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Influence of Fuselage and Rotor Damping and Robustness Analysis of a Dynamic Vibration Absorber Applied to a Helicopter

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This paper presents the evaluation of a spring-mass-damping vibration system (SMDVS) attached to the fuselage of a helicopter with hinged blades. The dynamic behavior of the system is altered by the SMDVS, which consequently changes the stability characteristics of the aircraft when it is on the ground, i.e.: ground resonance instabilities. The SMDVS has potential use in helping alleviate the exponential growth of the aircraft dynamic response at the unstable regions if considered as a dynamic vibration absorber (DVA). The system is considered stable if its dynamic behavior has a negative exponential growth. The DVA parameters (mass, stiffness, and damping) are of great relevance for the system stability (helicopter+DVA), and can contribute to the mitigation of ground resonance instabilities. The helicopter parameters, specially the rotor and fuselage damping, can help mitigate the instabilities. The first purpose of this work is to use Coleman's Method to assess the influence of fuselage and rotor damping on ground resonance instabilities. The stability of the system is evaluated with different combinations of fuselage and rotor damping for two different helicopters: a helicopter without DVA and a helicopter with DVA. The results showed that in order to vanish the instabilities, the system must have a proper combination of damping. The second purpose of this work is to introduce uncertainties in the dynamic system related to hinge stiffness of one rotor blade and to use the μ -analysis to predict the smallest perturbations that make the system unstable for the two helicopter types. The results showed that for both helicopters, the proper combination of fuselage and rotor damping can produce a system so robust that it can lose one of its rotor blade stiffness and still remain stable. Also, the analysis showed that adding the DVA to the system does not change its robustness.

Keywords: Coleman's Method, helicopter ground resonance, dynamic vibration absorber, μ -analysis, robustness analysis.

INTRODUCTION

The ground resonance is a phenomenon that can occur in helicopters with hinged and hingeless blades and it happens when the fuselage oscillations excite one or more rotor modes, which creates a wobble of the rotor center of gravity. This makes the rotor effective mass couples with the vibration of the fuselage (fuselage mode), which in turn excite the rotor modes. The continuation of this process increase the displacements the rotor and the fuselage experience, leading to the aircraft destruction (Ganiev and Pavlov, 1973; Johnson, 1994). The earliest work on ground resonance was done by Coleman and Feingold (1957). Also, a study done by Wang and Chopra (1992) showed that when a least stable mode is improved due to blade dissimilarity, the other modes become less stable.

When it comes to analyzing the helicopter ground resonance instabilities, it is important to study all the relevant parameters of the system and their effects on the overall stability. The dynamic vibration absorber (DVA) parameters (mass, stiffness, and damping) are of great relevance to the development of the instabilities and can contribute to their mitigation. Nonetheless, the helicopter parameters also play a big role in this mitigation and are worth to be thoroughly studied. The two major parameters are the helicopter fuselage and rotor blades damping, especially their combination. The partial objective of this work is to assess the influence of fuselage and rotor damping in the ground resonance instabilities for a helicopter with DVA e a helicopter without DVA.

Dynamic systems are susceptible to failure and aging of its components that may appear randomly, compromising its nominal operation, which can lead to damage of the structure (Oliveira *et al*., 2015). Therefore it is important to perform a robustness analysis to assess how the system handles those effects. The final partial objective is to evaluate the influence of fuselage and rotor blade damping on instabilities, as well as to perform a robustness analysis in the system with DVA and compare it with the helicopter without DVA.

MECHANICAL MODEL

The helicopter studied in this work, shown in Fig. 1, has been simplified according to the model used in Sanches (2011) and Sanches *et al.* (2011,2012) and is similar to the one used in Oliveira *et al.* (2014).

The fuselage, modeled as a rigid body with mass m_f, has one translational movement $x(t)$ along its lateral direction. The spring attached along the x direction has stiffness K_x and the damping, modeled as a viscous damper, has coefficient C_x. The helicopter at its equilibrium position has the center of mass of the fuselage (point O) coincident with the origin of the inertial reference frame (X_o, Y_o, Z_o).

The rotor system comprises one rigid rotor hub and an assembly of $N_b = 4$ blades and it revolves at a speed Ω. The blades have mass m_b and moment of inertia I_{zb} around the z-axis (located at its center of mass). The blades have stiffness K_{bk} and damping coefficient C_{bk} at point B, which is where the rotor head and the fuselage are joined by a rigid shaft.

The radius of gyration is defined by the length b and a is defined as the rotor eccentricity. For each kth blade there is an in-plane lead-lag motion defined by $\varphi_k(t)$ and an azimuth angle defined as $\beta_k = 2\pi(k-1)/N_b$ with respect to the x-axis. The origin of the rotational reference frame (x, y, z) is parallel to the inertial one and is located at the geometric center of the rotor hub (coincident at point O). Aerodynamic forces on the blades are not taken into account. Such an assumption is quite realistic since the helicopter is on the ground and the vibration induced in ground resonance phenomenon has greater effects than aerodynamic forces generated by the rotor.

The spring-mass-damping vibration system has mass m_a, stiffness K_a and damping coefficient C_a. Note that it translates $x_a(t)$ with respect to the inertial frame. For each blade, its position in the inertial frame is given as:

$$x_{bk}(t) = a\,cos(\psi_k) + b\,cos(\psi_k + \varphi_k(t)) + x(t) \tag{1}$$

$$y_{bk}(t) = a\,sin(\psi_k) + b\,sin(\psi_k + \varphi_k(t)) + x(t) \tag{2}$$

where $\psi_k(t) = \Omega t + \beta_k$.

