

# NONLINEAR DYNAMICS OF A THREE-DIMENSIONAL GUYED MAST WITH UNCERTAIN GUYS PRETENSION

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*Abstract: The study of the nonlinear dynamic response of a guyed mast considering the uncertainty of the guys pretension is reported in this work. A computational model is constructed with the mast represented by an equivalent beam-column and the three guys at one level, by cables with an initial pretension and only having tensile capacity. The system of partial differential equations is discretized using the finite element method considering hermite elements for the mast (Bernoulli beam theory) and nonlinear cable elements for the guys. Also, the second order effect due to the axial loads on the mast is taken into account. A lateral dynamic load is applied on the mast. An ad hoc software developed by the first author is employed to solve the problem. Once the deterministic problem is stated, an uncertainty quantification is carried out. In this case, the stochastic variable is the pretension. Since the design value can be modified at the construction stage and during the service life, the pretension force is modeled as a random variable with a probability density function (PDF) derived from the Principle of Maximum Entropy (PME). The model herein presented contributes to attain a more realistic description of the structure, mainly regarding the three-dimensional representation and the sensibility to the variability of the guys pretensions.*

**Keywords:** *guyed mast, nonlinear dynamics, uncertain pretension, propagation of uncertainties*

## NOMENCLATURE

### Latin symbols

$E$  : Young modulus  
 $I$  : second order moment of the area  
 $A$  : area  
 $m$  : mass per unit length  
 $c$  : viscous damping coefficient  
 $u, v, w$  : displacements  
 $\theta$  : rotation in the  $\hat{i}$  direction  
 $D$  : sag of cable  
 $L_c$  : length of the cable  
 $Y$  : initial configuration of the cable  
 $n$  : number of elements  
 $t$  : time  
 $p, q$  : generic distributed forces  
 $M_t$  : generic distributed torsional moment  
 $N$  : axial force in the beam

### Greek symbols

$\varepsilon$  : lagrangian elongation of cables  
 $\omega$  : circular frequency

### Subscripts

$c$  : relative to cable  
 $b$  : relative to beam  
 $x, y, z$  : relative to Cartesian coordinates  
 $u, v, w$  : relative to displacements in the  $\hat{i}, \hat{j}$  and  $\hat{k}$  directions

## INTRODUCTION

Guyed masts are a structural typology extensively employed to support devices such as antennas for radio, TV and other types of telecommunication devices (Fig. 1 a). Its low cost offers clear advantages in the open country, where there are no restrictions on the position of the cable anchors. However, nowadays it is commonly found in urban areas.

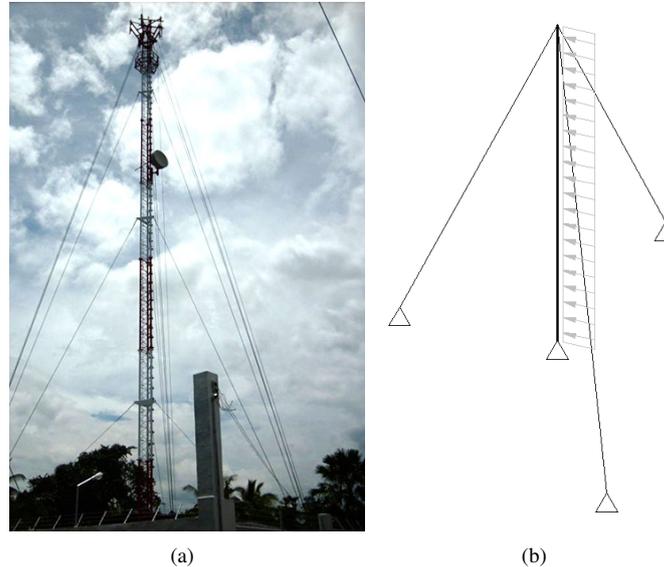
Despite the large potential of adverse impact (in particular, bad quality of the transmission in communications), the dynamic response is not studied in detail, with exception of special cases (e.g. Preidikman *et al.*, 2006; Shi, 2007; de Oliveira *et al.*, 2007; Saudi, 2014; Shi and Salim, 2015).

