SDRE Trajectory Tracking Control for a Hovercraft Autonomous Vehicle

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Abstract:
A Hovercraft is an amphibious vehicle lifted by a propeller that causes the effect of an air cushion between the vehicle and the surface. This way, a hovercraft becomes a fast and versatile vehicle to be used in different kinds of tasks such as rescues, environmental monitoring and coast guard patrolling. This paper presents the control problem formulation in order to track a reference trajectory of a Hovercraft dynamical model. For this purpose the SDRE (State Dependent Riccati Equation) control method is applied to the model of this dynamical system. The nonlinear control problem is formulated in order to minimize the cost functional. Numerical simulation are performed using Matlab®, so that, the equations of the system and the reference are integrated to generate data about the position, orientation and velocities of the Hovercraft. The results confirm that the control method succeeded in controlling the Hovercraft in all proposed cases.

Keywords: Hovercraft autonomous vehicle; State-dependent Riccati equation; Nonlinear control, trajectory tracking control; error space-state form.

NOMENCLATURE

Latin symbols

$\dot{\mathbf{x}}$: linear global velocity relative to an Earth fixed referential
$\dot{\mathbf{y}}$: linear global velocity relative to an Earth fixed referential
$\dot{\mathbf{u}}$: linear surge acceleration of the vehicle
$\dot{\mathbf{v}}$: linear lateral or sway acceleration of the vehicle
$\dot{r}$ **:** angular acceleration of the vehicle
$\mathbf{e}$: error vector

Greek symbols

$\dot{\varphi}$: angular global velocity relative to an Earth fixed referential
τ_u : linear acceleration in surge direction
τ_r : rotational acceleration
β : quotient between damping and mass

Subscripts

r: relative to the reference dynamical system adopted

INTRODUCTION

A Hovercraft is an amphibious vehicle which floats on a cushion of pressurized air supplied by one or more fans and contained inside a cavity on a flexible structure known as skirt (Hinchey and Sullivan, 1993). Kuznetsov (2009) says that these fans create a pressure distribution in the opposite direction of the gravitational force, thus minimizing the vehicle contact with the ground or water. This causes a ground effect related to the navigation surface. The ground effect creates a thin air layer that lubricates the base of the vehicle, avoiding the contact with the surface of navigation and consequently reducing the drag forces. It provides the vehicle not only excellent performance on rough surfaces but also the high speed that other conventional marine vehicles cannot achieve (Chun and Jun, 2004).

Although, to plan some missions using Hovercrafts it is necessary to control the trajectory of the vehicle. To do this some controllers of high performance must be applied as pointed in Morales *et al.* (2015), finally Hovercrafts present some non-holonomic restrictions of movement that don't allow the application of simple feedback control laws

(Brockett, 1983).

In mobile robotics the vehicle trajectory tracking control can be separated in two different problems: the stabilization of the final position and the trajectory tracking of a reference (Rafikova and Kurka, 2012). The first problem is related to the Brockett's sufficiency condition because of the non-holonomic restrictions of movement, that is: any continuous control law couldn't ensure the asymptotic convergence of the vehicle to its resting configuration. And the second problem refers to the determination of a position or a reference trajectory where the robot should be stabilized by the minimization of the error between this reference and the actual trajectory of the system. To solve that problem, a suitable mechanism of control is a state feedback control presented by Rafikova and Kurka (2012).

A way to control this sort of systems is using a sub-optimal method of control, known as SDRE - (State Dependent Riccati Equation), which the principal advantage is that it's not necessary to linearize the system.

Then, the objective of this project is to formulate the tracking control trajectory problem of a Hovercraft, which has non-holonomic restrictions, applying the SDRE control method. In the section Methodology it is shown how the dynamical system is treated to allow the use of the control method, writing it in an error space-state system.

Numerical simulations were performed using Matlab®, so that the equations of the error space-state form, dynamical model of the vehicle and the reference were integrated to generate data about the position, orientation and velocities of the Hovercraft. In this section it will also be presented the graphs about these simulations that provide visualization of the behavior of the dynamical system during a determined interval of time, including the trajectory tracking of the vehicle.

