DINAME 2017 - Proceedings of the XVII International Symposium on Dynamic Problems of Mechanics
A. T. Fleury, D. A. Rade, P. R. G. Kurka (Editors), ABCM, São Sebastião, SP, Brazil, March 5-10, 2017

Optimum Design of a set of Dynamic Neutralizers to Passive Vibration Control Considering Physical Parameters, Location and Viscoelastic Material Variable

Francielly Elizabeth de Castro Silva[1], **Carlos Alberto Bavastri**[2]

[1]Uninter – Escola Superior Politécnica, Rua Luiz Xavier, n° 103, Centro, Curitiba, Paraná, Brasil, franciellye.castro@gmail.com

[2]Universidade Federal do Paraná, Centro Politécnico, Jardim das Américas, Curitiba, Paraná, Brasil, bavastri@ufpr.br

Abstract: Structures exposed to high level of vibrations produced by resonance or dynamic instability can fail or have, in general, their lifetime reduced. In some cases, the unwanted noise level affects the operation for which the equipment or structure was designed. In these cases, a feasible solution to reduce vibration levels and/or irradiated noise is to use viscoelastic dynamic neutralizers (VDNs). These devices have been used with considerable success in vibration control, due the high capability of viscoelastic materials to dissipate large vibration energy. The vibration and sound group (GVIBS) has a methodology that takes into account the natural frequency and the optimal position of one degree-of-freedom VDNs, to reduce vibration levels in linear dynamic structures, known as primary system, in a broadband of frequency. In this context, the present study aims to develop a methodology for optimal design of VDNs considering also, among other design variables, the viscoelastic material between in a set of characterized materials. The optimal configuration is obtained by the application of a hybrid optimization technique which uses Genetic Algorithms (considering continuous and discrete variables in the same design vector) in order to approximate the global minimum and, after that, a nonlinear programming method to perform a local search considering just the continuous variables. To evaluate the proposed methodology, simulations were performed on a metallic plate where one or more neutralizers are considered. A new proposal to calculate the VDNs mass, allows obtaining different values for different VDNs to control one or more modes, which turns vibration attenuation more efficient. The results of the optimal design of VDNs, for a given temperature, proves to be promising, and the inclusion of the viscoelastic material in the optimization process, proves to be an interesting aspect in vibration control, provided we can use the most adequate material for each situation (environment temperature, frequency band, primary system, among others).

Keywords: Passive vibration control, dynamic neutralizers, viscoelastic material, optimization.

INTRODUCTION

Dynamic Vibration Neutralizers (DVN), often called Dynamic Vibration Absorbers - are mechanical devices to be attached to another mechanical system, or structure, called 'primary system', with the purpose of reducing, or controlling vibrations and sound radiation.

The first reference to these devices was made by for Frahm in 1909. Since then, a great quantity of work has been done to understand and generalize the DVNs theory starting from the design of a single degree-of- freedom undamped system (Ormondroyd and Den Hartog, 1928) and proceeding to much more complex structures. Among the first works carried out in this area were those by Frahm (1909) and Den Hartog (1956), which used DVNs to reduce rolling motion of ships. Systems that have been studied in detail include single-degree-of-freedom DVNs applied to particular positions of uniform beams, with particular boundary conditions (Jacquot, 1978 and Çandır and Özgüven, 1986), as well as DVNs distributes (Manikahally and Crocker, 1991 and Esmailzadeh and Jalili, 1998).

Viscoelastic dynamic vibration neutralizers (VDVN or simply VDN) are easy to build and apply to structures of any sizes and shapes. This is in part possible thanks to the modern technology considering viscoelastic materials, which makes it easy to mold them in any shape and to tailor it to meet almost any specifications.

In Espíndola and Silva (1992), a general theory for the optimum design of neutralizer systems, when applied to a generic structure, in any amount, was derived. That theory has been applied to the optimum design of systems of viscoelastic neutralizers (Espíndola and Bavastri, 1995a and 1997; Espíndola *et al.*, 1998 and Espíndola *et al.*, 2005). This theory is based on the concept of equivalent generalized mass and damping parameters for the absorbers and on an equivalent Den Hartog methodology for one-degree-of-freedom primary system - as introduced by author[1].

With this concept, it is possible to write down the equations for the movement of the composite system (primary plus neutralizers) in terms of the generalized coordinates (degrees-of-freedom), previously chosen to describe the configuration space of the primary system alone, in spite of the fact that the composite system has additional degrees-of-freedom introduced by the attached neutralizers. This fact was crucial in developing the present theory for it permits a coordinate transformation using the modal matrix of the primary system, which is an invariant during the optimization process. With this transformation it is possible to obtain the modal space for the composite system without solving a complex eigenvalues problem for the whole composite system at each step of the iterative process, which could make it computationally out of question.

In recent years, the concept of fractional derivative has been applied to the construction of parametric models for viscoelastic materials (Bagley and Torvik, 1979; Bagley and Torvik, 1986; Torvik and Bagley, 1987; Pritz, 1996; Liebst and Torvik, 1996; Rossikhin and Shitikova, 1998; Espíndola *et al.*, 2004; Espíndola *et al*., 2005b).

The choice of viscoelastic material is fundamental to obtain an efficient vibration control. Febbo *et al*. (2014) show a methodology for an optimal design of VDNs to control a cubic nonlinear system, for a determined temperature, considering previously the type of viscoelastic material.

Tavares (2005) applied the methodology developed by Espíndola and Silva (1992) and Espíndola and Bavastri (1997) in the optimal project of VNDs considering the optimal positions of the devices on a metallic plate. In that work, a hybrid technique was used through Genetic Algorithm (GA) for searching of the positions (for each individual of the GA) and - after optimizing and using nonlinear programming - to find the optimal frequencies of the devices, for each generation, thus elevating its computational coasts.

Considering that the optimal neutralizer parameters (natural frequency, position and viscoelastic material) can be written in the optimization problem as continuous and discrete variables simultaneously, is necessary to employ a more robust optimization method capable of handling this kind of problem, and thus providing a better final solution.

In this context, this paper aims to develop an optimal design methodology of VDNs using a hybrid optimization technique through Genetic Algorithms (GA) and nonlinear programming, to obtain the optimal parameters of the control devices.

General Ideas an Definitions

In the present work, the expression of 'primary system' stands for the system or structure prior to the attachment of the set of neutralizers. The primary structure, or primary system, considered in this paper may be of any shape, no matter how irregular or geometrically complex may be.

