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Control of Multiple Mobile Robots in Dynamic Formations

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Abstract: This work deals with the control of multiple mobile robots in trajectory, while maintaining a formation, through the use of State-Dependent Riccati Equation control method. Three robots with differential drive are used in a scheme in which one is considered the leader and the other two are considered followers. By changing formation parameters, this work seeks to achieve two different formations, V-shapped and Echelon formation, very common in the military field. Simulations are performed in LabVIEW software, demonstrating the successful application of the control method in mobile robot tracking problems while maintaining formations.

Keywords: Multiple Robot Control, Leader-follower problem, SDRE control, Nonlinear control

INTRODUCTION

The control of multiple mobile robots in formation is a theme that became the target of several recent studies, due to its wide range of applications, both civilian and military. The idea is to control a robot so that it converges to a defined path while maintaining a formation with other robots. The term swarm is often used to describe robot formations that are synchronized in a geometric layout, which may or may not be variable with time.

Among the various approaches for the control of multiple robots in formations, the most common ones are virtual structure, behavior-based methods and leader-follower (also called master-slave). The first method treats all robots in formation as a single unit, the virtual structure, in which robots are disposed in a desired layout in respect to themselves and a reference frame (Lewis and Tan, 1997). Behavior-based control is the assignment of complex behaviors for each robot, such as speed synchronization, the ability to avoid collisions and centralization of the formation (Balch and Arkin, 1999). Finally, on the follower-leader method used in this work, one or more robots are designated as leaders, while the others, which are called followers, receive the position information about the leader and converge to its trajectory while maintaining a desired distance and orientation relative to it (Dai and Lee, 2012; Park and Yoo, 2015). To adjust its position in respect to the leader, the follower robot calculates track points based on the position and velocity information obtained from the leader. This method is known for simplifying the inclusion of new robots to the group, thus making the system easily scalable (Consolini *et al, 2008).* Rafikova *et al* (2016) proposed a reminiscent of the leader-follower approach that allowed a robot to follow a chaotic trajectory using the State Dependent Riccati Equation (SDRE) control method to achieve synchronization.

One approach to control methods of multiple robots the navigation in formations is the tracking method, which requires a group of interconnected robots to reach a target or a reference path simultaneously. This approach can be used to solve a set of problems, such as navigation while maintaining formation problems, rendezvous problems and dynamic trajectories tracking problems, like the one presented in this work. It has several civil and military applications for surveillance, reconnaissance missions, access to information on battlefields, etc. (Wang and Xin 2011). Similar complex problems of cooperative tracking with dynamic references, using different control methods, were addressed in (Chung and Slotine, 2007; Khoo *et al*, 2009; Rodriguez-Angeles and Nijmeijer, 2004; Sun and Mills, 2002)

The present work deals with the control of a team of three mobile robots using the SDRE control method to solve a leader-follower dynamic trajectory tracking problem. A controller is designed to make the robots navigate synchronized with one another in order to achieve and maintain two different formations. Computational simulations are shown to validate the results and the efficiency of the control method in mobile robots tracking problems.

METHODOLOGY

Robotic Model

This study uses mobile robots with differential drive and rear support. The model has two identical wheels, positioned in the same axis, restricted to move only around it and independently controlled by motors. The rolling of the wheels is considered pure, so that there are no drift during movement. Such features have a restrictive nature, preventing the movement of the system in some directions at an initial moment, characterizing it as nonholonomic.

The system analysis is carried in a kinematic level. It is assumed that the center of mass is located in the geometric center of the robot, hence eliminating possible unbalances and consequent Coriolis and centripetal acceleration effects.

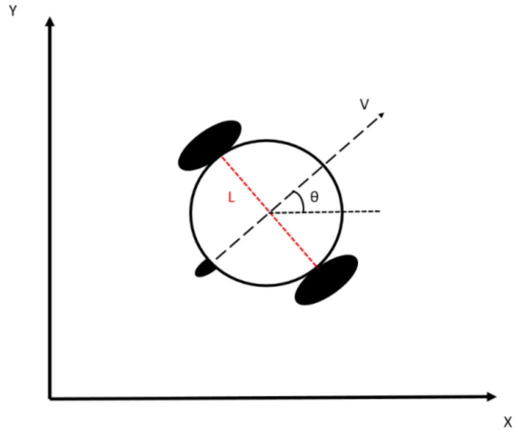

Figure 1 – Model of a differential drive robot.

