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DYNAMIC MODELLING OF THE MANIPULATOR OF A MINI-HYDRAULIC EXCAVATOR IN LOAD CONDITION

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Abstract: Small hydraulic excavators are versatile machines used in a wide range of operations such as digging, removal of debris, the transportation of cargo, ground and in general earthmoving. Operating a hydraulic excavator in certain environments is a difficult and dangerous task, especially in hazardous environments subject to natural disturbances or inadequate health conditions for human work. Because of these conditions, the control for automation of excavators or its components has been the subject of many studies in recent years. In a near future we intend to develop a control system involving all the degrees of freedom to the handler of a mini - excavator. Therefore, within the control methodology in mind, we need a complete mathematical model of the kinematics and dynamics of this mechanism. This work includes an extensive bibliographic review of this problem, where we observe that the majority of work only studies the digging operation and that there are few studies devoted to dynamic modeling of the manipulator in the loaded condition. The work also proposes a path for the simulation of the dynamic models. In a tutorial approach and looking for the most efficient modeling method, the manipulator movements were modelled using three different methods: the Lagrange method, the method of Gibbs-Appell in the form proposed by Kane, and the principle of virtual work. The work also includes comparison and validation by simulating a computational model obtained from the CAD geometry handler, created by commercial software for dynamic analysis. Results are discussed and evaluated and suggestions for future work are enclosed.

Keywords: Excavator, hydraulic manipulator, kinematics analysis, dynamic analysis, computational model

INTRODUCTION

Hydraulic excavators are versatile machines used in various types of operations, such as excavating, removing debris, cargo, ground, and earthworks in general. Operating a hydraulic excavator in certain environments is a difficult task, especially when it comes to risk environments subject to natural disturbances or inadequate health conditions for human work. In recent years, the control for the automation of an excavator has been the subject of many studies seeking for high efficiency and improved safety.

To develop a workable control system for the manipulator of a hydraulic excavator it is necessary a complete model of the kinematics and dynamics of the mechanism, with a number of degrees of freedom suitable for the operation that we want to accomplish.

At first, we present a review of literature describing the works that have focused on the study of the manipulator. After that, we formalize the kinematics and dynamics of a chosen manipulator. We determine the length of the actuators according to the generalized coordinates, to perform the transformation of the motor torques to hydraulic forces generated by the actuators. Under the rules of the equipment, we specify the move operation load. At last we present results, their discussions and the completion of the work.

LITERATURE REVIEW

Vähä and Skibniewski (1993) developed a model of the dynamics of the manipulator in the digging operation, with three degrees of freedom, using a Newton-Euler approach. Although the extension of the work, some of the assumptions made in relation to the tensor of inertia and mass center position of the bodies are however not realistic.

The focus of the work of Singh (1995) was the development of tactical plans for robotic digging, thus in his manipulator dynamic model, he presented only the torques resulting from the action of gravitational forces.

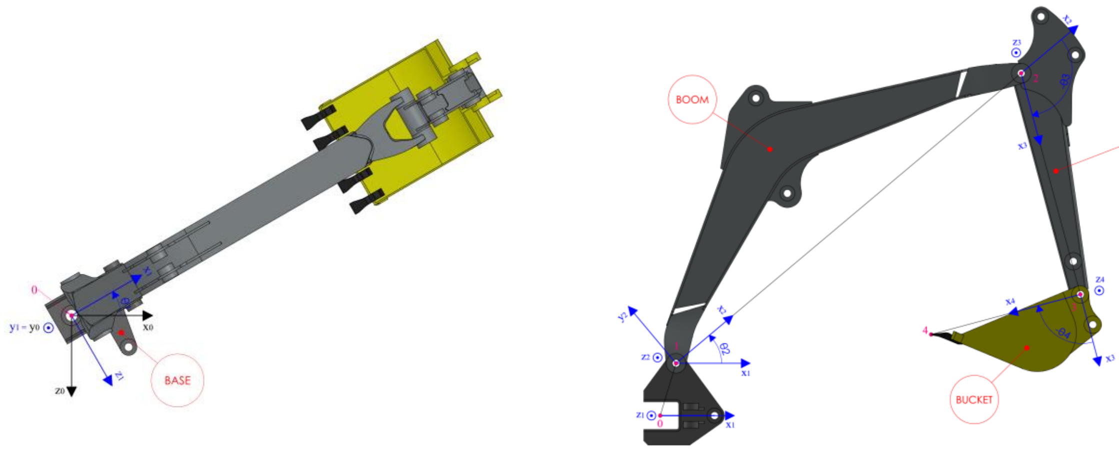
Koivo *et al.* (1996) developed a model of the manipulator dynamics also using the Newton-Euler approach and implementing the model in three degrees of freedom, overcoming the shortcomings of the dynamic model presented by Vähä and Skibniewski (1993). This dynamic model is one of the most complete and is suitable to the manipulator controller design of a hydraulic excavator during the digging operation in unattended mode. This work also presents an example of using the dynamic model along with a PD (Proportional and Derivative) controller to control the unattended mode of a hydraulic excavator.