Figure 1(a) illustrates a vibration mode shape and a set of neutralizers of viscoelastic nature positioned on them, which can be applied to any vibration mode or any structure with a general shape. Figure 1(b) shows a particular ordinary absorber. Between its rigid mass m_a and the structure itself lies a viscoelastic spring which is a piece of viscoelastic material, perhaps with some metal inserts.

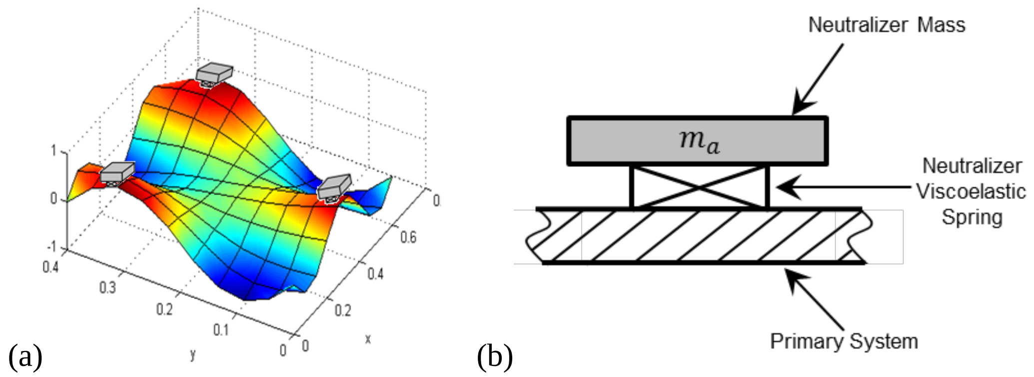


Figure 1 - (a) Primary structure with absorbers attached to it and (b) a particular neutralizer.

Equivalent Generalized Quantities for a Neutralizer

The simple absorber (the one of degree of freedom absorber) has a single lump mass (m_a) connected to the primary structure through a resilient device (a "spring", see Fig. 1), presumably of viscoelastic nature, with complex stiffness $K_c(\Omega)$ equal to (Espíndola and Silva, 1992)

$$K_c(\Omega, T) = \vartheta G_c(\Omega) = \vartheta G(\Omega)[1 + \eta(\Omega)]. \tag{1}$$

In the above equation, $G_c(\Omega)$ is the complex shear modulus of the viscoelastic material, $G(\Omega)$ is the dynamic shear modulus, $\eta(\Omega)$ is the so called 'loss factor'; Ω is the circular frequency and ϑ is a geometric factor, depending on the shape and metal inserts of the viscoelastic spring. In the present article, a four parameter model based on fractional derivatives is used to determine the optimum design of vibration neutralizers. The shear complex modulus can be written as

$$G_c(\Omega) \approx \frac{G_0 + G_\infty (i\Omega b)^\alpha}{1 + (i\Omega b)^\alpha}, \tag{2}$$

where G_0 and G_∞ are the so called 'complex lower and upper asymptotes', respectively, α is the fractional order of the derivative appearing in the constitutive differential equation for the viscoelastic material, and b is the relaxation time constant of the material. The relaxation time is highly sensitive to temperature and is normally expressed as $b = b_0 s(T)$, where b_0 is b, computed in the so called reference absolute temperature T_0. An expression normally used to compute the shift function is $log_{10} s(T) = -\theta_1(T - T_0)/(\theta_2 + T - T_0)$, where θ_1 and θ_2 are constants to be determined experimentally. T is the environmental absolute temperature. For the sake of simplicity, the letter T will be omitted in the present notation.

It is also appropriate to write the complex shear modulus as follows:

$$G_c(\Omega) = G(\Omega)[1 + \eta(\Omega)], \tag{3}$$

where $G(\Omega)$ is the real part of the complex shear modulus called dynamic elasticity modulus, and $\eta(\Omega) = Im(G_c(\Omega))/Re(G_c(\Omega))$. The loss factor is a measure of the ability of the material to convert deformation energy into heat.

In Fig. 2, $Q(\Omega)$ and $F(\Omega)$ stand for Fourier Transforms of the basis displacement $q(t)$ and the applied force $f(t)$, respectively. This apllied force is a result of the interaction between the absorber and the point of the primary structure where it is attached.

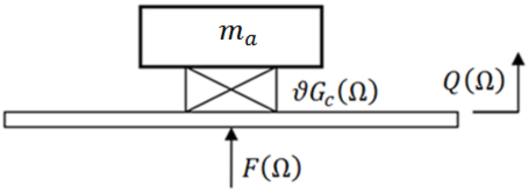


Figure 2 – Scheme of a simple (single degree of freedom) neutralizer.

It is a simple matter to verify that the interaction force $F(\Omega)$ at the attachment (massless) plate "feels" the neutralizer as a dynamic stiffness given by

$$K_a(\Omega) = \frac{F(\Omega)}{Q(\Omega)} = \frac{m_a \Omega^2 \vartheta G_c(\Omega)}{m_a \Omega^2 - \vartheta G_c(\Omega)}. \tag{4}$$

The anti-resonant frequency of the absorber is defined as the one that, in the absence of damping, makes the denominator of Eq. 4 equal to zero:

$${\Omega_a}^2 = \frac{\vartheta G(\Omega_a)}{m_a}. \tag{5}$$

In Eq. (5), Ω_a stands for the anti-resonant frequency of the neutralizer and $\vartheta G(\Omega_a)$ is the stiffness of the viscoelastic spring at the anti-resonant frequency Ω_a. Defining: $r_a(\Omega) = G(\Omega)/G(\Omega_a)$., Eq. (4) can be rewritten as:

$$K_a(\Omega) = -m_a \Omega^2 \frac{\left[\Omega_a^{2r_a(\Omega)} - \Omega^2\right]\Omega_a^{2r_a(\Omega)} + \left[\Omega_a^{2r_a(\Omega)}\eta(\Omega)\right]^2}{\left[\Omega_a^{2r_a(\Omega)} - \Omega^2\right]^2 + \left[\Omega_a^{2r_a(\Omega)}\eta(\Omega)\right]^2} + i\Omega m_a \frac{\Omega^{3\Omega_a 2r_a(\Omega)}\eta(\Omega)}{\left[\Omega_a^{2r_a(\Omega)} - \Omega^2\right]^2 + \left[\Omega_a^{2r_a(\Omega)}\eta(\Omega)\right]^2}. \tag{6}$$

Now imagine a single-degree-of-freedom system in which the mass $m_{eq}(\Omega)$ is connected to a fixed reference through a viscous dashpot of constant $c_{eq}(\Omega)$. If a Fourier transformable force $f(t)$ is applied to the mass, it will respond with movement $x(t)$. The ratio of the input force to the output displacement, in the frequency domain, will be $k(\Omega) = \frac{F(\Omega)}{X(\Omega)} = -\Omega^2 m_{eq}(\Omega) + i\Omega c_{eq}(\Omega)$. If this equation is now compared with Eq. 6, one can conclude that the primary structure 'sees' the absorber at the point of attachment as a mass $m_{eq}(\Omega)$ connected to a viscous dashpot constant $c_{eq}(\Omega)$, the other end of this dashpot being connected to the fixed reference. Figure 3 shows this interpretation.