The linear velocity of the robot can be described as the average of the right and left wheels velocities:

$$\mathbf{v} = \frac{\mathbf{v}_r + \mathbf{v}_l}{2} \tag{1}$$

The robot's position in the X and Y axis can be described by the projection of the robot's linear velocities on these axes:

$$\frac{dx}{dt} = \dot{x} = \mathbf{v}cos(\theta) \tag{2}$$
$$\frac{dy}{dt} = \dot{y} = \mathbf{v}sin(\theta) \tag{3}$$

The angular velocity of the robot, which corresponds to the variation of the rotation angle θ over time, can be described as the difference of the angular velocity of each wheel:

$$\frac{d\theta}{dt} = \dot{\theta} = \boldsymbol{\omega} = \frac{\mathbf{v}_r - \mathbf{v}_l}{2r} \tag{4}$$

Thus, the robot's kinematic model can be described as:

$$\begin{aligned} \dot{x} &= \mathbf{v}cos(\theta) \\ \dot{y} &= \mathbf{v}sin(\theta) \\ \dot{\theta} &= \boldsymbol{\omega} \end{aligned} \tag{5}$$

Where $\dot{x}$ and $\dot{y}$ describe the robot's velocity on the Cartesian coordinate system and $\dot{\theta}$ indicates its rotation in respect to the X axis, or in other words, the orientation of the robots on the plane (Rafikova, 2010). Therefore, for a set of *n* robots, the system's model can be described as:

$$\begin{aligned} \dot{x}_i &= \mathbf{v_i}\cos(\theta_i) \\ \dot{y}_i &= \mathbf{v_i}sin(\theta_i) \\ \dot{\theta}_i &= \boldsymbol{\omega}_i \end{aligned} \tag{6}$$

Where $i = 1, 2, 3, \ldots, n$ and $i = 1$ represents the leader robot.

The Problem of Tracking of Multiple Mobile Robots

The problem of tracking of multiple mobile robots consists on the stabilization of a system with two or more identical robots in a reference trajectory, as depicted on Figure 2.

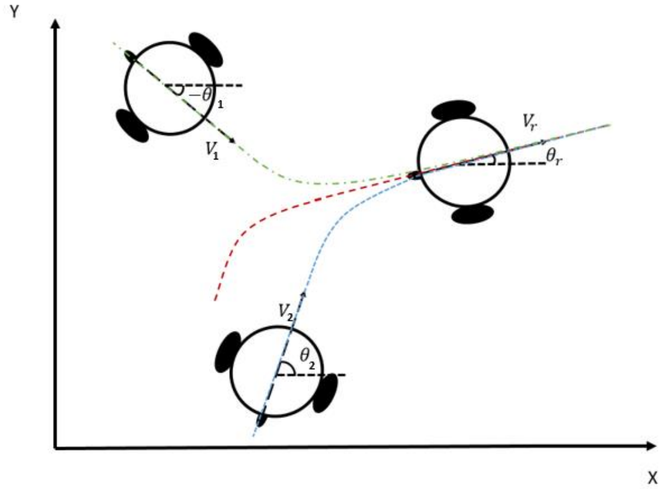

Figure 1: Multiple robots converging to a reference trajectory.

The reference trajectory, to which all robots converge, is described as being a robot itself, called reference robot, whose kinematic model is identic to Equation (7) of the other robots.

$$\begin{aligned} \dot{x}_r &= \mathbf{v_r} cos\theta_r \\ \dot{y}_r &= \mathbf{v_r} sin\theta_r \\ \dot{\theta}_r &= \omega_r \end{aligned} \quad (7)$$

