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Novel modular modeling methodology applied to the problem of a pipe conveying fluid

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Abstract: This paper proposes an extension of a novel modular modeling approach, originally developed for lumped parameter systems, to the derivation of FEM-discretized equations of motion of one-dimensional distributed parameter systems. The novel methodology is characterized by the use of a recursive algorithm based on projection operators that allows any constraint condition to be considered a posteriori*. This leads to a modular approach in which a system can be conceived as the top member of a hierarchy in which the increase of complexity from one level to the parent one is associated to the enforcement of some constraints. For lumped parameter systems this allows the implementation of modeling procedures starting from already known mathematical models of subsystems. In the case of distributed parameter systems, such a novel methodology not only allows to explore subsystem-based modeling strategies, but also makes it possible to propose formulations in which compatibility and boundary conditions are treated as constraints. The benchmark chosen to explore these further possibilities is the extensively studied problem of a cantilevered pipe conveying fluid. Supposing a solid pipe that respects the hypotheses of the Linear Theory of Elasticity, but is subject to geometric nonlinearities, and assuming an axial plug-flow, a Hamiltonian derivation of the discretized equations of motion is performed without an* a priori *enforcement of neither compatibility nor boundary conditions. After that, proposing a formulation that reconciles the use of the Finite Element Method along with the modular modeling algorithm for one-dimensional distributed parameter systems, some numerical simulations are performed in order to address qualitatively this novel methodology.*

Keywords: Analytical Mechanics, Mathematical Modeling, Finite Element Method, Modular Modeling, Pipe Conveying Fluid.

INTRODUCTION

The formalisms and strategies adopted in the mathematical modeling of dynamic systems are still very dependent on the hypotheses adopted. Slight modifications in a model may sometimes lead to a complete rederivation of the corresponding dynamic equations. In order to explore the possibility of using already known mathematical models, either of subsystems or of the system ifself under some simplifying assumptions, as the starting point of a modeling procedure, Orsino (2016) proposed a novel modular methodology for lumped parameter systems in which, a system is assumed to be the hierarch of an arrangement of mathematical models organized in terms of complexity. The system of equations of motion associated to parts of a given system, when it is assumed that there is no interaction among them, is less complex than the model of the system itself, for instance. Thus, in the hierarchical structure conceived, the increase of complexity is directly associated to the enforcement of constraints.

The application of variational principles in the derivation of mathematical models of dynamic systems typically lead to conditions of the form (with "·" standing for the inner product of the corresponding linear space):

$$\delta q \cdot f = 0 \tag{1}$$

Whenever all the variations δq can be assumed to be arbitrary and independent, equation (1) is equivalent to stating that $f = 0$. This is not the case, however, when due to constraints, the variations δq can not be set arbitrarily. Due to the assumption that variations are infinitesimals, there must be a linear operator A, associated to the constraints of the system (Orsino, 2016; Orsino and Hess-Coelho, 2015), such that δq must lie in the kernel of A, i.e.:

$$A\delta q = 0 \tag{2}$$

Let S be a linear operator whose image coincides with the kernel of A. Taking $\delta q = S\delta\theta$, equation (2) is automatically satisfied for any chosen $\delta\theta$, and therefore, equation (1) is satisfied if and only if:

$$S \cdot f = 0 \tag{3}$$

In this case S acts as a projection operator onto the kernel of A that makes it possible to derive the equations of motion associated to the system, $S \cdot f = 0$, starting from the model associated to the unconstrained system (i.e., when the variations δq can be assumed to be arbitrary and independent), $f = 0$.

Consider that the system is conceived as a hierarchical structure of models. Assume that, at level r, the equations of motion are given by $f_r = 0$ and the set of all the constraints that should be satisfied by the model lead to the following conditions to be satisfied by the variations: $A_r \delta q = 0$. Let S_r be a projection operator onto the kernel of A_r and consider that at the parent level, $r+1$, the extra constraints to be enforced lead to further invariants involving the variations, which are given by the equation $\tilde{A}_{r+1}\delta q = 0$. Therefore, at level $r+1$, the set of all constraints are associated to a linear operator $A_{r+1} = (A_r, \tilde{A}_{r+1})$, such that, $A_{r+1}\delta q = (A_r \delta q, \tilde{A}_{r+1}\delta q) = (0,0) = 0$. Orsino (2016) proves that if C_{r+1} is a projection operator onto the kernel of $B_{r+1} = \tilde{A}_{r+1}S_r$, it can be stated that $S_{r+1} = S_r C_{r+1}$ is a projection operator onto the kernel of A_{r+1}. Therefore, the equations of motion associated to the level $r+1$ of the hierarchy are given by:

$$f_{r+1} = C_{r+1} \cdot f_r = 0 \tag{4}$$

Thus, the proposed algorithm leads to a recursive methodology in which the effect of constraints in the dynamics of a given system can be included *a posteriori* and modularly by obtaining a sequence of projection operators associated to the hierarchy of models proposed.

