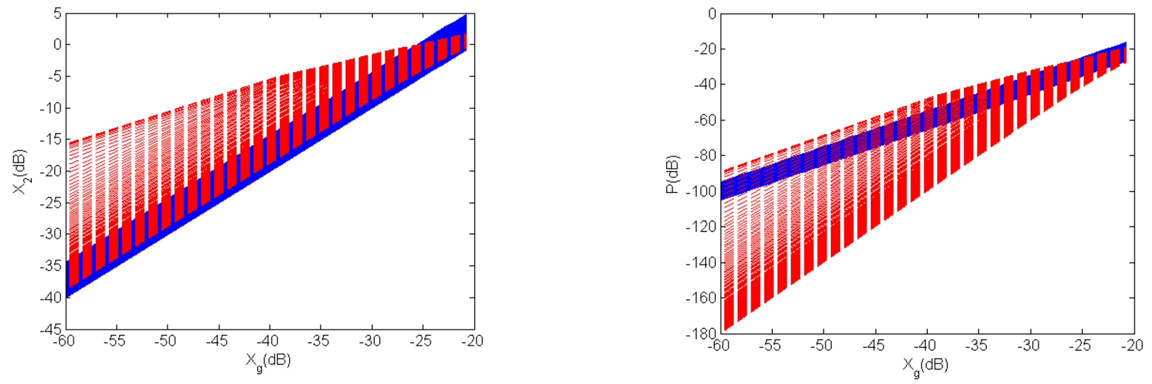
(a) (b)

Figure 7 - (a) Amplitude of the response in dB (b) harvested power in dB, as a function of the excitation amplitude in dB, linear (blue solid line), and nonlinear (red dashed line)



(a) (b)

Figure 8- Harvested power as a function of mistuning and excitation amplitude (a) linear electrical damping (b) nonlinear electrical damping

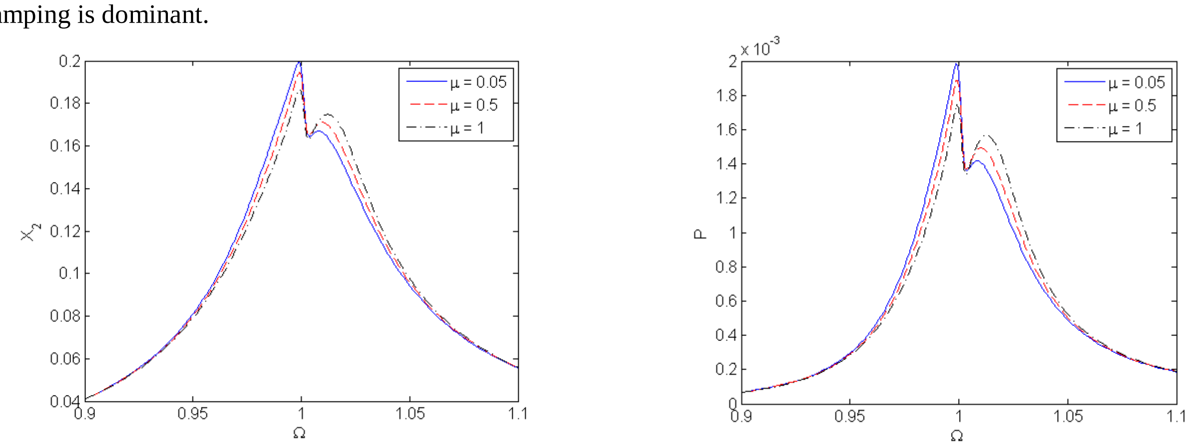


(a) (b)

Figure 9 - (a) Amplitude of the response in dB (b) harvested power in dB, as a function of the excitation amplitude in dB, linear (blue solid line), and nonlinear (red dashed line) for different mistuning

Case 2. Nonlinear stiffness and linear electrical damping

For nonlinear stiffness, HBM is used in MANLAB, which is a free source code to obtain the response amplitudes. Different nonlinear stiffness values, μ, are considered and the response and the harvested power are obtained at $X_g = 0.01$. At low excitation level therefore, the effect of the nonlinear stiffness is not evident as the linear damping is dominant.

