THE USE OF A PNEUMATIC INERTER FOR VIBRATION ISOLATION

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Abstract: This article presents the analysis of a pneumatic vibration isolator due to harmonic base excitation and shock base excitation. Vibration isolators have a broad range of applications and they are found on most machines and vehicles. This vibration isolator Inerter or inertial isolator consists of a two terminal fixing mechanism that reacts to acceleration. Inerter devices can be mechanical with levers, bars, beams, masses, rack with flywheel or pneumatic with tubes, chambers, and bellows filled with fluid. The main advantage of an Inerter is that it presents a region where transmissibility is very small, almost zero, after passing the resonance frequency. On the other hand, the disadvantage is that, the mechanism stabilizes in a continuous value of transmissibility at high frequencies. It is analyzed a device proposed by Goodwin, consisting of a flexible bellow without stiffness, which supports the body suspended to be isolated. The flexible bellow communicates with a rigid chamber through a rigid metal duct that is closed and fully filled with a gaseous fluid (air). Thus, the article presents the viscous damper behavior (mass-spring-damper) relative to the vibration isolator Inerter (mass-spring-damper-Inerter) for the harmonic response, numerical simulations of shock response, frequency spectrum, transmissibility and time histories. The method of Shock Response Spectra (SRS-maximax) considers the higher displacements of the system. It will be analyzed the influence of the pneumatic system parameters (metal ducts and flexible bellow dimensions) in the stiffness, damping, and inertance relative to frequency. The Inerter has shown efficiency in performing the vibration isolation in the regions after the resonance frequency. The analysis of the pneumatic parameters has shown the influence of each one on the system behavior.

Keywords: Pneumatic Inerter, Vibration Isolator, Shock Response

INTRODUCTION

Suspension and isolation mechanisms of vehicles, aircraft and industrial machinery have become complex systems to control undesired vibration and provide the required system performance. Some vehicles and aircraft use tires with a high profile. In these systems occurs vibration phenomena that is difficult to control. The spring effect of the high-profile tires is not damped and, it generates unwanted oscillations in the suspension system. The bounce effect (jump, rebound) can be observed during aircraft landings, when the tires touch the ground and, the landing gear successively hits the track.

According to Papageorgiou *et al.* (2009), the fact is that, the mass element in combination with the spring and damper is insufficient to carry all the mechanical impedance of a passive suspension system (Smith and Wang, 2002). The bounce and spring effects of high-profile tires may be avoided by using an inertial vibration isolator of Inerter type (Smith and Walker, 2002).

The vibration isolator Inerter or inertial isolator consists of a two terminal fixing mechanism that reacts to acceleration. A unit of mass (kilogram) represents the inertance parameter. The Inerter device can be mechanical with levers, bars, beams and masses or rack, and flywheel or pneumatic with tubes, chambers, and bellows filled with fluid. The main advantage of an Inerter is that it presents a region where transmissibility is very small, almost zero, after passing the resonance frequency. On the other hand, the disadvantage is that, the mechanism stabilizes in a continuous value of transmissibility at high frequencies. This device has smaller dimensions and weight than classical mass dampers, allowing a more efficient suspension assembly, light and compact, (Smith and Wang, 2004). The Inerter can be configured according to the tire natural frequencies and resonances, reducing the vibrations acting on the vehicle. According to Chen *et al.* (2009), the electrical analog of the Inerter is the inductor (Mathematical Ideal). It will be analyzed the mechanism proposed by Goodwin consisting of a pneumatic Inerter. Thus, the article presents the viscous damper behavior (mass-spring-damper) relative to the vibration isolator Inerter (mass-spring-damper-Inerter) for the harmonic response, numerical simulations of shock response, frequency spectrum, transmissibility and time histories. The method of Shock Response Spectra (SRS-maximax) considers the higher displacements of the system. It will be analyzed the influence of the pneumatic system parameters (metal ducts and flexible bellow dimensions) in the stiffness, damping, and inertance relative to frequency. The Inerter has shown efficiency in performing the vibration isolation in the regions after the resonance frequency. The analysis of the pneumatic parameters has shown the influence of each one on the system behavior.