This is the first of a series of papers in which an extension of this modular modeling methodology for distributed parameter systems will be developed. For these systems, not only subsystem-based modeling strategies can be explored, but also novel formulations for considering the effects of compatibility and boundary conditions can be proposed. Indeed, these conditions lead to identities that can be treated as constraint equations, which indicates that they can be included *a posteriori* if an appropriate form of the modular modeling methodology is used. Such an approach tends to reduce the dependency that the modeling procedure adopted for a given system has with respect to the considered simplifying hypothesis. Also, it allows to perform modeling steps without requiring *a priori* consideration of constraint conditions which, in many cases, are the main source of complexity in the derivation of the associated equations of motion.

Once the objective is to propose a novel modeling methodology, it is adequate to choose a benchmark case study for which several existing modeling approaches have been applied and whose dynamic behavior has been extensively studied. The chosen study case should be complex enough to highlight some advantages of applying the new approach, but, at the same time, should avoid complexities unrelated to the modeling procedure itself. In this text, the novel methodology is applied to obtain the discretized equations of motion of a classical one-dimensional problem of Fluid-Structure Interaction, for which a comprehensive treatise (with an extensive literature review) has been written by Païdoussis (2014): the problem of a pipe conveying fluid. With several studies performed along the last eight decades, this problem is characterized by an interesting dynamic behavior concerning transitions to unstable regimes according to the relations among the physical parameters of the pipe and the characteristics of the internal axial flow.

In this paper, the novel modeling approach is applied to a cantilevered pipe, constituted by a homogeneous hollow solid cylinder that respects the hypotheses of the Linear Theory of Elasticity. The nonlinearities of the model are purely geometric, due to compatibility conditions. It is also supposed that there is an internal axial plug-flow, i.e., the boundary layer adjacent to the inner wall is neglected so that the velocity of the flow can be assumed to be constant at any point of a cross-section perpendicular to the axis of the pipe. Moreover, the environment external to the pipe can be considered as vacuum. Along the modeling procedure, neither compatibility nor boundary conditions are taken into consideration *a priori*; they are interpreted as constraints, whose effect can be computed *a posteriori* by the modular methodology. The following section presents a generalized discrete Hamiltonian formulation for the problem when a Galerkin-like discretization is introduced. After that, comes a section in which a specialized formulation based on the Finite Element Method (FEM) is derived. The subsequent section shows some results associated to numerical simulations performed with an algorithm that reconciles the use of FEM along with the novel modular modeling algorithm. At the final section, a brief discussion and conclusive remarks are drawn.

As a general remark on notation, it is worth mentioning that the symbols adopted in this text try to follow as much as possible both the notations from Orsino (2016) and from Païdoussis (2014). When a given variable stands for a matrix, the lowercase notation will be associated to column-matrices and the uppercase notation to matrices that can only be represented by two-dimensional arrays. Moreover, when a and b stand for matrices, the operators "$\cdot$" and "$\otimes$" are defined as follows:

$$a \cdot b = a^{\mathrm{T}} b \qquad \text{and} \qquad a \otimes b = a b^{\mathrm{T}} \tag{5}$$

GENERAL DISCRETE FORMULATION

Consider the pipe as a hollow homogeneous cylinder whose nondeformed configuration corresponds to its center line as a straight line parallel to the direction of the local gravitational field, which is given by the unit vector $\hat{\mathbf{e}}_x$. Assume a planar motion for the system, i.e., any motion perpendicular to the plane of curvature (a.k.a. out-of-plane motions) can be neglected. Let $\hat{\mathbf{e}}_y$ be the unit vector normal to the plane of motion, and adopt $\hat{\mathbf{e}}_z = \hat{\mathbf{e}}_x \times \hat{\mathbf{e}}_y$. Consider that the coordinate system whose origin is the center of the inlet of the pipe in the nondeformed configuration and whose Cartesian basis is defined by the unit vectors $(\hat{\mathbf{e}}_x, \hat{\mathbf{e}}_y, \hat{\mathbf{e}}_z)$, is rigidly attached to an inertial reference frame. Suppose also that the diameter of the pipe is small compared to its length, so that the Kirchhoff-Love beam model can be adopted. The position vector of any point along the center line of the pipe is denoted by $\mathbf{r} = x\hat{\mathbf{e}}_x + z\hat{\mathbf{e}}_z$, with x and z being parametric functions of time and of a single spatial coordinate, which can be either the arc-length coordinate of the center line, s, or the x-coordinate of the corresponding point in the nondeformed configuration, x_0. Adopting a Galerkin-like discretization, it can be stated that for both x and z, there is a column-matrix of linearly independent real-valued functions of a real variable, which is either s or x_0, and another column-matrix of time-dependent generalized coordinates, q_x and q_z respectively, such that x and z can be approximated as follows:

