

Regularization of the inverse problem of damage identification using the modified sequential algebraic algorithm

Josué Costa Baptista¹, Roberto Aizik Tenenbaum¹, Filipe Otsuka Taminato¹

¹ Laboratory of Instrumentation in Dynamics, Acoustics and Vibration, State University of Rio de Janeiro, Nova Friburgo, Rio de Janeiro, 28625570 Brazil, josuecostabaptista@gmail.com, ratenenbaum@gmail.com, filipeot@gmail.com

Abstract: The modified sequential algebraic algorithm (MSAA) is a routine developed to solve the longitudinal acoustic wave propagation problem in inhomogeneous media. This method accurately predicts the echo generated by the inhomogeneities, particularly when the acoustic impedance variation profile in the damaged part of the structure is non-smooth. Used for solving the inverse problem by a stochastic optimization approach, MSAA shows to be efficient to identify rough profiles of impedance with low noise level in the input data. As noise level increases, the inverse problem turns out to be 'ill-posed' and often yields inappropriate solutions. The purpose of this paper is to apply a regularization scheme to reduce the error in the damage estimation. In the numerical examples 40 parameters are successfully identified for a signal to noise ratio of 20 dB. The regularization procedure decreased the maximum absolute error to levels lower than 6%, and for the roughest impedance profile, the mean squared error was reduced by more than one order of magnitude and the error propagation avoided.

Keywords: structural damage identification, acoustic wave propagation, inverse problems, regularization, noisy data

INTRODUCTION

Equipment and structures, even if well designed, may be damaged due to environmental conditions, such as corrosion, overload, cyclic stress, misapplication, and aging. These damages may lead to failure, yielding risk to people lives. So, in this context, the detection of damage is crucial for preventive maintenance of such equipment and structures.

Structural health monitoring (SHM) techniques are broadly applied to increase safety and to reduce maintenance costs in structures. SHM is related to the implementation of strategies for detection, localization, and quantification of damage severity in structures (Silkorsky, 2000).

Most work reported in structural damage detection deals with methods based on the structure vibrational behavior (Rucevskis and Wesolowski, 2010), (Doebbling *et al.*, 2010). Although the vibration approach has been applied with success in several damage identification problems (Manson and Barthorpe, 2010), (Wait *et al.* 2005), it is widely recognized that, for small inhomogeneities, these methods are not effective, since high frequency contents, resulting from small amplitude damages, might not to be reflected on the structure modal properties.

The acoustic wave propagation in the non-homogeneous media approach has been applied as an alternative to solve the damage identification problem, especially in slender structures, such as bars, beams, and columns, responding to small changes in the acoustic impedance (Gangadharan *et al.*, 2009), (Tenenbaum *et al.*, 2011), (Grabowska *et al.*, 2008). One of the advantages of this method, besides the high sensitivity to subtle damages, is the fact that it deals directly in the time domain (Tenenbaum and Zindeluk, 1992a), being computationally straightforward.

In a recent paper, Taminato *et al.* (2015) published an improved algorithm, called *modified sequential algebraic algorithm* (MSAA), to solve the direct problem of plane wave propagation in non-homogeneous media. In the sequel, the authors presented an optimization procedure to solve the damage identification problem and showed that it succeeds, even if the acoustical impedance profile is non-smooth.

Due to the sequential nature of the MSAA, the identification procedure estimates one parameter at a time, which, on one hand, strongly speeds up the optimization process, and, on the other hand, yields to some error propagation when noisy data is involved. Aiming at reducing the error propagation and to improve the solution of the inverse problem with a realistic level of additive noise in the input signals, two regularization schemes are employed in this paper: the Tikhonov (Tikhonov and Arsenin, 1977) and the total variation (Rudin *et al.*, 1992) regularizations.

As it will be shown in the numerical examples section, the results of the identification of non-smooth impedance profiles with a medium level of additive noise on the input data presents maximum and mean squared errors between actual and identified impedance profiles about one order of magnitude lower than the results found by Taminato *et al.* (2015).

PROBLEM FORMULATION

Mathematical model for the direct problem

Characteristic variables approach

The longitudinal plane wave propagation in inhomogeneous media is described by the following hyperbolic second order differential equation: (Gordon and Sanday, 1977)

$$\sigma_{tt} - c^2 \left[\sigma_{xx} + \left(\frac{A'}{A} - \frac{\rho'}{\rho} \right) \sigma_x + \rho \left(\frac{A'}{\rho A} \right)' \sigma \right] = 0, \quad (1)$$

where $\sigma(x, t)$ is the stress field, depending on position x and time t ; $A(x)$ is the transversal cross section area; $\rho(x)$ is the material density; $c(x) = (E(x)/\rho(x))^{1/2}$ is the longitudinal stress wave speed; and $E(x)$ is the Young modulus. The prime denotes total derivative and the subscripts stand for partial derivatives.