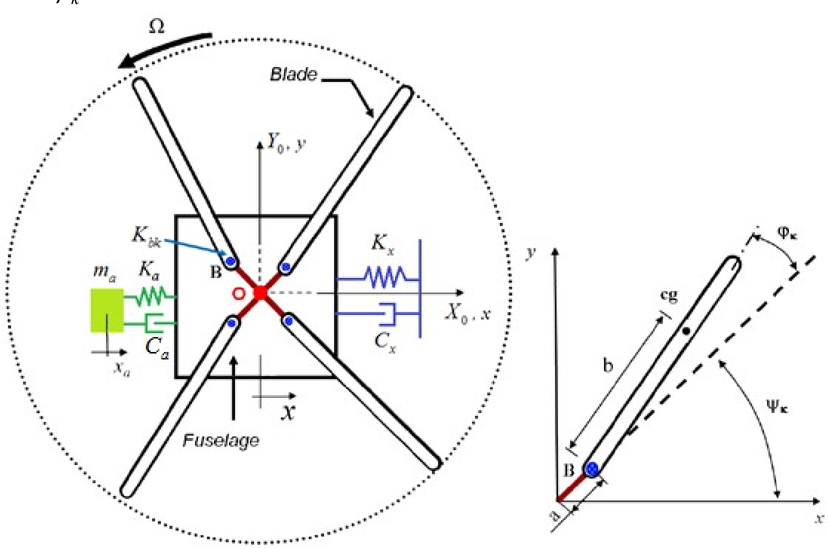


Figure 1 - Sketch of the mechanical model: (a) general view and (b) blade view

Equations of Motion

The mechanical system equations of motion are obtained by applying Lagrange's equations of the kinetic (T) and potential V energies and the virtual work (δW) of the external forces and moments applied to the system. Sanches (2011) derived the energies expressions for the helicopter with hinged blades. The ones used in this work are slightly different for additional terms must be considered concerning the effects of the spring-mass-damping vibration system (SMDVS). The kinetic energy (T_a), potential energy (V_a) and virtual work of the SMDVS are given as:

$$T_a = \frac{1}{2} m_a \left(\dot{x}_a^2(t) \right) \tag{3}$$

$$V_a = \frac{1}{2} K_a \left[x(t) - x_a(t) \right]^2 \tag{4}$$

$$\delta W_a = -C_a \left(\dot{x}(t) - \dot{x}_a(t) \right) \delta x + C_a \left(\dot{x}(t) - \dot{x}_a(t) \right) \delta x_a \tag{5}$$

The equations of motion in the matrix form are expressed as:

$$\mathbf{M}(t)\ddot{\mathbf{u}}(t)+\mathbf{G}(t)\dot{\mathbf{u}}(t)+\mathbf{K}(t)\mathbf{u}(t)=\mathbf{F_{ext}}(t) \tag{6}$$

where $\mathbf{u}(t)=\{x(t) \quad x_a(t) \quad \varphi_1(t) \quad \varphi_2(t) \quad \varphi_3(t) \quad \varphi_4(t)\}^T$ corresponds to the degrees of freedom of the system.

The matrices $\mathbf{M}$, $\mathbf{G}$, and $\mathbf{K}$ correspond to the mass, damping, and stiffness matrix respectively. $\mathbf{F_{ext}}$ is equal to zero since all blades possess the same inertial and geometrical properties. The matrices have periodic terms, as shown by Eq. (7), (8), and (9).

$$\mathbf{M}(t)=\begin{bmatrix} 1 & 0 & -r_{m1}\sin(\psi_1) & -r_{m2}\sin(\psi_2) & -r_{m3}\sin(\psi_3) & -r_{m4}\sin(\psi_4) \\ 0 & 1 & 0 & 0 & 0 & 0 \\ -r_{b1}\sin(\psi_1) & 0 & 1 & 0 & 0 & 0 \\ -r_{b2}\sin(\psi_2) & 0 & 0 & 1 & 0 & 0 \\ -r_{b3}\sin(\psi_3) & 0 & 0 & 0 & 1 & 0 \\ -r_{b4}\sin(\psi_4) & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{7}$$

$$\mathbf{G}(t)=\begin{bmatrix} r_{ca}r_{mdva}+r_{cf} & -r_{ca}r_{mdva} & -2\Omega r_{m1}\cos(\psi_1) & -2\Omega r_{m2}\cos(\psi_2) & -2\Omega r_{m3}\cos(\psi_3) & -2\Omega r_{m4}\cos(\psi_4) \\ -r_{ca} & r_{ca} & 0 & 0 & 0 & 0 \\ 0 & 0 & r_{cb1} & 0 & 0 & 0 \\ 0 & 0 & 0 & r_{cb2} & 0 & 0 \\ 0 & 0 & 0 & 0 & r_{cb3} & 0 \\ 0 & 0 & 0 & 0 & 0 & r_{cb4} \end{bmatrix} \tag{8}$$