$$x = n_x(s) \cdot q_x = \bar{n}_x(x_0) \cdot q_x \quad \text{and} \quad z = n_z(s) \cdot q_z = \bar{n}_z(x_0) \cdot q_z \tag{6}$$

where $n_x(s)$ and $n_z(s)$ are projection functions in the deformed configurations and $\bar{n}_x(x_0)$ and $\bar{n}_z(x_0)$, respectively, are their corresponding forms in the nondeformed configuration. Denoting by U the magnitude of velocity of the internal plug-flow with respect to the pipe, it can be stated that the velocities associated to a point on the central line of the pipe at a given cross section (defined by the coordinate s) of the pipe ($\mathbf{v}_\text{p}$) and of the flow ($\mathbf{v}_\text{f}$) with respect to an inertial reference frame are given by:

$$\mathbf{v}_\text{p} = \frac{\partial \mathbf{r}}{\partial t} = n_x(s) \cdot \dot{q}_x \hat{\mathbf{e}}_x + n_z(s) \cdot \dot{q}_z \hat{\mathbf{e}}_z \tag{7}$$

$$\mathbf{v}_\text{f} = \frac{\partial \mathbf{r}}{\partial t} + U\frac{\partial \mathbf{r}}{\partial s} = (n_x(s) \cdot \dot{q}_x + U n_x'(s) \cdot q_x)\hat{\mathbf{e}}_x + (n_z(s) \cdot \dot{q}_z + U n_z'(s) \cdot q_z)\hat{\mathbf{e}}_z \tag{8}$$

The application of the McIver's (1973)[1] extended form of Hamilton's principle to this problem leads to the following expression (Païdoussis, 2014):

$$\delta \int_{t_1}^{t_2} L \,\mathrm{d}t + \int_{t_1}^{t_2} \delta \tilde{W}_\text{S} \,\mathrm{d}t = 0 \tag{9}$$

$$\delta \tilde{W}_\text{S} = -\iint_\text{S} p(\delta \mathbf{r} \cdot \hat{\mathbf{n}}) \,\mathrm{dS} + \iint_\text{S} \rho_\text{f}(\delta \mathbf{r} \cdot \mathbf{v})(\mathbf{v}_\text{S} - \mathbf{v}) \cdot \hat{\mathbf{n}} \,\mathrm{dS} \tag{10}$$

In these expressions, L represents the Lagrangian of the system and $\delta \tilde{W}_\text{S}$ stands for the sum of two surface integrals computed in the inlet and outlet surfaces of the pipe, which are generically denoted by S: the first associated to the virtual work due to external pressure, p, and the second associated to the flux of momentum due to the flow, with ρ_f standing for the density of the fluid, $\mathbf{v}$ standing for the velocity of the fluid particles that are crossing the surface S and $\mathbf{v}_\text{S}$ standing for the the velocity of a point of the surface that instantly coincides with this particle.

Denoting by d_p the internal diameter of the pipe, defining $m_\text{f} = \rho_\text{f} \pi d_\text{p}^2/4$ as the mass of fluid per unit of length of the pipe, and assuming that the cross-section of the pipe corresponding to the inlet surface is clamped and that, at the outlet, $p = 0$, it can be stated that:

$$\delta \tilde{W} = -m_\text{f} U \left(\frac{\partial \mathbf{r}}{\partial t}(\ell) + U \frac{\partial \mathbf{r}}{\partial s}(\ell) \right) \cdot \delta \mathbf{r}(\ell) \tag{11}$$

Moreover, the Lagrangian of the system is given by $L = T - V$ with, T corresponding to the kinetic energy of the system and V to the potential energy, whose expressions, considering the hypotheses adopted, are:

$$T = \frac{1}{2} \int_0^\ell \left(m_\text{p} \mathbf{v}_\text{p} \cdot \mathbf{v}_\text{p} + m_\text{f} \mathbf{v}_\text{f} \cdot \mathbf{v}_\text{f} \right) \mathrm{d}x_0 \tag{12}$$

$$V = \frac{1}{2} \int_0^\ell \left(EA\varepsilon^2 + EI(1+\varepsilon^2)\kappa^2 \right) \mathrm{d}x_0 - \int_0^\ell (m_\text{p} + m_\text{f}) \mathbf{g} \cdot \mathbf{r} \,\mathrm{d}x_0 \tag{13}$$