Equation (1) allows no closed form solution. The characteristic variables are then introduced to substitute Eq. (1) by a system of two first order differential equations in the characteristic plane (Tenenbaum and Zindeluk, 1992b) as:

$$U_r + \frac{\dot{Z}}{4Z} V = 0; \quad (2)$$

$$V_s - \frac{\dot{Z}}{4Z} U = 0, \quad (3)$$

where $Z = \rho c A$ is the *generalized acoustic impedance*, $U(r, s)$ and $V(r, s)$ are, respectively, the progressive and regressive stress components propagating along the r and s directions, defined as (Robinson and Treitel, 1980):

$$r = t + \tau; \quad (4)$$

$$s = t - \tau,$$

and \dot{Z} is the derivative of Z , regarding the independent variable τ , called *travel time*, and defined as

$$\tau(x) = \int_0^x \frac{d\xi}{c(\xi)}. \quad (5)$$

One of the advantages of Eqs. (2–3) is that it is an uncoupled system of equations. To solve this system, adequate boundary conditions must be provided in the characteristic plane (r, s) . Let us consider, for instance, the probing of the slender structure at its extremity $x = 0$. In this case:

$$\begin{aligned} U(s, s) &= F(s) = f(t); \\ V(r, 0) &= 0, \end{aligned} \quad (6)$$

where $f(t)$ is the excitation pulse at the free end of the structure, a progressive wave at the boundary $r = s$ ($x = \tau = 0$). The second boundary condition is the Sommerfeld radiation condition, indicating there are no other perturbations inside the structure (Pierce, 1991). Due to the inhomogeneity, a regressive wave will be observed at $x = 0$, which is the output echo $g(t) = V(s, s)$.

Modified Sequential algebraic algorithm

For the sake of completeness, the kernel of the formulation presented by “Taminato et al., 2015” will be briefly reviewed in this paper.

Considering the propagation of a longitudinal acoustic wave in a slender structure (a bar, for instance), whose main dimension under interest (in direction x) is L , the structure is discretized in N sections of the same size $\Delta x = L/N$. From Eq. (5), $\Delta \tau = \Delta x/c$. In the plane (τ, t) , the characteristic curves $r = \text{constant}$ and $s = \text{constant}$, are mutually orthogonal straight lines, resulting in angles $\pm 45^\circ$ with the axis τ and t . The temporal discretization will be $\Delta t = \Delta \tau$, such that $\Delta r = \Delta s = 2\Delta \tau$.

The structure is excited at its end $x = 0$ by a pulse $f(t)$ sampled at each time interval $2\Delta t$. The corresponding discretized excitation pulse is $F_j = f(2j\Delta t)$. The discretized echo, observed at the same station, will then be $G_j = g(2j\Delta t)$. The total time interval considered is $(0, 2L/c)$, which is the time spent by the acoustic wave to cover twice the structure’s useful length.

The discretized generalized acoustic impedance, Z_i will be then considered, and the reflection and transmission coefficients are given, respectively, as

$$R_i = \frac{Z_{i+1/2} - Z_{i-1/2}}{Z_{i+1/2} + Z_{i-1/2}}; \quad T_i = \frac{2Z_{i+1/2}}{Z_{i+1/2} + Z_{i-1/2}}, \quad (7)$$

satisfying the relationship

$$T_i = 1 + R_i. \quad (8)$$

As it is shown in Taminato *et al.* (2015), the stress components inside the characteristic mesh, by using a second order finite differences scheme are given as:

$$\bar{U}_{n+1}^p = \left(\frac{1 - \frac{R_{n-p}^2}{4}}{1 + \frac{R_{n-p}^2}{4}} \right) \bar{U}_n^p - \left(\frac{R_{n-p}}{1 + \frac{R_{n-p}^2}{4}} \right) \bar{V}_n^p, \quad (9)$$

$$\bar{V}_n^{p+1} = \left(\frac{1 - \frac{R_{n-p}^2}{4}}{1 + \frac{R_{n-p}^2}{4}} \right) \bar{V}_n^p + \left(\frac{R_{n-p}}{1 + \frac{R_{n-p}^2}{4}} \right) \bar{U}_n^p, \quad (10)$$

where \bar{U} and \bar{V} are the progressive and regressive stress components, respectively.