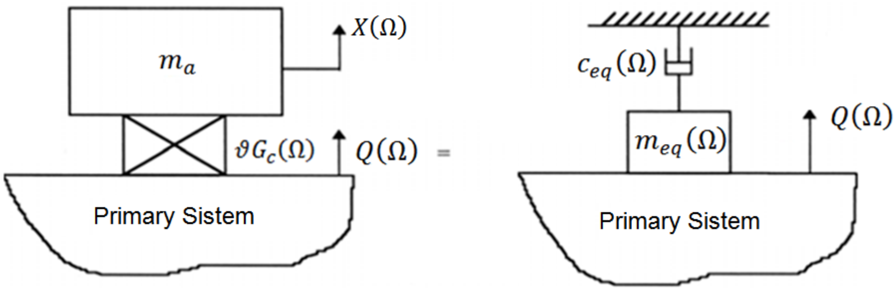


Figure 3 – Dynamically equivalent system

The expression for the equivalent parameters for the viscoelastic absorber was defined by Espíndola and Silva (1992) and given by

$$m_{eq}(\Omega) = m_a \frac{r_a(\Omega)\left\{r_a(\Omega)[1+\eta^2(\Omega)] - {\varepsilon_a}^2\right\}}{[{\varepsilon_a}^2 - r_a(\Omega)]^2 + [r_a(\Omega)\eta(\Omega)]^2}, \tag{7}$$

$$c_{eq}(\Omega) = m_a \Omega_a \frac{r_a(\Omega)\eta(\Omega){\varepsilon_a}^3}{[{\varepsilon_a}^2 - r_a(\Omega)]^2 + [r_a(\Omega)\eta(\Omega)]^2}, \tag{8}$$

where $\varepsilon_a = \Omega/\Omega_a$.

Compound System Response

Considering a primary system modeled as a n-degree-of-freedom structure, both its damping and mass matrices will still in order n x n after the attachment of the neutralizer, despite the fact that p simple absorbers will add p new degrees of freedom to the compound system. The stiffness matrix is completely unchanged by the attachment of neutralizers. Thus, if p such neutralizers, with equivalent masses and equivalent damping constants, are attached to the n-degree-of-freedom primary system at the generalized coordinates, the motion equations can be written, in the frequency domain, as follows (see Espíndola and Bavastri, 1995)

$$\left[-\Omega^2\tilde{\mathbf{M}} + i\Omega\tilde{\mathbf{C}} + \mathbf{K}\right]\boldsymbol{Q}(\Omega) = \boldsymbol{F}(\Omega), \tag{9}$$

where the modified mass and damping matrices are given by

$$\tilde{\mathbf{M}} = \mathbf{M} + \begin{bmatrix} 0 & & & \cdots & & 0 \\ & \ddots & & & & \\ & & m_{eq1}(\Omega) & & & \\ & & & \ddots & m_{eqp}(\Omega) & \\ & & & & & \ddots \\ 0 & & & & & 0 \end{bmatrix} = \mathbf{M} + \mathbf{M_e} \tag{10}$$

and

$$\tilde{\mathbf{C}} = \mathbf{C} + \begin{bmatrix} 0 & & & \cdots & & 0 \\ & \ddots & & & & \\ & & c_{eq1}(\Omega) & & & \\ & & & \ddots & c_{eqp}(\Omega) & \\ & & & & & \ddots \\ 0 & & & & & 0 \end{bmatrix} = \mathbf{C} + \mathbf{C_e}, \tag{11}$$

where $\mathbf{C}$ and $\mathbf{M}$ are the ordinary damping and mass matrices of the primary system, respectively; and matrices $\mathbf{M_e}$ and $\mathbf{C_e}$ are diagonal; and the elements are complex functions of the frequency.

The next step is to solve the eigenvalue problem $\mathbf{K\Phi} = \mathbf{\Lambda M}\boldsymbol{\phi}$, involving the ordinary mass and stiffness matrices of the primary system, and to define the modal matrix $\mathbf{\Phi}$, containing $\hat{n} \ll n$ eigenvectors Φ_{r_k}, $k = 1, \dots, \hat{n}$. It is assumed that the corresponding band of frequencies $\left[\Omega_{r_1}, \dots, \Omega_{r_{\hat{n}}}\right]$ covers the band of frequencies within which vibrations are to be reduced. The eigenvectors are orthonormalized, so that $\mathbf{\Phi^T M \Phi} = \mathbf{I}_{\hat{n}}$ and $\mathbf{\Phi^T K \Phi} = \mathbf{\Lambda}_{\hat{n}} = \begin{bmatrix} \ddots & & \\ & {\Omega_{r_k}}^2 & \\ & & \ddots \end{bmatrix}_{\hat{n}\,\mathrm{x}\,\hat{n}}$.

Now, applying the transformation

$$\boldsymbol{Q}(\Omega) = \mathbf{\Phi}\,\boldsymbol{P}(\Omega), \tag{12}$$

to Eq. 9, and pre-multiplying by $\mathbf{\Phi}^{\boldsymbol{T}}$, and assuming proportional hysteretic damping in the primary system, this rewrites the Eq. (9) as follows

$$\left[-\Omega^2\left[\mathbf{I}_{\hat{n}}+\widehat{\mathbf{M}}_{\mathbf{a}}(\Omega)\right]+i\Omega\left[\begin{bmatrix}\ddots & & \\ & \eta_k\Omega_k & \\ & & \ddots\end{bmatrix}+\widehat{\mathbf{C}}_{\mathbf{a}}(\Omega)\right]+\boldsymbol{\Lambda}_{\hat{n}}\right]\boldsymbol{P}(\Omega)=\boldsymbol{\Phi}^T\boldsymbol{F}(\Omega). \tag{13}$$