In these equations m_p stands for the mass per unit of length of the pipe, EA for the axial rigidity, EI for the flexural rigidity. Also, ε is the normal strain and κ is the curvature, both measured at the center line of the pipe, which can be computed by the following expressions:

$$(\varepsilon + 1)^2 = \left(\frac{\partial x}{\partial x_0} \right)^2 + \left(\frac{\partial z}{\partial x_0} \right)^2 = \left(\frac{\mathrm{d}\bar{n}_x}{\mathrm{d}x_0}(x_0) \cdot q_x \right)^2 + \left(\frac{\mathrm{d}\bar{n}_z}{\mathrm{d}z_0}(x_0) \cdot q_z \right)^2 \tag{14}$$

$$\kappa^2 = \left(\frac{\partial^2 x}{\partial s^2} \right)^2 + \left(\frac{\partial^2 z}{\partial s^2} \right)^2 = (n_x''(s) \cdot q_x)^2 + (n_z''(s) \cdot q_z)^2 \tag{15}$$

[1] As a matter of fact, after Casetta and Pesce (2013) stated a more general extended form of Hamilton's principle for non material volumes, a question arouse if McIver's form would suffice for the problem. In Casetta and Pesce (2013), the extended Hamilton's principle is obtained with an extra term in equation (10), related to flux of kinetic energy, what enables recovering the Lagrange's equation for non material volumes derived by Irschik and Holl (2002) some years earlier. The question on McIver's form sufficiency was answered by Kheiri and Paidoussis (2014), for the usual case in which the pipe is already completely full of liquid, as in the present paper. However, in the present author's opinion, in the transient cases of liquid filling, starting from an empty pipe state, or of two-phase plug-flows, such a sufficiency has to be discussed further.

with the prime notation standing for derivatives with respect to s.

Consider a cantilevered pipe (clamped inlet, free outlet) and assume that the effect of axial vibrations is negligible, such that the inextensibility condition is satisfied. Thus, $\delta s = \delta x_0$, which leads to the following identity:

$$\left(\frac{\partial x}{\partial x_0}\right)^2 + \left(\frac{\partial z}{\partial x_0}\right)^2 = 1 \quad \text{or} \quad \left(\frac{\partial x}{\partial s}\right)^2 + \left(\frac{\partial z}{\partial s}\right)^2 = 1 \tag{16}$$

Therefore, according to equation (14), $\varepsilon = 0$ and:

$$(n_x'(s) \cdot q_x)^2 + (n_z'(s) \cdot q_z)^2 = 1 \tag{17}$$

$$q_x \cdot (n_x'(s) \otimes n_x'(s))\delta q_x + q_z \cdot (n_z'(s) \otimes n_z'(s))\delta q_z = 0 \tag{18}$$

Note that equation (18) can be rewritten as $A_x \delta q_x + A_z \delta q_z = 0$. Therefore, the inextensibility condition is in fact a constraint, which, by the modular methodology, can be included *a posteriori*. Thus, the remaining of the derivation presented in this section will be performed letting this condition aside. Despite the similarities between the expressions presented below and the derivations of a linearized model, this does not mean that any further simplification has been made. The model still applies to any planar motion of the pipe, apart the inextensibility condition, no matter how significant are the effects of the geometric nonlinearities.

After the foregoing considerations it can be stated that, after discretization:

$$\delta \int_{t_1}^{t_2} T \, \mathrm{d}t = \int_{t_1}^{t_2} \sum_{j=x,z} \left(\delta \dot{q}_j \cdot (M_j \dot{q}_j - B_j^{\mathrm{T}} U q_j) - \delta q_j \cdot (B_j U \dot{q}_j + W_j U^2 q_j)\right) \mathrm{d}t \tag{19}$$

Integrating by parts and considering that $\delta q_x = \delta q_z = 0$ at the time instants t_1 and t_2:

$$\delta \int_{t_1}^{t_2} T \, \mathrm{d}t = -\int_{t_1}^{t_2} \sum_{j=x,z} \left(\delta q_j \cdot (M_j \ddot{q}_j + (B_j - B_j^{\mathrm{T}}) U \dot{q}_j + (W_j U^2 - B_j^{\mathrm{T}} \dot{U}) q_j)\right) \mathrm{d}t \tag{20}$$

with the matrices M_j, B_j and W_j defined as follows:

$$M_j = \int_0^\ell (m_{\mathrm{p}} + m_{\mathrm{f}})(n_j \otimes n_j) \, \mathrm{d}s, \quad B_j = -\int_0^\ell m_{\mathrm{f}} (n_j' \otimes n_j) \, \mathrm{d}s \quad \text{and} \quad W_j = -\int_0^\ell m_{\mathrm{f}} (n_j' \otimes n_j') \, \mathrm{d}s \tag{21}$$