Defining \bar{R}_i and \bar{T}_i

$$\bar{R}_i = \frac{R_i}{1 + \frac{R_i^2}{4}}; \quad \bar{T}_i = \frac{1 - \frac{R_i^2}{4}}{1 + \frac{R_i^2}{4}}, \quad (11)$$

which satisfy the relationship

$$\bar{R}_i^2 + \bar{T}_i^2 = 1, \quad (12)$$

as the *apparent reflection* and *apparent transmission* coefficients, respectively, Eqs. (9–10), can be rewritten in terms of the new coefficients, yielding to:

$$\bar{U}_{n+1}^p = \bar{T}_{n-p} \bar{U}_n^p - \bar{R}_{n-p} \bar{V}_n^p, \quad (13)$$

$$\bar{V}_n^{p+1} = \bar{T}_{n-p} \bar{V}_n^p + \bar{R}_{n-p} \bar{U}_n^p. \quad (14)$$

Equations (13–14) provide the longitudinal wave propagation solution of the *modified sequential algebraic algorithm*, with the following boundary conditions:

$$\begin{aligned} \bar{U}_{n+1}^n &= F_n; \\ \bar{V}_n^0 &= 0, \end{aligned} \quad (15)$$

leading to the discretized output echo $\bar{G}_n = \bar{V}_n^n$.

Mathematical model for the inverse problem

The inverse problem considered in this work can be stated as follows. The sampled measurements of the regressive wave \bar{G}_j , $j = 1, 2, \dots, N$, at time instant t_j , depends on the unknown vector of the discretized acoustic impedance in the damaged region, \mathbf{Z} , defined as:

$$\mathbf{Z} = (Z_1, Z_2, \dots, Z_N). \quad (16)$$

The damage identification is cast as an optimization problem in which the objective is to minimize the sum of the square of the residual difference between the experimental data, \bar{G}_j and the echo, $G_j(\mathbf{Z})$ calculated by the model of the forward problem

$$\mathfrak{F}(\mathbf{Z}) = \sum_{j=1}^N [G_j(\mathbf{Z}) - \bar{G}_j]^2. \quad (17)$$

Aiming at obtaining realistic synthetic experimental data, a disturbance in the signal is introduced as

$$\bar{G}_j = G_j^{\text{exact}}(\mathbf{Z}_{\text{exact}}) + \varepsilon \Gamma_j, \quad (18)$$

where the exact echo $G_j^{\text{exact}}(\mathbf{Z}_{\text{exact}})$ is computed from the solution of the forward problem, using the exact parameters $\mathbf{Z}_{\text{exact}}$, ε simulates the standard deviation of the measurement error, and Γ is a computer generated pseudo-random number in the range $[-1, 1]$. This is equivalent to introduce Gaussian additive noise to the output signal, whose signal-to-noise ratio (SNR) will depend on the echo amplitude, as it will be shown in the section Numerical Results.

Luus-Jaakola heuristic

To take advantage of the sequential structure of the MSAA, the Luus-Jaakola (LJ) optimization technique (Luus and Jaakola, 1973) is applied to estimate one parameter at a time, which reduces search complexity and results in a strong economy of computation time. Despite the sequential feature of MSAA, in the optimization's point of view, LJ works as a global approach, since it searches for the minima in the entire considered domain. Intending to reduce error propagation, a regularization scheme is introduced.

The head advantage, compared to differential evolution algorithm (DE) (Storn and Price, 1997), (Price et al., 2005), used by Taminato *et al.* (2015), is that LJ has only one parameter to be adjusted, namely the region contraction factor, while DE has three, specifically, the size of the population, the differential weight, and the crossover probability. Tenenbaum *et al.* (2012) tested different stochastic methods and concluded LJ is the most appropriate optimization method to be applied to this class of problems.

Tikhonov Regularization

Tikhonov regularization (TR) is one of the most popular regularization methods for stabilizing the solution of ill-posed problems. In the TR, the value of the regularization parameter α relies on the roughness of the damage. So, a too small value, below 0.1, yields the estimation of rough profiles, while the opposite supposes that the profile to be identified is smooth (Stephany *et al.*, 2009). Since rough impedance profiles are expected, the new estimations of LJ are weighted according to its roughness, measured by the 2nd order Tikhonov regularization function (Tikhonov and Arsenin, 1977), resulting in the regularized form of the least squares equation:

$$\mathfrak{F}(\mathbf{Z}) = \sum_{j=1}^N [G_j - G_j^{\text{exp}}]^2 + \alpha \sum_{j=2}^{N-1} [G_{j+1} + 2G_j + G_{j-1}]^2. \quad (19)$$

Total Variation Regularization

Total variation (TV) functional was first introduced in a pioneering work aiming at preserving sharp discontinuities (edges) in images (Rudin *et al.*, 1992). The TV regularization method allows a much broader class of functions to be the solution of the inverse problem, including functions with discontinuities (Strong and Chan, 2003), which is the purpose of this work, since one expects to improve the identification of rough impedance profiles. So, the regularized form of the least squares equation for the TV regularization is given by

$$\mathfrak{F}(\mathbf{Z}) = \sum_{j=1}^N [G_j - G_j^{\text{exp}}]^2 + \alpha \Delta x \sum_{j=2}^{N-1} [|G_j - G_{j+i}| + |G_j - G_{j-1}|], \quad (20)$$

where Δx is the size of the spatial discretization.

