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Nonlinear Free Vibration Analysis of Multi-Articulated Offshore Towers

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Abstract: Articulated towers are compliant offshore structures economically attractive in the oil industry. As the water depth increases the tower could be built-up with multi-framed structures, generally modeled as rigid bars linked by articulated joints. The restoring forces are included by the introduction of rotational springs in the structural model. When subjected to sea environmental loads, articulated towers may exhibit large displacements. A nonlinear analysis is thus required. In this work the nonlinear normal modes (NNMs) technique is used to derive a reduced order model to study the free nonlinear vibration of a tri-articulated tower. The modal surfaces are obtained by the asymptotic method. The obtained modal oscillators present only odd nonlinearities, and the harmonic balance method is then applied to obtain the frequency-amplitude curves analytically. The results show hardening behavior for the first mode and softening behavior for both second and third modes. These curves are compared with results obtained by the numerical solution of the original equations of motion via Runge-Kutta integration on time domain, at chosen points near the equilibrium point − used as reference solution. These comparisons show a good agreement between the proposed analytical solution and the reference one, revealing also the validity region for the asymptotic method for each mode. The domain of validity of the expansion used is also checked by comparing the phase portrait obtained using the modal oscillators and the numerical integration of the system of equations of motion, confirming the effectiveness of the reduced order model derived from the nonlinear modal analysis from this study. A comparison between the free vibration analysis and analysis considering hydrostatic forces is also carried out, revealing the influences on the structure′s natural frequencies.

Keywords: Offshore structures, nonlinear vibration, nonlinear normal modes, invariant manifolds, articulated towers.

INTRODUCTION

Articulated towers are a class of compliant structures suitable for many offshore applications. Compliant platforms are economically attractive in the offshore industry due to their reduced weight compared to conventional platform (Kirk and Jain, 1977; Nagamani and Ganapathy, 2000). While fixed offshore structures are designed to withstand the sea environmental loads without considerable displacements, compliant structures, having small natural frequencies, are designed to allow limited displacements. The structure does not supply resistance to horizontal forces due to wind, ocean currents or waves. However, restoring forces are provided through buoyancy, ballast, guy wires, axial piles, tendons and other devices (Bar-Avi and Benaroya, 1997; Sellers and Niedzwecki, 1992). Using this structural concept, the natural frequencies of compliant towers are designed to be lower than the wave excitation frequencies (Islam et al., 2009), avoiding the effects of dynamical amplification due to resonance with environmental loads.

A typical articulated tower consists of a single leg articulated tower supporting a platform on its top and fixed at the bottom. The support at the bottom of the tower can be a sole universal joint in the tower system, and the structure can also use intermediate universal joints, in a multi-hinged or multi-articulated tower configuration, which is more attractive to deep water systems (Zaheer and Islam, 2012; Nagamani and Ganapathy, 2000).

Due to their reduced stiffness and environmental loading conditions, compliant structures may present large displacements and rotations, which may lead to nonlinear effects on a more precise vibration analysis (Tabeshpour *et al.,* 2006). In general, discretization schemes, such as those based on the finite element method, are used to perform a structural nonlinear dynamic analysis of such problems. However, the large number of degrees-of-freedom necessary to describe precisely these problems, in face of the coupled terms generated by the considered nonlinearities, turns the analysis task cumbersome, limiting parametric analyses in initial design stages (Pesheck *et al.*, 2001). An alternative to overcome such difficulties is to use reduced order models.

The use of reduced order conceptual models became an attractive approach for performing nonlinear dynamic analysis of compliant offshore structures in the last decades. When analytically derived, such models and its solutions can be used to understand the dynamic behavior of offshore structures subjected to large displacements *(Falzarano et al.*, 2001; Happawana *et al.*, 1995).

Nonlinear normal modes (NNMs) are a very useful tool to derive reduced order models in nonlinear vibration analysis of structures and machines (Rosenberg, 1962; Shaw and Pierre, 1991; Vakakis, 1991). By capturing the nonlinear essence of the problem and by retaining explicitly the structural dynamical dependency of some physical parameters, they can result in analytical reduced order models that facilitate the parametric analysis, leading to a better understanding of the structure's nonlinear behavior. NNMs have emerged as an extension of their linear counterpart. However, they do not exhibit some of fundamental features of the linear modal analysis such as superposition and solution uniqueness (Gavassoni *et al.*, 2015). They provide a solid theoretical and mathematical tool for interpreting a wide class of nonlinear dynamic phenomena. Reduced order models obtained by the use of NNMs are able to capture the contribution of several modes, thus allowing a smaller number of modes to generate a precise model compared to standard reduction approaches such as those using linear modes (Gavassoni *et al.*, 2014).