Moreover, considering that under the adopted hypotheses $\varepsilon = 0$, the terms related to the potential energy can be expressed as follows:

$$\delta \int_{t_1}^{t_2} V \, \mathrm{d}t = \int_{t_1}^{t_2} \int_0^\ell (EI\kappa \, \delta\kappa - (m+M) g \, \delta x) \, \mathrm{d}s \, \mathrm{d}t = \int_{t_1}^{t_2} \sum_{j=x,z} \left(\delta q_j \cdot (K_j q_j - g_j)\right) \mathrm{d}t \tag{22}$$

with K_j and g_j defined by:

$$K_j = \int_0^\ell EI (n_j'' \otimes n_j'') \, \mathrm{d}s, \quad g_x = \int_0^\ell (m_{\mathrm{p}} + m_{\mathrm{f}}) g n_x \, \mathrm{d}s \quad \text{and} \quad g_z = 0 \tag{23}$$

Finally, the discretized form of the term related to the energy flow in the inlet and outlet surfaces of the pipe, is given by:

$$\int_{t_1}^{t_2} \left(m_{\mathrm{f}} U \left(\frac{\partial \mathbf{r}}{\partial t}(\ell) + U \frac{\partial \mathbf{r}}{\partial s}(\ell) \right) \cdot \delta \mathbf{r}(\ell) \right) \mathrm{d}t = \int_{t_1}^{t_2} \sum_{j=x,z} \left(\delta q_j \cdot (\Theta_j U \dot{q}_j + \Omega_j U^2 q_j)\right) \mathrm{d}t \tag{24}$$

with Θ_j and Ω_j computed as follows:

$$\Theta_j = (m_{\mathrm{f}} n_j \otimes n_j)\Big|_{s=\ell} \quad \text{and} \quad \Omega_j = (m_{\mathrm{f}} n_j \otimes n_j')\Big|_{s=\ell} \tag{25}$$

Thus, the generic mathematical model associated to the system, if neither compatibility nor boundary conditions are considered, is given by:

$$M_j \ddot{q}_j + (B_j - B_j^{\mathrm{T}} + \Theta_j) U \dot{q}_j + ((W_j + \Omega_j) U^2 - B_j^{\mathrm{T}} \dot{U} + K_j) q_j = g_j \quad \text{for} \quad j = x, z \tag{26}$$

FEM FORMULATION

In order to be able to perform numerical simulations using the derived mathematical model, the Finite Element Method (Ibrahimbegovic, 2009) is going to be applied to complete the derivation of the discretized model. Thus, the pipe will be modeled as being constituted by a finite number ν of two node elements, such that the k-th element is delimited by the nodes $(k-1)$ and k, each of them lying in the approximated center line of the pipe. For each node, let s_k stand for the the arc-length coordinate of the node k, and define:

$$(x_k, x_k', z_k, z_k') = \left(x(t, s_k), \frac{\partial x}{\partial s}(t, s_k), z(t, s_k), \frac{\partial z}{\partial s}(t, s_k) \right) \tag{27}$$

Moreover, define the column-matrix of generalized coordinates of the model, represented in tuple form, as follows:

$$q = (x_0, x_0', z_0, z_0', \dots, x_k, x_k', z_k, z_k', \dots, x_\nu, x_\nu', z_\nu, z_\nu') \tag{28}$$

Assume that, for each element, there is a continuous, bijective and monotonically increasing function, $\xi : [s_{k-1}, s_k] \to [-1, +1]$, such that for $s_{k-1} \leq s \leq s_k$:

$$j = \hat{n}_{k,0,-1}(\xi(s))\, j_{k-1} + \hat{n}_{k,1,-1}(\xi(s))\, j_{k-1}' + \hat{n}_{k,0,+1}(\xi(s))\, j_k + \hat{n}_{k,1,+1}(\xi(s))\, j_k' \quad \text{for} \quad j = x, z \tag{29}$$

Let, for instance:

$$s(\xi) = \frac{s_k - s_{k-1}}{2}\xi + \frac{s_k + s_{k-1}}{2} \quad \Rightarrow \quad \frac{\mathrm{d}s}{\mathrm{d}\xi}(\xi) = \frac{s_k - s_{k-1}}{2} = \frac{\ell_k}{2} \quad \Rightarrow \quad \int_{s_{k-1}}^{s_k} (\cdot)\,\mathrm{d}s = \frac{\ell_k}{2}\int_{-1}^{+1} (\cdot)\,\mathrm{d}\xi \tag{30}$$

In order to satisfy the conditions: $j(s_{k-1}) = j_{k-1}$, $j'(s_{k-1}) = j_{k-1}'$, $j(s_k) = j_k$ and $j'(s_k) = j_k'$, for $j = x, z$, the following interpolating polynomials can be adopted:

$$\begin{cases} \hat{n}_{k,0,-1}(\xi) = \frac{1}{4}\left(2 - 3\xi + \xi^3\right) & \hat{n}_{k,0,+1}(\xi) = \frac{1}{4}\left(2 + 3\xi - \xi^3\right) \\ \hat{n}_{k,1,-1}(\xi) = \frac{1}{8}\ell_k(\xi - 1)^2(\xi + 1) & \hat{n}_{k,1,+1}(\xi) = \frac{1}{8}\ell_k(\xi - 1)(\xi + 1)^2 \end{cases} \tag{31}$$

Replacing these interpolating polynomials expressions in equations (21, 23, 25), and considering the last two identities in equation (30) the corresponding block diagonal matrices can be computed.

