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INVESTIGATION OF THE LIMITS OF THE MODAL DAMPING ASSUMPTION FOR STRUCTURES WITH NONLINEAR LOCALIZED DAMPERS

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Abstract: In many engineering area, quite often vibration is not desirable and the interest lies in reducing it by dissipation of energy or damping. Damping presents one of the most important physical aspects to model and estimate, since it plays a large role in determining the performance of a dynamic system and the amplitude of vibrations. The structural damping is well known and is typically responsible for a light level of damping compared to the joint damping. Damping in jointed interfaces is still not well mastered because of the complexity of the dissipation by friction, which includes nonlinear and localization effects. So, when eigenmodes are coupled with damping due to the nonlinear behaviour and the localization effects, as in the case of the most assembled structures, the Rayleigh assumption of proportional damping can lead to significant errors. The aim of this paper is the investigation of the representation assessment of the damping in a modal framework, in the case of localized dissipation. In order to quantify the effects of the nonlinear and the modal coupling on the dynamic responses, a quantification/correction method based on perturbation approach using asymptotic expansion of the frequency responses is proposed. The limitations of the proportional damping are then inspected and the correction of error estimations of modal damping due to the neglect of coupling terms is illustrated. A double-beam structure with a nonlinear interface is proposed to illustrate the validity and effectiveness of the proposed method.

Keywords: Modal damping, Localized dissipation, Perturbation method

INTRODUCTION

Problems involving vibrations occur in many areas of mechanical, civil and aerospace engineering (Caignot *et al.*, 2010). Vibrations are often undesirable and the interest lies in reducing them by introducing dampers. Since the publication of Rayleigh (1896), a large body of literature can now be found on damping problems. Although the topic of damping is an age-old problem, the demands of modern engineering have led to a steady increase of interest in recent years (Ahmadian & Jalali, 2007), (Bograd *et al.*, 2011), (Adhikari, 2013), (Hammami *et al.*, 2016).

Damping presents one of the most important physical aspects to model and estimate, since it plays a large role in determining the amplitude of vibrations. Indeed, there are mainly two different kinds of damping in a structure (Mead, 1999) : (1) *structural damping* due to the internal dissipation in the material and (2) *joint damping* due to dissipation by friction between joint interfaces. The first kind of damping (*structural damping*) is widely known today for homogeneous structures but it is still the subject for research in composite structures. A number of experimental studies (Goodman & Klumpp, 1956), (Beards & Williams, 1977), (Ungar, 1973) have demonstrated that the damping resulting from interface effects is typically higher than the material damping in metallic structures. Consequently, the second kind of damping (*joint damping*) in the joint interface must be modeled in order to predict the vibration levels accurately. Damping in jointed interfaces is still not well mastered because of the complexity of the dissipation by friction which includes nonlinear behavior and localization effects (Sinou & Jezequel, 2007). Therefore, the eigenmodes will be coupled with damping. So the assumption of proportional damping can have limitations and may lead to significant errors in the prediction response in many cases of assembled structures.

Although the development of many sophisticated dissipation models, damping is still not well mastered because of the neglect of coupling terms in the equations of motion of the discrete system. To the best of our knowledge, the impacts of nonlinear coupling and the modal coupling are still unknown and have yet to be identified in the literature.

The aim of this paper is to investigate the limits of modal damping assumption in both cases of linear and nonlinear localized dissipation. In order to quantify and reduce the effects of the nonlinear and the modal coupling on the dynamic responses, an estimation and correction method based on perturbation approach using expansion of the responses is proposed (Krifa *et al.*, 2016). The proposed numerical example is a double-beam structure with a nonlinear interface, where accurate estimations of exact solutions are available.

PROPORTIONAL DAMPING ASSUMPTION

Damping is generally modeled under Rayleigh's (1896) or Caughey's assumptions (1965). It is expressed as a linear combination of the mass and stiffness matrices,

$$C = \alpha_1 M + \alpha_2 K \tag{1}$$

The Rayleigh's (proportional) damping is a sufficient condition to obtain a diagonal generalized damping matrix ($\beta = \phi^T C\phi$, where ϕ is the modal basis of the associated conservative system). Caughey introduced a necessary and sufficient condition

$$CM^{-1}K = KM^{-1}C \tag{2}$$

which leads to a diagonal generalized damping matrix β. In the general case, the damping matrix is expressed as a power series of terms ($M^{-1}K$) in the form

$$C = M \sum_{j=0}^{N-1} \alpha_j (M^{-1}K)^j \tag{3}$$

A further generalized form of proportional damping was proposed by Adhikari (2001).

$$C = M\alpha_3(M^{-1}K) = K\alpha_4(K^{-1}M) \tag{4}$$

where $\alpha_i(\bullet)$, $i = 3,..4$ are general functions.