Other authors' works show that guyed structures have special sensitivity to the type and amplitude of the excitation (Lenci and Ruzziconi, 2009; Wei *et al.*, 2011; Wei *et al.*, 2016), even avoiding the resonance effects. After the derivation

of the equations of motion of a cable-stayed beam, the in-plane and out-of-plane eigenvalue problems are solved by Wang *et al.* (2014). Also, nonlinear modes are studied along with the contribution of the coupling term.

A study on this regard, related to mechanical systems is reported by Bellizi and Sampaio (2012). The smooth decomposition method combined with the Petrov-Galerkin projection for structure-preserving model reduction is used to analyze second-order discrete nonlinear structural systems under random excitation. Nonlinear mechanical systems under random excitation with homogeneous and non-homogeneous mass distribution were considered.

In a recent work by the authors (Ballaben *et al.*, 2015) the nonlinear dynamic response of plane guyed structures is analyzed through reduced order models, with consideration of uncertainties in structural parameters.



**Figure 1 – Guyed mast. a) Typical guyed tower for mobile signal transmission; b) Model under study.**

In the present study and using the classical extended Hamilton's principle, the equations of motion that govern the vibrations of the system are obtained. The nonlinear model of the cable follows the approach reported by Gattulli and Lepidi (2007). Then, and after the statement of the weak form, the governing system is discretized by finite elements.

The FEM approximation to the complete model in dynamic nonlinear analysis is computationally expensive. Then, an ad hoc, nonlinear optimized code for guyed structures were developed by the authors within a finite element environment. With this tool, the time-expensive Monte Carlo simulations are possible within acceptable runtimes.

Although the guy pretension is determined at the design stage, it can change during the construction procedure and also along the structure service life with respect to the design value, affecting the system performance and even its stability (Margariti and Gantes, 2015). As the guy pretension is a significant parameter of the structure, its variation is a relevant issue and the introduction of uncertainty appears adequate. In this work, the initial tension of the guys is considered stochastic. The probability density distribution (PDF) is selected by means of the Principle of Maximum Entropy (Shannon, 1948). All the values of pretension studied are within the ranges suggested by the standards.

It was found that the effect of the random initial pretension ( $H$ ) in the response is very dissimilar and heavily influenced by the nonlinear behavior of the guys. When low values of  $H$  are analyzed, bifurcations of the PDF of the displacements are observed, while for the higher values of  $H$ , an almost deterministic response is obtained.

## STUDIED MODEL

In this section, the studied model is presented. First, the equations of motion for cables and beams are stated. Then, some comments about the nonlinear finite element discretization and solver methods are given. Finally, the specific details, constants, constraints and loads of the structure under study are listed.

### Equations of motion

Next, the equation of motion of nonlinear cables and beams with the addition of the second order effect are presented. Further details in the derivation of the equations can be found in Ballaben *et al.* (2015).

The following assumptions are made: a) both the cable and the beam-column are considered as homogeneous one-dimensional elastic continua obeying a linear stress-strain relationship; b) the equilibrium configuration for the inclined cable is described through a quadratic parabola under the assumptions of small sag to length ratio; c) axial extensions of the cable are described by the Lagrangian strain of the centerline; d) the flexural, torsional and shear stiffness of the cable are neglected; e) the shear strain of the beam-column is assumed negligible; f) the nonlinearity of the problem arises from the cable formulation; g) a second order effect due of the axial load (assumed constant) is accounted for in the beam-column equation. Under these assumptions and using the classical extended Hamilton's principle, the equations of

motion governing the transverse vibration of cables (Eq. 1) and beams (Eq. 2) are obtained:

$$\begin{aligned} u_c : m_c \ddot{u}_c + c_c \dot{u}_c - EA_c \varepsilon' &= p_u, \\ v_c : m_c \ddot{v}_c + c_c \dot{v}_c - [Hv'_c + EA_c(Y' + v'_c)] \varepsilon' &= p_v, \\ w_c : m_c \ddot{w}_c + c_c \dot{w}_c - [Hw'_c + EA_c w'_c] \varepsilon' &= p_w. \end{aligned} \quad (1)$$

$$\begin{aligned} u_b : m_b \ddot{u}_b - EA_b u_b'' &= q_u; \quad GI_x \theta'' + M_t = 0, \\ v_b : m_b \ddot{v}_b + EI_z v_b'''' - Nv_b'' &= q_v, \\ w_b : m_b \ddot{w}_b + EI_y w_b'''' - Nw_b'' &= q_w. \end{aligned} \quad (2)$$