The invariant manifold based definition of nonlinear normal modes is used in this work along with asymptotic methods in order to derive analytical reduced order models to study the nonlinear vibration of a tri-articulated offshore tower, based on the work of Shaw and Pierre (1991). The equations of motion are derived using the Euler-Lagrange equation. The reduced order model is used to investigate the fundamental tower behavior under nonlinear free vibration. The validation of the expansion is verified by the comparison of the phase portrait and the amplitude-frequency relation obtained via the reduced order model and the numerical integration of the original system of equations of motion.

STRUCTURAL MODEL

In this study, is adopted the model of a tri-articulated tower based on rigid members linked by universal joint connections, according to the basic model studied by Sellers and Niedzwecki (1992). The universal joints restrict the degrees-of-freedom of the system but do not supply the tower with any restoring force when it is displaced by wave, currents or winds. The restoring forces are provided by buoyancy, ballast and mooring systems, as shown in Fig. 1(a). The restoring forces resulting from ballast and mooring are modelled as rotational springs, as illustrated in the model presented in Fig. 1(b), while the buoyancy effect is described on the energetic formulation. The spring stiffness is denoted by k_i. The deck and facilities loads are considered by a single mass, m, located at the top of the tower. Each structural member is considered to be infinitely rigid, and has its length denoted by l_i and cross sectional area equals to A_i, while the specific weight of the rigid members material is ρ_i.

This model results in a three-degree-of-freedom problem, corresponding to the angles of rotation, θ_i, at the base of each member. By employing the Lagrange formulation, the nonlinear vibration of the problem is mathematically described by three coupled nonlinear differential equations in terms of the generalized coordinates θ_i as described as follow.

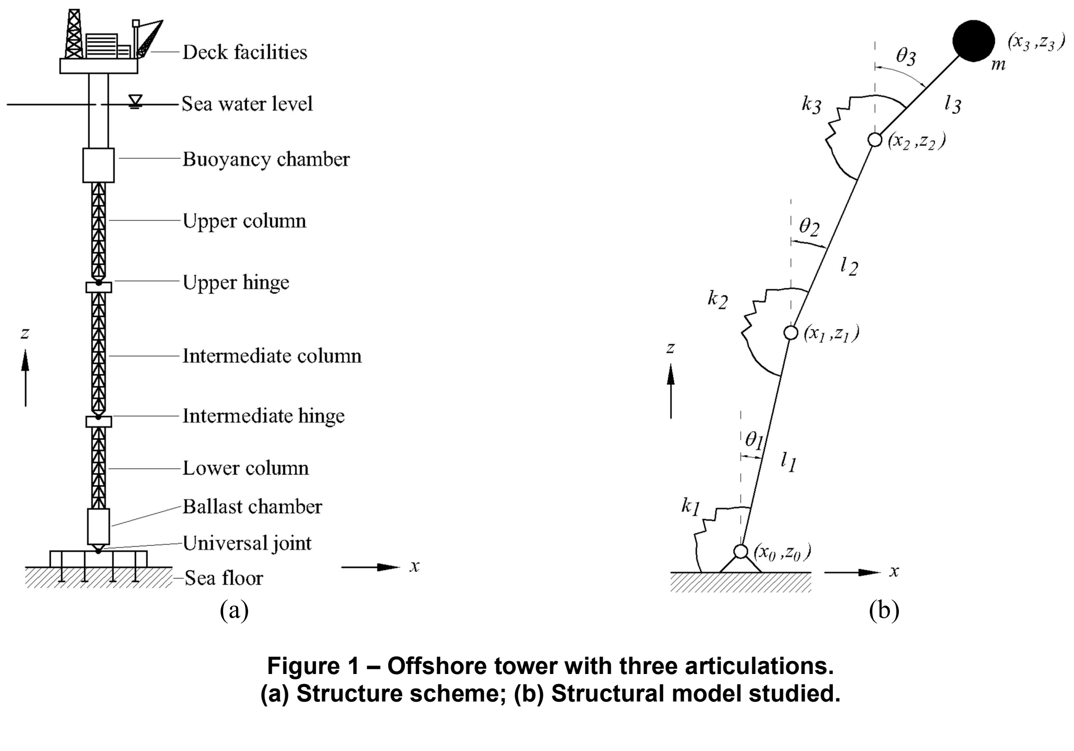


Figure 1 – Offshore tower with three articulations.
(a) Structure scheme; (b) Structural model studied.

Mathematical Formulation

The equations of movement are obtained in relation to an equilibrium position, described mathematically considering a static analysis of the problem.