The proportional damping assumption is not always well-founded in reality, but leads to a diagonal generalized damping matrix that minimizes computational costs. The physical meaning of Caughey's and Rayleigh's assumptions is that dissipation in the structure is uniformly distributed so that modes are decoupled with respect to damping effects. But in real behavior, the big amount of damping is localized in interfaces of assembled structures. The present study will investigate the limits of the proportional damping assumption for structures with non-proportional linear and nonlinear dissipative interfaces.

In order to estimate the modal damping of a structure, the Modal Strain Energy method (MSE) can be used. It was firstly suggested by Ungar and Kerwin (1962), and has been used since to address viscoelastic damping problems of sandwich structures by Johnson and Kienholz (1982). Later, a modified MSE was proposed (Hu *et al.*, 1995) in order to improve the estimation of modal damping. The objective of the modal strain energy is to determine the damping factor corresponding to each vibration mode of the structure. It is based on the concept of the dissipated energy in the interfaces for which the close form expression of the loss factor is the ratio between dissipated energy E_ν^d and maximal potential energy E_ν^p, over a cycle of periodic vibration (Krifa *et al.*, 2015), as shown in this equation

$$\xi_\nu = \frac{1}{4\pi}\frac{E_\nu^d}{E_\nu^p} \quad ; \quad \nu = 1,2,... \tag{5}$$

QUANTIFICATION METHOD INVOLVING NONLINEAR LOCALIZED DISSIPATION

Proposed method

We propose an extension of perturbation technique (Bogoliubov & Mitropolski, 1961), (Nayfeh, 2011) for a multi-degree of freedom (MDOF) system. The scope of the proposed method (Krifa *et al.*, 2016) is that it takes into account the effects of coupling including linear and nonlinear terms. The *perturbation method* is applicable to problems in which a small positive non-dimensional bookkeeping parameter ε is associated with the coupling term of the differential equation. If the solution of the linearized problem is periodic, and if ε is small ($\varepsilon \ll 1$), we can expect the perturbed solution to be periodic also. The perturbation method will be explained more fully here after. The equations governing the dynamic response of the MDOF system can be expressed in the following form,

$$M\ddot{y} + f_D(y,\dot{y}) + Ky = f_E(t) \tag{6}$$

where $f_E(t) = F_E\cos(\Omega t)$ is a harmonic excitation with an excitation frequency Ω, M and K are respectively the mass and the stiffness matrix of the system. $f_D(y,\dot{y})$ is the dissipated force vector of the system, which includes the material dissipation and principally the dissipation due to the interfaces between the substructures.

Let's consider ϕ and Λ respectively the modal base and the spectral matrix associated to the conservative system $K\phi = M\phi\Lambda$, where : $\phi^T M\phi = I, \phi^T K\phi = \Lambda$.

By projection on the modal base Φ truncated at the first m modes and multiplying on the left by Φ^T, Equation (6) in modal coordinates q yields

$$\ddot{q} + F_D(q,\dot{q}) + \Lambda q = F_E(t) \tag{7}$$

where $F_D(q,\dot{q}) = \phi^T f_D(q,\dot{q})$ is the modal nonlinear forces due to friction contacts and $F_E(t) = \phi^T f_E(t)$ is the generalized excitation force. The above system presents m coupled nonlinear differential equations because of the presence of the dissipated force $F_D(q,\dot{q})$. In order to investigate both modal coupling effect and the nonlinear effect, a double perturbation method is considered.

This method consists in decomposing the dissipation force into two parts according to the type of dissipation : *linear* and *nonlinear*. Each part is also decomposed into two parts : *uncoupled* and *coupled* terms. So, the dissipation force can be expressed as follows :

$$\phi^T f_D(q,\dot{q}) = \underbrace{\underbrace{\phi^T {f_D}^{UL}(q,\dot{q})}_{Uncoupled} + \varepsilon_2 \underbrace{\phi^T {f_D}^{CL}(q,\dot{q})}_{Coupled}}_{Linear} + \underbrace{\varepsilon_1 \left(\underbrace{\phi^T {f_D}^{UNL}(q,\dot{q})}_{Uncoupled} + \varepsilon_2 \underbrace{\phi^T {f_D}^{CNL}(q,\dot{q})}_{Coupled} \right)}_{Nonlinear} \tag{8}$$

where $\phi^T {f_D}^{UL}, \phi^T {f_D}^{CL}$ are respectively the uncoupled and coupled *linear* dissipation force, and $\phi^T {f_D}^{UNL}$,$\phi^T {f_D}^{CNL}$ are respectively the uncoupled and coupled *nonlinear* dissipation force. ε_1 and ε_2 are two small positive non-dimensional bookkeeping parameters.