Here  $(*)' = d(*)/dx_{c,b}$  and  $\dot{(*)} = d(*)/dt$ ,  $H$  is the component along the chord of the mean static cable pretension  $T$  in the cable (due to the small slope,  $H$  and  $T$  are practically equal),  $Y$  is the initial configuration of the cable and, due the hypothesis of small sag ( $D$ ) to span ratio,  $Y(x_c) = 4D(x_c/L_c - (x_c/L_c)^2)$ . Finally,  $\varepsilon = u_c + Y'v'_c + 1/2v_c'^2 + 1/2w_c'^2$  is the elongation of the cable. Both equations are completed with suitable boundary conditions.

### Weak formulation and finite element discretization

A general form of weak formulation for the Eqs. (1) and (2) writes as

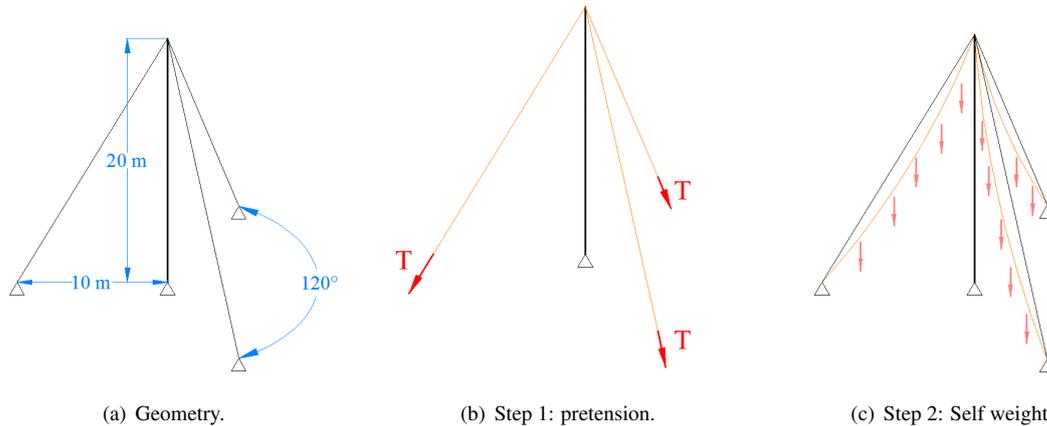
$$M(\ddot{v}, \phi) + C(\dot{v}, \phi) + K_L(v, \phi) + K_{NL}(v, \phi) + BC(v, \phi) = F(v, \phi), \quad (3)$$

where  $M$ ,  $C$ ,  $F$  are the mass, damping and external force operators respectively.  $K_L$  and  $K_{NL}$  are the linear and nonlinear stiffness operators, and  $BC$  is the boundary condition operator.  $\phi$  denotes the admissible functions and  $v$  are solutions of the Eqs. (1) and (2).

After stating the weak formulation, the system is discretized by means of an ad hoc non linear finite element (NLFEM) formulation. The column is modeled using a two nodes, 6-DOF (transverse and axial displacements and slope at each node) beam elements (hermite interpolation functions for the transverse displacements and their derivatives and linear interpolation functions for the axial displacements and the torsional rotations) and the cable using a three nodes, 6-DOF (axial and transverse displacements at each node) cable element (cuadratic interpolation functions).

The nonlinear dynamic response is obtained using the Newton-Raphson method for the iterations and the Newmark method for the time integration.

As an initialization and before the dynamic run, the pretension of the guys is applied through deformation of the cables (step 1); then, the self-weight of the guys is activated (step 2). Steps 1 and 2 are depicted in Fig. 2 (b) and (c). The position of the anchors and the initial pretension are checked (step 3). If the error is less than 0.5%, the program uses this deformed/stressed state as the initial state of the dynamic analysis. Otherwise, the initial length of the cable is modified (step 4) and steps 1 to 4 are repeated until the error meets the prescribed tolerance.