NUMERICAL RESULTS

Damage scenarios

To evaluate the efficiency of the regularization procedures applied on the MSAA to solve damage identification problems, three inhomogeneities were modeled. One is considered smooth (Fig. 1a) and the two other are considered rough (Figs. 1b and 1c). Damage 1 is a rectilinear impedance profile; Damage 2 and 3 are impedance profiles generated randomly with a maximum reduction, compared to the nominal impedance (normalized as 1), of 25% and 50%, respectively. For all damage scenarios, it is assumed that the number N of discrete points to be identified is equal to 40.

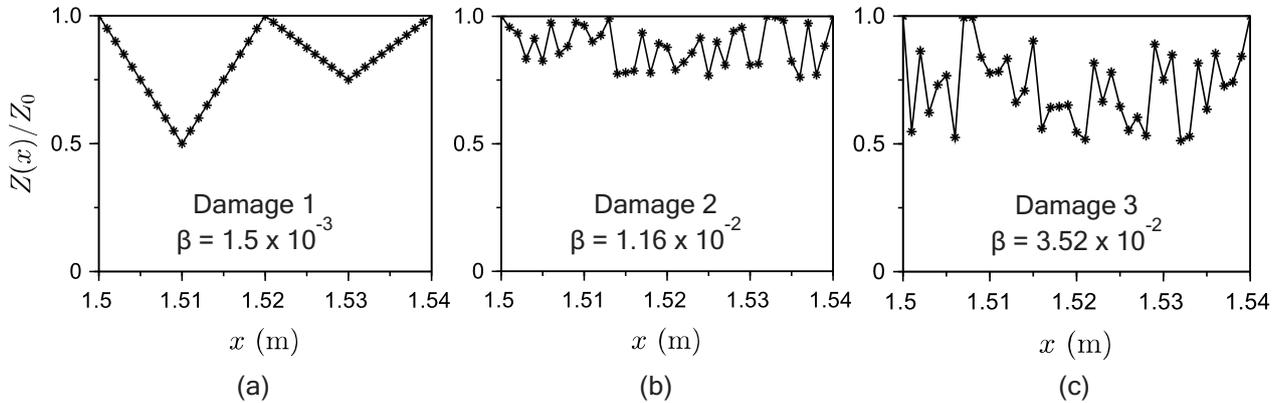


Figure 1 – Scenarios of damage for smooth (a) and rough (b) and (c) acoustic impedance profiles.

The dimensionless parameter β , defined by Taminato *et al.* 2015 as the average of the quadratic derivatives of the normalized acoustic impedance and given by

$$\beta = \frac{A_0}{N} \sum_{i=1}^N \left(\frac{d\bar{Z}}{dx} \right)_i^2, \quad (21)$$

was used to evaluate the roughness of the examined impedance profiles.

In Eq. (21), $\bar{Z} = Z/Z_0$ is the normalized acoustic impedance, Z_0 being the nominal (undamaged) acoustic impedance and A_0 being the nominal cross-section area of the structure. When $\beta < 5 \times 10^{-3}$ the profile is considered smooth; when $\beta > 5 \times 10^{-3}$ the profile is considered rough. The computed value of β for Damages 1, 2 and 3 are: 1.5×10^{-3} , 1.16×10^{-2} and 3.52×10^{-2} , respectively.

For simulation purposes, the structure material is considered steel ($E = 7.1 \times 10^{10}$ Pa, $\rho = 2840$ kg/m³, $c = 5 \times 10^3$ m/s). The discretization in space is $\Delta x = 1$ mm and, in time, it is $\Delta t = \Delta x/c = 0.2$ μ s.

In the next subsections, the results obtained by applying the Luus-Jaakola optimization to the non-regularized and regularized approaches on the recovery of the generalized acoustic impedance profile, $Z(\tau)$, using noisy data with SNR as shown in Table 1. This noise level is greater than the one usually found in actual experimental setups (Tenenbaum and Zindeluk, 1992a). Noiseless problems are not addressed in this work, since Taminato *et al.* (2015) have already shown the efficiency of the identification with noiseless data.