From Equation (8) one can distinguish three particular cases :

(a) when $\varepsilon_1 = 0$, the parameter $\varepsilon_2 \ll 1$ allows to study the effects of the linear modal coupling.

(b) when $\varepsilon_2 = 0$, the parameter $\varepsilon_1 \ll 1$ allows to study the effects of the uncoupled nonlinearity.

(c) when both parameters $\varepsilon_1 \neq 0$ and $\varepsilon_2 \neq 0$, both effects of *modal coupling* and *nonlinearity* can be investigated.

Substituting Equation (8) into (7) yields

$$\ddot{q} + \Lambda q + \phi^T {f_D}^{UL}(q,\dot{q}) + \varepsilon_2 \phi^T {f_D}^{CL}(q,\dot{q}) + \varepsilon_1 \left(\phi^T {f_D}^{UNL}(q,\dot{q}) + \varepsilon_2 \phi^T {f_D}^{CNL}(q,\dot{q})\right) = \phi^T f_E(t) \tag{9}$$

For the k^{th} mode, Equation (9) yields

$$\ddot{q}_k + {\omega_k}^2 q_k + \phi^T f_{D_k}^{UL}(q_k,\dot{q}_k) + \varepsilon_2 \phi^T f_{D_k}^{CL}\left(q_{l\neq k},\dot{q}_{l\neq k}\right) + \varepsilon_1 \left(\phi^T f_{D_k}^{UNL}(q_k,\dot{q}_k) + \varepsilon_2 \phi^T f_{D_k}^{CNL}\left(q_{l\neq k},\dot{q}_{l\neq k}\right)\right) = \phi^T f_{E_k}(t) \tag{10}$$

For $\varepsilon_1 \ll 1$ and $\varepsilon_2 \ll 1$, one assumes that the perturbation solution of Equation (10) is sought in the form

$$q = \sum_{i,j} \varepsilon_1^i \varepsilon_2^j q^{(i,j)} = q^{(0,0)} + \varepsilon_1 q^{(1,0)} + \varepsilon_1 \varepsilon_2 q^{(1,1)} + \varepsilon_2 q^{(0,1)} + \cdots \tag{11}$$

where $q^{(0,0)}$ is a Linear Uncoupled response that corresponds to the proportional assumption, $q^{(0,1)}$ is a Linear Coupled response, $q^{(1,0)}$ is a NonLinear Uncoupled response and $q^{(1,1)}$ is a NonLinear Coupled response.

Substituting Equation (11) into (10) and regrouping terms of the same order give

Order (0,0) , $\varepsilon_1 = 0$ and $\varepsilon_2 = 0$: Proportional damping assumption

$$\ddot{q}_k^{(0,0)} + {\omega_k}^2 {q_k}^{(0,0)} + \phi^T f_{D_k}^{UL}({q_k}^{(0,0)}, \dot{q}_k^{(0,0)}) = \phi^T f_{E_k}(t) \tag{12}$$

Order (0,1), $\varepsilon_1 = 0$ and $\varepsilon_2 \neq 0$: Effect of modal coupling
Order (1,0) , $\varepsilon_1 \neq 0$ and $\varepsilon_2 = 0$: Effect of uncoupled nonlinearity
Order (1,1) , $\varepsilon_1 \neq 0$ and $\varepsilon_2 \neq 0$: Effect of coupled nonlinearity

$$\begin{aligned} &\ddot{q}_k^{(1,1)} + {\omega_k}^2 {q_k}^{(1,1)} + \phi^T f_{D_k}^{UL}({q_k}^{(1,1)}, \dot{q}_k^{(1,1)}) = -\phi^T f_{D_k}^{CL}({q_{l\neq k}}^{(1,0)}, \dot{q}_{l\neq k}^{(1,0)}) \\ &-\phi^T f_{D_k}^{UNL}({q_k}^{(0,1)}, \dot{q}_k^{(0,1)}) - \phi^T f_{D_k}^{CNL}({q_{l\neq k}}^{(0,0)}, \dot{q}_{l\neq k}^{(0,0)}) \end{aligned} \tag{13}$$

From the order (1,1), the double perturbation method takes into account both the nonlinearity and modal coupling effects. The main advantage of this method consists in solving linear uncoupled equations at each order. The nonlinear terms are taken into account in the right hand side of the resulting equation at each order and depends only on the solutions at the previous orders.