**Figure 2 – Geometry of the studied guyed mast and first two steps to get the initial deformed/stressed configuration for the dynamic/uncertainty studies.**

To improve the runtime, the initial configurations as well as rotation, mass and linear stiffness matrices were first solved and preallocated. Thus, the solver only needs to recalculate the linearized stiffness matrix of the cable elements and the residual vectors at each iteration. Also, a parallelization of the algorithm is implemented.

### Problem under study

The studied problem consists in a 20 m height guyed mast, as depicted in Fig. 2 (a), with one level of three cables at the top. The anchors of the cables are separated 10 m from the mast and  $120^\circ$  of each other. The mast is fixed at the

base and it is modeled using 5 beam elements with consideration of the second order effect. The cables are pinned at the anchor point and each cable is discretized with 5 three-node nonlinear cable elements.

The dynamic load (designed to obtain maximum column displacements within the limits of the Bernoulli beam theory) is applied on the mast, and consists in a uniformly distributed load of 400 N, with a time variation given by  $f(t) = 0.5 \cos(3\pi t) + 1.1$ .

The assumed values of the constants for the problem are detailed in Tab. 1.

**Table 1 – Values of constants and parameters of Eqs. (1) and (2)**

Properties	Value
$E$ (GPa)	209
$I_x = I_y = I_z$ (m <sup>4</sup> )	$3 \times 10^{-5}$
$A_b$ (m <sup>2</sup> )	$1.5 \times 10^{-3}$
$m_b$ (kg/m)	11.77
$A_c$ (m <sup>2</sup> )	$7.85 \times 10^{-5}$
$m_c$ (kg/m)	0.62
$H$ (N)	500 - 13500

### UNCERTAINTY QUANTIFICATION STUDIES

The uncertainty quantification is performed considering the randomness in the guys tension  $\mathbb{H}$  (corresponding to the deterministic parameter  $H$ ).

The Principle of Maximum Entropy (PME) (Shannon, 1948; Cursi and Sampaio, 2015) allows to determine the best PDF that suits with the imposed constraints, and introduces no unwarranted information, *i.e.* supplied information is equal to the removed uncertainty (Kapur and Kesavan, 1992).

The PME states that, subjected to known constraints, the PDF which best represents the current state of knowledge is the one with largest entropy. PME addresses the problem in a statistically sound way. The approach is systematic and allows to handle data which is limited or coming from different sources.

The measure of uncertainty of a random variable  $X$  is defined by the following expression

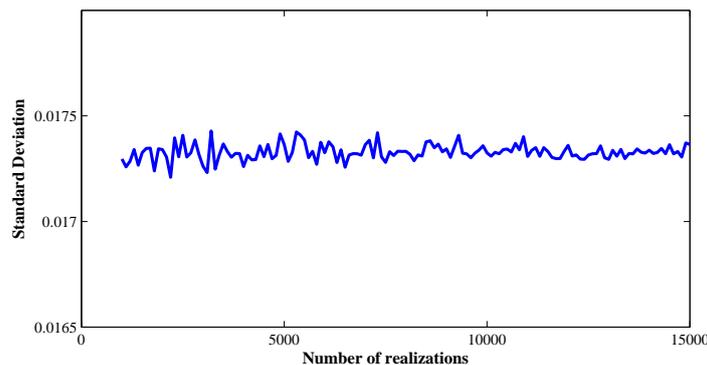
$$S(f_X) = - \int_D f_X(X) \log(f_X(X)) dX, \tag{4}$$

in which  $f_X$  stands for the PDF of  $X$  and  $D$  is its domain. The maximization problem is frequently solved using the Lagrange’s method, with a multiplier accounting for each constraint.

Assuming the constraints of positiveness and bounded second moment, the PME leads to a gamma PDF. The gamma distribution (Eq. 5) with parameters  $a$  and  $b$ , where  $E(X) = ab$  ;  $\sigma_X^2 = ab^2$ , is given by the expression:

$$f(x|a, b) = \frac{1}{b^a \Gamma(a)} x^{a-1} e^{-\frac{x}{b}}. \tag{5}$$

Afterwards, Monte Carlo simulations are performed in order to find the influence of the guys tension  $\mathbb{H}$  with the selected PDF, on the structural response. To achieve significant statistical results, a convergence study on the standard deviation was first performed to determine the minimum number of realizations of the Monte Carlo simulations. Figure 3 shows the typical outcome of a convergence study. It is possible to observe that, at least, a number of 7000 realizations are necessary to achieve acceptable convergent results. However, the convergence study results are highly dependent on the selection of parameters (here, different values of  $E(\mathbb{H})$ ) used on the gamma distribution used to model  $\mathbb{H}$ , and number of realizations must be checked case by case.