FREQUENCY RESPONSE

Analytical development

The analyzed model has a mass m suspend by an elastic element with stiffness coefficient k and damping coefficient c. Figure 1(a) shows the mechanism proposed by Goodwin (1965), consisting of a flexible bellow with the area base A_0 without stiffness, supporting the suspended body to be isolated. The flexible bellow communicates with a rigid chamber through a rigid metal duct length (L_1 and area A_1). The mechanism is composed of the flexible bellows, pipes and a rigid chamber, being a closed system and fully filled with a gaseous fluid. Every fluid has a specific property, in this modeling, the fluid stored inside the pneumatic system has a specific density ρ, the volumetric module B, and the fluid viscosity coefficient μ. Figure 1(b) shows the classical model, mass-spring-damper with an Inerter element model similar to Goodwin's (1965) but with discrete elements.
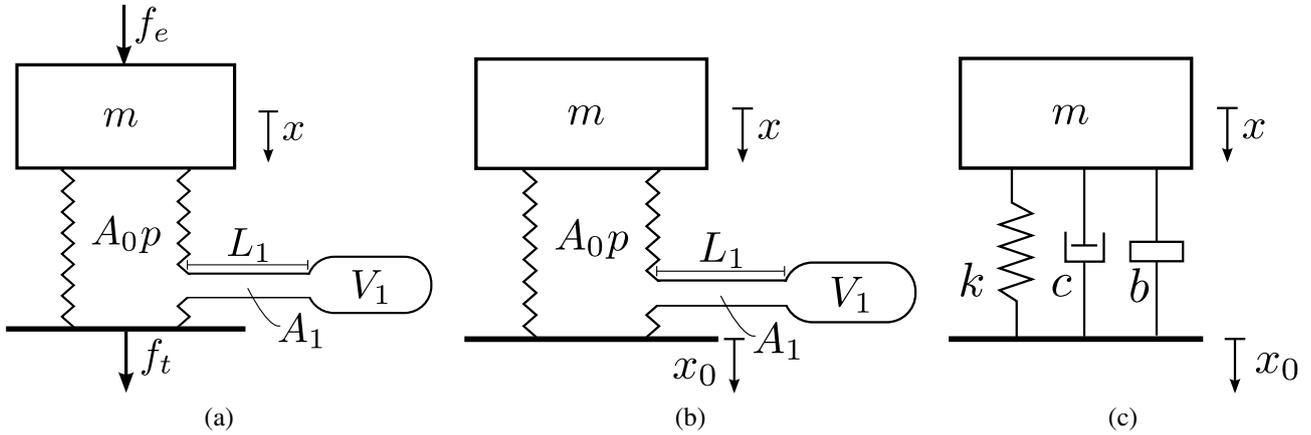
Figure 1 – System of one degree of freedom (a) Goodwin Isolator (force transmissibility), (b) Goodwin Isolator (Displacement Transmissibility) and (c) representation using elements of stiffness, damping and inertance

The mechanism studied by Goodwin (1965) consisted of the analysis of force transmissibility, which can be similarly analyzed by the model shown in 1(a). Assuming the external force excitation of the type $f_e = F_0 e^{j\omega t}$, the balance of forces on the mass can be written as Eq. (1):

$$m\ddot{x} = F_0 e^{\omega t} - A_0 p \tag{1}$$

Considering that, A_0 is the area of the bellows, p is the fluid pressure inside the bellows and L_1 is the length of the duct, the balance of the forces can be written as Eq. (2):

$$\rho A_1 L_1 \frac{A_0}{A_1} \ddot{x} = A_0 p - 8\pi\mu L_1 \frac{A_0}{A_1} \dot{x} - \frac{A_0}{V_1} B A_1 x \tag{2}$$

It is convenient to rewrite the Eq. (2) multiplying it by the ratio of the areas A_0/A_1, so we have:

$$\rho A_1 L_1 \left(\frac{A_0}{A_1}\right)^2 \ddot{x} = A_0 p - 8\pi\mu L_1 \left(\frac{A_0}{A_1}\right)^2 \dot{x} - \frac{A_0^2 B}{V_1} x \tag{3}$$