METHODOLOGY

In this section it is formulated the control problem studying the mathematical dynamic model of the vehicle and the reference chosen. After that, it is applied the SDRE method of control at the error space-state system.

Modelling of the dynamical System

The chosen hovercraft model can be found in Fantoni *et al.* (1999), it considers the dynamics of a sub-actuated hovercraft, which possesses 3 degrees of freedom and only two actuators, two propellers, in this case. The body shape is considered circular and symmetric in respect to *x* and *y* direction, as shown in figure 1. The damping terms are neglected. This hovercraft dynamic model is obtained through the study of a ship dynamic model developed in (Fossen, 1994) and (Pettersen and Nijmeijer, 1998). The equations are of the following form:

$$
\begin{aligned}
\dot{\mathbf{x}} &= \mathbf{u}\,cos(\varphi) - \mathbf{v}\,sin(\varphi) \\
\dot{\mathbf{y}} &= \mathbf{u}\,sin(\varphi) + \mathbf{v}\,cos(\varphi) \\
\dot{\varphi} &= r \\
\dot{\mathbf{u}} &= \mathbf{vr} + \tau_{\mathbf{u}} \\
\dot{\mathbf{v}} &= -\mathbf{ur} - \beta\mathbf{v} \\
\dot{r} &= \tau_{r}
\end{aligned}
\tag{1}
$$

where x, y and φ refers to the position and orientation of the Hovercraft regarding to an Earth fixed referential and **u**, **v**, and *r* refer to the linear velocities of surge, sway and angular velocity of the vehicle.

As shown in Figure 1 the propellers are positioned in the vehicle center of mass and are actuated by two motors actuated independently.
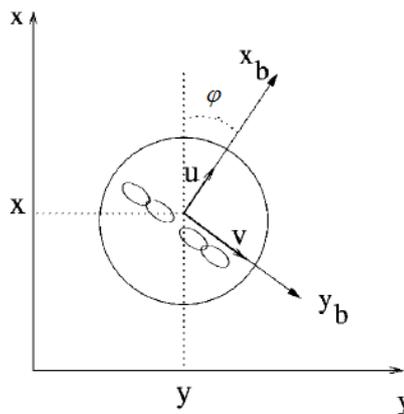

Figure 1 - Schematic representation of the vehicle. In: Fantoni *at al.*, 1999

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SDRE Control Method

The State Dependent Riccati Equation control method is a suboptimal control of a nonlinear system (2) through the minimization of the functional (3) as shown in Cloutier (1997).

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}) + \mathbf{B}(\mathbf{x})\mathbf{u} \tag{2}$$

$$\mathbf{x} = \mathbf{x_0}$$

$$J[\mathbf{u}] = \frac{1}{2}\int_{t_0}^{\infty} \mathbf{x^T Q(x)x} + \mathbf{u^T R(x)u}.dt \tag{3}$$

Placing a system (2) in a state dependent form though a non unique parameterization:

$$\dot{\mathbf{x}} = \mathbf{A}(\mathbf{x})\mathbf{x} + \mathbf{B}(\mathbf{x})\mathbf{u}, \tag{4}$$

It is possible to obtain a controller in a form:

$$\mathbf{u} = -\mathbf{R}^{-1}(\mathbf{x})\mathbf{B^T}(\mathbf{x})\mathbf{P}(\mathbf{x})\mathbf{x} \tag{5}$$

Solving a state dependent algebraic Riccati equation:

$$\mathbf{P(x)A(x)} + \mathbf{A^T(x)P(x)} - \mathbf{P(x)B(x)R^{-1}(x)B^T(x)P(x)} + \mathbf{Q(x)} = 0 \tag{6}$$