Also, if these interpolating polynomials are replaced in (17, 18), constraint equations representing the inextensibility condition can be written down. Particularly, the equations corresponding to the imposition of this condition to the nodes ($\xi = -1, +1$) and to the midpoint of each element ($\xi = 0$) are given respectively by:

$$(x_k')^2 + (z_k')^2 = 1, \quad \text{for} \quad k = 0, 1, \dots, \nu \tag{32}$$

$$\left(\frac{3}{2}\frac{(x_k - x_{k-1})}{\ell_k} - \frac{1}{2}\frac{(x_{k-1}' + x_k')}{2}\right)^2 + \left(\frac{3}{2}\frac{(z_k - z_{k-1})}{\ell_k} - \frac{1}{2}\frac{(z_{k-1}' + z_k')}{2}\right)^2 = 1, \quad \text{for} \quad k = 1, \dots, \nu \tag{33}$$

Furthermore, due to the boundary conditions of the problem, which were not already taken into consideration during the modeling procedure, it can be stated that the following constraint equations must be satisfied:

$$x_0 = 0, \quad x_0' = 1, \quad z_0 = 0 \quad \text{and} \quad z_0' = 0 \tag{34}$$

Thus, in this paper the system will be represented by a hierarchy with four levels, as ilustrated in Figure 1: at the bottom level (level 0), the mathematical model described by equations (26) with the corresponding coefficient matrices obtained by the Finite Element Method according to the foregoing considerations; at level 1 there is the enforcing of the constraints corresponding to the boundary conditions, equations (34); at level 2, the constraints due to the imposition of the inextensibility condition at the nodes, equations (32), are also enforced; finally, at the top level of this hierarchy (level 3), the constraints associated to the inextensibility condition applied to the midpoint of each element, equations (33), are additionally enforced. Note that the model at level 2 might provide satisfactory results once both boundary and compatibility conditions are being considered; it is expected, however, for the model at the top level of the structure to provide even more realistic results.

Level 3
Enforcement of the inextensibility condition at the midpoint of each element, as described by equations (33).

Level 2
Enforcement of the inextensibility condition at each node, as described by equations (32).

Level 1
Enforcement of constraints due to boundary conditions at the clamped edge (node 0), described by equations (34).

Level 0
Accelerations given by the FEM-discretized equation (26).
No boundary or inextensibility conditions imposed.

Figure 1 – Representation of the problem of a pipe conveying fluid as a hierarchy of four levels.

NUMERICAL SIMULATIONS

In order to promote a qualitative assessment of the model derived in the previous sections, a numerical simulation is proposed. Before choosing the simulation parameters it is convenient to redefine the variables of the system, in order to be able to write down the associated equations of motion in terms of nondimensional quantities. Basically, such a procedure involves finding nondimensional variables to replace time (t), the arc-length coordinate of the center line (s) and the associated Cartesian coordinates (x and z). Using the asterisk as a superscript to denote the corresponding nondimensional variables, the definitions adopted will follow the convention proposed by Yoshizawa et. al. (Yoshizawa et. al., 1985; Yoshizawa et. al., 1986):

$$t = t^* \sqrt{\frac{(m_\mathrm{p} + m_\mathrm{f})\ell^4}{EI}}, \quad s = s^* \ell, \quad x = x^* \ell \quad \text{and} \quad z = z^* \ell \tag{35}$$

In this convention the length scale of the problem is defined by the total length of the pipe, such that all the length variables of the model must lay in the interval $[0,1]$. Also, in this convention the time scale of the problem is defined by a period associated to the bending stiffness effects. From now on, whenever any mention to any these variables is done, it should refer to the nondimensional forms, so that the asterisk notation can be simply ommited. Assuming that the magnitude of the relative velocity of the plug-flow with respect to the pipe U is independent of the time and of the state of the pipe and performing these variable replacements, it can be stated that the model depends on three nondimensional independent parameters only:

$$\upsilon = U \sqrt{\frac{m_\mathrm{f} \ell^2}{EI}}, \quad \beta = \frac{m_\mathrm{f}}{m_\mathrm{p} + m_\mathrm{f}} \quad \text{and} \quad \gamma = g\ell^3 \frac{m_\mathrm{p} + m_\mathrm{f}}{EI} \tag{36}$$