The error vector **e**(t) is now introduced as the relative difference between the state of a controlled robot and the state of the reference robot

$$\mathbf{e_i}(t) = [e_{1i} \;\; e_{2i} \;\;\; e_{3i}]^T \quad (8)$$

which expands in:

$$\mathbf{e_i}(t) = \begin{bmatrix} x_i & - & x_r & - & d_x \\ y_i & - & y_r & - & d_y \\ \theta_i & - & \theta_r & - & d_\theta \end{bmatrix} \quad (9)$$

where x_i, y_i and θ_i are the coordinates of a given controlled robot, x_r, y_r and θ_r are the coordinates of the reference robot and d_x, d_y and d_θ are constants that influences the convergence of the robots. If the constants are null the controlled robot converges to the reference trajectory, otherwise the robot converges to distances on the x and y axis and an orientation angle θ in respect to the reference robot, thus executing the desired formation.

For any pair leader-follower, the deviation system can be described as:

$$\begin{aligned} \dot{e}_{1i} &= \mathbf{v_r} cos(\theta_r)\cos(e_{3i}) - \mathbf{v_r} sin(\theta_r) sin(e_{3i}) + u_{1i} cos(\theta_r) cos(e_{3i}) - u_{1i} sin(\theta_r) sin(e_{3i}) - \mathbf{v_r} cos(\theta_r) \\ \dot{e}_{2i} &= \mathbf{v_r} \sin(\theta_r)\, cos(e_{3i}) - \mathbf{v_r} sin(e_{31}) cos(\theta_r) + u_{1i} sin(\theta_r) cos(e_{3i}) + u_{1i} \cos(\theta_r) sin(e_{3i}) - \mathbf{v_r} sin(\theta_r) \\ \dot{e}_{3i} &= u_{2i} \end{aligned} \quad (10)$$

with $i = 2, \ldots, n$.

In matrix for, the error system can be presented as:

$$\dot{\mathbf{e}} = \mathbf{A}(\mathbf{e})\mathbf{e} + \mathbf{B}(\mathbf{e})\mathbf{u} \quad (11)$$

in which the matrices **A(e)** and **B(e)** are given by:

$$\mathbf{A}(\mathbf{e}) = \begin{bmatrix} 0 & 0 & \frac{\mathbf{V_1} cos(\theta_r)(\cos(e_{3i})-1) - \mathbf{V_1} \sin(\theta_r)\sin(e_{3i})}{e_{3i}} \\ 0 & 0 & \frac{\mathbf{V_1} sin(\theta_r)(\cos(e_{3i})-1) - \mathbf{V_1} \cos(\theta_r)\sin(e_{3i})}{e_{3i}} \\ 0 & 0 & 0 \end{bmatrix} \quad (12)$$

$$\mathbf{B}(\mathbf{e}) = \begin{bmatrix} \cos(e_{3i})\cos(\theta_r) - \sin(\theta_r)\sin(e_{3i}) & 0 \\ \sin(\theta_r)\,cos(e_{3i}) + \cos(\theta_r)\sin(e_{3i}) & 0 \\ 0 & 1 \end{bmatrix} \tag{13}$$

SDRE Control Method (State-Dependent Riccati Equation)

Let the dynamic nonlinear system be described as:

$$\dot{\mathbf{x}} = f(\mathbf{x}) + \mathbf{B}(\mathbf{x})\mathbf{u}, \tag{14}$$

with *f(0) = 0.*

Using the Optimal Control Theory, the goal is to find the optimal control **u** that minimizes the cost functional:

$$\mathbf{J}[\mathbf{u}] = \int_0^\infty [\mathbf{e}^T\mathbf{Q}(\mathbf{x})\mathbf{e} + \mathbf{u}^T\mathbf{R}\mathbf{u}]dt$$
(15)

For **Q(x)** *c*ontinuous and positive-definite.