$$\mathbf{K}(t)=\begin{bmatrix} \omega_a^2 r_{mdva}+\omega_x^2 & -\omega_a^2 r_{mdva} & \Omega r_{m1}\sin(\psi_1) & \Omega r_{m2}\sin(\psi_2) & \Omega r_{m3}\sin(\psi_3) & \Omega r_{m4}\sin(\psi_4) \\ -\omega_a^2 & \omega_a^2 & 0 & 0 & 0 & 0 \\ 0 & 0 & \Omega^2 r_{a1}^2+\omega_{b1}^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & \Omega^2 r_{a2}^2+\omega_{b2}^2 & 0 & 0 \\ 0 & 0 & 0 & 0 & \Omega^2 r_{a3}^2+\omega_{b3}^2 & 0 \\ 0 & 0 & 0 & 0 & 0 & \Omega^2 r_{a4}^2+\omega_{b4}^2 \end{bmatrix} \tag{9}$$

$$\mathbf{F_{ext}}(t)=\{0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0\}^T \tag{10}$$

where for $k=1,...,4$:

$$r_{mk}=\frac{bm_{bk}}{m_f+\sum_{i=1}^{N_b}m_{bi}} \qquad r_{mdva}=\frac{m_a}{m_f+\sum_{i=1}^{N_b}m_{bi}}$$

$$r_{bk}=\frac{bm_{bk}}{b^2m_{bk}+I_{zb_k}} \qquad r_{ak}^2=ar_{bk}$$

$$r_{cf}=\frac{C_x}{m_f+\sum_{i=1}^{N_b}m_{bi}} \qquad r_{cbk}=\frac{C_{bk}}{b^2m_{bk}+I_{zb_k}}$$

$$r_{ca}=\frac{C_a}{m_a} \qquad \omega_a^2=\frac{K_a}{m_a}$$

$$\omega_x^2=\frac{K_x}{m_f+\sum_{i=1}^{N_b}m_{bi}} \qquad \omega_{bk}^2=\frac{K_{bk}}{b^2m_{bk}+I_{zb_k}}$$

Helicopter Numerical Data

In this work two helicopters are to be considered: one helicopter with no DVA (Helicopter Type 1) and one helicopter with DVA (Helicopter Type 2). The fuselage and rotor properties are the same for both types. A study on the influence of a DVA attached to the fuselage of a helicopter is presented in Sanches *et al.* (2014). The DVA attached in Helicopter Type 2 has its parameters defined with a similar optimization technique described also in Sanches *et al.* (2014) to find DVA mass m_a and natural frequency ω_a, which defines the design vector. The damping coefficient C_a is determined considering the optimal damping ζ_{opt} obtained by Rade and Steffen (2011). Thus:

$$C_a = 2\zeta_a m_a \omega_a \qquad (11)$$

$$\left(\zeta_a\right)_{opt} = \sqrt{\frac{3\mu}{8\left(1+\mu\right)^3}}$$

where $\mu = m_a / m_f$

The searched optimal values of m_a and ω_a (design vector) are determined from the analysis of the Pareto distribution. The objective functions were the exponential growth of the dynamical system and the DVA mass, since high mass values are not beneficial for aeronautical applications. Since the process is used to find the DVA mass m_a and the damping depend on it, at each step in the process the damping C_a is computed. Table 1 shows numerical data for Helicopter Type 1 and Tab. 2 shows numerical data for Helicopter Type 2 after optimization process.

Table 1 - Numerical Data for Helicopter Type 1.

Fuselage	Rotor		
$m_f = 2902.9$ kg	$m_{bk} = 31.9$ kg	$a = 0.2$ m	$b = 2.5$ m
$\omega_x = 6.0\pi$ rad/s	$\omega_{bk} = 3.0\pi$ rad/s	$I_{zbk} = 259$ kg.m^2	
$C_x = 2284.9$ Ns/m	$C_{bk} = 172.8$ Nms/rad		

Table 2 - Numerical Data for Helicopter Type 2.

Fuselage	Rotor			DVA
$m_f = 2902.9$ kg	$m_{bk} = 31.9$ kg	$a = 0.2$ m	$b = 2.5$ m	$m_a = 16.1$ kg
$\omega_x = 6.0\pi$ rad/s	$\omega_{bk} = 3.0\pi$ rad/s	$I_{zbk} = 259$ kg.m^2		$\omega_a = 6.02\pi$ rad/s
$C_x = 2284.9$ Ns/m	$C_{bk} = 172.8$ Nms/rad			$C_a = 22.67$ Ns/m

INFLUENCE OF FUSELAGE AND ROTOR DAMPING

According to the literature (Coleman and Feingold, 1957; Wang and Chopra, 1992; Sanches *et al.,* 2012), the ground resonance phenomenon is characterized by the dynamic coupling of the poorly damped cyclic rotor modes with that from the fuselage. The fact of adding a SMDVS in the fuselage changes its modal characteristics. The fuselage consists, afterwards, into a dynamic system represented by two lumped mass/spring/damping elements, having two resonance frequencies and two modes of vibration.