Control Problem Formulation

Considering the vehicle system (1) and a reference system of the form:

$$
\begin{aligned}
\dot{\mathbf{x}}_{\mathbf{r}} &= \mathbf{u_r}cos(\varphi_r) - \mathbf{v_r}sin(\varphi_r) \\
\dot{\mathbf{y}}_{\mathbf{r}} &= \mathbf{u_r}sin(\varphi_r) + \mathbf{v_r}cos(\varphi_r) \\
\dot{\varphi}_r &= r_r \\
\dot{\mathbf{u}}_{\mathbf{r}} &= \mathbf{v_r}\mathbf{r}_r + \boldsymbol{\tau}_{\mathbf{ur}} \\
\dot{\mathbf{v}}_{\mathbf{r}} &= -\mathbf{u_r}\mathbf{r_r} - \beta\mathbf{v_r} \\
\dot{r}_r &= \tau_{rr}
\end{aligned}
\tag{7}
$$

An error is defined as the difference between the (1) and (7):

$$
\begin{bmatrix} e_1 \\ e_2 \\ e_3 \\ e_4 \\ e_5 \\ e_6 \end{bmatrix} = \begin{bmatrix} x - x_r \\ y - y_r \\ \varphi - \varphi_r \\ u - u_r \\ v - v_r \\ r - r_r \end{bmatrix} \tag{8}
$$

Substituting and deriving (8) the system yields:

$$
\begin{aligned}
\dot{e}_1 &= \left[\left(\frac{-1 + cos e_3}{e_3}\right)\left(\mathbf{u_r}cos(\varphi_r) - \mathbf{v_r}sin(\varphi)_r\right) - \left(\frac{sin(e_3)}{e_3}\right)\left(\mathbf{u_r}sin(\varphi_r) + \mathbf{v_r}cos(\varphi_r)\right)\right]e_3 \\
&\quad + \left[cos(e_3)cos(\varphi_r) - sin(e_3)sin(\varphi_r)\right]e_4 - \left[sin(e_3)cos(\varphi_r) - cos(e_3)sin(\varphi_r)\right]e_5 \\
\dot{e}_2 &= \left[\left(\frac{-1 + cos(e_3)}{e_3}\right)\left(\mathbf{u_r}sin(\varphi_r) + \mathbf{v_r}cos(\varphi_r)\right) + \left(\frac{sin(e_3)}{e_3}\right)\left(\mathbf{u_r}cos(\varphi_r) - \mathbf{v_r}sin(\varphi)\right)\right]e_3 \\
&\quad + \left[sin(e_3)cos(\varphi_r) - cos(e_3)sin(\varphi_r)\right]e_4 + \left[cos(e_3)cos(\varphi_r) - sin(e_3)sin(\varphi)_r\right]e_5 \\
\dot{e}_3 &= e_6
\end{aligned}
\tag{9}
$$

$$\dot{e}_4 = (e_6 + r_r)e_5 + \mathbf{v_r}e_6 + \mu_1$$
$$\dot{e}_5 = -(e_6 + r_r)e_4 - \beta e_5 - \mathbf{u_r}e_6$$
$$\dot{e}_6 = \mu_2$$

Then, the system in the error space-state could be written in the matricial form:

$$
\begin{bmatrix} \dot{e}_1 \\ \dot{e}_2 \\ \dot{e}_3 \\ \dot{e}_4 \\ \dot{e}_5 \\ \dot{e}_6 \end{bmatrix} = \mathbf{A(e)} \begin{bmatrix} e_1 \\ e_2 \\ e_3 \\ e_4 \\ e_5 \\ e_6 \end{bmatrix} + \mathbf{B(e)} \begin{bmatrix} 0 \\ 0 \\ 0 \\ \mu_1 \\ 0 \\ \mu_2 \end{bmatrix} \quad , \tag{10}
$$

where the matrices $\mathbf{A(e)}$ and $\mathbf{B(e)}$ are:

$$
\mathbf{A(e)} = \begin{bmatrix}
0 & 0 & a_1 & a_3 & -a_4 & 0 \\
0 & 0 & a_2 & a_4 & a_3 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & a_5 & \mathbf{v_r} \\
0 & 0 & 0 & -a_5 & -\beta & -\mathbf{u_r} \\
0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
\tag{11}
$$

$$
\mathbf{B(e)} = \begin{bmatrix}
0 & 0 \\
0 & 0 \\
0 & 0 \\
1 & 0 \\
0 & 0 \\
0 & 1
\end{bmatrix}
$$