The $\widehat{\mathbf{M}}_{\mathbf{a}}(\boldsymbol{\Omega})$ and $\widehat{\mathbf{C}}_{\mathbf{a}}(\boldsymbol{\Omega})$ are written as

$$\widehat{\mathbf{M}}_{\mathbf{a}}(\Omega)=\begin{bmatrix}\sum_{i=1}^{p} m_{eq_i}\Phi_{r_i1}^2 & \sum_{i=1}^{p} m_{eq_i}\Phi_{r_i1}\Phi_{r_i2} & \cdots & \sum_{i=1}^{p} m_{eq_i}\Phi_{r_i1}\Phi_{r_i\hat{n}} \\ \vdots & \vdots & \ddots & \vdots \\ \sum_{i=1}^{p} m_{eq_i}\Phi_{r_i\hat{n}}\Phi_{r_i1} & \sum_{i=1}^{p} m_{eq_i}\Phi_{r_i\hat{n}}\Phi_{r_i2} & \cdots & \sum_{i=1}^{p} m_{eq_i}\Phi_{r_i\hat{n}}^2\end{bmatrix}, \tag{14}$$

$$\widehat{\mathbf{C}}_{\mathbf{a}}(\Omega)=\begin{bmatrix}\sum_{i=1}^{p} c_{eq_i}\Phi_{r_i1}^2 & \sum_{i=1}^{p} c_{eq_i}\Phi_{r_i1}\Phi_{r_i2} & \cdots & \sum_{i=1}^{p} c_{eq_i}\Phi_{r_i1}\Phi_{r_i\hat{n}} \\ \vdots & \vdots & \ddots & \vdots \\ \sum_{i=1}^{p} c_{eq_i}\Phi_{r_i\hat{n}}\Phi_{r_i1} & \sum_{i=1}^{p} c_{eq_i}\Phi_{r_i\hat{n}}\Phi_{r_i2} & \cdots & \sum_{i=1}^{p} c_{eq_i}\Phi_{r_i\hat{n}}^2\end{bmatrix}, \tag{15}$$

Specification of Absorber Masses

For primary systems with only one degree of freedom, the recommended ratio between the absorber mass (m_a) and primary structure mass (m_{ps}) by Den Hartog (1956) for one-degree-of-freedom of the primary system is $\mu = m_a/m_{ps} =$ 0,1 to 0,25. The use of the modal mass ratio concept has been proposed by Espíndola and Silva (1992) for a multiple-degree-of-freedom system (primary system) as

$$\mu_k=\frac{\sum_{i=1}^{p}\Phi_{r_ik}^2\, {m_{a_i}}^{(k)}}{m_k}. \tag{16}$$

where ${m_{a_i}}^{(k)}$ is the mass of the i$^{\text{th}}$ neutralizer, for the k$^{\text{th}}$ mode taken inside the frequency band. m_k stands for the k$^{\text{th}}$ modal mass of the primary system, which, in case of orthonormalized eigenvectors, is equal to one. So, given μ_k, one for each of the modes of interest, a set of equations is established and ${m_{a_i}}^{(k)}$, $i = 1,\dots,p$ and $k = 1,\dots,\hat{n}$ are computed by singular values decomposition of the system matrix associated with Eq. (16). The matrix of the system shown in Eq. (16) is of the order of $\hat{n}$ x $\hat{n}p$. Developing Eq. (16), we have

$$\begin{bmatrix}\left[\Phi_{k_11}^2 \;\cdots\; \Phi_{k_p1}^2\right] & 0 \;\cdots\; 0 & 0 & 0 \;\cdots\; 0 \\ 0 \;\cdots\; 0 & \left[\Phi_{k_12}^2 \;\cdots\; \Phi_{k_p2}^2\right] & 0 & 0 \;\cdots\; 0 \\ 0 \;\cdots\; 0 & 0 \;\cdots\; 0 & \cdots & 0 \;\cdots\; 0 \\ 0 \;\cdots\; 0 & 0 \;\cdots\; 0 & 0 & \left[\Phi_{k_1\hat{n}}^2 \;\cdots\; \Phi_{k_p\hat{n}}^2\right]\end{bmatrix}\begin{Bmatrix}{m_{a_1}}^{(1)} \\ \vdots \\ {m_{a_p}}^{(1)} \\ \hline {m_{a_1}}^{(2)} \\ \vdots \\ {m_{a_p}}^{(2)} \\ \hline \vdots \\ \hline {m_{a_1}}^{(\hat{n})} \\ \vdots \\ {m_{a_p}}^{(\hat{n})}\end{Bmatrix}=\begin{Bmatrix}\mu_1 \\ \mu_2 \\ \vdots \\ \mu_{\hat{n}}\end{Bmatrix}, \tag{17}$$

or in a matrix form

$$\boldsymbol{\Phi}_{\hat{n}\text{ x }\hat{n}p}^2\,\mathbf{M}_{a_{\hat{n}p\text{ x }1}}=\boldsymbol{\mu}_{\hat{n}\text{ x }1}, \tag{18}$$

where the over written in the mass vector in Eq. (17) represents the mode to be controlled.

Solving the linear system, it is possible to find the modal masses for each neutralizer, through the minimum norm, finding the pseudoinverse of matrix $\boldsymbol{\Phi}^{\mathbf{2}}$. Therefore, for a frequency broadband control, where one or more neutralizers are considered, the mass of each neutralizer can be written as

$$m_{a_i} = \frac{\sum_{j=1}^{\hat{n}} m_{a_i}{}^{(j)}}{\hat{n}}. \qquad (19)$$

Optimization Strategy

The optimal VDN parameters are obtained through the minimization of the Euclidian norm of vector $\widehat{\boldsymbol{P}}(\Omega)$, whose coordinates are the maximum absolute values of the modal coordinates of the composed system in the analyzed frequency band and the mass minimization of neutralizers (Eq. 19), simultaneously, given by

$$\text{Minimize} \quad fobj(\boldsymbol{x}) = \left\| \max_{\Omega_{a1}<\Omega<\Omega_{a2}} \widehat{\boldsymbol{P}}(\Omega,\boldsymbol{x}) \right\|_2 + \left(w_1 \sum_{j=1}^{p} m_{a_j} \right)^2 \qquad (20a)$$

$$\text{Subject to} \quad x_i{}^l \le x_i \le x_i{}^u \qquad (20b)$$

where $\boldsymbol{x} = (\boldsymbol{\Omega_a}, \widetilde{\boldsymbol{x}}, M_V)$, with $\boldsymbol{\Omega_a} = \left[\Omega_{a_1}, \dots, \Omega_{a_p}\right]$, $\widetilde{\boldsymbol{x}} = [\tilde{x}_1, \dots, \tilde{x}_p]$ and M_V, where Ω_{a_j} is the j^{th} natural frequency; $\tilde{x}_j$ is the j^{th} fixing position of the neutralizers at the primary system; and M_V is the viscoelastic material of the neutralizers, with known values of the four fractional parameters **(**G_0, G_∞, α and b**)**; Ω_{a1} and Ω_{a2} correspond to the minimum and maximum frequency range of interest; and w_1 is a weighing factor defined by Tavares (2005) as:

$$w_1 = \frac{\left\| \max_{\Omega_{a1}<\Omega<\Omega_{a2}} \widehat{\boldsymbol{P}}(\Omega,\boldsymbol{x}) \right\|_2}{0{,}02 m_{sp}}. \qquad (21)$$