Particular case of linear localized dissipation

The aim of this section is to present the formulation of the perturbation method in the particular case of linear damping.

It is a common practice to approximate the nonlinear behavior with an equivalent linear damping and not conduct a nonlinear analysis as is adressed by Bandstra (1983). Because of its simplicity, the equivalent viscous model will be considered here (Petyt, 2010), (Jalali, 2016). From Equation (6), one can deduces the discrete form of the damped linear vibration problem where $f_D(y,\dot{y}) = C\dot{y}$, and C is the viscous damping matrix.

Assuming that the viscous damping matrix C is a non-proportional matrix, which represent the case of most assembled structures where the dissipation is principally distributed in the interfaces, then the generalized damping matrix $\beta = \phi^T C \phi$ will be a full matrix. In the particular case of linear dissipation, for simplification reasons, the following notations will be considered : $\varepsilon_1 = 0$ and $\varepsilon_2 = \varepsilon$, the order $(0,0) = (0)$, the order $(0,1) = (1)$ and the order $(0,n) = (n)$, ect...

Given $\varepsilon \in [0,1]$, the perturbation method consists in expressing the generalized damping matrix β in the form:

$$\beta = \beta_1 + \varepsilon \beta_2 \tag{14}$$

where β_1 is the diagonal part of the matrix β and β_2 contains the off-diagonal terms with zeros on the diagonal.

If $\varepsilon = 0$, then Equation (14) gives $\beta = \beta_1$, which corresponds to the case of proportional damping. Otherwise, if $\varepsilon = 1$ then Equation (14) gives $\beta = \beta_1 + \beta_2$, corresponding to the general case of localized damping without added assumptions.

For $\varepsilon \ll 1$, one assumes that the perturbation solution of the Equation (6) is sought in the form

$$y_n(t) = y^{(0)} + \varepsilon y^{(1)} + \varepsilon^2 y^{(2)} + \cdots + \varepsilon^n y^{(n)} \tag{15}$$

Assume that the perturbation terms $y^{(i)}$ of the i^{th} order projected on the modal basis are expressed as follow :

$$y^{(i)} = \phi q^{(i)} \tag{16}$$

Each perturbation term $y^{(i)}$, which is complex, is expressed as the multiplication of the modal basis ϕ ,which is real, and the modal coordinate $q^{(i)}$ which is complex.

Now, the perturbation terms $q^{(i)}$ of the i^{th} order need to be derived.

$$q^{(0)} = (-\omega^2 I + j\omega\beta_1 + \Lambda)^{-1} \phi^T f_E \tag{17}$$

The response $q^{(0)}$ corresponds to the initial unperturbed system. This response is calculated once in the beginning of the procedure and the remaining higher order terms $q^{(i)}$ are expressed as a function of $q^{(0)}$.

In general, the n^{th} order term in ε allows $q^{(n)}$ to be expressed as a function of $q^{(n-1)}$ or even $q^{(0)}$ as follows:

$$q^{(n)} = -j\omega\left(-\omega^2 I + j\omega\beta_1 + \Lambda\right)^{-1} \beta_2 q^{(n-1)} = \psi q^{(n-1)} = \psi^n q^{(0)} \tag{18}$$

where $\psi = -j\omega(-\omega^2 I + j\omega\beta_1 + \Lambda)^{-1}\beta_2$ is a matrix depending on β_1 and β_2.

The 0^{th} order term with $(\varepsilon = 0)$ yields the unperturbed FRF resulting from a diagonal generalized damping and is given by the classical expression:

$$y_0(\omega) = \phi q^{(0)} = \phi(-\omega^2 I + j\omega\beta_1 + \Lambda)^{-1} \phi^T f_E \tag{19}$$

Moreover, the perturbed response of order n can be expressed as:

$$y_n(\omega) = \phi(1 + \varepsilon\psi + \varepsilon^2\psi^2 + \cdots + \varepsilon^n\psi^n) q^{(0)} \tag{20}$$

It should be noted that the matrix ψ is calculated once for each frequency ω. The main advantage of the expression (20) is its capacity to calculate, with a very low computational cost, any order of the perturbation to improve accuracy. The low cost is explained, first, by the inversion of diagonal matrix β_1 in Equation (18), second, whatever the size of the model governed by Equation (6), the matrix is equal in size to the number of retained modes. To validate the perturbation method, the results will be compared to the direct reference method using full generalized damping matrix $\beta = \beta_1 + \varepsilon\beta_2$.