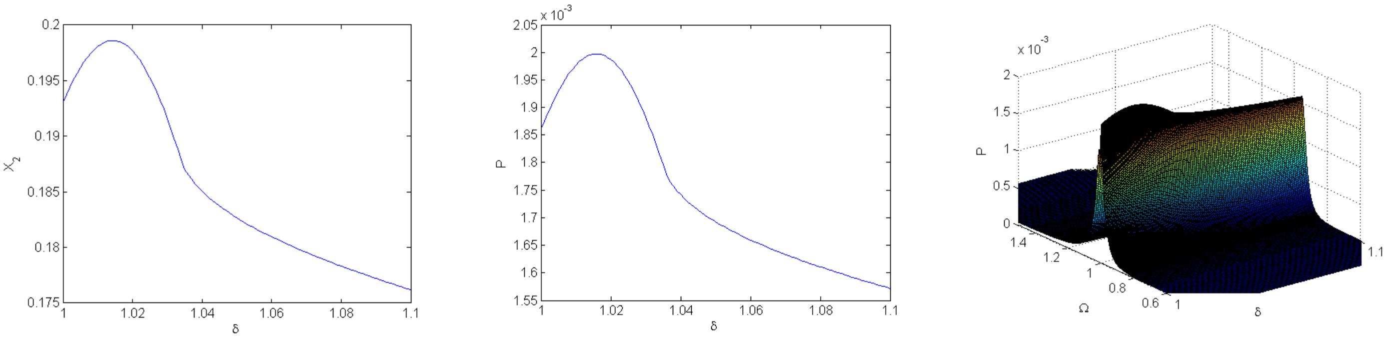


(a) (b)

Figure 10- Nonlinear stiffness and linear damping $c_e = 0.05\,Ns/m$ with $\delta = 1$ (a) amplitude of the responses, (b) harvested power, solid line ($\mu = 0.05$), dashed line ($\mu = 0.5$), and dashed-dotted ($\mu = 1$)

Effect of mistuning

The effect of mistuning is considered and the maximum amplitude of the displacement and the power harvested are obtained using numerical simulation. Figures 11(a) and (b) show that at $\delta = 1.017$, the maximum harvested power is achieved. The contour of the power as a function of mistuning and frequency is also shown in Fig.11(c).

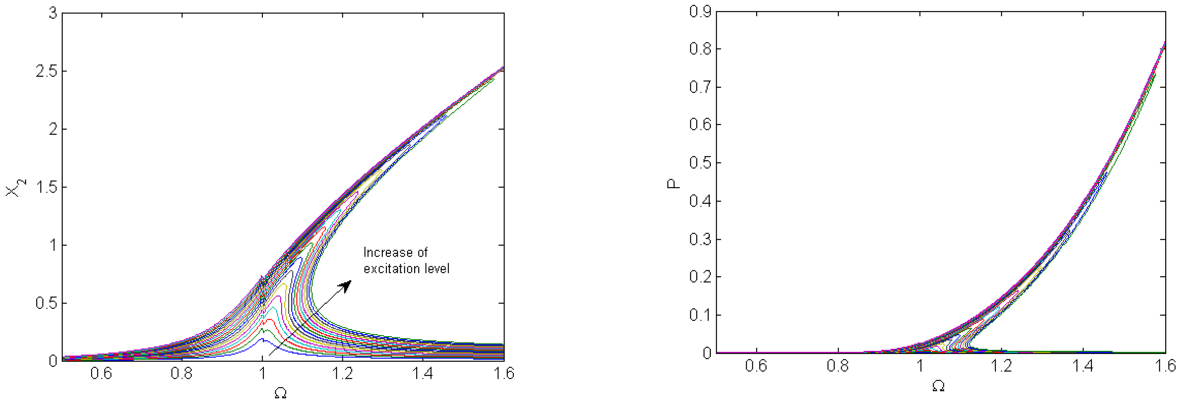


(a) (b) (c)

Figure 11- Nonlinear stiffness with linear electrical damping with $X_g = 0.01$ (a) maximum displacement (b) maximum power harvested (c) contour of power

Effect of the excitation amplitudes.

The excitation level is varied between 0.01 to 0.1 and the response and the harvested power are obtained using harmonic balance method. Obviously, by increasing the excitation level, the nonlinear behavior is more evident and the harvested power increases.