Rafikova *et al.* (2016) and Mracek and Cloutier (1998) affirm that the solution of this problem can be found through solving the Hamilton-Jacobi-Bellman equation associated to this system. Since the solution of the HJB equation is usually hard to find, one can approximate it using a State Dependent Riccati Equation of the form:

$$\mathbf{P}(\mathbf{x})\mathbf{A}(\mathbf{x}) + \mathbf{A}(\mathbf{x})^{\mathbf{T}}\mathbf{P}(\mathbf{x}) - \mathbf{P}(\mathbf{x})\mathbf{B}(\mathbf{x})\mathbf{R}(\mathbf{x})^{-\mathbf{1}}\mathbf{B}(\mathbf{x})^{\mathbf{T}}\mathbf{P}(\mathbf{x}) + \mathbf{Q}(\mathbf{x}) = \mathbf{0} \tag{16}$$

Equation 12 results in a suboptimal controller, however, it makes it much easier problem to be solved, compared to the resolution through the HJB equation. The control law is given by:

$$\mathbf{u} = -\mathbf{R}(\mathbf{x})^{-1}\mathbf{B}(\mathbf{x})^T\mathbf{P}(\mathbf{x})\mathbf{x} \tag{17}$$

where **P(*x*)** is the solution to the Riccati equation.

The system can now be written as:

$$\dot{\mathbf{x}} = f(x) + \mathbf{B}(\mathbf{x})\mathbf{u}, f(0) = 0 \tag{18}$$

where *f(x)* = **A(x)x**.

This is known as State-Dependent Coefficients form. The matrices **A(x)** and **B(x)** are system state functions and thus coefficients in the Riccati equation. For any solution obtained this way, the SDRE method comes down to the resolution of a LQR at each sampling instant. In this work, the computational time needed to compute each instant can be as small as 1 ms. To ensure the existence of the SDRE controller, **A(x)** must be a controllable parameterization of the nonlinear system for a given region if **[A(x), B(x)]** is controllable for every **x** of that region (Çimen, 2010; Mracek and Cloutier, 1998)

RESULTS AND NUMERICAL SIMULATIONS

The concept of mobile robots formation can be considered as physical distances in respect to the leader or adjacent agents that each robot must respect. One may then introduce parameters in the *x* and *y* coordinates of each robot in order to ensure that these distances, and therefore, the formation are maintained. In this work, formation parameters called *dx* and *dy* have been introduced. These parameters act on each follower robot ensuring a predetermined distance from the leader in each axis.

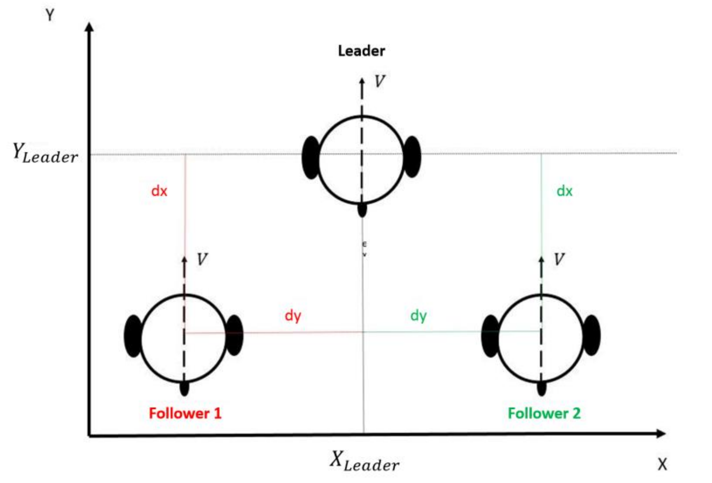

Figure 3 – Formation parameters.

Simulations were carried using the software LabVIEW, in which the user uses function blocks for programming. The Follower 1, Leader and Follower 2 were positioned, initially, on the points (0.5, 2.5), (2.5, 2.5) and (4.5, 2.5) on the XY plane, respectively. The main function of the program is to define the initial conditions of the robots, obtain the appropriate information of the leader and apply the proposed control on the followers, alongside with the desired positioning restrictions.

For the application of the control method the weighted matrices **Q** and **R** of the Ricatti equation are design parameters for the controller and thus can be chosen arbitrarily: In the present work, for simplicity, these matrices were chosen as:

$$\mathbf{Q} = \begin{bmatrix} 10 & 0 & 0 \\ 0 & 10 & 0 \\ 0 & 0 & 10 \end{bmatrix} \quad (19)$$

$$\mathbf{R} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad (20)$$

Figures 4 to 7 refer to simulations of the robots navigating in V-shaped formation with linear velocity of 0.2 m/s, and null angular velocity.