The proposed perturbation method will be illustrated with a numerical example in order to investigate the importance of modal coupling and the impact of the localized damping level on the frequency response of the structure.

NUMERICAL SIMULATIONS

As shown in Figure 1, the considered structure is composed of two beams. Each beam is discretized into ten 2D beam finite elements (2 dofs per node), so the full model has 40 dofs. The excitation force is applied to node number 1 of the first beam. This model represents two steel substructures assembled by a nonlinear joint.

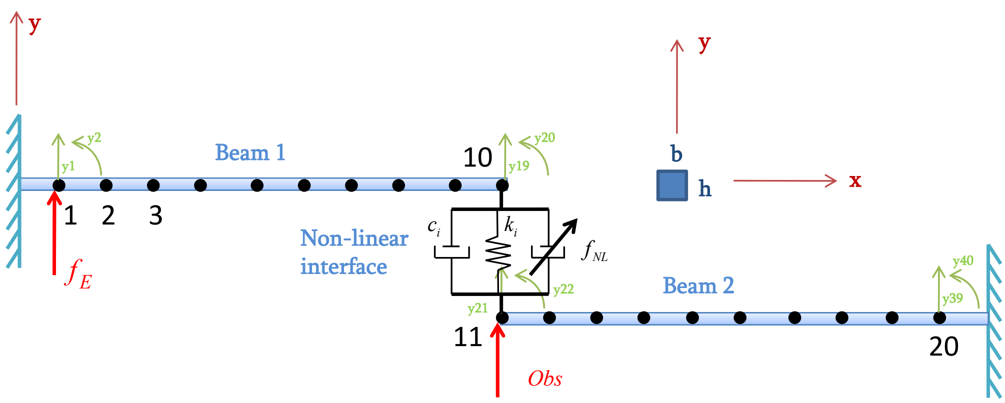

Figure 1 – Double-beam structure

Table 1 – Physical and geometric properties of the structure and interface parameters

E	Young's modulus (GPa)	210	k_i	Linear stiffness (N/m)	$1e4$
ρ_s	Mass density (kg/m^3)	7800	c_i	Viscous damper (N/(m/s))	20
h	Beam thikness (m)	0.02	L_1	Length of the first beam (m)	0.701
b	Beam width (m)	0.05	L_2	Length of the second beam (m)	0.700
ν	Poisson ratio	0.3	α	scalar parameter of the nonlinear damping (Ns/m^3)	$\alpha \in [0 - 1e12]$

The details of the simulation data are summarized in Table 1. The joint is represented by a nonlinear lumped model composed of spring element k_i and a viscous damper element c_i and a Van der Pol model governed by the parametric scalar α.

The dissipation force is expressed by the following equation:

$$f_D(t,\dot{u}) = f_{LN} + f_{NL} = c_i \times \dot{y}_{rel} + \alpha {y_{rel}}^2 \dot{y}_{rel} \tag{21}$$

where y_{rel} is the relative displacements, $\dot{y}_{rel}$ is the relative velocity at the interface. Figure 2 shows the first two mode shapes for the undamped system of two beams with one interface. From this Figure, we can see that the second mode will have a larger modal damping because it dissipates the most energy, compared to the first one, due to the large relative displacements at the interface.

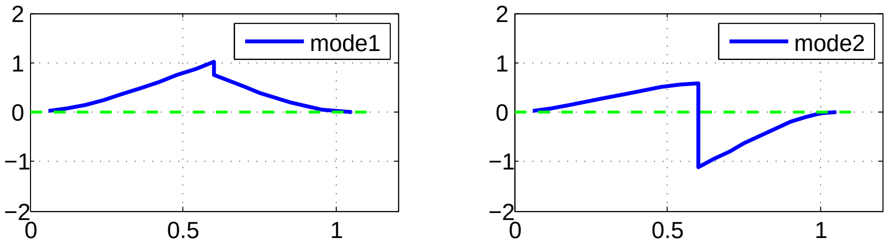

Figure 2 – Mode shapes of the double-beam with interface

The modal damping results and eigenfrequencies calculated by the state space method are presented in Table 2. The influence of extra-diagonal damping coefficients on the dynamic responses of the damped system will then be studied using the proposed perturbation method for both cases of *nonlinear* and *linear* localized dissipation.