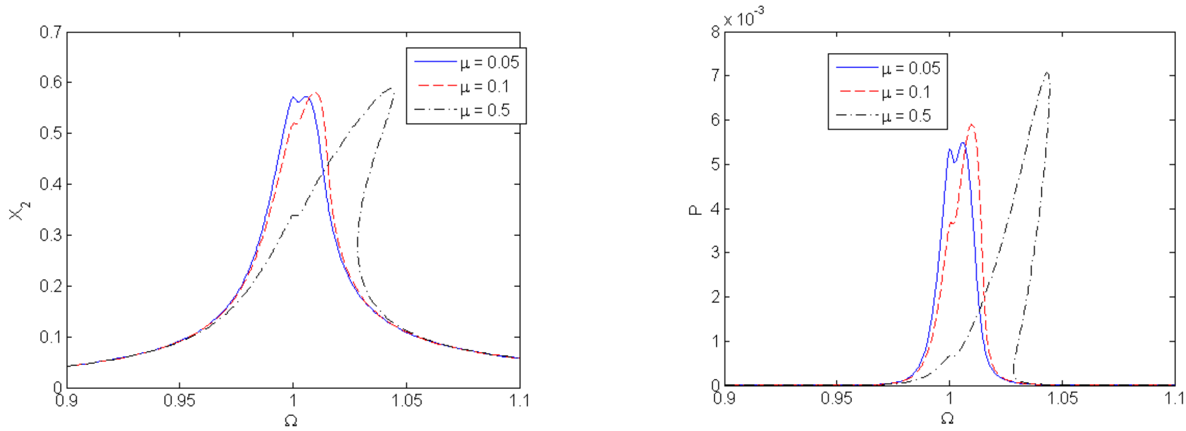


(a) (b)

Figure 12- Nonlinear stiffness and linear damping $c_e = 0.05\, Ns/m$ with $\delta = 1$ (a) amplitude of the responses, (b) harvested power, as a function of frequency for different excitation levels

Case 3. Nonlinear stiffness and nonlinear electrical damping

Since at low excitation level, the nonlinear damper has much higher response amplitude, the effect of nonlinear stiffness becomes evident as can be seen in Fig. 13. As the nonlinear stiffness increases, the frequency response bends to the right and the amount of the maximum harvested power also increases.

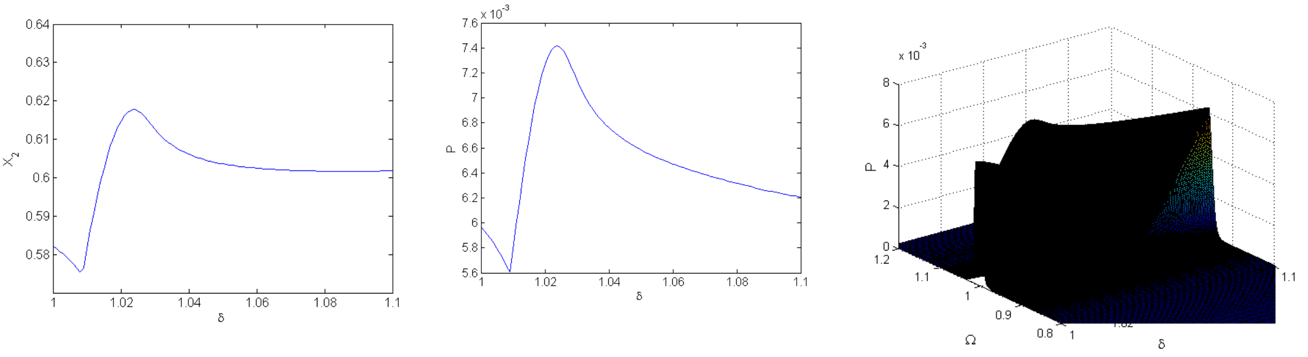


(a) (b)

Figure 13- Nonlinear stiffness and nonlinear damping $c_{eN} = 0.0667\, Ns^3/m^3$ with $\delta = 1$ (a) amplitude of the responses, (b) harvested power, solid line ($\mu = 0.05$) , dashed line ($\mu = 0.5$),and dashed-dotted ($\mu = 1$)