The total potential energy of the tower, Π, is equal the sum of the elastic potential energy of the tower, U, and the potential energy due the external loads, V, described as follows:

$$\Pi = U + V \tag{1}$$

$$U = \sum_{i=1}^{3} [k_i(\theta_i - \theta_{i-1})], \theta_0 = 0 \tag{2}$$

$$V = -W \tag{3}$$

The potential energy due to external loads, V, is equal the negative of the external loads work, as shown in Eq. (4). The forces considered in the elastic analysis are the platform and facilities weight, the columns weight and buoyancy forces:

$$W = W_{plataform} + W_{columns} + W_{buoyancy} \tag{4}$$

The work done by the platform weight, $W_{platform}$, is given by:

$$W_{plataform} = mg \sum_{i=1}^{3} [l_i(1 - cos\theta_i)] \tag{5}$$

where g stand for the gravity acceleration constant.

The work performed by the weight of the column rigid members, $W_{columns}$, is equal to:

$$W_{columns} = g \sum_{i=1}^{3} \rho_i l_i A_i \left[\sum_{j=i}^{3} l_j(1 - cos\,\theta_j) + \frac{l_i(1 - cos\,\theta_i)}{2} \right] \tag{6}$$

where ρ_i is the specific mass of the columns material.

The work performed by the buoyancy forces acting on the columns of the structure, $W_{buoyancy}$, is given by:

$$W_{buoyancy} = -g \sum_{i=1}^{3} \rho_w l_i A_i \left[\sum_{j=i}^{3} l_j(1 - cos\,\theta_j) + \frac{l_i(1 - cos\,\theta_i)}{2} \right] \tag{7}$$

where ρ_w is the specific mass of water.

The total kinetic energy of the tower. T, is equal the sum related to three dynamic displacements: due the displacement of the platform and facilities mass, $T_{platform}$, the displacement of the columns masses, $T_{columns}$, and due the considered added mass effect, under the consideration that the structural members take movement totally underwater, T_{added}. These are described as follow:

$$T_{plataform} = \frac{1}{2}\left[\left(\sum_{i=1}^{3} l_i\,cos\,\theta_i \right)^2 + \left(\sum_{i=1}^{3} -l_i sen\theta_i \right)^2 \right] \tag{8}$$

$$T_{columns} = \frac{1}{2}\rho_i \left[\sum_{i=1}^{3} l_i A_i \left[\left(\sum_{k=1}^{i-1} l_k\,cos\,\theta_k + \frac{1}{2} l_i\,cos\,\theta_i \right)^2 + \left(\sum_{k=1}^{i-1} -l_k\,sin\,\theta_k - \frac{1}{2} l_i \mathrm{sen}\theta_i \right)^2 \right] + \frac{A_i l_i^3}{12}\left(\dot{\theta}_i\right)^2 \right] \tag{9}$$

$$T_{added} = \rho_w \sum_{i=1}^{3} \frac{\pi d_0^{\,2}}{8} C_{A,i} \left[\left(\sum_{k=1}^{i-1} l_k\,cos\,\theta_k \dot{\theta}_i + \frac{l_i\,cos\,\theta_i \dot{\theta}_i}{2} \right) - \left(\sum_{k=1}^{i-1} l_k sin\theta_k \dot{\theta}_i - \frac{l_i\,sin\,\theta_i \dot{\theta}_i}{2} \right) \right] cos\,\theta_i \dot{\theta}_i \tag{10}$$

where $C_{A,i}$ is the added mass coefficient.

The Lagrangean obtained for the considered system is given by:

$$Lg = T - \Pi \tag{11}$$

Through the application of variational techniques to Eq. (11), the global dynamic of the system is obtained and governed by three equations of movement, whose are obtained with use of:

$$\frac{\partial Lg}{\partial \theta_i} - \frac{d}{dt}\left(\frac{\partial Lg}{\partial \dot{\theta}_i}\right) = 0, i = 1..3 \quad (12)$$

NONLINEAR NORMAL MODES

The nonlinear modal analysis performed in this work uses the invariant manifold approach proposed by Shaw and Pierre (1993). According to this approach, a nonlinear normal mode is a motion that takes place on a two-dimensional invariant manifold, *i.e.*, a surface in the system phase space, which is tangent, at the equilibrium position, to the eigenspace formed by the linear modes of the linearized problem (Pesheck, 2000). The motion corresponding to a single nonlinear mode can be parameterized by the displacement and velocity of a single degree-of-freedom of the system, which is called the master pair. All remaining degrees-of-freedom, called slave coordinates, are related to the master pair via the constraint equations. The procedure for obtaining the NNM reduced order models presented in sequence is based on the work of Gavassoni *et al.* (2015).