Eq. (3) can be simplified to:

$$A_0 p = b\ddot{x} + c\dot{x} + kx \tag{4}$$

Where, $b = \rho A_1 L_1 \left(\frac{A_0}{A_1}\right)^2$, $k = \frac{A_0^2 B}{V_1}$ and $c = 8\pi\mu L_1 \left(\frac{A_0}{A_1}\right)^2$. Substituting eq. (4) in eq. (1), results

$$(m+b)\ddot{x} = -c\dot{x} - kx + F_0 e^{\omega t} \tag{5}$$

If harmonic motion of the type $x(t) = X e^{j\omega t}$ is assume, then it is possible to write the force transmissibility as a function of the excitation frequency

$$\frac{F_t}{F} = \frac{-\omega^2 b + j\omega c + k}{-\omega^2 (m+b) + j\omega c + k} \tag{6}$$

This equation represents the force transmissibility of the system shown in figure 1(a) and it is equivalent to the transmissibility of the system under base excitation shown in figure 1(b), such that

$$\frac{X}{X_0} = \frac{-\omega^2 b + j\omega c + k}{-\omega^2 (m+b) + j\omega c + k} \tag{7}$$

The system in equation one can be written in non-dimensional form, adopting the mass ratio $\mu = b/m$, the damping factor $\xi = c/2m\omega_n$, where $\omega_n = \sqrt{k/m}$ and the frequency ratio $\Omega = \omega/\omega_n$

$$T = \frac{-\Omega^2\mu + 2j\xi\Omega + 1}{-\Omega^2(1+\mu) + 2j\xi\Omega + 1} \tag{8}$$

Comparison of the system with and without Inerter

The transmissibility of the traditional spring-mass isolator is compared to the transmissibility of the spring-mass-inerter isolator in figure 2. In this case damping has been neglected such some characteristic frequency points are identified.
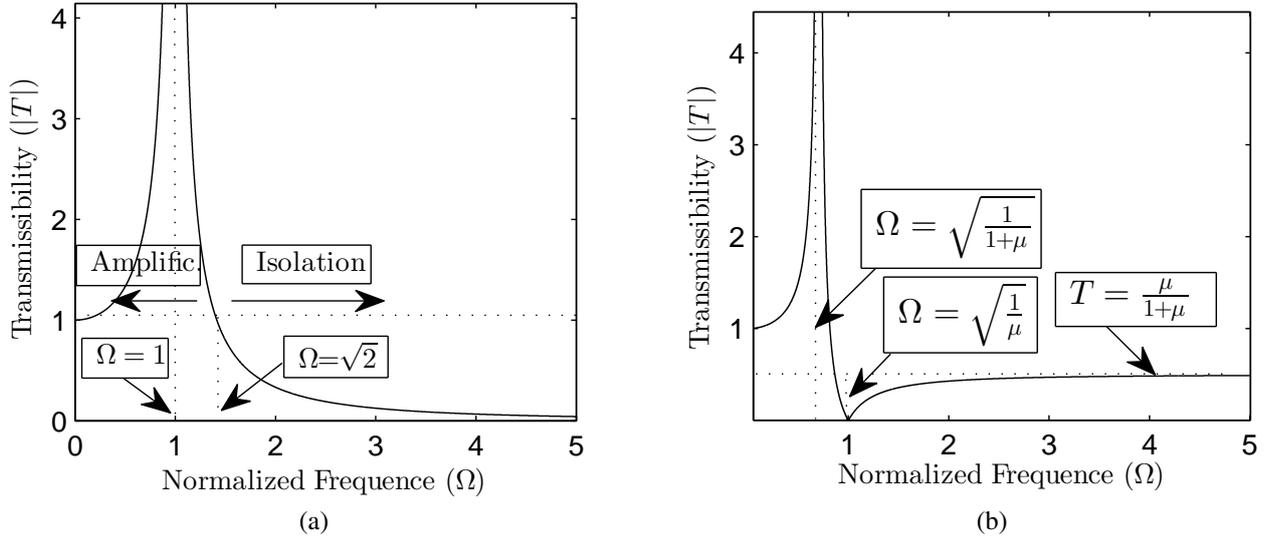
Figure 2 – Transmissibity as a function of the excitation frequeny, (a) classical spring-mass, (b) spring-mass-inerter