and the coefficients of the matrix $\mathbf{A(e)}$ are:

$$
a_1 = \left[\left(\frac{-1+\cos(e_3)}{e_3} \right)(\mathbf{u_r}\cos(\varphi_r) - \mathbf{v_r}\sin(\varphi)) - \left(\frac{\sin(e_3)}{e_3} \right)(\mathbf{u_r}\sin(\varphi_r) + \mathbf{v_r}\cos(\varphi_r)) \right]
$$

$$
a_2 = \left[\left(\frac{-1+\cos(e_3)}{e_3} \right)(\mathbf{u_r}\sin(\varphi_r) + \mathbf{v_r}\cos(\varphi_r)) + \left(\frac{\sin(e_3)}{e_3} \right)(\mathbf{u_r}\cos(\varphi_r) - \mathbf{v_r}\sin(\varphi)) \right]
\tag{12}
$$

$$a_3 = \cos(e_3 + \varphi_r)$$
$$a_4 = \sin(e_3 + \varphi_r)$$
$$a_5 = (e_6 + r_r)$$

Then the state dependent algebraic Riccati equation (13) should be solved each time step which is possible by numeric non- iterative approaches by using the stable eigenvectors of the Hamiltonian matrix.

$$
\mathbf{P(e)A(e)} + \mathbf{A^T(e)P(e)} - \mathbf{P(e)B(e)R^{-1}(e)B^T(e)P(e)} + \mathbf{Q(e)} = 0 \ , \tag{13}
$$

to obtain the control vector:

$$
\mathbf{u} = -\mathbf{R^{-1}(e)B^T(e)P(e)e} \tag{14}
$$

NUMERICAL SIMULATION

After obtaining the equations of the error space-state of the dynamical system it was possible to perform numerical simulations. First it was done the numeric integration using the fourth order Runge-Kutta integrator (ode45) in Matlab®. Function *lqr* was used to solve the control problem. This command tests the controllability of the system and performs the numeric solution of the algebraic Riccati equation according to (Arnold 1984). Computation cost of this solution is low and adequate to real-time systems. The initial conditions of the problem were presented in the figures subtitle, the parameter $\beta = 1.2$ was adopted as it is proposed in Morales, *at al.*(2015) for all calculations. First, the system was simulated applying the initial condition of surge direction $\tau_{ur}= 0.5$ to the straight line trajectory. To the circular trajectory it was set $\tau_{ur}= 0.28$.

The matrices $\mathbf{Q(e)}$ and $\mathbf{R(e)}$ to solve the *lqr* were chosen as

$$\mathbf{Q(e)} = \begin{bmatrix} 5 & 0 & 0 & 0 & 0 & 0 \\ 0 & 5 & 0 & 0 & 0 & 0 \\ 0 & 0 & 5 & 0 & 0 & 0 \\ 0 & 0 & 0 & 5 & 0 & 0 \\ 0 & 0 & 0 & 0 & 5 & 0 \\ 0 & 0 & 0 & 0 & 0 & 5 \end{bmatrix} \tag{15}$$

$$\mathbf{R(e)} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

This choice is empiric and this values are selected to ensure stability and to minimize the overshut of the system. The physical parameters of the Hovercraft are presented in the Tab.1 and were obtained by Aguiar, (2003):

Table 1 – Physical parameters of the Hovercraft

Parameter	Dimension
Mass	5.5 kg
J (rotacional inertia)	0.047 Kgm²
l (moment arm)	0,123 m
d_v	5.5 Kg/s
d_w	0.41Kg/s
Deep	25.4 cm
Wide	35.6 cm
High	18.1 cm

Linear trajectory

Here, the purpose is that the Hovercraft tracks the trajectory of a virtual reference that was set as a straight line with the orientation of 45º. The initial conditions to the Hovercraft are *[x, y, φ, **u**, **v**, r] = [5,-5,1,0, 0,0]* and for the reference are *[xr, yr, φr, **ur**, **vr**, rr] = [0,0, π/4,0.5,0,0]*.