In the present study, the vector of the design variable consists of the natural frequency ($\boldsymbol{\Omega_a}$) (which varies continuously and with binary codification), the neutralizer position ($\widetilde{\boldsymbol{x}}$) (which varies discretely with integer codification) and the viscoelastic material type M_V (which also varies discretely with integer codification). The genetic programming algorithm considers each individual as a possible optimal configuration. In the present work, it is codifyed through the following chromosome (design variable vector):

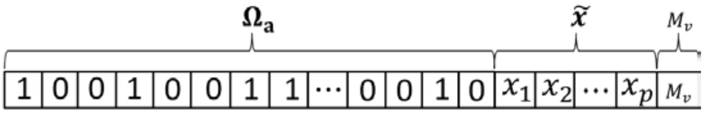


Figure 4 – Vectoc of designs variable

The viscoelastic materials used in the optimal design of neutralizers have the follow properties (see Tab. 1):

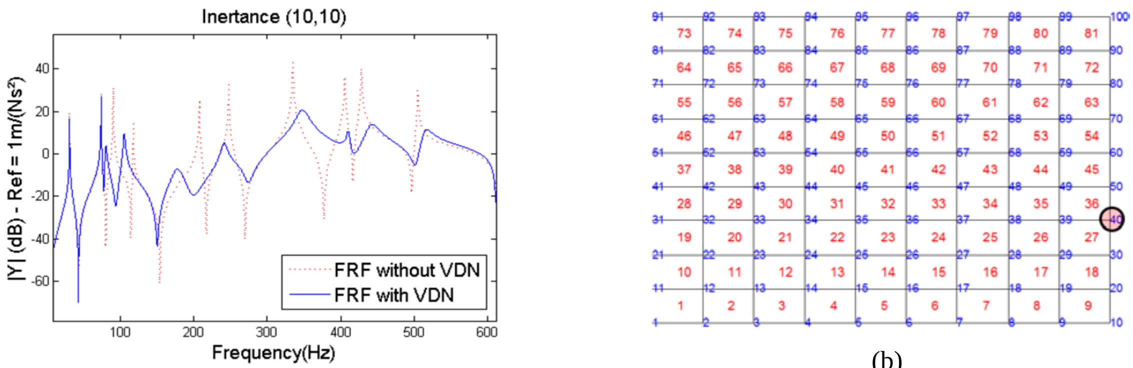
Table 1 – Viscoelastic materials properties.

Viscoelastic Materials	Temperature (K)	G_0 (Pa)	G_∞ (Pa)	β	φ	θ_1	θ_2
Natural Rubber	273	$2.79e^6$	$8.16e^8$	0.297	0.000355	9.74	148
Neopren	273	$4.55e^6$	$4.18e^8$	0.319	0.00274	5.09	46.5
Butilic Rubber	273	$1.76e^5$	$2.41e^8$	0.424	0.00424	9.91	119
C1002	285,7	$6.45e^5$	$9.1e^8$	0.536	0.00208	28.13	276
C2003	314,9	$2.37e^6$	$3.21e^9$	0.479	0.00160	126.94	1157

The general algorithm of the computational structure, used in the present work has the following parameters as input data for the optimal design of neutralizers: frequency range of interest; modes to be controlled, number of neutralizers, and working temperature (and, in special cases, the environmental temperature).

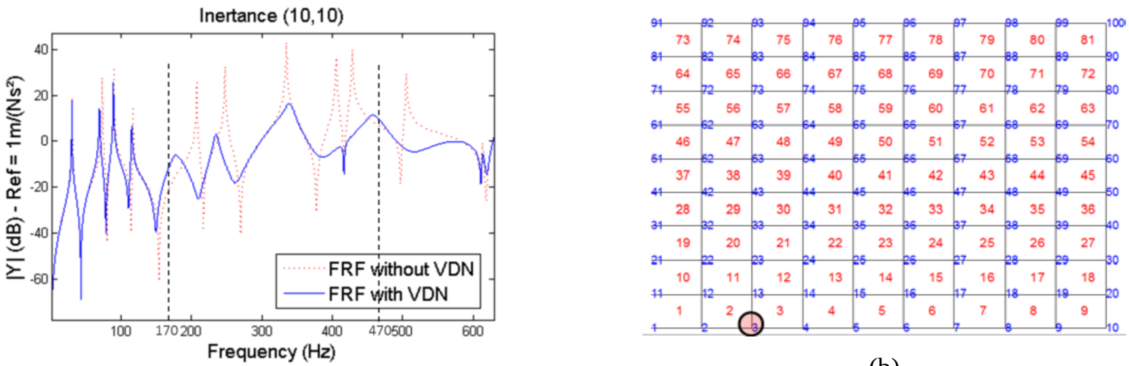
Numerical Example

This section presents the results for the vibration control of a plane plate using the proposed methodology. The plate has $\rho = 7850\,\text{kg/m}^3$ of density, the elasticity modulus is $\text{E} = 207\,\text{GPa}$ and the Poisson coefficient $\nu = 0{,}3$, with dimensions 0,6 x 0,4 x 0,005 (m). The frequency band to be controlled is 170 Hz to 470 Hz. The modes corresponding to this frequency band are 5 to 9. This methodology was buit considering one, two and four VDNs at each analysis and considering 243 K, 273 K, and 303 K as environmental temperatures, to evaluate the optimum 'combination' of viscoelastic material, positions and frequencies of the VDNs. Figs. 5 to 13 (a) and (b) show the inertance at response point 10 and excitation 10 with and without the neutralizers and the positions of the VDNs on the plate, respectively. Table 2 shows the results of each analysis.