It is well known that the maximum peak of transmissibility for the spring-mass isolator occurs at its natural frequency ω_n. For the case of the spring-mass-inerter, this occurs at frequency $\omega = \sqrt{k/(m+b)}$. The interesting feature of the system with the inerter element, occurs at frequency $\omega = \sqrt{k/b}$, known as notch frequency. this occurs because the inerter adds a zero in the transmissibility function shown in equation 7.

In the classical model the isolation occurs from the excitation frequency $\omega = \sqrt{2}\omega_n$. From that point, the transmissibility decreases with the increasing excitation frequency, tending to zero when the frequency of excitation tends to infinity Smith (2002).

When the model with Inerter is analyzed at high frequencies, $\lim T(\omega)_{\omega\to\infty}$, it has the equation of transmissibility written as eq. 9.

$$T = \frac{\mu}{1+\mu} \tag{9}$$

Figure 2(b) shows the main characteristic of the Inerter: the anti-resonance. The anti-resonance decreases the value of the transmissibility of the system at very low values near zero. After this point, the transmissibility increases again and tend to the value $T = \frac{\mu}{1+\mu}$. The transmissibility of the system with inerter is lower than the classical system in the anti-resonance and higher than the classical system to high-frequency excitation.

Figure 3(a) shows the influence of the mass ratio μ in relation to the natural frequencies of the system.

This analysis used an harmonic excitation force, $x_0 = Xe^{j\omega t}$. According to the increase of the mass ratio, occurred the reduction of the natural frequency, ω_n, of the system.

Figure 3(b) shows the system frequency response when the inertance of the system are varied. The ordinate represents the displacement of the sprung mass, the abscissa shows the normalized frequency.

The arrows indicate the trend of the curves according to the increase of the inertance; each curve has a value of inertance ($\mu = 0,10 : 0,05 : 0,25$).
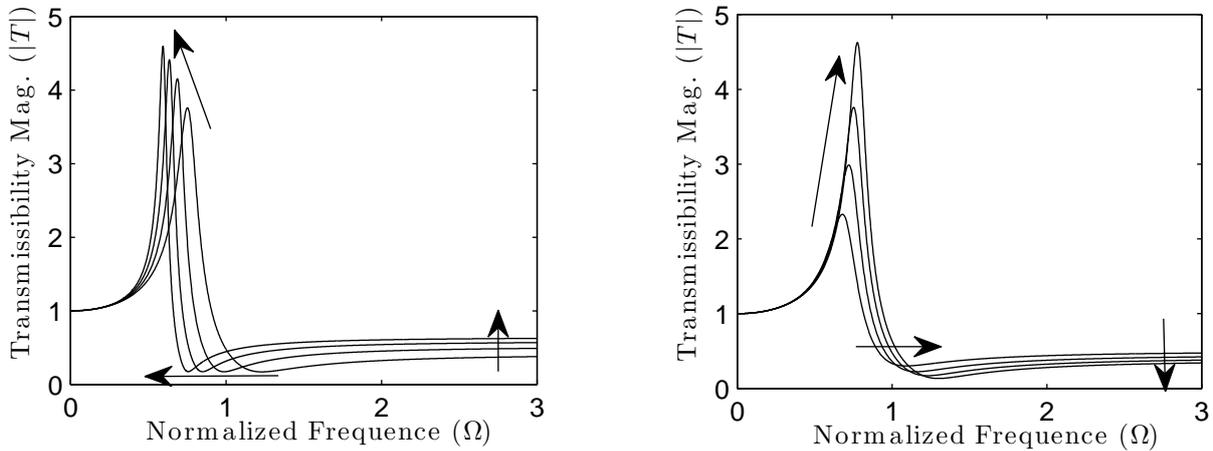
PARAMETERS VARIATION

In this section, parametric analysis of the the parameters A_0, A_1, L_1 and ρ is performed to check how these parameters change the transmissibility, the natural frequency and the anti-resonance of the system in function of the frequency ratio.