The constraint equations determine the invariant manifold geometry for a given nonlinear mode. In the case of a constant constraint function, the manifold geometry is characterized by a plane in the phase system, and the mode is called a similar mode (Nayfeh *et al.*, 1996), otherwise the mode is called non-similar and the modal surface is curvilinear. This work uses the asymptotic mode to derive the nonlinear modes. To use this method the set of nonlinear equations of motion must be re-written as Cauchy first order equations (Falzarano *et al.*, 2001):

$$\begin{aligned} \{\dot{\theta}\} &= \{y\} \\ \{\dot{y}\} &= \{f\} \end{aligned} \quad (13)$$

where θ_i are the coordinates (rotations) and y_i are the corresponding velocities. The generalized force vector $\{f\}$ consists of nonlinear moments that in general depend upon $\{\theta\}$ and $\{y\}$.The dot above the variables stands for its first time derivative. Since the system of equations of motion resulting by the application of Eq. (12) are not in the Cauchy form, *i.e.*, they are not uncoupled in the inertia terms, Cramer's rule is applied to perform the decoupling.

Then a displacement-velocity pair is arbitrarily chosen as the master pair. As an example the first degree-of-freedom displacement-velocity pair, θ_1 e y_1, is chosen as the master pair:

$$\begin{aligned} u &= \theta_1 \\ v &= y_1 \end{aligned} \quad (14)$$

The other slave pairs are represented in terms of u and v by the constraint functions (P_i, Q_i):

$$\begin{aligned} \theta_i &= P_i(u,v), i = 1..3 \\ y_i &= Q_i(u,v), i = 1..3 \end{aligned} \quad (15)$$

where, particularly, $P_1(u,v)=u$, $Q_1(u,v)=v$.

The next step is to eliminate the explicit time dependence of the equations. This is done by performing the time derivatives of the constraint equations. By replacing the time derivative resulting from the chain rule into the equations of motion, Eq. (13), and by the use of the master pair definitions, Eq. (14), and the slave relations, Eq. (15), the following system of six second order partial differential equations is obtained:

$$Q_i(u,v) = \frac{\partial P_i(u,v)}{\partial u} v + \frac{\partial P_i(u,v)}{\partial v} f_1(u, P_2(u,v), ..., P_n(u,v); v, Q_2(u,v), ..., Q_n(u,v)), i = 1..3 \quad (16)$$

$$\begin{aligned} f_i(u,P_2(u,v),...,P_n(u,v);v,Q_2(u,v),...,Q_n(u,v)) &= \frac{\partial Q_i(u,v)}{\partial u} v + \frac{\partial Q_i(u,v)}{\partial v} f_1(u,P_2(u,v),...,P_n(u,v);... \\ ...v,Q_2(u,v),...,Q_n(u,v)), i = 1..3 \end{aligned} \quad (17)$$

The determination of the restraint equations leads to an order reduction of the problem, since their substitution in the original equations of motion results in a single degree-of-freedom nonlinear modal oscillator. Except for a few special cases, there is no closed exact solution for the differential partial equations that govern the invariant manifold expressed by Eq. (16) and Eq. (17). The solution can be derived in an analytical form using a Taylor series around the equilibrium configuration, taken here for simplicity as $\{\theta_0\} = 0$. Accordingly, the constraint functions can be written up to third order terms as:

$$P_i(u,v)=a_{1i}u+a_{2i}v+a_{3i}u^2+a_{4i}uv+a_{5i}v^2+a_{6i}u^3+a_{7i}u^2v+a_{8i}uv^2+a_{9i}v^3, i=1..3 \tag{18}$$

$$Q_i(u,v)=b_{1i}u+b_{2i}v+b_{3i}u^2+b_{4i}uv+b_{5i}v^2+b_{6i}u^3+b_{7i}u^2v+b_{8i}uv^2+b_{9i}v^3, i=1..3 \tag{19}$$

The substitution of Eq. (18) and Eq. (19) into the equation of motion results in an algebraic system of equations in terms of the constraint equations coefficients a_{ij} and b_{ij} and they can be sequentially solved. The so-obtained solution is valid only locally (Nayfeh *et al.*, 1996), and the validity domain is not known a priori, being determined only by comparison with numerical solutions of the original problem.