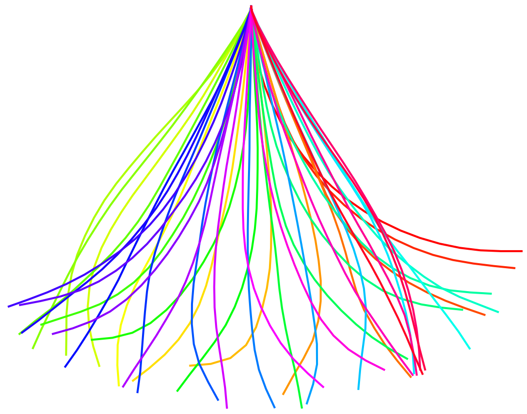
υ stands for the nondimensional magnitude of the relative velocity of the flow (with respect to the pipe) according to the space and time scales adopted, β is associated to the relative contribution of the flow to the total inertial forces and γ is associated to the relative magnitude of the gravitational forces with respect to the flexural ones. In the simulation performed, the physical parameters are selected such that $\upsilon = 2.5$, $\beta = 0.16$ and $\gamma = 2^{13} = 8192$. The chosen value for γ makes the gravitational force effects dominant in the dynamics of the system. Thus, its response should somehow resemble the dynamic response of a multiple pendulum, which indeed happens, as it can be noticed in the results presented in Figure 2.

The pipe is divided in 21 elements (22 nodes) whose nondimensional lengths are $\ell_1 = 1/80$, $\ell_2 = 3/80$ and $\ell_k = 1/20$, for $k \geq 3$. The initial configuration is taken such that the nodes of the first 20 elements lay in a 90 degree circular arc and the last two nodes remain horizontally aligned. The simulations are performed using a routine that involves, for each time step, the use of equations (26) to obtain the values of $\ddot{q}$ associated to the unconstrained system (level 0 of the hierarchy), followed by a constraint enforcement algorithm, based on the modular modeling methodology and on Udwadia-Kalaba equations[2] (Udwadia and Kalaba, 2002; Udwadia and Kalaba, 1992) that estimates the corresponding values of $\ddot{q}$ satisfying the boundary and inextensibility conditions, by a constraint stabilization algorithm based on Baumgarte's (1972) technique. This latter technique improves the estimation. An explicit integration algorithm is used to compute the approximate state at the next time step, using the 4th order Runge-Kutta method. The simulation shown in Figure 2 ran in two stages: the first covering the nondimensional time interval $[0, 0.125]$ with a nondimensional time step of $1/2^{15}$ and the second covering the interval $[0.1, 0.3]$ with nondimensional time step of $1/25000$ (approximately 30% greater), using the result obtained in the first stage as initial configurations at the instant 0.1. As it can be noticed, particularly in Figures 2e and 2f, there is no incompatibility among the results obtained in these two stages in the intersection between the two nondimensional time intervals. It is also worth noting that the results indicate that the imposed constraints are satisfied, representing adequately, for this discretized model, the boundary (Figure 2b) and inextensibility (Figure 2d) conditions.

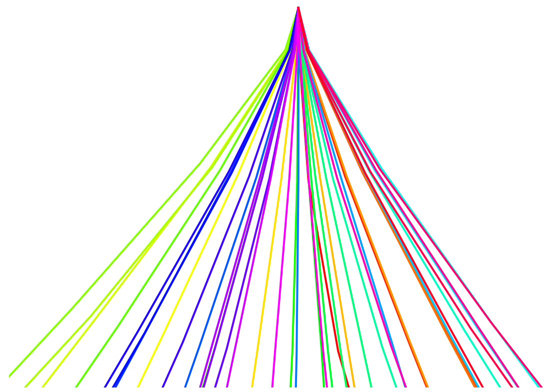
CONCLUSIONS

This paper showed that, provided an adequate restatement of the problem, the modular modeling methodology can be useful in the derivation of discretized equations of motion for a distributed parameters system. Whenever it is possible to take advantage of not considering boundary or compatibility conditions since the early stages of the modeling procedure, such a strategy could be adopted, once the modular modeling methodology allows an *a posteriori* computation of the effects associated to these conditions in the dynamic behavior of a system. The hierarchical conception of a system not only can simplify the procedure for obtaining its equations of motion, but also makes modeling procedures less dependent on the hypotheses adopted, allowing even the reutilization of the lower levels of the hierarchy for the modeling of different systems. For the case study presented, a cantilevered pipe conveying fluid, the application of the modular methodology was particularly encouraging, once the only source of nonlinearities, according to the hypotheses, was the inextensibility condition: adopting redundant coordinates, the FEM-discretization stage required to obtain the model at the level 0 of the hierarchy becomes identical to a linear formulation. Both boundary and inextensibility conditions were treated as