Table 1 – Signal-to-Noise ratio

	ϵ	SNR (dB)
Damage 1	0,01	25
Damage 2	0,01	18
Damage 3	0,01	25

In Figs. 2 to 4, the vertical axis is the identified generalized impedance profile $Z(\tau)$, normalized by the nominal impedance profile of the structure, Z_0 . The horizontal axis indicates the distance along the slender structure, counting from the signal capture station. To compare the identified (estimated) with the exact impedance profile, the absolute error (AE), given by

$$AE = |Z_{\text{exact}} - Z_{\text{estimated}}|, \tag{22}$$

and the mean squared error (MSE), given by

$$MSE = \frac{1}{N} \sum_{i=1}^N (Z_{\text{exact}}^i - Z_{\text{estimated}}^i)^2, \tag{23}$$

are computed.

The input pulse is, for all cases, a normalized rectangular one with a length much greater than the damage length. This is a common incoming pulse type used in actual pulse-echo experimental tests (Tenenbaum and Zindeluk, 1992a).

Non-regularized identification

Figure 2 presents the identification results by using the MSAA-LJ technique, without regularization, in addition to the corresponding absolute and mean squared errors (AE and MSE, respectively) for Damages 1, 2, and 3. The identification of Damages 1 and 2 succeed without noticeable error propagation, producing a maximum absolute error (MAE) of about 2% for both cases. However, looking at the reconstruction of Damage 3, which is more critical, remarkably, the MSAA-LJ technique does not attain good performance, being evident the presence of error propagation with increasing x , raising AE to approximately 30% by the end of the damage.

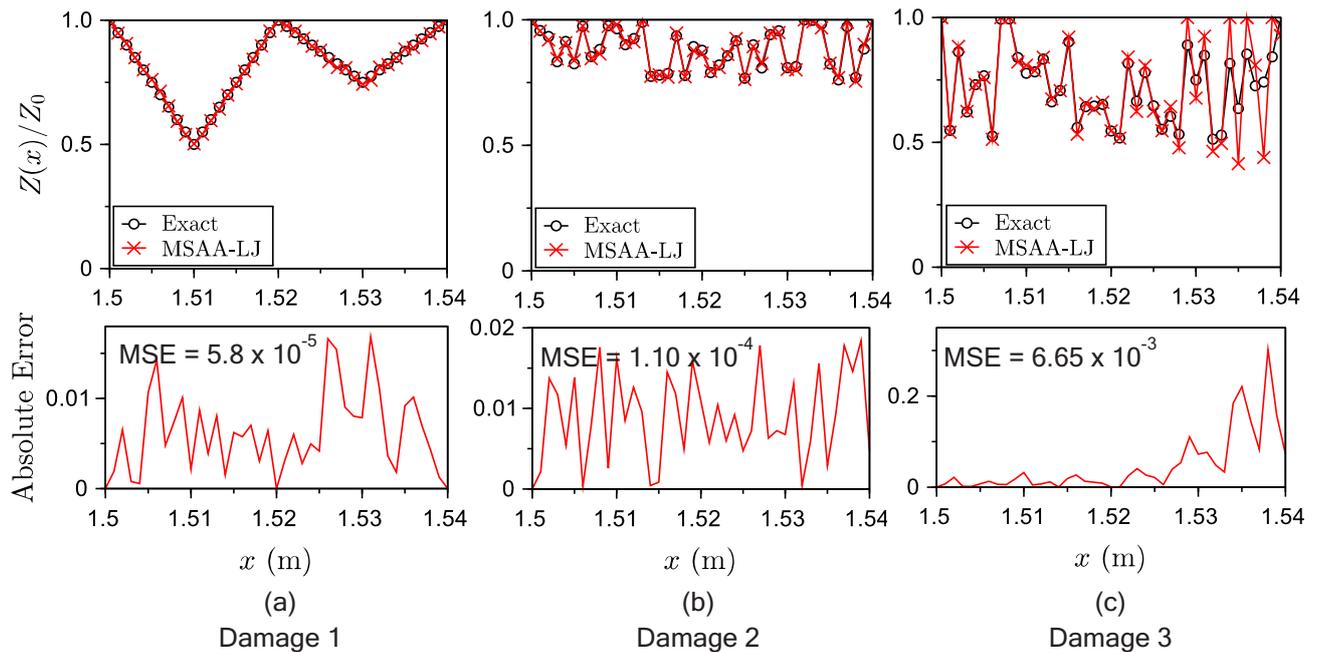


Figure 2 – Acoustic impedance profile identification by using the MSAA-LJ technique in Damages 1, 2 and 3 (top) and corresponding AE (bottom).

Regularized identification

Figure 3 summarizes the results attained by the MSAA-LJ technique with Tikhonov regularization (MSAA-LJ-TR), in addition to the corresponding AE and MSE for Damages 1, 2, and 3. Comparing Figures 2 and 3, it becomes unmistakable that applying Tikhonov's regularization slightly deteriorates the MSE in the identification of Damage 1 and 2. However, this reduction of accuracy is not a truly problem because the MAE is still inferior to 3% for both Damages. The biggest advantage in applying the TR appears in the identification of Damage 3, where it reduces the MSE of more than one order of magnitude, from 6.65×10^{-3} to 2.97×10^{-4} , and the error propagation seems to disappear.