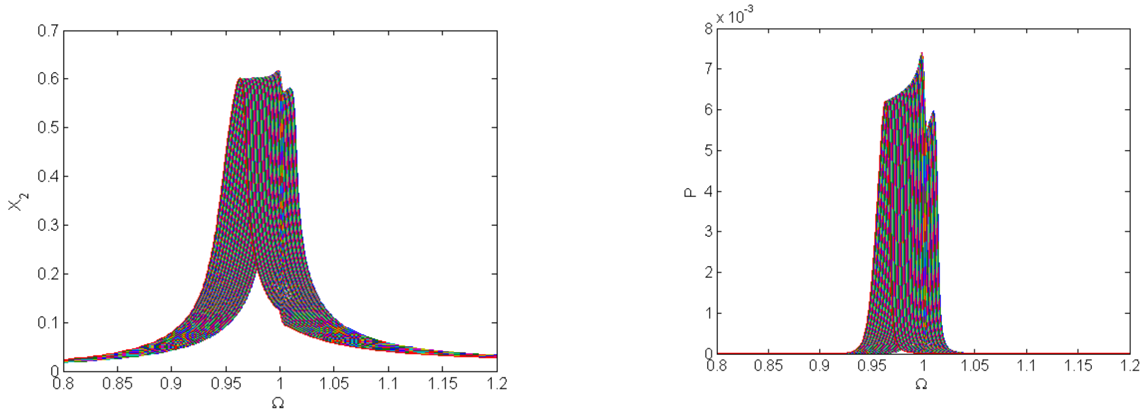
Effect of mistuning

For the system with both nonlinear stiffness and damping, the effect of mistuning is plotted in Fig. 14. The maximum displacement is maximum when the mistuning is about 1.025. The harvested power is 7.4mW, which is much higher than the previous case as shown in Fig. 14(b) and (c). Figure 15 shows the variation of displacement and the power for different values of mistuning between 1 and 1.1.



(a) (b) (c)

Figure 14- Nonlinear stiffness $\mu = 0.1$ with nonlinear electrical damping with $X_g = 0.01$ (a) maximum displacement (b) maximum power harvested (c) contour of power

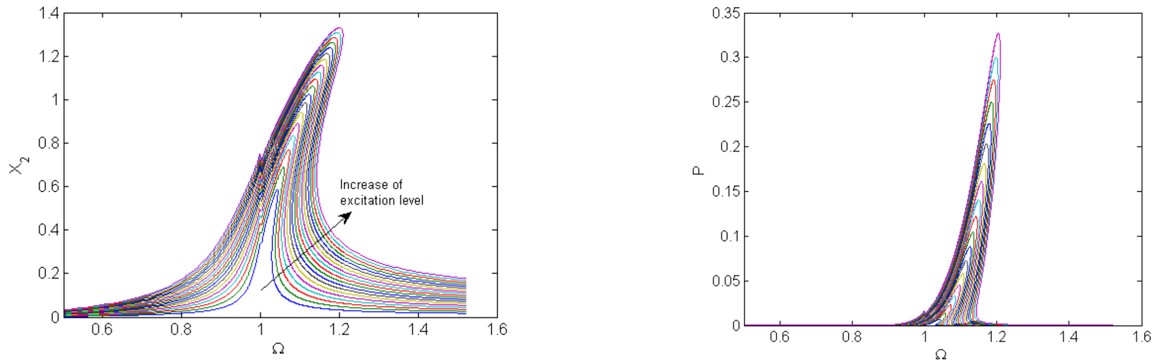


(a) (b)

Figure 15- Nonlinear stiffness $\mu = 0.1$and nonlinear damping $c_{eN} = 0.0667 Ns^3/m^3$ with $X_g = 0.01$ (a) amplitude of the responses, (b) harvested power, as a function of frequency for different mistuning values between 1 and 1.1

Effect of the excitation amplitudes

The excitation level is varied between 0.01 to 0.1 and the response and the harvested power are obtained using harmonic balance method. By increasing the excitation amplitude the harvested power increases as shown in Fig. 16.



(a) (b)

Figure 16- Nonlinear stiffness and nonlinear damping $c_{eN} = 0.0667 Ns^3/m^3$ with $\delta = 1$ (a) amplitude of the responses, (b) harvested power, as a function of frequency for different excitation levels

DISCUSSION

For the weakly coupled nonlinear system, the mistuning can have significant effect on the vibration response. The amplitude of the displacement can increase due to the mistuning even in the presence of nonlinearities. This could be of potential benefit for functionalizing mistuning to harvest energy from weakly coupled periodic systems. The use of nonlinear damping can extend the dynamic frequency range and the harvested power increases for small excitation amplitudes as shown in Fig 7. There is an optimum value of mistuning, which maximizes the power. However, the nonlinear damper is not as robust as the linear damper when there is mistuning as demonstrated in Fig. 9.

The nonlinear stiffness also increases the harvested power. However, one should consider the jump and bifurcation phenomena as they can limit the practical application of the device. The inclusion of nonlinear damping and nonlinear stiffness

CONCLUSIONS

A parametric study is carried out to investigate the effects of the parameters such as nonlinear damping, nonlinear stiffness, excitation levels and the mistuning on the harvested power. Nonlinear damper has the benefit of extending the dynamic range of the harvester. An optimum value of mistuning can increase the harvested power. Nonlinear stiffness is also included in the model to represent the hardening effect due to electromagnetic field. Nonlinear stiffness increases the maximum harvested power. The performance in terms of the harvested power for different excitation levels and mistuning is obtained.

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