The simulations of tracking a linear trajectory in fig. 3 shows the error space-state system related to the position and orientation of the Hovercraft converging to zero in less than 10 s, which means that after this time it is expected that the Hovercraft will be tracking the same trajectory that the reference with the same orientation. In fig. 4 the error related to the velocities of the Hovercraft is presented and converges to zero after 12 seconds.
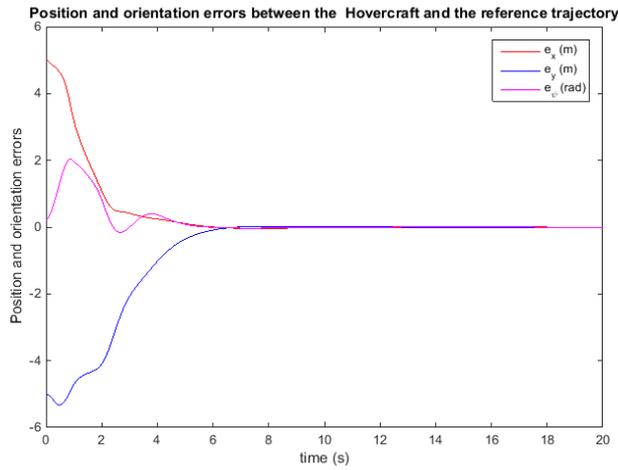
Figure 3 – Simulation of the error space-state system for position and orientation
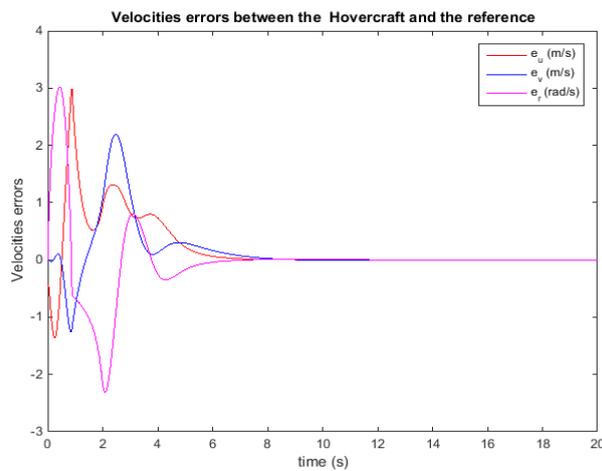

Figure 4 –Error space-state system for velocities

In figure 5 both hovercraft and reference trajectories are presented on the plane. It is possible to see that the Hovercraft follows the desired trajectory of a straight line with the inclination of 45º. The figure 6 presents a zoomed detail of the convergence of the hovercraft to the trajectory. From this figure it is clear that the hovercraft trajectory starts at different set of initial conditions than a reference and converges successfully towards it.
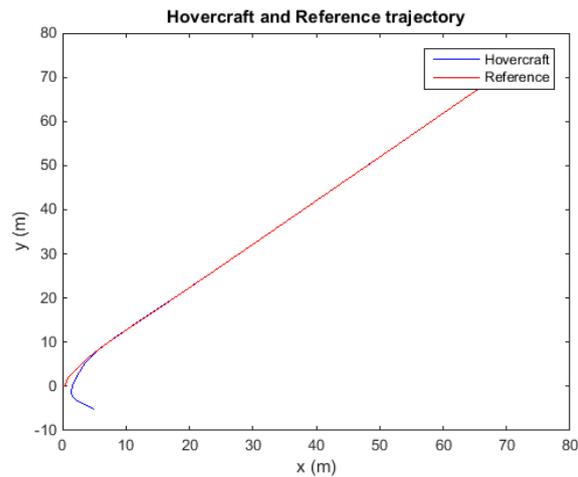
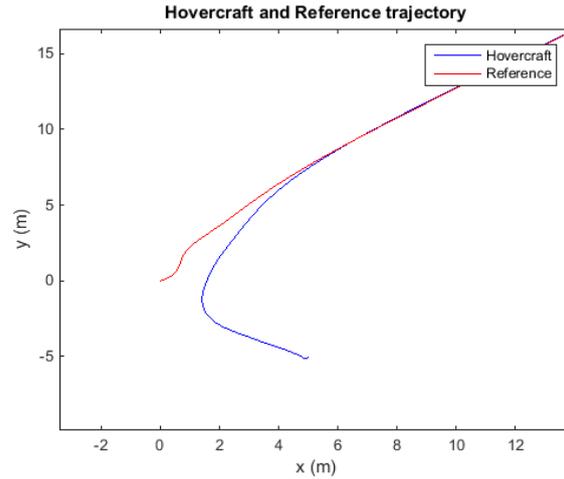**Figure 5 – Trajectories of the vehicle and the reference**