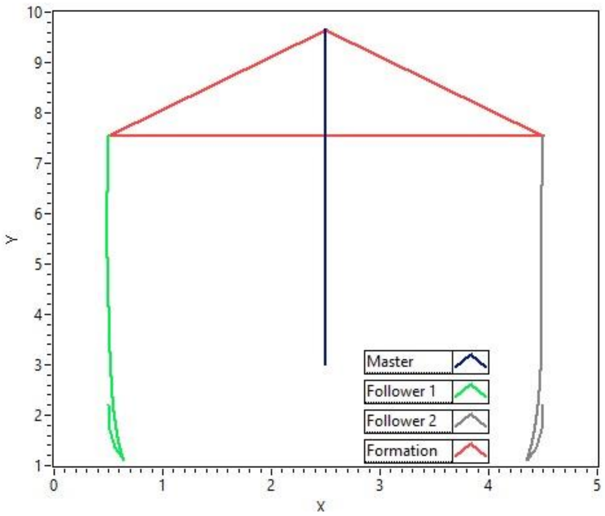

Figure 4 – Robots in V-shaped formation.

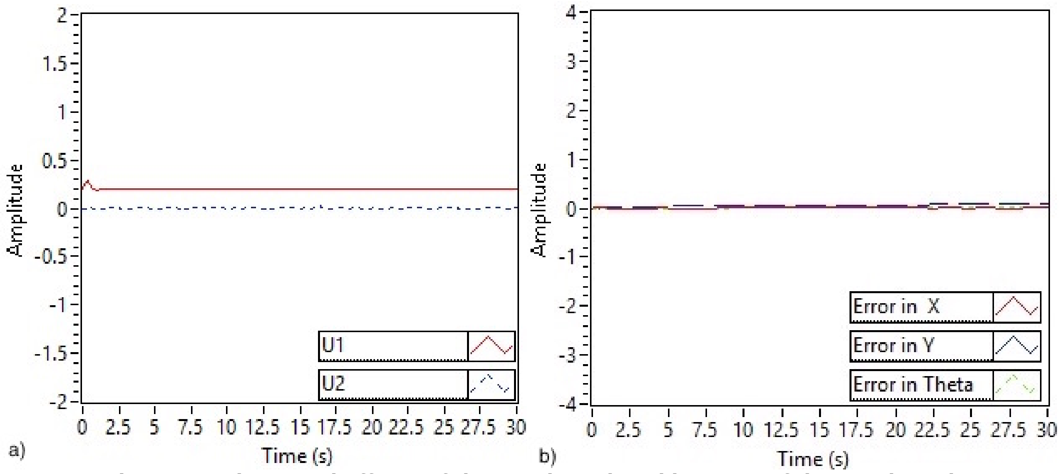

Figure 5 – a) Control efforts of the Leader robot; b) Errors of the Leader robot.

Without angular velocity the V-shaped formation moves forward, as seen in Figure 4. The followers respect the distance of 2 m on the X axis and 2 m in the Y axis as defined by *dx* and *dy* parameters. Figure 5a) shows the control efforts of the leader, where U1 is the control effort on the linear velocity of the robot and U2 is the control on the angular speed. Both reflect the velocities of 2 m / s and 0 rad / s initially set.