In order to assess the influence of fuselage and rotor damping on ground resonance instabilities, Coleman's Transformation is used. It introduces modal coordinates of an isotropic rotor (i.e., all the blades have the same properties) in the equations of motion which were written considering the blades displacement on the rotor rotating frame. For instance,

$$\eta = -\left(\frac{2}{N_b}\right)\sum_{k=1}^{N_b}\varphi_k \sin\left(\psi_k\right) \qquad (12)$$

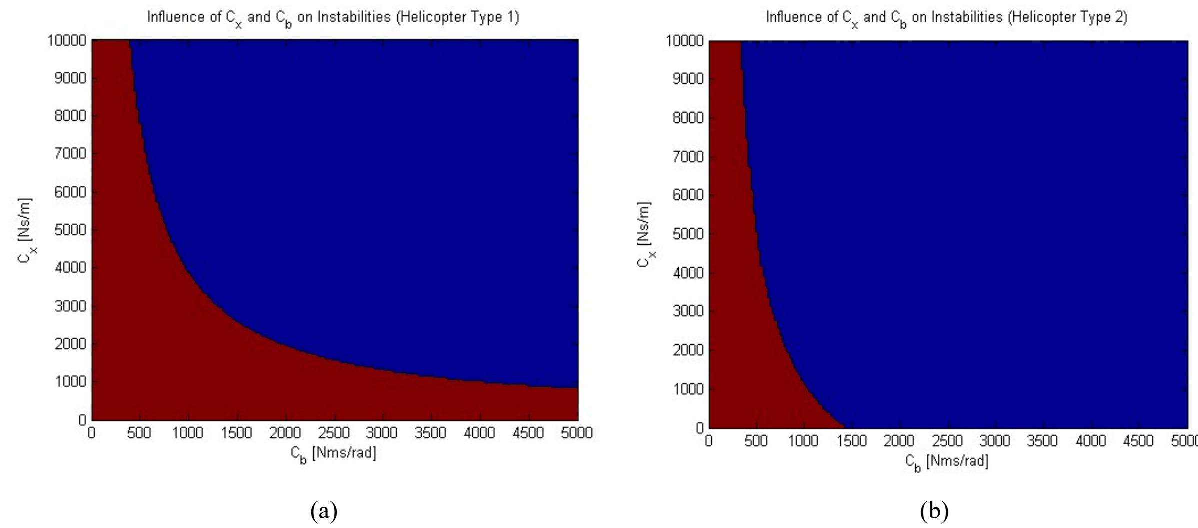
$$\theta = -\left(\frac{2}{N_b}\right)\sum_{k=1}^{N_b}\varphi_k \cos\left(\psi_k\right)$$

and they correspond to the sine and cosine components, respectively, of a four blade rotor mode shapes. These components correspond to the lateral and longitudinal shifting of the rotor center of mass and are called cyclic modes (Sanches, 2011) and their interaction with fuselage modes leads to unstable oscillations. By applying Coleman's Transformation to Eq.(6), one may have the following equations of motion:

$$\mathbf{M_C}\ddot{\mathbf{z}}\left(\mathbf{t}\right)+\mathbf{G_C}\dot{\mathbf{z}}\left(\mathbf{t}\right)+\mathbf{K_C}\mathbf{z}\left(\mathbf{t}\right)=\mathbf{0_{5x1}} \qquad (13)$$

in which the mass, damping, and stiffness matrices are not time dependent. The dynamic system described in Eq.(13) is stable if the real part of all the system eigenvalues ρ are negative for all revolving speeds Ω.

The fuselage and rotor blades damping effect is to mitigate the vibrations of the system. However, the instabilities of the helicopter can vanish, resulting in a system with no positive real part in its eigenvalues, depending on the damping values used for the simulation. Figure 2 shows the stability analysis for different combinations of C_x and C_b for the helicopter without DVA (Helicopter Type 1) and for helicopter with DVA (Helicopter Type 2). The analysis is done for the rotor speed comprised between $0 \le \Omega \le 10$ Hz. In this simulation, the values for C_x were varied from 0 to 10000 Ns/m with 220 points, while the values for C_b were varied from 0 to 5000 Nms/rad with 200 points.

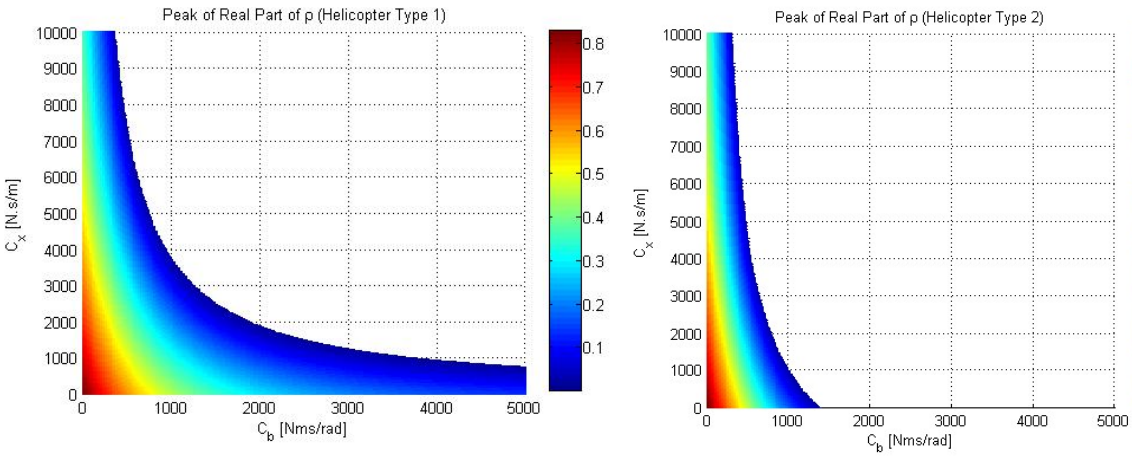


Legend: Blue = Stable Region; Red = Unstable Region

Figure 2 - Influence of Fuselage and Rotor Damping on Instabilities: (a) Helicopter Type 1; (b) Helicopter Type 2