Table 2 – Frequencies and modal damping for the first six modes

Frequency (Hz)	35.96	45.19	214.23	265.10	598.92	741.57
Modal damping $\xi_\nu(\%)$	1.43	4.21	0.54	0.50	0.19	0.17

Case 1 : Nonlinear localized damping

From Figure 3, it can be seen that, the order (0,0) which corresponds to the proportional damping hypothesis has a 14.23 % response level error. This error decreases if we consider the linear coupling. The order (0.1) presents an error equals to 6.75% . Taking into account the uncoupled nonlinearity, the response is clearly corrected and the error of the order (1.0) is reduced to 3.14%. Finally, if the coupled nonlinearity is considered, the order (1,1) presents in this case 2.98% of error with respect to the nonlinear reference model.

The Table 3 shows that the error increases with the increase of the value of the nonlinear parameter α for all the orders. For the case of few nonlinearity , when $\alpha = 1e11$, the error drops from 14.23% to 2.98%, and from 21% to 3.5% in the case of $\alpha = 3e11$. The order does not completely compensate the dynamic response error when $\alpha = 4.5e11$. But the proposed method is of high interest in weakly nonlinear structures. In this case the order (1,1) is sufficient, when $\alpha \leq 3e11$, in order to get an accurate result of dynamic response (less than 4% of level error) using the perturbation method. Finally, the proposed method is able to predict the maximal level error due to the neglect of coupling linear and nonlinear terms. Also, the perturbation method is able to reduce these errors and generate an accurate result in the case of non-proportional nonlinear dissipation.

Table 3 – Amplitude Error (%)

Nonlinear parameter $\alpha \times 1e11$	1	1,5	3	4.5
perturbation order (0,0)	**14.23**	**15.68**	**20.98**	**25.45**
(0,1)	6.75	8.46	14.11	19.06
(1,0)	3.14	3.70	5.79	10.02
(1,1)	**2.98**	**3.33**	**3.5**	**9.20**

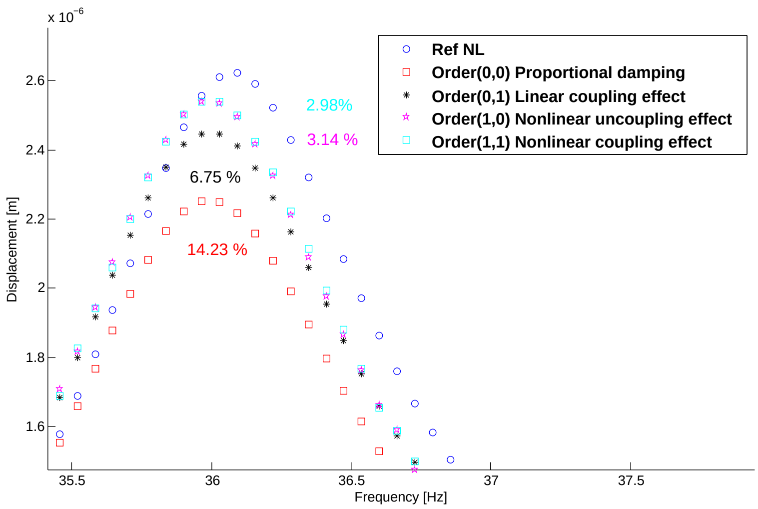

Figure 3 – Frequency response of the nonlinear model, nonlinear coupling parameter $\alpha = 1e11$, focus on the first mode: ○ Reference, □ Proportional damping, ∗ Order (0,1) - Linear coupling effect, ⋆ Order (1,0) - nonlinear uncoupling effect, □ Order (1,1) - nonlinear coupling effect

Case 2 : Linear localized damping

Impact of modal coupling

Now, the study will deal with the specific case of linear localized dissipation. So, the aim of this section is to investigate the impact of modal coupling, which is due to the presence of the extra-diagonal terms in the generalized damping matrix. The impact of the neglect of these terms can be quantified and corrected using the proposed method.