In Figure 4(a) , the parameter ρ specific density of the fluid was varied. The resonance frequency decreased whereas the transmissibility at that point increased. The anti-resonance frequency decreased and the transmissibility value remained constant. At high frequencies, the values of Transmissibility increased.
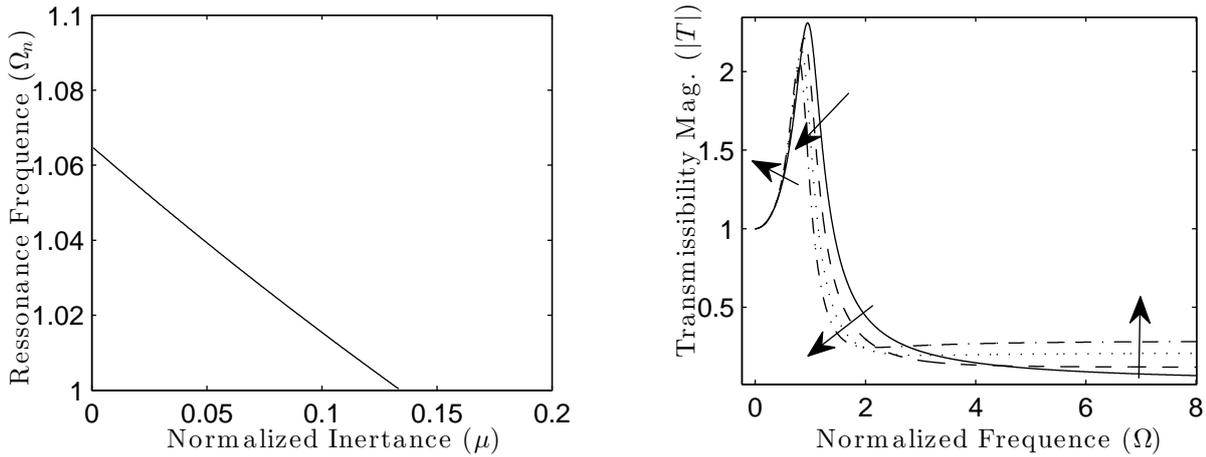
Figure 3 – Frequency response with harmonic base excitation (a) Natural frequency in function of inertance (b) Displacement in function of inertance and in function of excitation frequency

In Figure 4(b), the parameter A_1 area of the rigid duct was varied. The resonance frequency and the transmissibility increased at that point. The anti-resonance frequency increased and the transmissibility value decreased. At high frequencies, the values of Transmissibility decreased.