The procedure to obtain reduced order models using the NNMs can be summarized in six main steps *(Kerschen et al.*, 2009):

1) Choose the master coordinates pair, rewritten as u and v;
2) Express slaved coordinates as functions $P_i(u,v)$ and $Q_i(u,v)$;
3) Use the invariant manifold technique to eliminate time dependence;
4) Approximate a local solution using polynomial expansion of P_i and Q_i in terms of u and v;
5) Substitute expansions into time-independent partial differential equations governing the geometry of the manifold, and solve polynomial expansion of P_i and Q_i;
6) Replace the slaved coordinates with their expansions, thus eliminating them from the system.

LINEAR MODAL ANALYSIS

The numerical example developed in this work uses parameters from physical experiments (Han and Benaroya, 2000; Kuchnicki and Benaroya, 2002) as summarized on Tab. 1.

Table 1 – Numerical parameters used in analysis.

Material	Aluminium
Column specific mass, ρ	2770.000 kg/m³
Platform massa, m	0.236 kg
Torsional stiffness constant, k	38.800 Nm/rad
Columns lenght, l	1.270 m
Columns outer diameter, d_0	0.025 m
Cross-sectional area, A_i	1.110 10^{-4} m²
Water specific mass, ρ_w	999.000 kg/m³
Inertia coefficient, C_M	2.000

The equations of movement of the system are derived using data from Tab. 1 on the application of the methodology presented on section 2. The dynamic equations of motion obtained after the application of Lagrange equations, given by Eq. (12), are shown in Eq. (20)-(22). One can notice that those equations are coupled in terms of displacement, velocity and acceleration:

$$\begin{aligned}&-77.600\theta_1+38.800\theta_2+10.698\sin\theta_1-2.013\cos\theta_1\left(-\sin\theta_1\dot\theta_1^2+\cos\theta_1\ddot\theta_1-\sin\theta_2\dot\theta_2^2+\cos\theta_2\ddot\theta_2-\sin\theta_3\dot\theta_3^2\right.\\&\left.+\cos\theta_3\ddot\theta_3\right)+2.013\sin\theta_1\left(-\cos\theta_1\dot\theta_1^2-\sin\theta_1\ddot\theta_1-\cos\theta_2\dot\theta_2^2-\sin\theta_2\ddot\theta_2-\cos\theta_3\dot\theta_3^2-\sin\theta_3\ddot\theta_3\right)-1.633\cos\theta_1^2\ddot\theta_1\\&-1.633\sin\theta_1^2\ddot\theta_1-0.052\ddot\theta_1-1.633\cos\theta_1\left(-\sin\theta_1\dot\theta_1^2+\cos\theta_1\ddot\theta_1-\sin\theta_2\dot\theta_2^2+\cos\theta_2\ddot\theta_2\right)+1.633\sin\theta_1\left(-\cos\theta_1\dot\theta_1^2\right.\\&\left.-\sin\theta_1\ddot\theta_1-\cos\theta_2\dot\theta_2^2-\sin\theta_2\ddot\theta_2\right)\end{aligned} \tag{20}$$

$$\begin{aligned}&-77.600\theta_2+38.800\theta_1+38.800\theta_3+7.696\sin\theta_2-2.013\cos\theta_2\left(-\sin\theta_1\dot\theta_1^2+\cos\theta_1\ddot\theta_1-\sin\theta_2\dot\theta_2^2+\cos\theta_2\ddot\theta_2\right.\\&\left.-\sin\theta_3\dot\theta_3^2+\cos\theta_3\ddot\theta_3\right)+2.013\sin\theta_2\left(-\cos\theta_1\dot\theta_1^2-\sin\theta_1\ddot\theta_1-\cos\theta_2\dot\theta_2^2-\sin\theta_2\ddot\theta_2-\cos\theta_3\dot\theta_3^2-\sin\theta_3\ddot\theta_3\right)\\&-1.633\cos\theta_2\left(-\sin\theta_1\dot\theta_1^2+\cos\theta_1\ddot\theta_1-\sin\theta_2\dot\theta_2^2+\cos\theta_2\ddot\theta_2\right)+1.633\sin\theta_2\left(-\cos\theta_1\dot\theta_1^2-\sin\theta_1\ddot\theta_1-\cos\theta_2\dot\theta_2^2\right.\\&\left.-\sin\theta_2\ddot\theta_2\right)-0.052\ddot\theta_2\end{aligned} \tag{21}$$

$$\begin{aligned}&38.800\theta_2-38.800\theta_3+4.492\sin\theta_3-2.013\cos\theta_3\left(-\sin\theta_1\dot\theta_1^2+\cos\theta_1\ddot\theta_1-\sin\theta_2\dot\theta_2^2+\cos\theta_2\ddot\theta_2-\sin\theta_3\dot\theta_3^2\right.\\&\left.+\cos\theta_3\ddot\theta_3\right)+2.013\sin\theta_3\left(-\cos\theta_1\dot\theta_1^2-\sin\theta_1\ddot\theta_1-\cos\theta_2\dot\theta_2^2-\sin\theta_2\ddot\theta_2-\cos\theta_3\dot\theta_3^2-\sin\theta_3\ddot\theta_3\right)-0.052\ddot\theta_3\end{aligned} \tag{22}$$