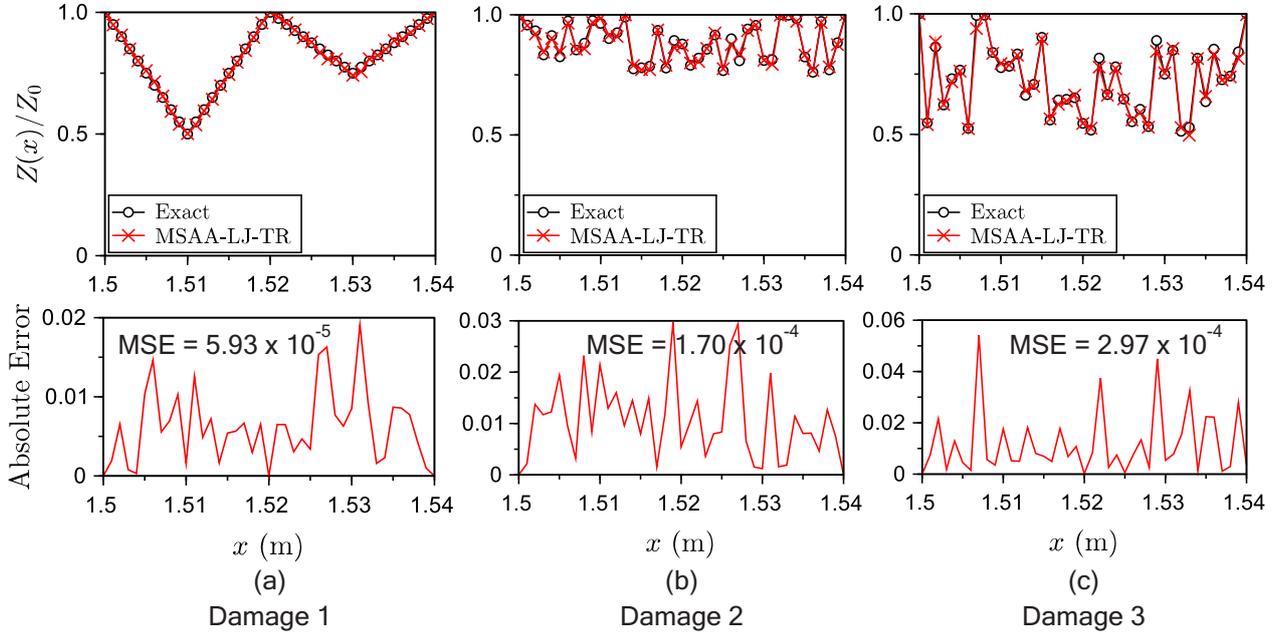


Figure 3 – Acoustic impedance profile identification by using the MSAA-LJ-TR technique in Damages 1, 2 and 3 (top) and corresponding AE (bottom).

Figure 4 shows the identification results obtained by the MSAA-LJ technique with total variation regularization (MSAA-LJ-TVR), besides the corresponding AE and MSE for Damages 1, 2, and 3. Comparing Figs. 2 (MSAA-LJ) and 4 (MSAA-LJ-TVR), the application of TVR deteriorates the identification of Damages 1 and 2, although, even with this harm, the MAE is still lower than 5%. The advantage of applying the TVR appears in the recovery of Damage 3, where it reduces the MSE of more than one order of magnitude and avoids error propagation.