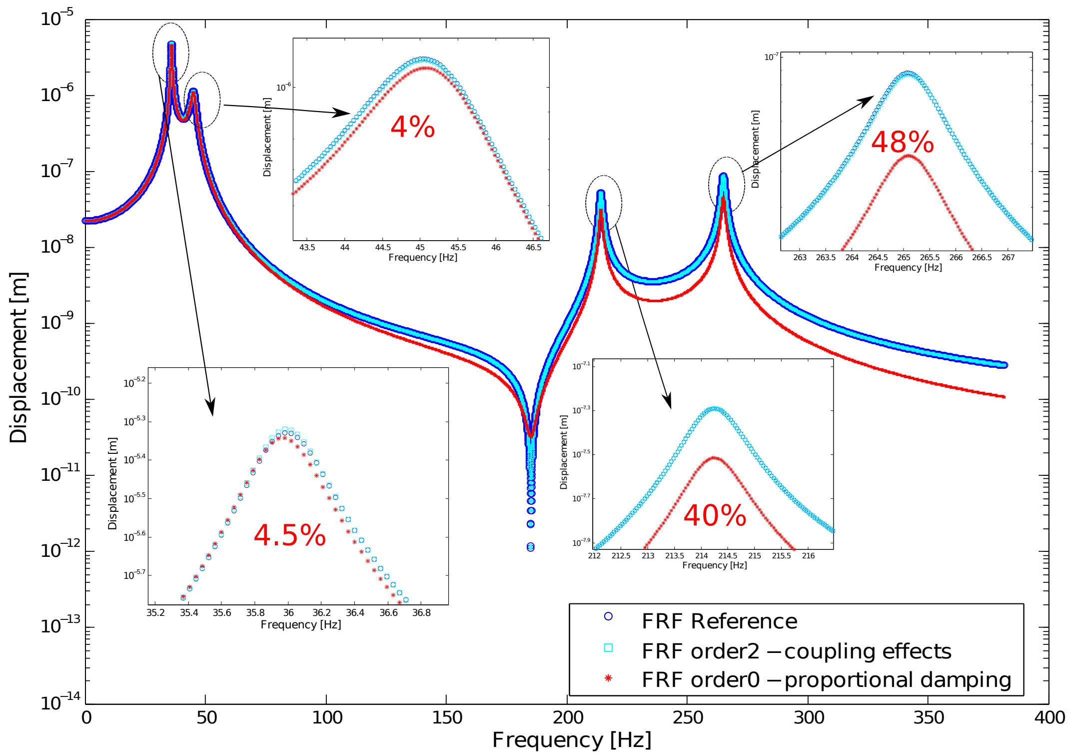

Figure 4 – Comparison of FRFs at interface dof 21 for $\varepsilon = 0.01$. (a) Modes 1 & 2, (ε_y^{Order0} = 4.5%, 4%). (b) Mode 3, (ε_y^{Order0} = 40%). (c) Mode 4, (ε_y^{Order0} = 48%). *Reference method*: FRF based on an inversion of full matrices. *Order0 : FRF based on the proportional assumption given by Eq. (19). Order2 : FRF based on the perturbed method taking into account the coupled terms using Eq. (20).*

Figure 4 shows the comparison of the FRFs between the unperturbed and perturbed systems with a perturbation coefficient of $\varepsilon = 0.01$, respectively for the first four modes.

The estimation of the error in the responses shows that the contribution of extra-diagonal terms is significant when comparing amplitudes between the Reference FRF and the FRF based on the proportional assumption (order 0). The amplitude errors are equal to 4.5%, 4%, 40% and 48%, respectively, for modes 1, 2, 3 and 4. These results illustrate that the proportional damping assumption can yield results with acceptable errors, as it is the case for the first two modes

(Figure 4). However, neglecting the contribution of the off-diagonal terms can lead to significant amplitude errors, as is the case for modes 3 and 4 (Figure 4). The errors for the two first modes are compensated by the level of the damping factors compared to the two following modes. Therefore the proportional damping approach is not satisfactory when the damping needs to be modeled properly for a broad frequency range.

Impact of the localized damping level

In this section, the effect of the localized damping level on the damping prediction will be examined. Five numerical simulations are performed with increasing levels of viscous damping ($c_i = 10, 20, 30, 40$ and $50\ Ns/m$) leading to increasing dissipated energies.

Considering the first mode, one can investigate the influence of the localized damping level. Table 4 shows that the error increases with the increase of the value of the localized dissipation damper c_i for 3 orders. The order 2, which take into account the coupling terms, is able to reduce the errors. It is important to note that increasing the perturbation order improves the convergence without increasing the computational cost, when using Equation (20).