In order to decouple the system formed by Eq. (20)-(22), Cramer's rule is applied, so the system can be rewritten in Cauchy form. The linearization of the obtained system of decoupled equations of motion results on the linear decoupled system:

$$\begin{aligned}60{,}214\theta_1 - 60{,}803\theta_2 + 21{,}095\theta_3 = 0\\ 80{,}745\theta_1 - 121{,}070\theta_2 + 62{,}065\theta_3 = 0\\ 20{,}010\theta_1 - 77{,}521\theta_2 + 56{,}539\theta_3 = 0\end{aligned} \quad (23)$$

The solution of the eigenvalue problem return the natural frequencies and vibration modes for the considered structural model, and compound the modal linear analysis for the studied problem. The three natural vibration frequencies for the system results ω_{01} = 0,426 rad/s, ω_{02} = 6,047 rad/s and ω_{03} = 14,180 rad/s. Seeking a better understanding of the model behavior, are calculated also the natural vibration frequencies without the consideration of buoyancy and added mass effects, resulting on the following natural vibration frequencies: ω_{01} = 0,617 rad/s, ω_{02} = 9,240 rad/s and ω_{03} = 21,273 rad/s. By simple comparison, it can be seen that the consideration of hydrostatic effects leads to a lowering on the natural frequencies of the structure, *i.e.*, the effective structure stiffness is reduced when the movement takes place underwater. The resulting eigenvectors from Eq. (23) correspond to the linear normal modes of vibration for the system, whose configurations are graphically shown in Fig. 2.

NONLINEAR MODAL ANALYSIS

For the construction of the invariant manifold geometry, it is chosen as master pair the displacement and velocities for the first DOF of the system, θ_1 and $\dot{\theta}_1$; the coordinates and velocities related to the other degrees-of-freedom of the system are rewritten with use of the restriction functions, related to the master pair, respectively:

$$\begin{aligned}\theta_1 = u, \dot{\theta}_1 = v\\ \theta_2 = P_2(u,v), \dot{\theta}_2 = Q_2(u,v)\\ \theta_3 = P_3(u,v), \dot{\theta}_3 = Q_3(u,v)\end{aligned} \quad (24)$$

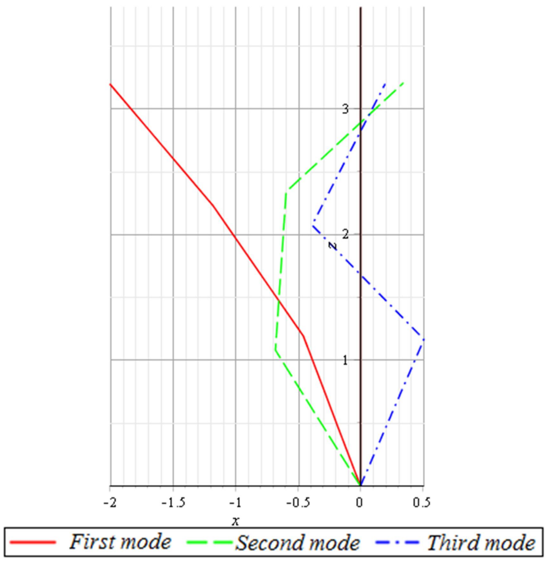


Figure 2 – Linear normal modes of vibration.

Applying the formulation shown in section 3, are obtained three 1-DOF oscillators, written in terms of the modal displacements for the chosen master pair:

$$\ddot{u} + 0{,}182u - 0{,}260uv^2 + 0{,}412u^3 = 0 \quad (25)$$

$$\ddot{u} + 36{,}572u + 6{,}378uv^2 - 191{,}968u^3 = 0 \quad (26)$$

$$\ddot{u} + 201{,}070u + 32{,}802uv^2 - 6155{,}259u^3 = 0 \quad (27)$$

These 1-DOF oscillators are the nonlinear normal modes of vibration for the system. It is necessary to identify the correspondence between the equations and the corresponding modes of vibration. This can be performed analyzing the constant factor multiplying the linear displacement term, u, of each 1-DOF oscillator, that must be equal to the square of the natural vibration frequency of the corresponding mode, obtained on the linear modal analysis, i. e., ω_i^2. It can be then stated that Eq. (25), Eq. (26) and Eq. (27) are the 1-DOF oscillators for the first, second and third nonlinear normal modes of vibration for the modeled system.