**Figure 3 – Convergence Study.**

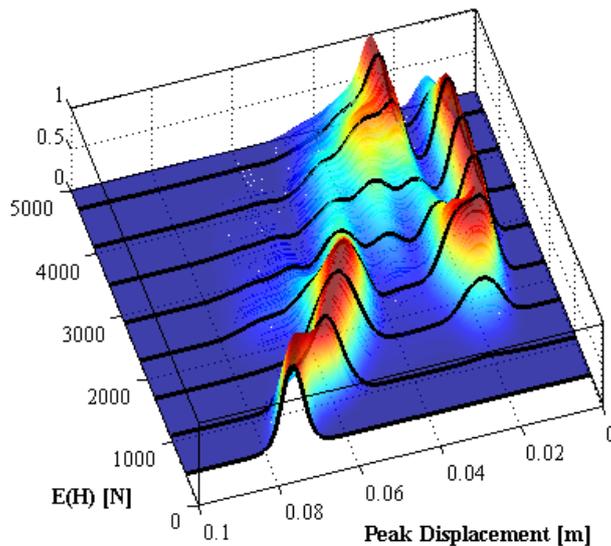
The range of values of the initial pretension was chosen following the standard code CIRSOC 306 (1995). Once the realizations are finished, the PDF graphs are constructed using the *ksdensity* function of MATLAB, that estimates the PDF of a set of data using the *kernel* method. The bandwidth for the kernel function (here a normal function is used), is optimized in MATLAB, and it is useful when the target PDF is normal, but can give wrong results when that condition is not fulfilled. In this work, after several tests, and following an engineering criterion, the authors adopted a bandwidth of  $5 \times 10^{-3}$ m, that gives as result smooth PDFs and allows to observe multimodalities, when the modes are separated enough to be of engineering interest.

The efficiency of the stochastic computational model is strongly dependent on the structural model and on the statistical tools. Regarding the first, the size and complexity conditions determine the time in which the deterministic model is solved. Without optimization, a dynamic study on a nonlinear finite element model of this size (102 degrees of freedom), is nearly impossible, due that, in top of this, the stochastic study requires thousands of realizations (*i.e.* solution of the deterministic model).

## RESULTS

In this section the deterministic and statistical response of the column are analyzed. The selected variable is the displacement at the top of the mast, which is of special interest because it correspond to a common location of antennas and, since it is the point of anchorage of the guys, it is the best choice to observe the nonlinear interaction between cables, column and loads. The direction observed corresponds with the direction of the load.

In Fig. 4 the variation of the PDF of the peak displacements is depicted, with a 3D plot. The initial tension is assumed a random variable with a gamma distribution and the mean value of the distribution also varies in order to cover the whole range of values of  $\langle H \rangle$  observed. Also, some of the PDF are delineated to improve the understanding of the plot and the complexities of the results. It can be seen that the statistical response of the system is very sensitive to changes of the  $E(\langle H \rangle)$ . In Fig. 5, three PDF of peak displacements, obtained for  $E(\langle H \rangle) = 500, 2100$  and  $4800$  N are shown. These PDF



**Figure 4 – 3D view evolution of PDF of peak displacements at the tip of the tower with  $E(H)$**

correspond to slices of the surface represented in Fig. 4, and are included here as examples of the variety of statistical results that are observed. Fig. 6 is a top view of Fig. 4. Warmer colors indicate a higher probability. From this figure, it is clear that very different statistical behaviors are observed for the range of initial pretension studied. Particularly, multimodalities are present in approximately the 70% of the cases observed. The 30% of the remaining cases of  $E(H)$  corresponds to the extreme cases of cable tension: the loosest cases, where the cable tension is so low that it provides a negligible stiffness to the system, and the response of the column is near to the fixed-free boundary condition, and the cases of higher tension, where the response of the column is near the fixed-pinned boundary condition. Both extreme situations offer linear-like changes in the statistical responses to the variation on the initial pretension. On the other hand, when  $E(H)$  is within the range of 1000 to 4500 N, there is an interaction between cables and column and the nonlinear behavior of the guys produces a richer variety of statistical results. The origin of the variations in the statistical results is partially deterministic and partially stochastic. In Fig. 7, a bifurcation study for the displacements at the top is depicted. It can be seen that bifurcations from periodic to nonperiodic motions (among others) are present when the initial tension of the guys is varied. Such behavior contributes strongly to the variability and diversity of the stochastic response.