It can be seen from Fig. 2 that the combination of fuselage and rotor blade damping have direct influence on the system instability. The blue region represents the stable portion, while the red part represents the unstable one. It can be seen that the behavior of those regions are deeply different between both helicopters, especially with respect to the values for C_b. The stable region for Helicopter Type 2 is achieved with a smaller value for C_b, with its maximum being at $C_b = 1407$ Nms/rad when $C_x = 0$, which is greater than the one for the Helicopter Type 1. This means that the DVA is good for the overall system stability, and by applying it to the fuselage, the helicopter can make use of smaller values for damping, which makes the rotor system more compact and less complex. Also, one can see that the stability of the system is more sensitive to variations of C_b, since with small values of that damping even if the values for C_x are high, the system is still unstable.

Another important analysis is concerning the maximum value (peak) of the real part of the eigenvalues, since it plays a big role on the ground resonance phenomenon. The smaller the peak, the smaller is the exponential growth of the helicopter dynamic time response. As shown in Fig. 3, the critical portion, i.e., the highest peaks of real part are present in the region which combines small values for both fuselage and rotor blade damping. Although for Helicopter Type 2 the values for peaks are smaller, both helicopters exhibit the same behavior.



(a) (b)

Figure 3 - Peak of Real Part of Eigenvalue: (a) Helicopter Type 1; (b) Helicopter Type 2

STABILITY ROBUSTNESS ANALYSIS

Dynamic systems are susceptible to variations, which can be caused by failure and aging of its components that may appear randomly, and may compromise its nominal operation. In the case of the helicopter system, the blade stiffness may have variations in its natural frequency, which can change the system (helicopter+DVA) dynamics in a way that it may be catastrophic. This topic aims in analyzing the robustness of the entire system, by changing the properties of one rotor blade, specially the differences that may appear between Helicopter Type 1 (without DVA) and Helicopter Type 2 (with DVA damping).

General Background

As discussed before, the simplifications made by Coleman consider all blades with similar mechanical properties, therefore it is no longer valid for the analysis in this section and Floquet theory is used instead. However, predicting the ground resonance for a wide range of dissimilar blade configurations means analyzing each point individually on a grid of parametric space generating (Sanches *et al.*, 2013), in which the Floquet theory results in high computational costs.

The stability of Linear Time Invariant (LTI) systems with parametric uncertainties have been analyzed by using the standard μ-analysis method (Balas *et al.*, 2013). The helicopter model, Eq.(6) is a continuous Linear Time Periodic (LTP) dynamic system. In Kalender *et al.* (2008), the robustness analysis of LTP systems is considered by transforming it into a LTI system and applying μ-analysis. In order to apply those methods, one need first to cast the LTP system into the Linear Fractional Transformation (LFT) form. In order to do so, the lifting procedure is applied to provide a LFT, which leads to uncertainty structures with highly-repeated parameters. In order to perform the parametric robustness analysis, one must consider the following LTP system $\delta(\Delta)$ (Oliveira *et al.*, 2015):

$$
\begin{array}{l}
r(t) \rightarrow \boxed{\begin{array}{l}\dot{w}(t) = \mathbf{A}(t)w(t) + \mathbf{B}(t)r(t) \\ y(t) = \mathbf{C}(t)w(t) + \mathbf{D}(t)r(t)\end{array}} \rightarrow y(t) \\
r(t) \leftarrow \boxed{r(t) = \Delta y(t)} \leftarrow y(t)
\end{array}
\tag{14}
$$

where $w(t) \in \mathbb{R}^n$ are the state variables, and $r(t)$ and $y(t)$ are inputs and output, respectively, of the LFT system.

The diagonal matrix Δ comprises bounded real unknown parameters which represent the uncertainties:

$$
\Delta = \text{diag}\left[\delta_1, \delta_2, \ldots, \delta_k\right] \tag{15}
$$

The system must be nominally stable when $\Delta = 0$. The primary purpose of the parametric robustness analysis is to find the smallest uncertainties δ_k which destabilize the closed-loop system $\delta(\Delta)$. Floquet's method can be applied to perform this analysis, it is very CPU time-consuming, which is why the μ-analysis, a much faster technique, is used on the LFT model. A comparison between both methods is shown in Oliveira *et al.* (2015), in which the authors conclude that both yield the same results.

Ground Resonance Parametric Analysis

The uncertainties introduced in the dynamic system in Eq.(6) are related to blade hinge stiffness and are presented as a function of in-plane lead-lag resonance frequency squared. In this work, only uncertainties in the 1st blade are accounted for the robustness analysis. By assuming $\bar{\omega}_{b1}$ as the nominal natural frequency of the blade, the modified frequency ω_{b1} is given by:

$$\omega_{b1}^2 = \left(1+\delta_1\right)\bar{\omega}_{b1}^2 \tag{16}$$

The first robustness analysis is to be done by choosing different combinations of C_b and C_x that gives a specific peak (h) of the real part of the eigenvalues, admitting all the combinations remain in the same line in the stability region. In order to do so, first the curve that gives combinations of both damping that yield the same peak (h) has to be found. For this work, the peak was chosen to be $h = -0.5$, which is comprised in the stable region.