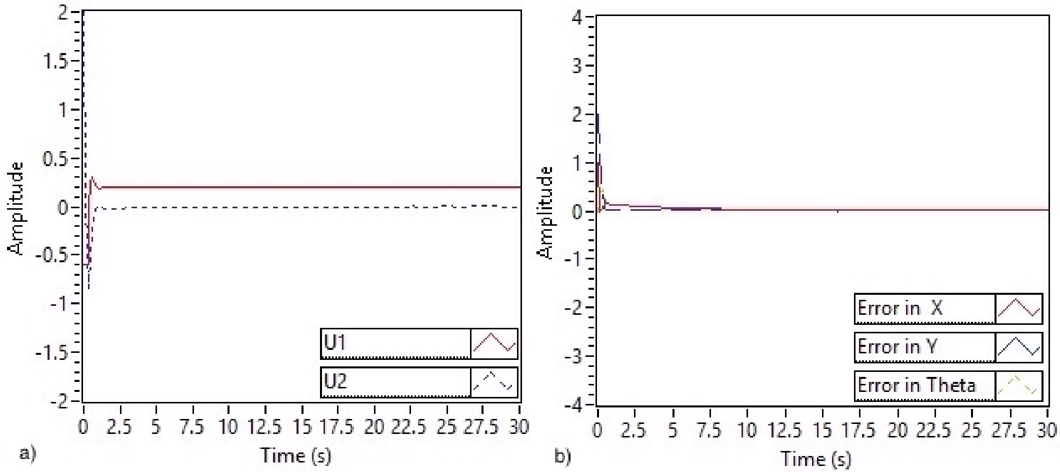

Figure 6 – a) Control efforts of Follower 1; b) Errors of Follower 1.

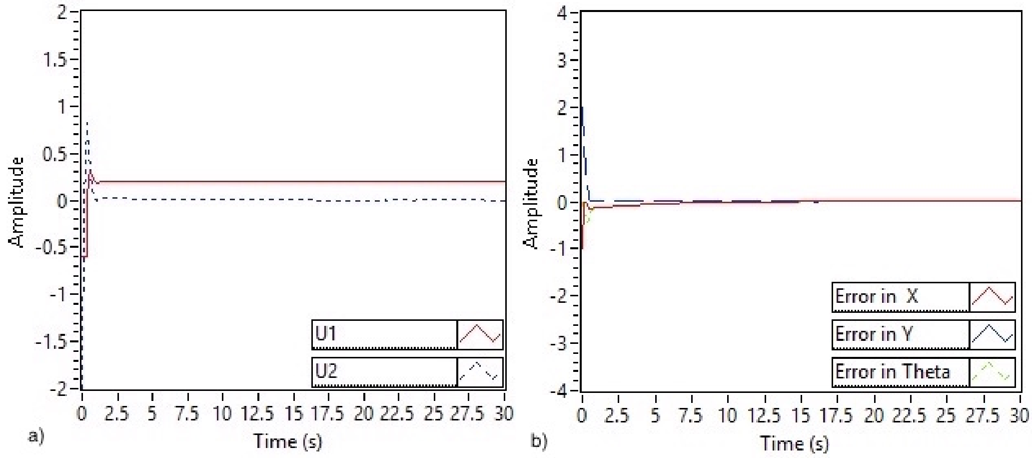

Figure 7 – a) Control efforts of Follower 2; b) Errors of Follower 2.

Figures 6a) and 7c) show the fast stabilization on the control efforts of the followers, of approximately 1 s. The limitations imposed to control efforts create a delay to this time of convergence, however without these limits there would be no practical application. Figures 5b), 6b) and 7b) show the stabilization of the errors of each robot. It is observed that the errors of the leader converge to zero extremely fast and the errors of the followers take about 10 s to converge to zero. The robot leader develops a small error in the three axes due to the errors in numerical integrations performed by the software.

Figures 8 to 11 refer to simulations of the robots navigating in Echelon formation. Initially the robots had a linear velocity of 0.2 m/s and null angular velocity. After a few seconds an angular velocity of 0.1 rad/s was introduced in order to make the robot formation turn left.