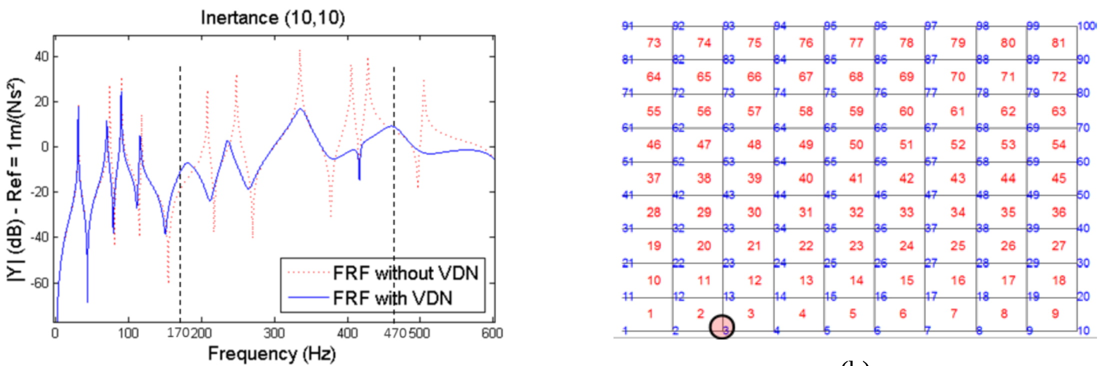
(a) (b)

Figure 5 - (a) Modes to control 5, 6, 7, 8 and 9 with 1 VDN for temperature 243 K. (b) VDN position on the plate.



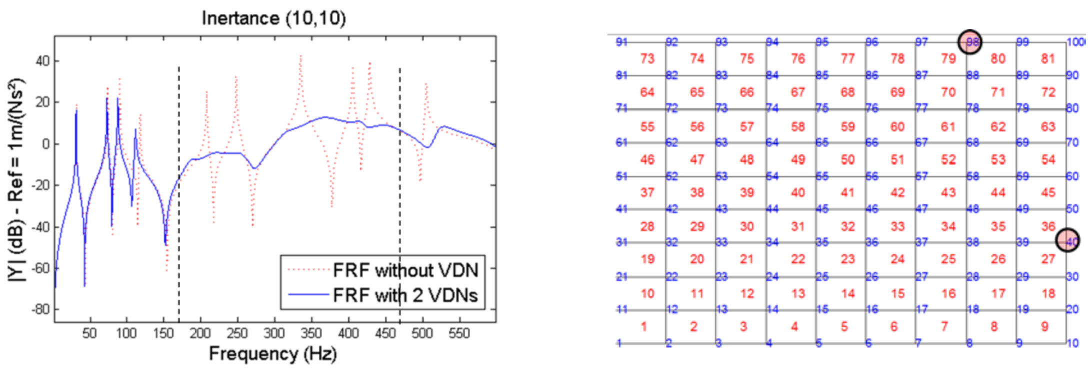
(a) (b)

Figure 6 - (a) Modes to control 5, 6, 7, 8 and 9 with 1 VDN for temperature 273 K. (b) VDN position on the plate.



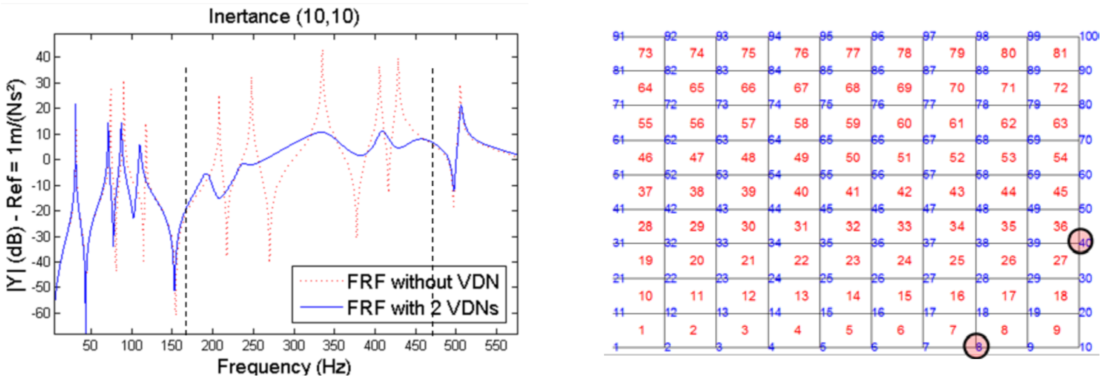
(a) (b)

Figure 7 - (a) Modes to control 5, 6, 7, 8 and 9 with 1 VDN for temperature 303 K. (b) VDN position on the plate.



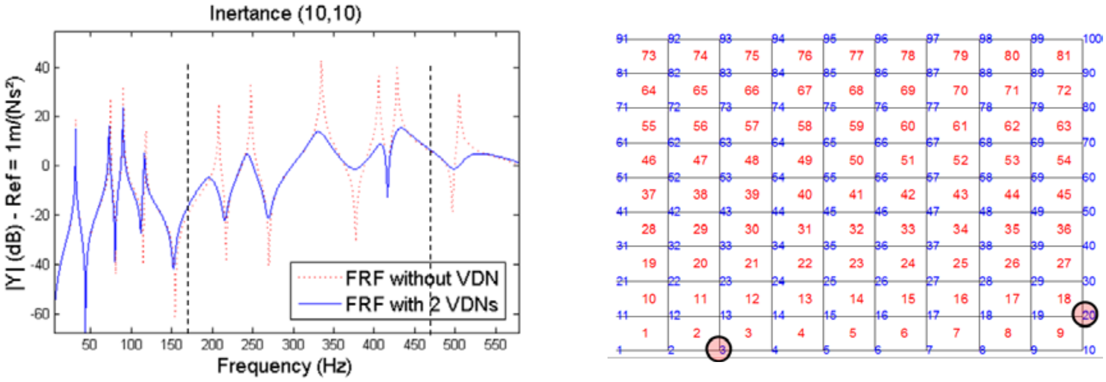
(a) (b)

Figure 8 - (a) Modes to control 5, 6, 7, 8 and 9 with 2 VDNs for temperature 243 K. (b) VDNs positions on the plate.