When performing the robustness analysis via the μ -analysis tools, one must select the rotor speed. For purposes of this work, four different speeds were considered: 2 Hz, 4 Hz, 6 Hz, and 8 Hz. The combinations C_b and C_x along the curve for $h = -0.5$ as well as their respective minimal perturbation δ_1 that destabilizes the helicopter at each rotor speed are shown in Tab. 3 for Helicopter Type 1 and in Tab. 4 for Helicopter Type 2.

Table 3 - Perturbations δ_1 for Helicopter Type 1.

		$\Omega = 2$ **Hz**	$\Omega = 4$ **Hz**	$\Omega = 6$ **Hz**	$\Omega = 8$ **Hz**
C_b [Nms/rad]	C_x [Ns/m]	δ_1	δ_1	δ_1	δ_1
1290	7688.5636	-1.0586	-1.2821	-1.6038	-2.0627
1668	6261.2820	-1.0586	-1.2827	-1.6039	-2.0627
2047	5515.3003	-1.0586	-1.2829	-1.6040	-2.0628
2426	5049.8542	-1.0586	-1.2830	-1.6040	-2.0628
2805	4741.7484	-1.0586	-1.2831	-1.6040	-2.0628
3184	4518.1112	-1.0586	-1.2832	-1.6040	-2.0628
3563	4344.1412	-1.0586	-1.2832	-1.6040	-2.0628
3942	4217.9519	-1.0586	-1.2832	-1.6041	-2.0628
4321	4119.8877	-1.0586	-1.2832	-1.6041	-2.0628
4700	4025.2589	-1.0586	-1.2833	-1.6041	-2.0628

Table 4 - Perturbations δ_1 for Helicopter Type 2.

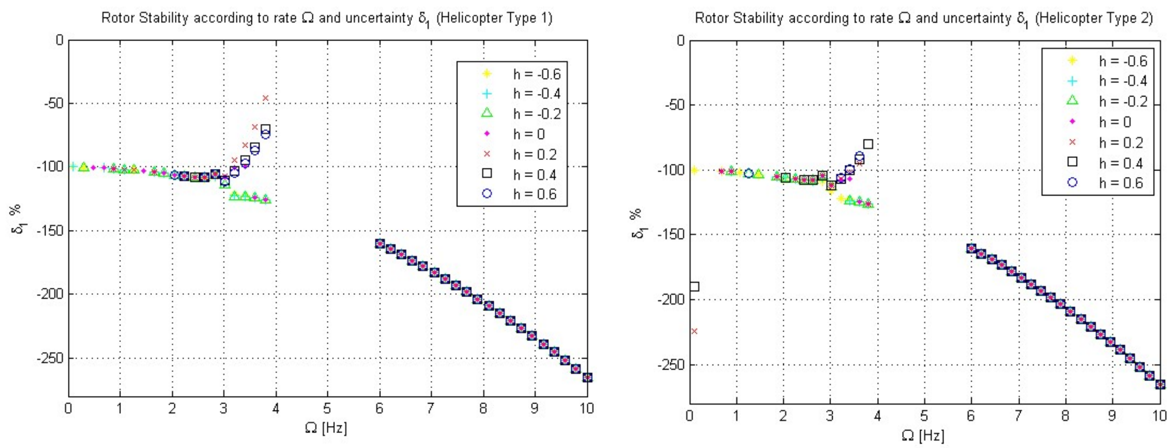
		$\Omega = 2$ **Hz**	$\Omega = 4$ **Hz**	$\Omega = 6$ **Hz**	$\Omega = 8$ **Hz**
C_b [Nms/rad]	C_x [Ns/m]	δ_1	δ_1	δ_1	δ_1
1131	7031.7050	-1.0586	-1.2828	-1.6040	-2.0628
1533	5126.2450	-1.0586	-1.2835	-1.6041	-2.0628
1935	4218.1842	-1.0585	-1.2837	-1.6042	-2.0629
2337	3733.5303	-1.0585	-1.2838	-1.6042	-2.0629
2740	3386.5867	-1.0585	-1.2839	-1.6042	-2.0629
3142	3167.5168	-1.0585	-1.2839	-1.6042	-2.0629
3544	3016.2318	-1.0585	-1.2840	-1.6042	-2.0629
3946	2859.3988	-1.0585	-1.2840	-1.6042	-2.0629
4348	2803.9668	-1.0585	-1.2840	-1.6042	-2.0629
4750	2651.9956	-1.0585	-1.2840	-1.6042	-2.0629

It is important to note from Tab. 3 and Tab. 4 that for different combinations of C_b and C_x the minimal perturbation δ_1 which leads the helicopter unstable remains practically constant for the same speed and does not change when the DVA is considered. Also, one can see that as the speed increases, the perturbation δ_1 increases, which means that the system is less robust in low rotor speeds.

By taking into account Eq.(16), one can note that the system is extremely robust once the perturbations less than -1 lead to negative blade natural frequencies. This means the helicopter can lose one of its rotor blade stiffness and still be a stable system with those combinations of damping at these rotating speeds.