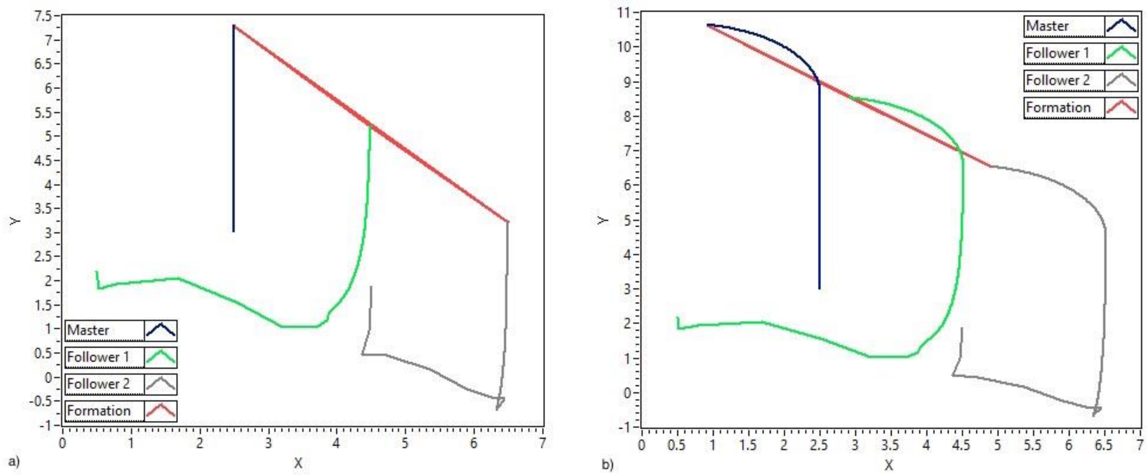

Figure 8 – Robots in Echelon formation.

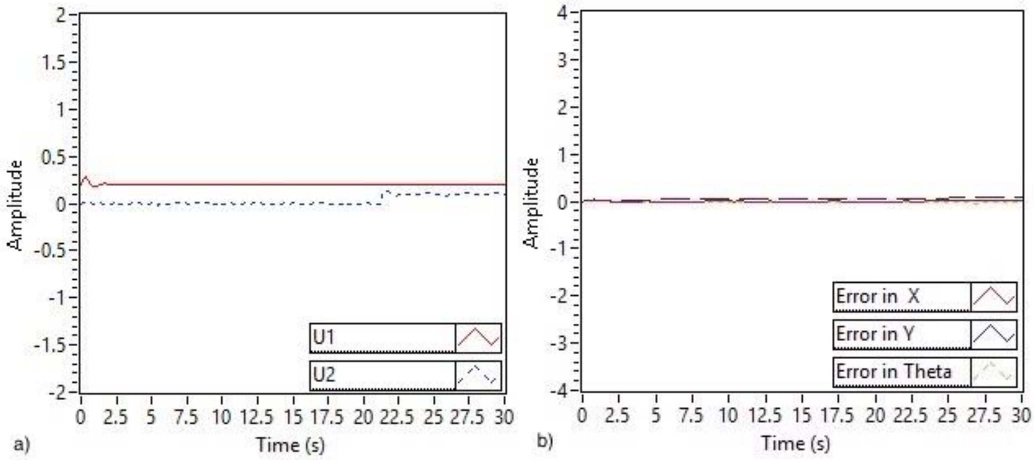

Figure 9– a) Control efforts of the Leader robot; b) Errors of the Leader robot.

Figure 8a) shows the robots achieving Echelon formation and Figure 8b) depicts the formation turning left due to the positive angular velocity. Figures 9a) shows that the control efforts of the leader stabilize quickly, and Figure 9b) shows that the errors of the leader also converge to zero very fast, with a small numerical error already mentioned before.