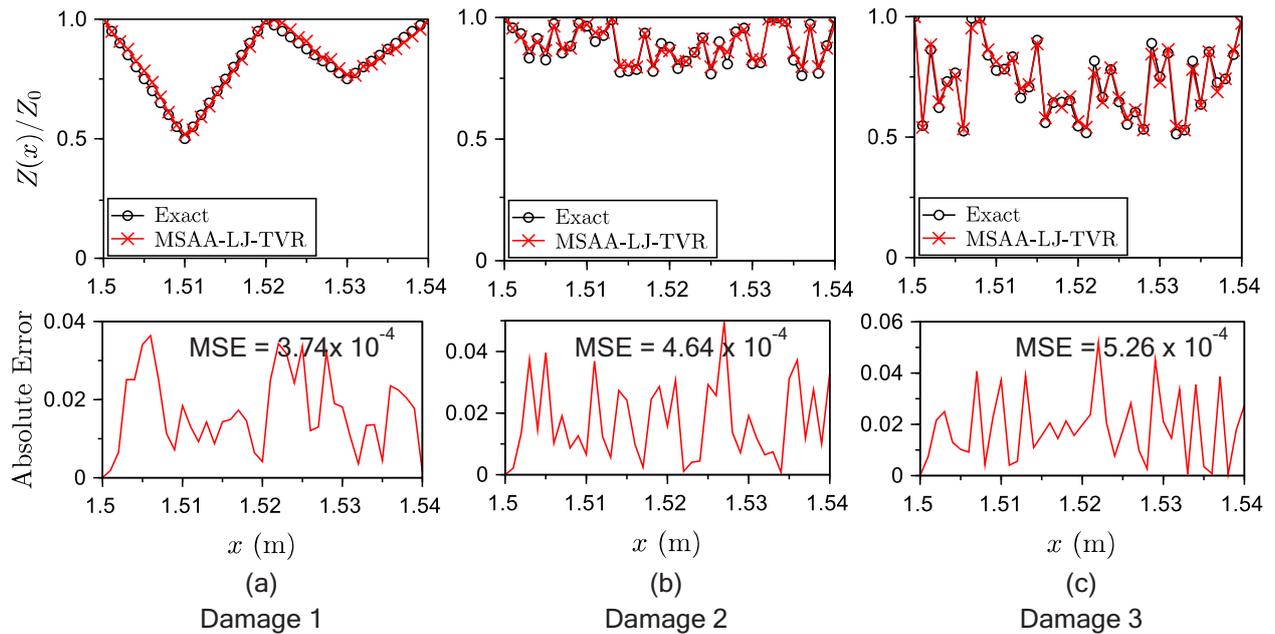


Figure 4 – Acoustic impedance profile identification by using the MSAA-LJ-TVR technique in Damages 1, 2 and 3 (top) and corresponding AE (bottom).

Table 1 summarizes the results obtained by each technique employed in this work. The identifications are compared with the mean squared error (MSE) and the maximum absolute error (MAE).

Table 2 – MSE and MAE of all optimization techniques

MSAA	Damage 1		Damage 2		Damage 3	
	MSE	MAE	MSE	MAE	MSE	MAE
LJ	5.80×10^{-5}	1.68 %	1.10×10^{-4}	1.84 %	6.65×10^{-3}	30.2 %
LJ-TR	5.93×10^{-5}	1.91 %	1.70×10^{-4}	2.97 %	2.97×10^{-4}	5.41 %
LJ-TVR	3.74×10^{-4}	3.64 %	4.64×10^{-4}	4.96 %	5.27×10^{-4}	5.26 %

Analyzing the results of the two regularizations implemented in this work, it is worth noting that Tikhonov and total variation regularizations achieve similar identifications. However, since Tikhonov regularization attain better results for Damages 1 and 2, it seems to be more appropriated to our problem. The application of a priori information, concerning the roughness of the damage to be identified, allowed Tikhonov’s regularization to achieve a fairly satisfactory recovery of rough damages (Damage 2 and 3), as expected, but also of the smooth one (Damage 1).

CONCLUSIONS AND REMARKS

In this work, an improvement of the technique of damage identification in slender structures with noisy data by using the forward *modified sequential algebraic algorithm* (MSAA), proposed by Taminato et al., 2015, is presented.

Since the MSAA technique shows to be relevant to solve the direct problem when non-smooth generalized impedance profiles are concerned, the main goal in this work is to reduce the error propagation found when the impedance profile roughness is critical and the noise level in the input data is significant.

Two regularization schemes, namely the Tikhonov (TR) and Total Variation (TVR), were applied to minimize the mean squared error in the impedance profile recovery. The first one showed to be more appropriate.

In the numerical examples, three damaged scenarios were considered, with Damage 1 being a smooth impedance profile and Damages 2 and 3 being non-smooth ones. A null average noise with signal-to-noise ratio of 20 dB is added to the input signals. Table 1 shows that the technique using, in the sequence, LJ and TR methods, leads to the better results.

Compared with LJ only, for instance, Damage 3 (the roughest one) had mean squared error between actual and recovered impedance profile reduced by one order of magnitude (from 6.65×10^{-3} to 2.97×10^{-4} for the TR and to 5.26×10^{-4} for the TVR) and the maximum absolute error among the profiles reduced from 30.2% to 5.41% for the TR and to 5.26% to the TVR.

Another important achievement is the fact that the error propagation, noticeable in Fig. 4(c) that shows the identification using MSAA-LJ, is prevented by using the MSAA-LJ-TR and MSAA-LJ-TVR techniques, as shown in Fig. 3(c) and Fig. 4(c). It is worth noting, however, that the identification of Damages 1 and 2 does not take advantage of the proposed technique, since the MSA and the MAE of both MSAA-LJ and MSAA-LJ-TR and MSAA-LJ-TVR techniques are almost the same.