It is important to note that even though the combinations fuselage and rotor dampings are different between Helicopter Type 1 and Helicopter Type 2, the robustness analysis yields practically the same results for all the four speeds. This means that by adding the DVA to the system does not change its robustness. From a practical point of view, this fact means only positive contributions of the DVA for the helicopter.

The previous analysis for both Helicopter Type 1 and Helicopter Type 3 was done considering only the peak of the maximum real part as $h = -0.5$. However, it is important to assess the different perturbations related to different peaks of real part (h). As seen before, the perturbations are practically constant along the curve for a specific peak of real part, which means that every combination of fuselage and rotor damping that lies in that curve will give the same perturbation as result. With that in mind, it is possible to analyze the minimum destabilizing perturbations for different h by choosing one set of fuselage and blade damping combination for each value of peak wanted. Here, the peaks chosen are -0.6, -0.4, -0.2, 0, 0.2, 0.4, and 0.6. In this case, the perturbations were evaluated in the range in which the helicopter is on the ground, previously defined as $0 \le \Omega \le 10$ Hz. The maximum perturbations allowed for the rotor stability according to the rotor speed for Helicopter Type 1 and Helicopter Type 2 are shown in Fig. 4.



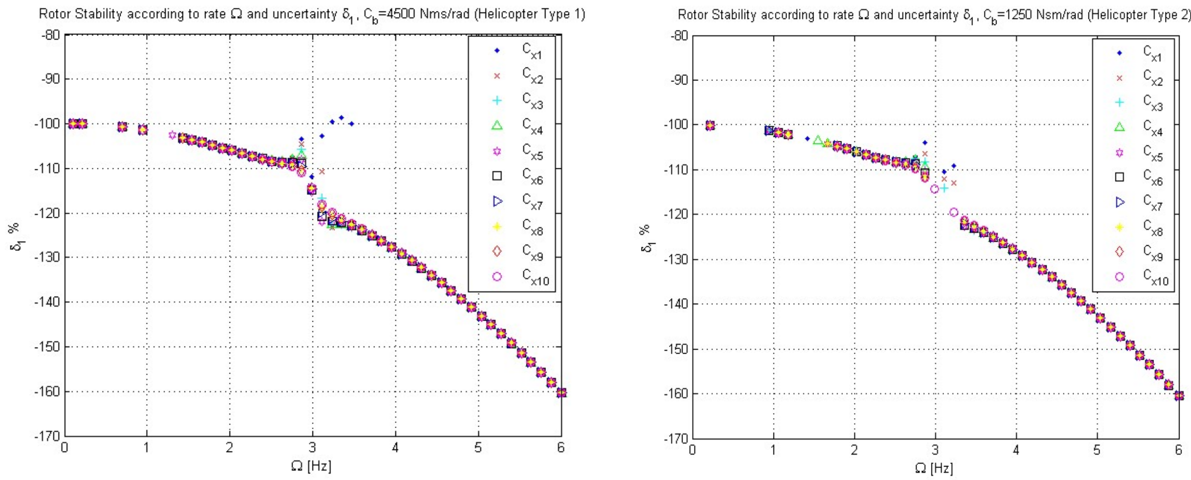
(a) (b)

Figure 4 - Rotor Stability According to Rotor Speed Ω and Perturbation δ_1: (a) Helicopter Type 1; (b) Helicopter Type 2

It is worth mentioning that the μ-analysis method can only be applied when the system is nominally stable. Therefore, the analysis is not performed in the range $4 \le \Omega \le 6$ Hz, which is the region where the system can be unstable for $h > 0$. Also, in order to be able to compare all the results, the analysis was not performed in that region for $h < 0$.

One can see from Fig. 4a that in the region $0 \le \Omega \le 3$ Hz, the perturbations are practically the same for all h, with some small differences. This means that the peak of the real part has almost no influence in the robustness of the system. However, in the range of $3 \le \Omega \le 4$ Hz it is possible to see that the perturbations are different for positive h and negative h. In fact, in the unstable region (positive h) the higher the peak, the more robust the system is, since the perturbations are bigger. In the region $6 \le \Omega \le 10$ Hz the perturbations are exactly the same for all values of h meaning that the peak has absolutely no effect in the robustness of the system. The same conclusions can be made for Helicopter Type 2 in Fig. 4b, the exception being the region $3 \le \Omega \le 4$ Hz, in which the perturbations are practically the same for all positive values of h. Also, by comparing the results for Helicopter Type 1 and Helicopter Type 2 in that region, it is possible to see that the perturbations for the helicopter with the DVA are greater than the ones for the helicopter without DVA, which means that Helicopter Type 2 is more robust than Helicopter Type 1 in that region.

The robustness analysis made for different values of peak of real part does not assess the influence of fuselage damping and rotor blade damping in the robustness of the system, since for each value of h a different set of C_b and C_x is considered. It is important to make a parametric analysis on both helicopters so conclusions can be made on that. In order to evaluate the influence of C_x in helicopters robustness, the rotor blade damping was considered fixed at $C_b = 4500$ Nms/rad for Helicopter Type 1and $C_b = 1250$ Nms/rad for Helicopter Type 2 and the fuselage damping was varied from $900 \text{ Ns/m} \le C_x \le 9000 \text{ Ns/m}$ with increments of 900 Ns/m, and the robustness analysis was performed in the range $0 \le \Omega \le 6$ Hz. The results are shown in Fig.5. For ease to see the results, for both cases the fuselage damping were given subscript index in which $C_{x1} = 900$ Ns/m, $C_{x2} = 1800$ Ns/m, and so on until $C_{x10} = 9000$ Ns/m.