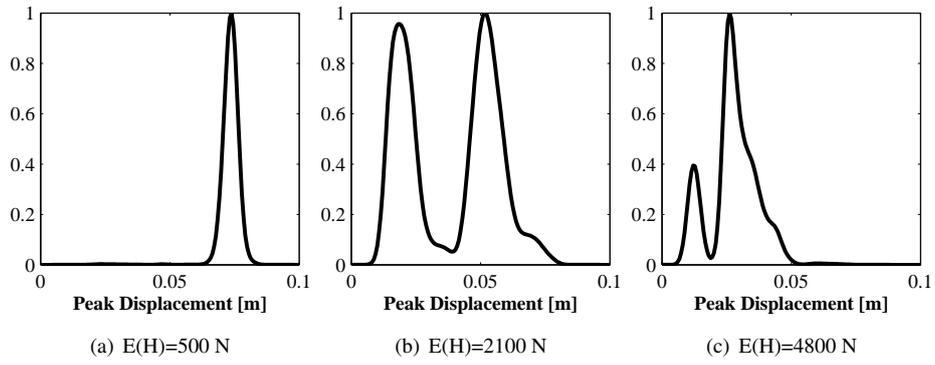


Figure 5 – Examples of PDF of peak displacements for different values of  $E(H)$

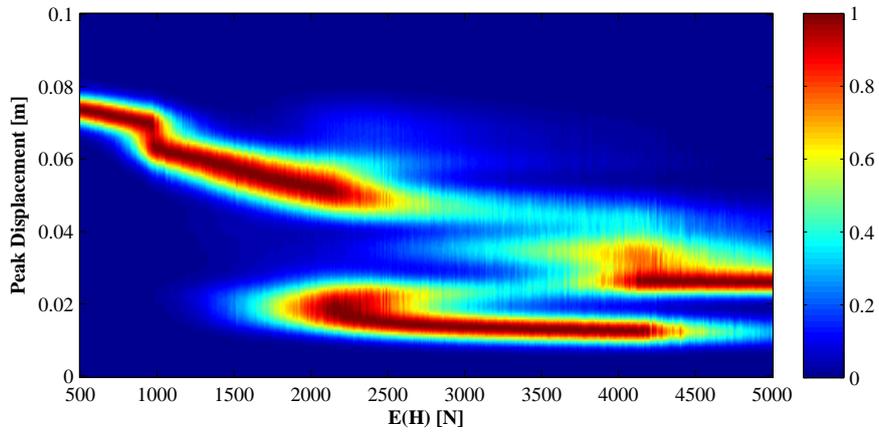


Figure 6 – PDF of peak displacements with  $\sigma = E(H)/5$  and  $E(H) = 500 : 1 : 5000$  N.