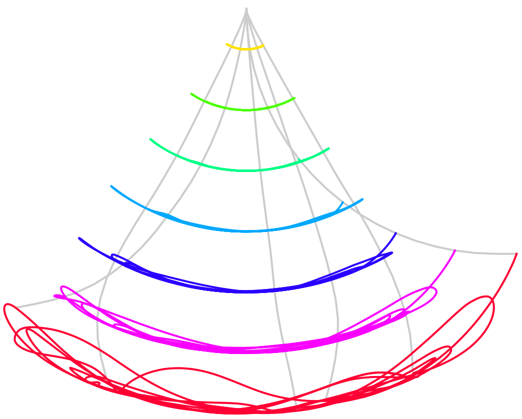
[2] In this case, the projection operator onto the kernel of B_{r+1} is obtained by $C_{r+1} = (I - B_{r+1}^+ B_{r+1})$, with I representing an identity operator and B_{r+1}^+ being the Moore-Penrose pseudo-inverse of the operator B_{r+1}.



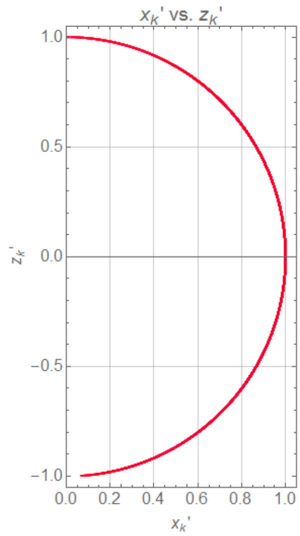
(a) Configuration snapshots of the pipe



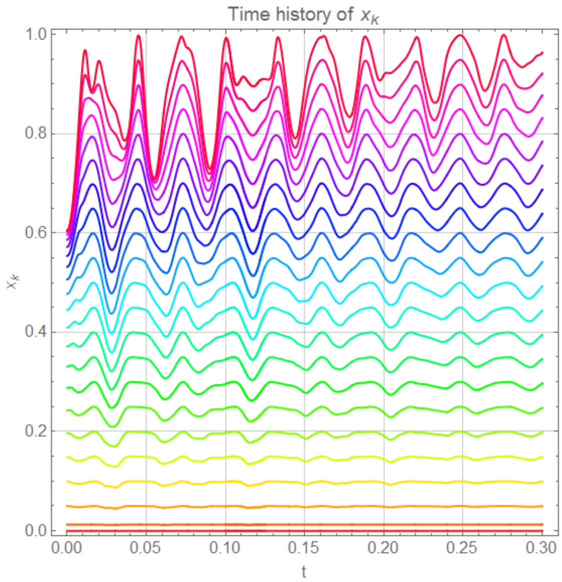
(b) Detail of the snapshots close to the clamped edge



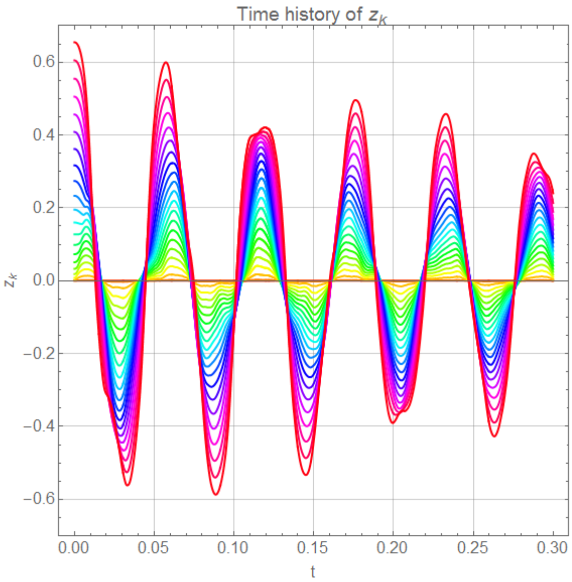
(c) Trajectories of some nodes



(d) x_k' versus z_k' for all the nodes



(e) Time histories of x_k, $k = 1, \dots, 22$



(f) Time histories of z_k, $k = 1, \dots, 22$

Figure 2 – Results of the numerical simulation (all the quantities represented are nondimensional)

constraints in the formulation of this problem, and the numerical simulation performed showed that, for the discretization adopted, the model leads to satisfactory results in which these conditions are adequately satisfied. Therefore, the results obtained in this paper motivate further investigations on the development of an extended form of the modular modeling methodology for applications to distributed parameters dynamic systems. Stability and dynamic analysis of the addressed problem are left for further papers.

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