Figure 6 – Detail: Trajectory tracking of the Hovercraft.

The control inputs are shown in Fig. 7. Black line represents the control input related to the linear acceleration applied to the system and the red line represents the time evolution of the control input related to the angular acceleration of the system. As it is seen, after 12 seconds they stabilize becoming constant and converging to the reference values as the trajectory converges more naturally.
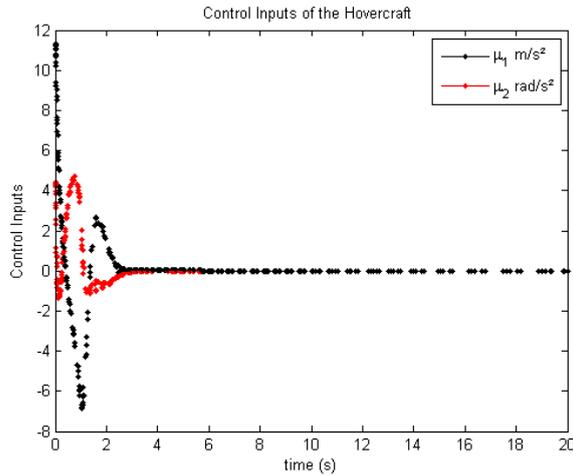

Figure 7 – Control inputs of the Hovercraft.

Circular trajectory

Simulations were performed to achieve tracking of a circular reference trajectory using this algorithm, adjusting only a few parameters. The parameters were adjusted as follows:

$$\dot{\varphi}_r = r_r = const$$
$$\varphi_r = r_r.t + c$$
$$\mathbf{u_r} = const \tag{16}$$
$$\mathbf{v_r} = 0$$
$$\dot{r}_r = \tau_r = 0$$

It means that to do a circle trajectory the surge velocity (u_r) is constant, the sway velocity (v_r) is null, and the angular velocity ($\dot{\varphi}_r$) is also constant. Then, it's possible to determine how is the angle of orientation of the reference at each time of the simulation using the second equation of the (16) . Initial conditions for the hovercraft trajectory were set as *[x, y, φ,* **u***,* **v***, r] = [2,1,0,0, 0,0]* and for the reference trajectory as *[xr, yr, φr,* **ur***,* **vr***, rr] = [0,0,π/2,1,0,0.5].*

Figure 8 present the error system states related to position and orientation of the vehicle with respect to time. The system starts to converge to zero in less than 10 seconds and completely after 23 seconds. Thus Hovercraft tracks the circular trajectory successfully after this time.
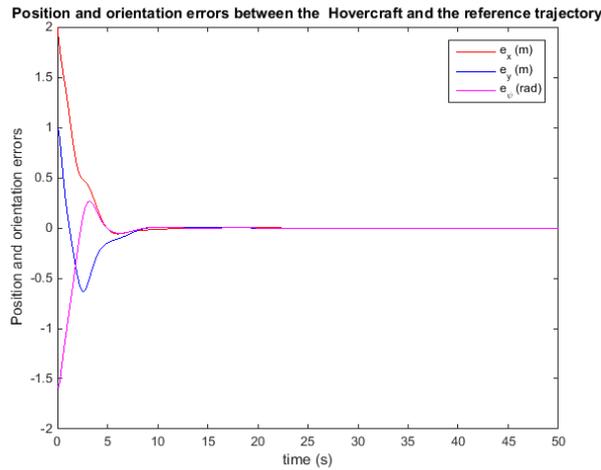
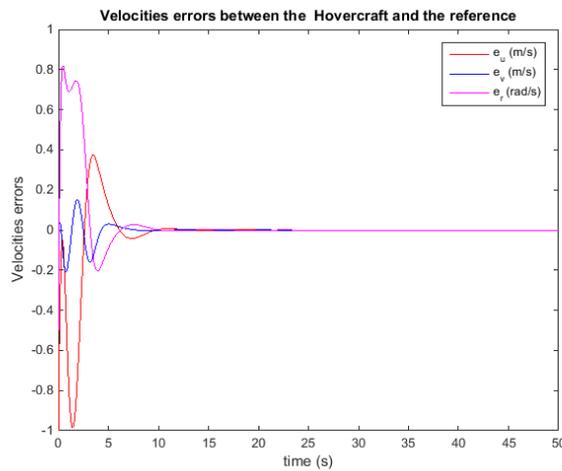**Figure 8 – Error space states for position and orientation of the system, performing circular reference**