Figure 4 – Crescent parameter variation (a)parameter ρ (b) parameter A_1

In Figure 5(a), the parameter L_1 the length of the rigid duct was varied. In Figure 5(b), the parameter A_0 the base area of the flexible bellows was varied. In these two cases, the system behaved similarly. The resonance frequency and the transmissibility decreased at that point increased. The anti-resonance frequency decreased and the transmissibility value increased. At high frequencies, the values of transmissibility increased.
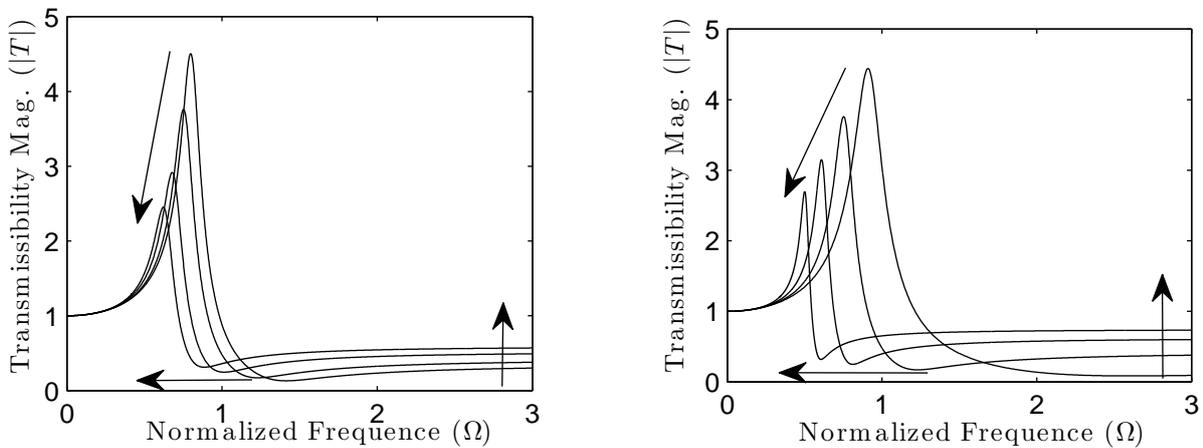
Figure 5 – Crescent parameter variation (a) parameter L_1 (b) parameter A_0

SHOCK RESPONSE ANALYSIS

For obtaining the shock response, the base of the model is excited with a half sine function, Eq. (10), called x_0, where t is the time variable, T is the period, A is the amplitude, ω is the angular frequency and x_0 is the displacement of the excitation eq.10.:

$$\begin{cases} x_0 = A\sin(\omega t), \text{ if } 0 \leq t \leq T/2 \\ x_0 = 0, \text{ otherwise} \end{cases} \tag{10}$$

The function of base displacement is represented by x_0. The model can present the effect of the jump or not, according to the considered frequency excitation.

For performing the analysis of the shock response, the parameters were adopted in a way that the system produced the sub-damped behavior Taskin *et al.* (2013).

Figure 6(a) shows the frequency spectrum of the classical model and the model with Inerter. The method used is the Shock Response Spectra (SRS-Maximax), where, it considers the absolute maximum displacement of each excitation frequency in the analysis range. The Shock Response is composed of two parts: one transient and the other of free vibration. In the interval of normalized frequency between $4 \leq \Omega \leq 15$, the model with Inerter showed lower displacement amplitude than the classic model.

Figure 6(b) shows the Time History from both systems, classical and with Inerter at the normalized frequency ($\Omega = 6,0$) and the inertance ($\mu = 0,20$).
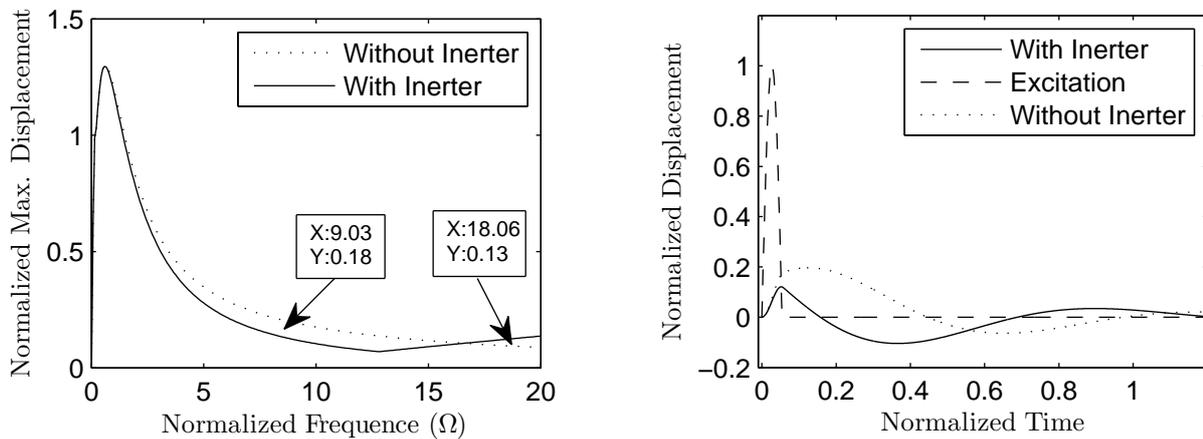
Figure 6 – (a) Maximum displacement amplitude (SRS) and (b) Time history ($\Omega = 6,0$; $\mu = 0,20$)

Figure 7(a) shows the Time History in the range that the model with Inerter presents smaller displacement amplitude than the classical model ($\Omega = 9,06$).