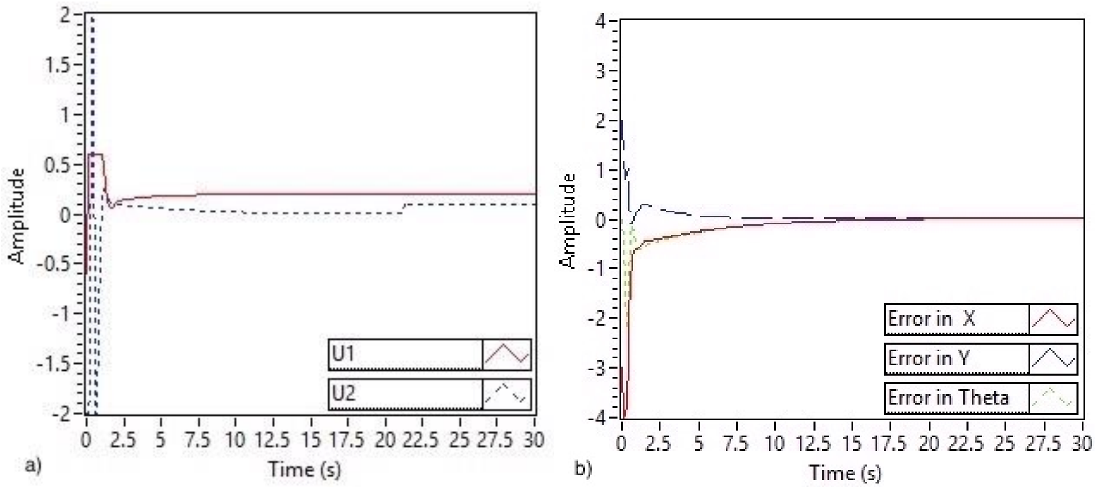

Figure 10– a) Control efforts of Follower 1; b) Errors of Follower 1.

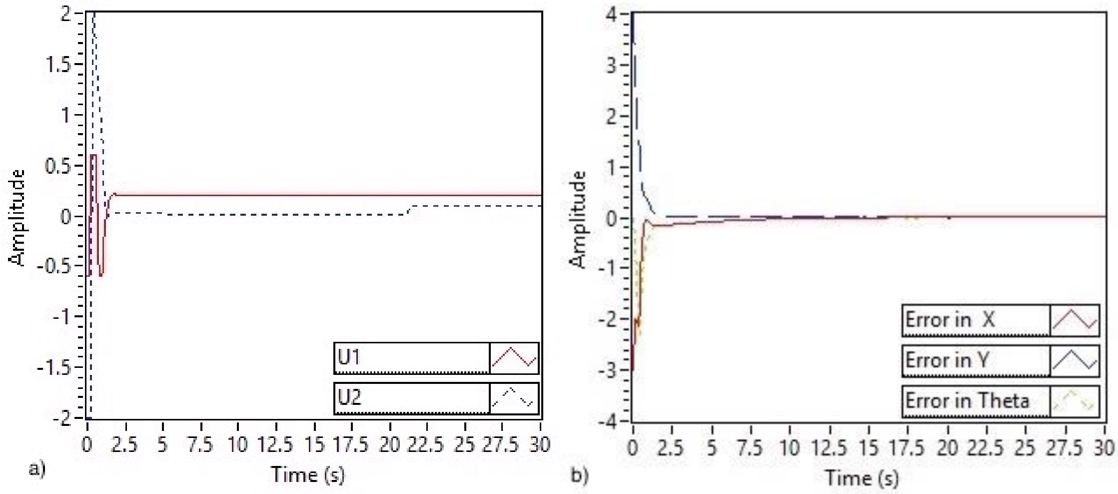

Figure 11– a) Control efforts of Follower 1; b) Errors of Follower 1.

Figures 10a) and 11a) show that the control efforts of the followers take a bit longer than the leader to stabilize, about 10 s for Follower 1 and 7 s for Follower 2. Figures 10b) and 11b) show that the errors of the follower robots converge to zero after approximately 19 s.

CONCLUSIONS

The SDRE control method was highly effective for the control of multiple mobile robots in formation, as it takes into account the non-linearities of non-holonomic dynamic system that represents the robot with differential drive The computational cost of the control application is low because the solution can be found through solving the Linear Quadratic Regulator instead of the Hamilton-Jacobi-Bellman equation. The proposed formations were achieved and maintained in all the simulated trajectories. The stabilization of errors and control efforts proved to be fast, less than 20 seconds for both cases. In more sophisticated robotic platforms with more powerful engines, this stabilization time can be decreased. Real time engineering applications are viable because of the small computational time needed to solve the controller at each sampling instant and the feasible control efforts, which can be limited in order to adapt to different robotic platforms.

The advantages of this approach are that it allows one to consider any robot or trajectory as a reference robot that is given in terms of velocities, which makes easy to modify the trajectory without having to address the *f(t)* function of the trajectory itself. The system is scalable, allowing the addition of other robots without many difficulties. The disadvantage of this approach are the lack of an anti-collision system. For future works, the practical application of this solution to the problem of mobile robots navigating in formation can be tested in order to ratify the theoretical results obtained in this work.

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