Figure 9 – Error space-states for velocities performing circular reference

Figure 9 present the error systems states related to the velocities of the system in forward (**u**), lateral (**v**) and rotational (*r*) directions of the system. As it is seen from this figure, the velocities error converges completely to zero after 25 seconds.

The Figure 10 shows the trajectory by the Hovercraft in blue and the reference trajectory in red. From this figure it is clear that the hovercraft controlled trajectory converges into a reference with success.
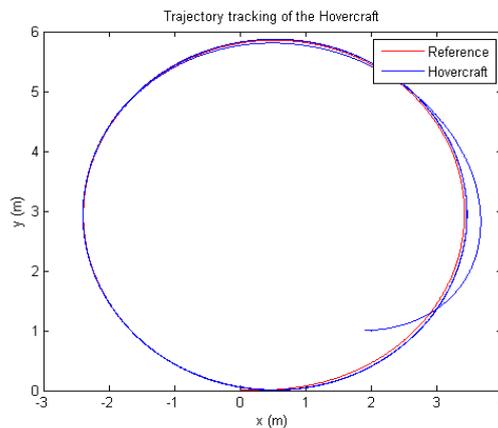

Figure 10 – Trajectory tracking of the Hovercraft.

The applied control inputs on the Hovercraft for the circular path are shown in Fig. 11. Black line represents the control input related to the linear acceleration applied to the system and the red line represents the time evolution of the

control input related to the angular acceleration of the system. As it is seen, after 12 seconds they stabilize becoming constant and converging to the reference values.
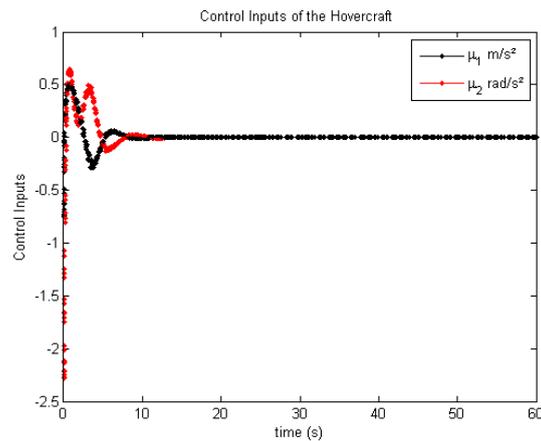

Figure 11 – Control inputs of the Hovercraft.

CONCLUSIONS

In this work the reference trajectory tracking control problem is formulated in which a dynamical system representing a hovercraft vehicle is tracked by the control effort toward a desired trajectory in a general form. The control method Stated Dependent Riccati Equation is applied to the solution of this problem and numerical simulations are presented regarding two basic trajectory regimes: a linear desired trajectory and a circular trajectory. By switching the reference velocities and some initial conditions and parameters it is possible to obtain different set of desired trajectories and track the main vehicle system towards them. The SDRE methods deals with the solution of the algebraic state-dependent Riccati equation by numerical methods based on the Hamiltonian matrix eigenvectors, which permits a step-by-step solution of the control problem and is suitable to real-time applications. The numerical simulations validate the successful trajectory tracking and the effectiveness of the control method.

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