ACKNOWLEDGMENTS

The authors acknowledge the Higher Education Personnel Improvement Coordination, CAPES, the Brazilian National Council for Scientific and Technological Development, CNPq, and Rio de Janeiro’s Foundation for Research Support, FAPERJ, for their financial support to this research.

REFERENCES

- Doebling S., Farrar C., Prime M. and Shevitz D., 1996, “Damage Identification and Health Monitoring of Structural and Mechanical Systems from Changes in their Vibration Characteristics: A Literature Review”. Los Alamos National Laboratory, Los Alamos, 1–127.
- Gangadharan R., Mahapatra D. R., Gopalakrishnan S., Murthy C. R. L. and Bhat M. R., 2009, “On the sensitivity of elastic waves due to structural damages: Time-frequency based indexing method”, *Journal of Sound and Vibration* **320**, 915–941.
- Gordon P. and Sanday S. C., 1977, “Elastic waves in heterogeneous bars of varying cross-section”, *J. Franklin Inst.* **330**(2), 129–145.
- Grabowska J., Palacz M. and Krawczuka M., 2008, “Damage identification by wavelet analysis”, *Mechanical Systems and Signal Processing* **22**, 1623–1635.
- Luus R. and Jaakola T. H. I., 1973, “Optimization by direct search and systematic reduction of the size of search region”, *AIChE Journal* **19**, pp. 760–766.
- Manson G. and Barthorpe, R. J., 2010, “Advanced feature selection for simplified pattern recognition within the damage identification framework”, *Shock and Vibration* **17**, 589–599.
- Price K. V., Storn R. M. and Lampinen J. A., 2005, “Differential Evolution. A Practical Approach to Global Optimization”, Springer, Berlin, 37–130.

- Pierce A. D., 1991, "Acoustics: An Introduction to Its Physical Principles and Applications", Acoustical Society of America, Woodbury, 177–178.
- Rucevskis, S. and Wesolowski, M., 2010, "Identification of Damage in a Beam Structure by Using Mode Shape Curvature Squares", *Shock and Vibration* **17**, 601–610.
- Rudin L.I., Osher S. and Fatemi E., 1992, "Nonlinear total variation based noise removal algorithms", *J. Physica D.* **60**, pp 259–268.
- Silkorsky, C., 2000, "Development of a Health Monitoring system for Civil Structure Using a Level IV Non-Destructive Evaluation Method". In *Structural Health Monitoring*, Fu-Kuo Chang (Editor), Technomic Publishing Company, Inc, Lancaster, 68–81.
- Stephany S., Becceneri J.C., Souto R.P., de Campos Velho H.F. and Silva Neto A.J., 2009, "A pre-regularization scheme for the reconstruction of a spatial dependent scattering albedo using a hybrid ant colony optimization implementation", *Appl. Math. Modell.* **39**, pp 561–572.
- Storn R. and Price K., 1997, "Differential evolution: a simple and efficient heuristic for global optimization over continuous spaces", *J. Global Optim.* **11**, 341–359.
- Strong D. and Chan T., 2003, "Edge-preserving and scale-dependent properties of total variation regularization", *Inverse Problems.* **19**, pp 165–187.
- Taminato F. O., Tenenbaum R. A. and Fernandes K. M., 2015, "A modified sequential algebraic algorithm applied to identify rough acoustic impedance profiles in slender structures", *J. Acoust. Soc. Am.* **183**, 1585–1594.
- Tenenbaum R. A., Fernandes K. M., Stutz L. T. and Silva Neto A. J., 2012, "Damage identification in bars with a wave propagation approach and a hybrid optimization method", *Shock and Vibration* **19**, 301–321.
- Tenenbaum R. A., Stutz L. T. and Fernandes K. M., 2011, "Comparison of vibration and wave propagation approaches applied to assess damage influence on the behavior of Euler Bernoulli beams", *Computers & Structures* **89**, 1820–1828.
- Tenenbaum R. A. and Zindeluk M., 1992, "A fast algorithm to solve the inverse scattering problem in layered media with arbitrary input", *J. Acoustic. Soc. Am.* **92**(6), 3371–3378.
- Tenenbaum, R. A. and Zindeluk M., 1992, "An exact solution for the one-dimensional elastic wave equation in layered media", *J. Acoust. Soc. Am.* **92**(6), 3364–3370.
- Tikhonov A. N. and Arsenin V. Y., 1977, "Solutions of Ill-Posed Problems", Winston and Sons - Washington.
- Robinson E. and Treitel S., 1980, "Geophysical Signal Analysis", Prentice Hall, New York, 295–320.
- Wait J. R., Park G. H. and Farrar C. R., 2005, "Integrated structural health assessment using piezoelectric active sensors", *Shock and Vibration* **12**, 389–405.

RESPONSIBILITY NOTICE

The authors are the only responsible for the material included in this paper.