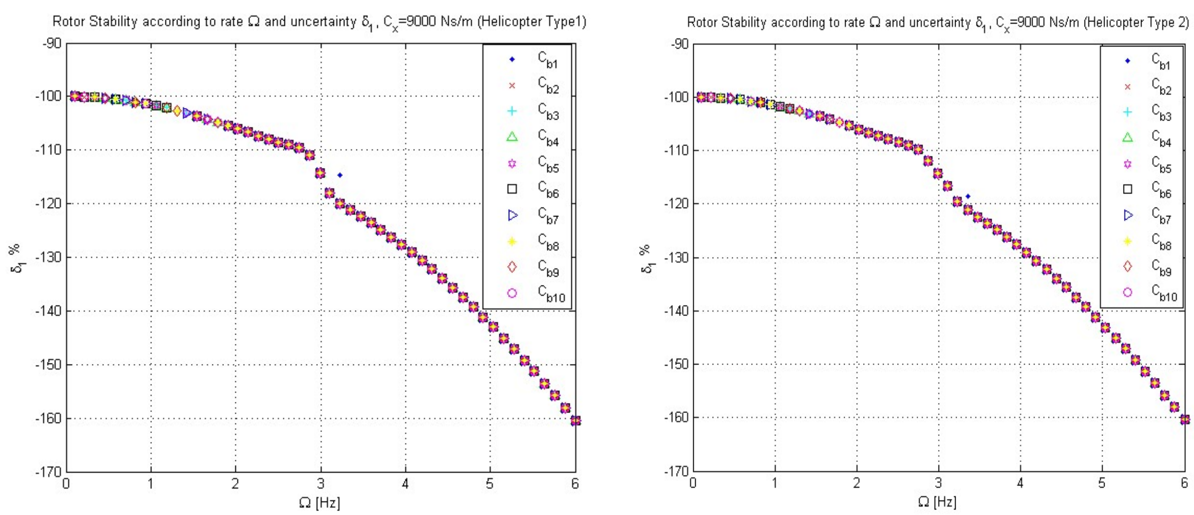


(a) (b)

Figure 5 - Rotor Stability According to Rotor Speed Ω and Perturbation δ_1: (a) Helicopter Type 1 $C_b = 4500$ Nms/rad ; (b) Helicopter Type 2 $C_b = 1250$ Nms/rad

One can see from Fig. 5 that the perturbations are practically the same in all the range of rotor speed, with the exception being in the region around $\Omega = 3 \text{ Hz} = 6\pi$ rad/s. If one may recall, $\Omega = 6\pi$ rad/s is the fuselage natural frequency. Since those analyses are done by varying the fuselage damping, its natural frequency may have influence on the result, since there is coupling of the imaginary part of the eigenvalues of fuselage and rotor blade. The conclusion of that analysis is that in that region around $\Omega = 3$ Hz, the robustness is sensitive to the fuselage damping, and different values of C_x yield different values of perturbations. Also, with the exception of that region, both helicopters have the same values for perturbation, meaning that the DVA has no effect on the robustness of the system, which corroborates with the conclusions made with the analysis of the influence of the peak of real part.

Now in order to evaluate the influence of rotor blade damping , the same procedure is made. For both helicopters the fuselage damping is fixed at $C_x = 9000$ Ns/m and the rotor blade damping is varied from $500 \text{ Nms/rad} \le C_b \le 4500$ Nms/rad . The results are shown in Fig. 6.



(a) (b)

Figure 6 - Rotor Stability According to Rotor Speed Ω and Perturbation δ_1: (a) Helicopter Type 1; (b) Helicopter Type 2

One can see from Fig. 6 that the perturbations are exactly the same in all the range of rotor speed, which means that the rotor blade damping has no influence on the robustness of both systems. Also, both helicopters have the same values for perturbation, which once again corroborates with the previous conclusions.

CONCLUSIONS

Electrical or mechanical devices might be attached to the fuselage of helicopters, which may be modeled as spring-mass-damping vibration systems and can interact with the helicopter ground oscillations, which might alter the stability characteristics of the helicopter. The influence of the SMDVS parameters added to the fuselage of a helicopter were under investigation in the present paper.

Firstly, the influence of fuselage and rotor blade damping, specially their combination, was evaluated. The results showed that in order to vanish the instabilities due to ground resonance, the system should have the proper combination of damping. Also, it was possible to see that the addition of the DVA is good for the overall system stability, since by applying it to the fuselage, the helicopter can make use of smaller values for damping, which makes the system more compact and less complex.

Lastly, a robustness analysis of the entire system was performed by changing the stiffness properties of one rotor blade for the helicopters with and without DVA to assess how the system handles those variations. The results showed that for both helicopters, as the rotor speed increases, the system become more robust. It was seen that with the proper combination of fuselage and rotor blade damping, the system can be so robust as to lose one of its rotor blade stiffness and still remain stable. Also, the analysis showed that in the region around the natural frequency of the fuselage, the robustness of the system is sensitive to the fuselage damping, since different values of it yield different values of perturbations.

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