One particular characteristic of nonlinear oscillatory systems, captured on a nonlinear analysis, is that the relation between vibration frequency and system energy input isn't linear, otherwise what occurs on linear systems. Such relation can be verified through different methods, one of them is the use of resonance curves, or the frequency-amplitude curves of the system. In these curves, with increasingly oscillation amplitudes, *i.e.*, an increase on the system energy input, can be verified a variation on the natural vibration frequencies of the system.

The resonance curves are obtained with application of another tool, the harmonic balance method, where the following substitution is adopted on the modal 1-DOF oscillators shown in Eq. (25)-(27):

$$u(t) = X_1 \mathrm{sen}(\omega t) \tag{28}$$

The dimensionless parameter Ω is also introduced, standing for the relation between the vibration frequency of the system and the natural vibration frequency of the system related to the analyzed NNM, that means, ω/ω_i. The resulting equations for the three 1-DOF oscillators, with such substitutions, are than written as:

$$-0,426X_1\Omega^2 + 0,729X_1^3 + 0,426X_1 - 0,028X_1^3\Omega^2 \tag{29}$$

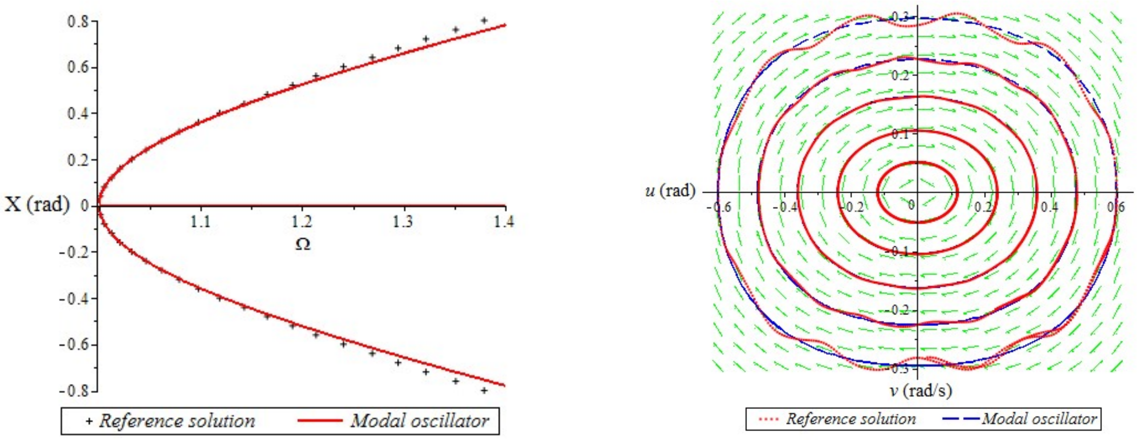
$$-6,047X_1\Omega^2 - 23,808X_1^3 + 6,047X_1 + 9,642X_1^3\Omega^2 \tag{30}$$

$$-14,180X_1\Omega^2 - 325,562X_1^3 + 14,180X_1 + 116,282X_1^3\Omega^2 \tag{31}$$

Equation (29) shows hardening behavior with increasing motion amplitudes, as shown in Fig. 3(a). Equations (30) and (31) show softening behavior with increasingly motion amplitudes on the system, as can be seen in Fig. 4(a) and Fig. 5(a). Together with the obtained solutions curves for the reduced order models, are shown dots on these three figures, corresponding to the numeric solutions from the integration of the full original equations of movement for the system, resulting from the application of Eq. (12), which are taken as reference solutions. The integration method used is the Runge-Kutta method.