Figure 7(b) shows the Time History in the range that the classical model presents smaller displacement amplitude than the model with Inerter ($\Omega = 19,02$).
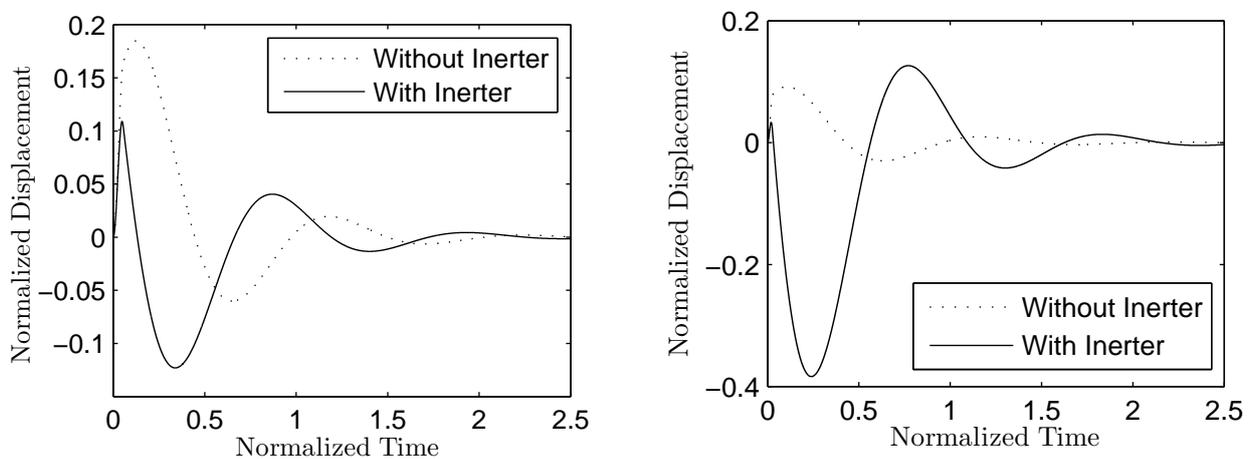
Figure 7 – Time History(a)Isolation ($\Omega = 9,06$) (b) Amplification ($\Omega = 18,06$)

CONCLUSIONS

The article presents the viscous damper behavior (mass-spring-damper) relative to the vibration isolator with Inerter (mass-spring-damper-Inerter) for the harmonic response. The model with Inerter presented a region where the transmissibility is very small, almost zero, after passing the resonance frequency. The viscous damper does not have this region.

The variation of A_0, A_1, L_1 and ρ of the pneumatic Inerter allows to visualize how the influence of them occur on the transmissibility and the points which the natural frequency and anti-resonance occur, according to the excitation frequency. The parameter A_0 alters the system's inertance, stiffness and damping. The parameters A_1 and L_1 change the inertance and damping of the system. The parameter ρ alters the system's inertance.

The increase of A_0 and L_1 has significantly decreased the transmissibility magnitude of the resonance frequency of the system unlike the parameters A_1 and ρ. Therefore, the best vibration isolation occurs when the coefficients of A_0 and L_1 are as large as possible, as well as the coefficients of A_1 and ρ are as small as possible.

The frequency spectrum of the comparison between the classical model and the model with Inerter (SRS-Maximax method), presented that in the interval of normalized frequency between $4 \leq \Omega \leq 15$ the model with Inerter showed lower displacement amplitude than the classic model.

Moreover, the Inerter is a mechanism that allows the system to be assembled with different settings, such as, in series, in parallel with the suspension elements (spring, damper, and other suspensions). The system analyzed is one of the numerous possible configurations to be performed.

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