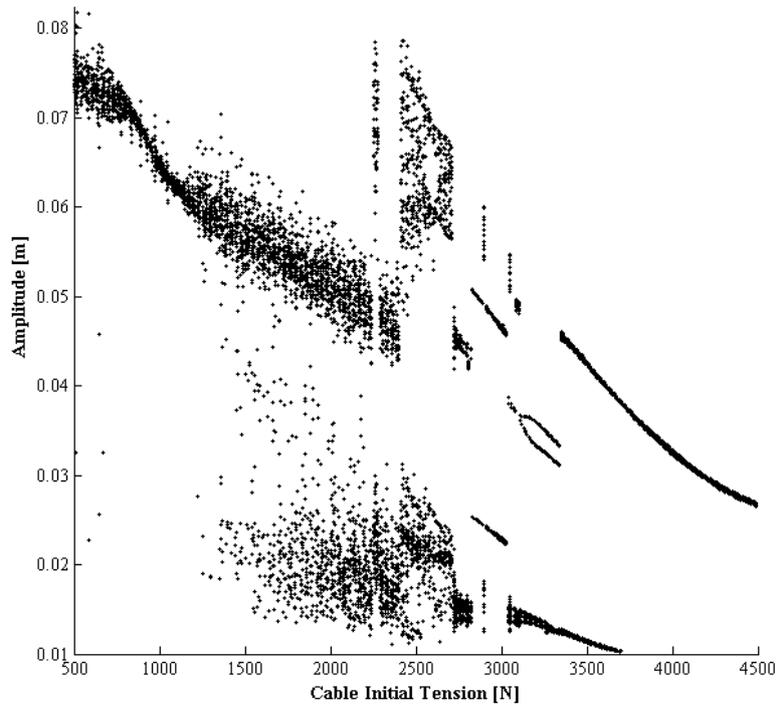
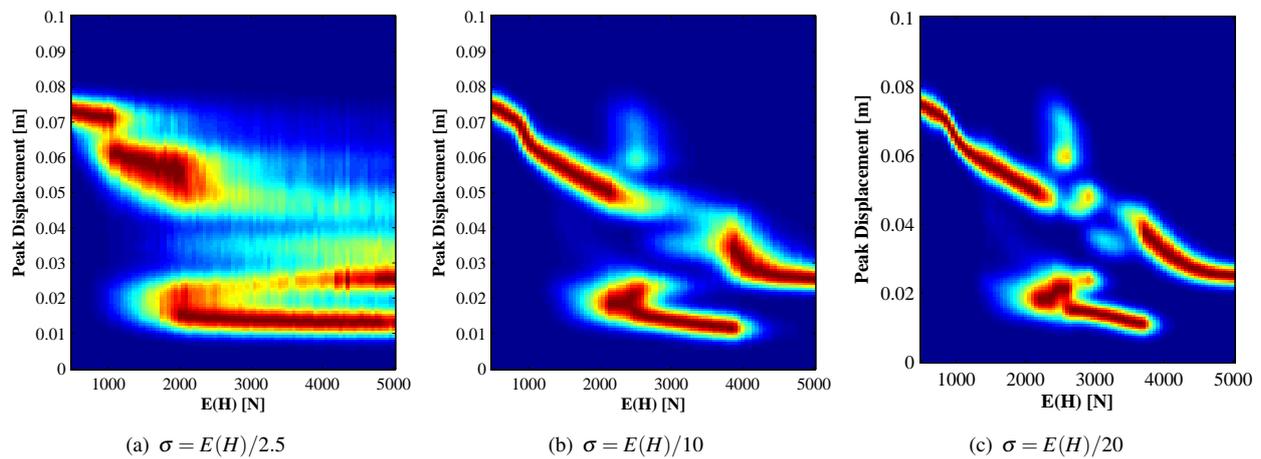


Figure 7 – Deterministic bifurcation plot.

The parameters used for modeling the stochastic parameters are also important in the results. The influence of the standard deviation ( $\sigma$ ) is depicted in Fig. 8. As expected, narrow PDFs of peak displacements are obtained when smaller values of  $\sigma$  are used, but multimodalities are still present.



**Figure 8 – Top view of PDF of peak displacements, varying  $E(H)$ . Influence of the standard deviation ( $\sigma$ ) on the statistical response.**

## FINAL COMMENTS

In this work, an optimized finite element formulation is used to study the nonlinear dynamic and stochastic response of a guyed mast, considering the initial tension of the guys with a gamma distribution as the random structural parameter.

For the ranges of initial tension suggested by the codes, a rich dynamic behavior is obtained, when the displacements at the top of the mast are studied. Particularly, bifurcations from periodic to nonperiodic motions (among others) are observed.

The variability in the deterministic dynamic response is reflected on the statistical response, which exhibits a variety of resulting PDFs when the mean value and standard deviation of the gamma distribution used to model the random guy initial tension are changed. Single and multimodal PDFs of peak displacements are found. The latter are present in approximately the 70% of the cases.

Finally, it worth to say that other variables can be of interest, *e.g.* the dynamic response of the guys, which are not analyzed here to keep the length of the paper within proper limits.

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