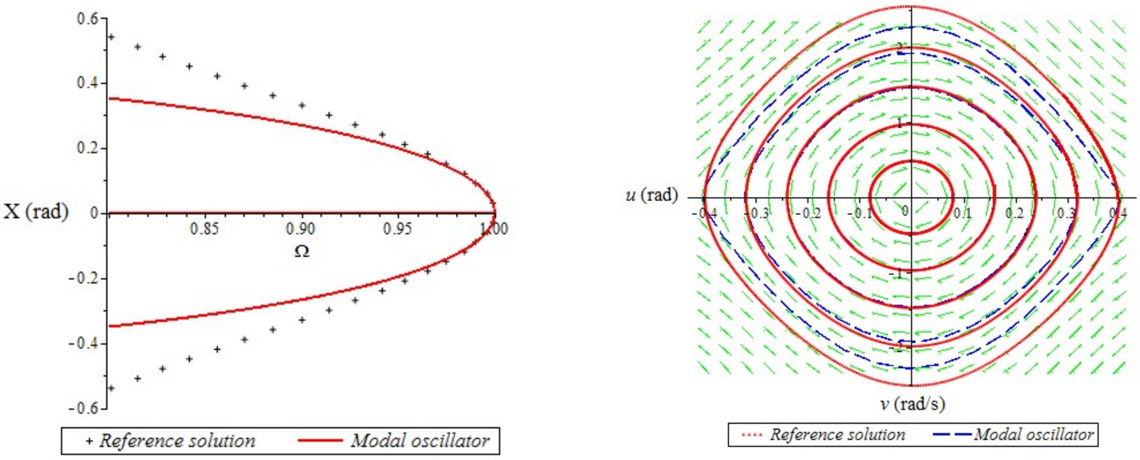
Another useful tool for analysis of the behavior of the reduced order model is the phase space of the system for each mode. Each orbit shown in Fig. 3(b), Fig. 4(b) and Fig. 5(b) is generated from a given initial condition for the structure, showing the relation between modal displacement, *u*, and velocity, *v*, predicted for the free vibration behavior of the structure. Also the solutions obtained with both the reduced order models and the numeric integration of original equations of movement are shown on the phase space figures.

On the frequency-amplitude curves can be observed the validity for the Taylor series expansions on the derivation of the reduced order models, for each NNM, as the solutions take distance from the initial equilibrium point, Ω = 1, neighborhood where the obtained models present the best agreement with the reference solution. Loss of precision can also be observed on the phase space portraits, as the solution moves away from the expansion equilibrium point, *u*= *0*, and the emergence of a caotic dynamic oscillatory behavior.



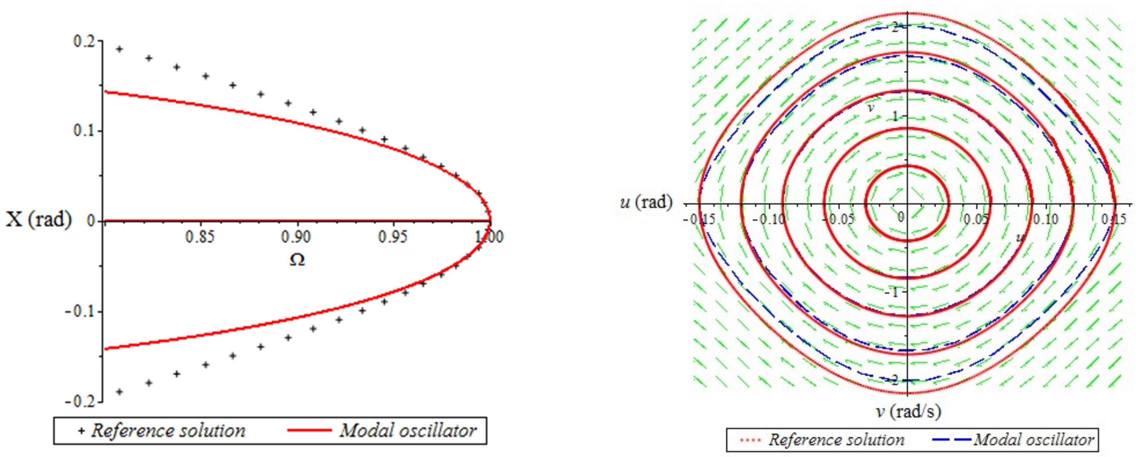
(a) (b)

Figure 3 – Modeled and reference solutions for the first NNM.
(a) Frequency-amplitude curve; (b) Phase space of the system.



(a) (b)

Figure 4 – Modeled and reference solutions for the second NNM.
(a) Frequency-amplitude curve; (b) Phase space of the system.



(a) (b)

Figure 5 – Modeled and reference solutions for the third NNM.
(a) Frequency-amplitude curve; (b) Phase space of the system.

CONCLUSIONS

The methodology presented and used in this paper allow the derivation of reduced order models for vibration problems that demand the consideration of nonlinearities.

The reduced order models allow fast parametric analysis for the oscillatory behavior of nonlinear systems, being one the greatest advantages on the study of nonlinear vibrations. Furthermore, these models show good agreement with reference solutions, as shown in this work. With the tools used in this paper, resonance curves and phase space of the system, it is possible to mark the domains of validity for the derived models, according to the design needs and the nature of the studied problem, with respect to the acceptable error of the solution.

This paper is part of a work in progress, the complete work will include a multimodal analysis of the problem, such as stability conditions and the consideration of current and wave effects on the structure.

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