Table 4 – Amplitude Error $(\%)$

Viscous damper $c_i[Ns/m]$	10	20	30	50
perturbation order (0)	**4.5**	**12.36**	**24.35**	**47.75**
(1)	2.29	5.16	9.4	25
(2)	**1.80**	**2.85**	**3.65**	**4.14**

Figure 5 shows, for the first four modes, the evolution of relative error between the MSE method, using proportional assumption, and the reference method. It can be seen that the error increases for these four modes with the increase of the localized dissipation. From Figure 5, we can deduce that the proportional hypothesis estimates the damping factor with unacceptable error ($\geq 40\%$) for all modes for high damping levels. And even for both modes 4 and 5 this assumption is no longer valid for low localized damping ($\varepsilon_y = 40\%$ for the 4^{st} mode and 48% for the 5^{st} mode), as seen in Figure 5.

In order to correct these level errors, the proposed method, which take into account the coupling terms, estimates the damping factor with acceptable accuracy ($\varepsilon_y < 2\%$) for low damping levels for all modes (Figure 6). Even for high localized damping the proposed method still valid ($\varepsilon_y < 4.5\%$), as seen in Figure 6.

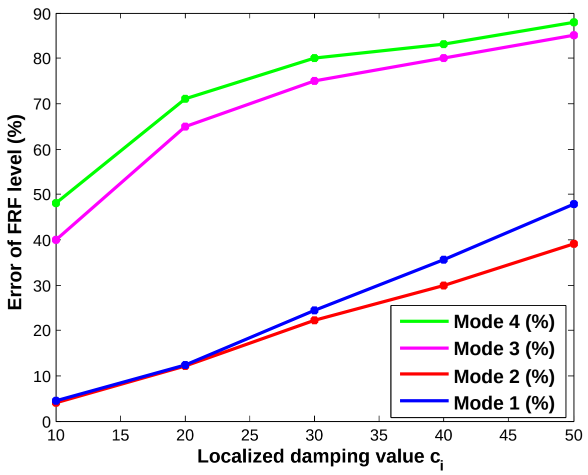

Figure 5 – Order 0 - Proportional assumption

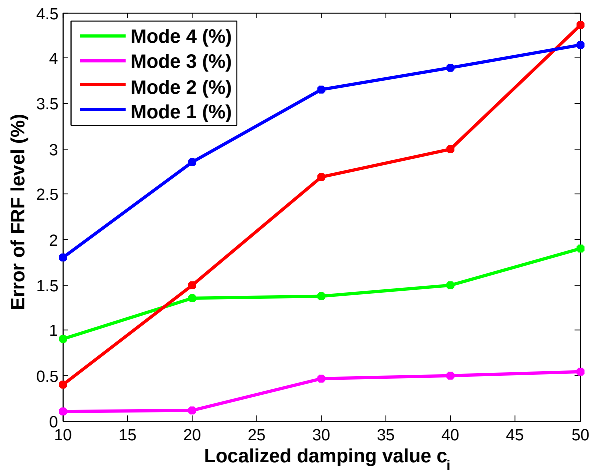

Figure 6 – Order 2 - Proposed method

This strategy reduces computation time while preserving a sufficient level of accuracy. Through an extended perturbation theory, we show that the coupling part of the restoring force can affect the frequency response level in the neighborhood of the resonances. The main advantage of the proposed method in this particular case is its capacity to correct the error induced from the neglect of the coupled terms with a reduced computational cost.

CONCLUDING REMARKS

The limitation of the modal damping assumption was firstly investigated when dealing with both linear and nonlinear localized dissipation. Then an estimation and correction method was proposed and illustrated with double-beam numerical example in order to investigate the importance of the coupling terms on the frequency response of the structure. Through an extended perturbation theory, we show that the coupling part of the restoring force can affect the frequency response level in the neighborhood of the resonances.

For a non-linear localized dissipation structure, the proposed method based on a double perturbation formulation is able to correct both amplitude and phase shift when switching to higher orders. For instance, the proposed method allows us to correct the errors : for mode 3, from 40% to 0.5% and for mode 4, from 48% to 1.8% in the case of low localized dissipation ($c_i = 10Ns/m$). Even for high level of localized dissipation, due to the proposed method, the error was reduced and didn't exceed $4,5\%$ for all modes.

In the specific case of linear localized dissipation, the proposed method is able to quantify and correct the error due to the modal coupling with a low computational cost due to the new matrix formalism. In the general case of nonlinear localized dissipation, the proposed method is accurate but the computational cost remains reasonable only for small size FEM. Finally, to apply the perturbation method for complex large FEM, it is advisable to work with metamodels which can be used locally to respresent interface dissipation, so that the nonlinear resolution taking into account the coupling effect becomes possible with a reasonable computational cost. Future work will include the development of adapted metamodels which are compatible with our proposed methodology in order to estimate the damping of nonlinear structures.

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