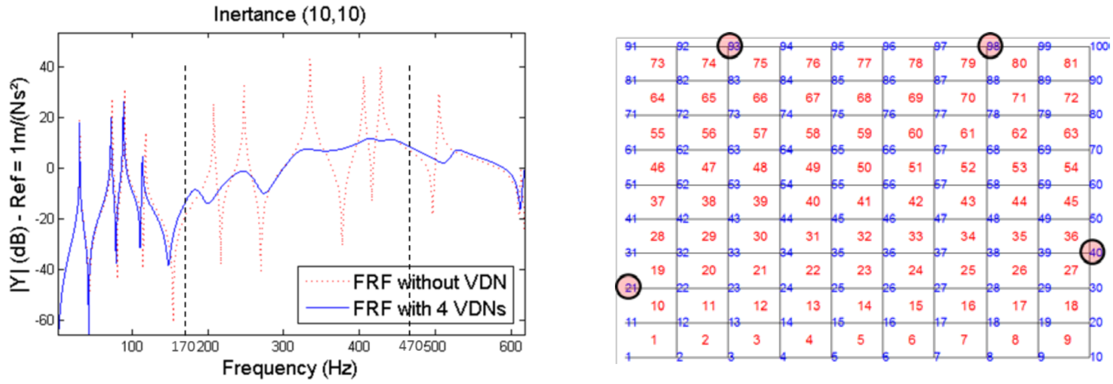
(a) (b)

Figure 9 - (a) Modes to control 5, 6, 7, 8 and 9 with 2 VDNs for temperature 273 K. (b) VDNs positions on the plate.



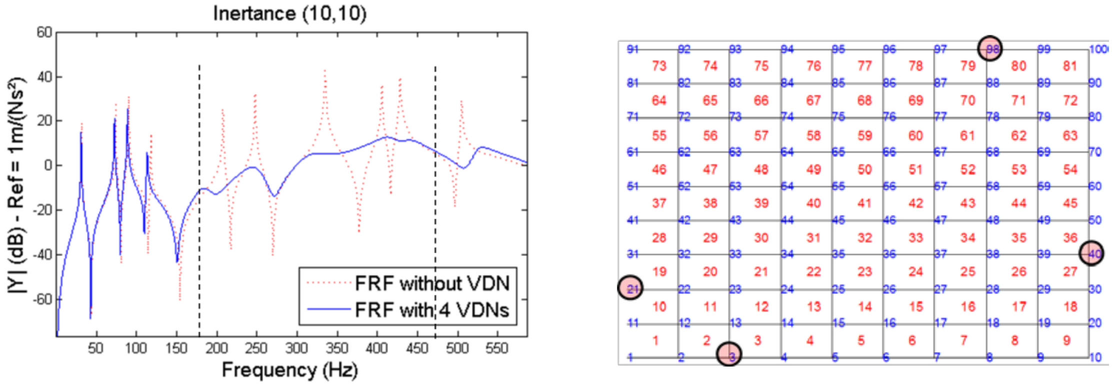
(a) (b)

Figure 10 - (a) Modes to control 5, 6, 7, 8 and 9 with 2 VDNs for temperature 303 K. (b) VDNs positions on the plate.



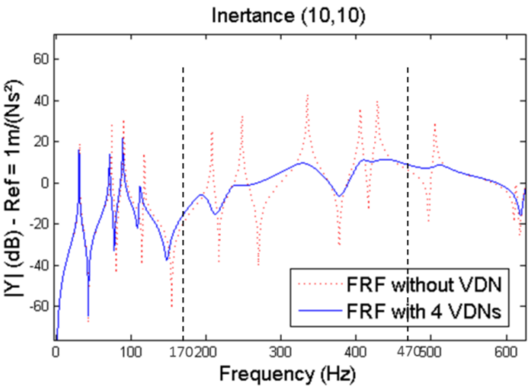
(a) (b)

Figure 11 - (a) Modes to control 5, 6, 7, 8 and 9 with 4 VDNs for temperature 243 K. (b) VDNs positions on the plate.

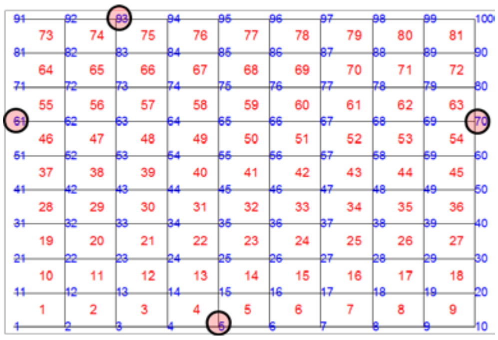


(a) (b)

Figure 12 - (a) Modes to control 5, 6, 7, 8 and 9 with 4 VDNs for temperature 273 K. (b) VDNs positions on the plate.



(a)



(b)

Figure 13 - (a) Modes to control 5, 6, 7, 8 and 9 with 4 VDNs for temperature 303 K. (b) VDNs positions on the plate.

Table 2 – VNDs results to control the modes 5, 6, 7, 8 and 9 with different temperatures.

VDN Data	1 VDN				2 VDNs				4 VDNs			
	Ω_a	x	M_v	m_a	Ω_a	x	M_v	m_a	Ω_a	x	M_v	m_a
T = 243 K	187.82	40	Natural Rubber	0.407	221.33 309.20	98 40	Natural Rubber	0.116 0.143	321.75 203.22 359.24 240.19	21 93 40 98	Natural Rubber	0.044 0.090 0.053 0.064
T = 273 K	248.01	3	Butilic Rubber	0.389	196.76 301.38	8 40	C1002	0.116 0.143	231.23 365.20 189.74 321.68	98 40 3 21	Neopren	0.066 0.063 0.070 0.053
T = 303 K	221.31	3	C1002	0.389	184.46 254.87	3 20	C1002	0.134 0.165	215.35 254.72 331.09 189.50	70 5 61 93	C2003	0.043 0.086 0.048 0.063

Table 2 shows the results of each analysis. Considering just one neutralizer, depending on the number of modes to be controlled, it could be very efficient for some, but not so much for others. This can be avoided introducing more neutralizers into the structure.

For the analyzed frequency band, natural rubber is the optimal viscoelastic material for 243 K (representing low environmental temperatures). For 273 K, three different optimal viscoelastic materials with similar behaviors were obtained. Finally, for 303K, which simulates high environmental temperature, the result was C1002 and C2003. As expected, in all cases, in a broadband frequency control, the optimization process shows that the viscoelastic material is working in the transition region, i.e., the region where the material has a higher degree of damping.

Conclusions

This article shows a general methodology to design a set of viscoelastic vibration neutralizers for passive vibration control.

As design variables, the present work assumed the natural frequencies, positions and the viscoelastic materials, which are described as continuum and discrete variables, simultaneously, and used the Genetic Algorithm to find the global optimal configuration and a non-linear programming for a local search. The choice of correct viscoelastic material in the algorithm provides a more adequate solution. It is possible to tell that a more efficient control can be achieved by this approach.

The theory is pretty general and bypasses any difficult related to the geometrical form of the primary system, just by assuming that the structure has a linear behavior.

A methodology to obtain the VDNs masses considers that the masses can vary depending on the device position, based on the modal mass of the primary system. Then, the optimization methodology allows searching a minimum mass of the set of neutralizers.

The Tab. 2 shows that the total mass does not increase along with the increase of the number of neutralizers, thus, the total mass for all analysis is approximately 0.4 kg (with one, two or four absorbers) for a primary system (metallic plate) with the weighting of 9.42 kg.

The efficacy of the neutralizers is shown in Figs. 5 to 13 that yielding a mean reduction of the peaks between 31.9 dB to 37.4dB, which proves the efficacy/effectiveness of this methodology.

REFERENCES

Bagley, R. L. and Torvik, P. J., 1979, "A generalized derivative model for an elastomer damper," The Shock and Vibration Bulletin, Vol. 49, No.2, pp. 135–143.

Bagley, R. L. and Torvik, P. J., 1986, On the fractional calculus model of viscoelastic behaviour," Journal of Rheology, Vol.30, No.1, pp. 133–155.

Çandir, B. and Ozguven, H. N., 1986, Dynamic vibration absorbers for reducing resonance amplitudes of hysteretically damped beams, in Proceedings of the Fourth International Modal Analysis Conference, Los Angeles, CA, Vol.l, No.2, pp. 1628–1635.

Den Hartog, J. P., 1956, Mechanical Vibrations, McGraw-Hill, New York.

Esmailzadeh, E. and Jalili, N., 1998, Optimum design of vibration absorbers for structurally damped Timoshenko beams, ASME Journal of Vibration and Acoustics, Vol.120, pp. 833–841.

Espíndola, J. J. and Bavastri, C. A., 1995, Modal reduction of vibrations by dynamic neutralizers in a frequency band a generalized approach, in: Proceedings of the Sixth International Symposium on Dynamic Problems of Mechanics, vol. 1, 06–10 March, Caxambu, Minas Gerais, Brazil, pp. 214–217.

Espíndola, J. J. and Bavastri, C. A., 1997, Reduction of Vibrations in Complex Structures with Viscoelastic Neutralizers: A Generalized Approach and a Physical Realization, DECT97, Sacramento, California.

Espíndola, J. J., Bavastri, C. A. and Teixeira, P. H., 1998, A hybrid algorithm to compute the optimal parameters of a system of viscoelastic vibration neutralizers in a frequency band, in: Proceedings of the Fourth International Conference on Motion and Vibration Control, vol. 2, Zurich, Switzerland, August, pp. 577–582.

Espíndola, J. J., Cruz, G. A. M., Lopes, E. M. O. and Bavastri, C. A., 2005, On the design of optimum systems of viscoelastic vibration neutralizers, in: E.P. Hofer, E. Reithmeier (Eds.), Modelling and Control of Autonomous Decision Support Based Systems, pp. 49–64.

Espíndola, J. J. and Silva, H. P., 1992, Modal reduction of vibrations by dynamic neutralizers, In: Proceedings of the 10th International Modal Analysis Conference, San Diego, CA, pp. 1367–1373.

Espíndola, J. J., Silva, H. P. and Lopes, E. M. O., 2005, A generalized fractional derivative approach to viscoelastic material properties measurements, Applied Mathematics and Computation, Vol.164, No.2, pp. 493–506.

Espíndola, J. J., Silva Neto, J. M., and Lopes, E. M. O., 2004, A new approach to viscoelastic material properties. Identification based on the fractional derivative model," in Proceedings of First IFAC Workshop on Fractional Differentiation and its Application, Bordeaux, France, pp. 19–21.

Espíndola, J. J., Silva Neto, J. M., and Lopes, E. M. O., 2005b, A generalized fractional derivative approach to viscoelastic material properties measurements," Applied Mathematics and Computation, Vol.164, No.2, pp. 493–506.

Febbo, M., Lopes, E. M. O. and Bavastri, C. A., 2014, Influence of temperature on optimum viscoelastic absorbers in cubic nonlinear systems, Journal of Vibration and Control, pp. 1-17.

Frahm, H., 1909, Device for Damping Vibration of Bodies, US Patent. No. 989959.

Jacquot, R. G., 1978, Optimal dynamic vibration absorbers for Timoshenko beams," M.Sc. Thesis, Sharif University of Technology, Tehran, Iran.

Kittis, L., 1996, Vibration Reduction over a Frequency Range, Journal of Sound and Vibration, v. 89, p. 559-569, 1983.

Liebst, B. S. and Torvik, P. J. Asymptotic approximations for systems incorporating fractional derivative damping, Journal of Dynamic Systems, Measurement, and Control, Vol.118, pp. 572–579.

Manikahally, D. N. and Crocker, M. J, 1991, Vibration absorbers for hysteretically damped mass-loaded beams, ASME Journal of Vibration and Acoustics, Vol.113, pp. 116–122.

Nashif, A. D., 1973, Materials for Vibration Control in Engineering, Shock and Vibration Bulletin, Vol.43, pp. 145-151.

Ormondroyd, J. and Den Hartog, J. P., 1928, Transactions of the American Society of Mechanical Engineers, The theory of the dynamic vibration absorber, Vol.50, pp. 9-22.

Rossikhin, Y. A. and Shitikova, V., 1998, Application of fractional calculus for analysis of nonlinear damped vibrations of suspension bridges," Journal of Engineering Mechanics, Vol.124, No.9, pp. 1029–1036.

Snowdon, J. C., 1968, Vibration and Shock in Damped Mechanical Systems, John Wiley & Sons Inc., New York.

Tavares, C. S., 2005, Projeto e Localização Otimos de Sistemas de Neutralizadores Dinâmicos Viscoelásticos Usando Algoritmos Genéticos, Master Thesis, Universidade Tecnológica Federal do Paraná, Curitiba, Paraná, Brasil.

RESPONSIBILITY NOTICE

The author(s) is (are) the only responsible for the material included in this paper.

ACKNOWLEDGMENTS

Castro F. E. S. acknowledges the financial support from Capes, Bavastri C. A. acknowledge the financial support from CNPq and PRH24.