Through an approach by Euler-Lagrange, Frankel (2004) obtained the equations of motor torque to the different joints of the manipulator of a hydraulic excavator. This study does not include the charge vector describing the interaction of the bucket with soil during digging.

Patel and Prajapati (2014) and Šalinić *et al.* (2014) developed models for the manipulator dynamics with three degrees of freedom, for the digging operation. Patel and Prajapati (2014) used a Euler-Lagrange approach whereas of Šalinić *et al.* (2014) the Kane method.

KINEMATICS OF THE MANIPULATOR

We consider the manipulator of a hydraulic excavator with four main bodies (base, boom, stick and bucket). To describe bodies' kinematics, we need an inertial frame called I and a system of rectangular coordinates (x_0, y_0, z_0), fixed in the center of the joint of the manipulator base, and also a local frame 1 solidary to the manipulator base, with origin coincident with the origin of the inertial frame and rectangular coordinates (x_1, y_1, z_1). This frame describes the position of the base relative to the inertial frame using the angular displacement θ_1, which is defined positive when moving in the counterclockwise sense, according to the right-hand rule. To describe the boom position relative to base frame and to describe the bucket position relative to stick frame, we need the local frames 2, 3 and 4, which are solidary to boom, stick and the bucket respectively. The rectangular coordinate systems (x_2, y_2, z_2), (x_3, y_3, z_3) and (x_4, y_4, z_4) with the angular displacements θ_2, θ_3 and θ_4 are relate to the frames 2, 3 and 4 respectively. The set of angular displacements ($\theta_1, \theta_2, \theta_3, \theta_4$) is the generalized coordinates of the system. For convenience, we assign a number to the center of the joints and links of the manipulator. The base joint center is the point 0, the boom joint center is the point 1, the stick joint center is the point 2, the bucket joint center is the point 3 and the tip of bucket teeth's is point 4. The base, boom, stick and bucket are the links 1, 2, 3 and 4 respectively. Figure 1 shows all frames, links, generalized coordinates and points defined above.



(a) (b)

Figure 1 – Frames, generalized coordinates and points.
(a): Representation in xz-plane; (b): Representation in xy-plane.

Next, we formalize the description of the orientation and the position of any point of the manipulator, relative to the inertial frame I. Starting with the description of the orientation, considering initially a frame j and a frame $j-1$, with $j = 1,\ldots,4$, where origin O_j coincide with origin O_{j-1}. The rotation matrix of j to $j-1$ is given by ${}^{j-1}\mathbf{R}_j$, that is the matrix of direction cosines (Baruh, 1999). This matrix has the property of being an orthogonal matrix, so: ${}^{j-1}\mathbf{R}_j^{-1} = {}^{j-1}\mathbf{R}_j^T = {}^{j}\mathbf{R}_{j-1}$ (Meirovitch, 2003).

Let ${}^{j-1}\boldsymbol{\omega}_j$ be the angular velocity vector of a frame j expressed in a frame $j-1$. To express this vector in frame I, it is necessary to perform the rotation of $j-1$ to I:

$$ {}^{I}\boldsymbol{\omega}_j = {}^{I}\boldsymbol{\omega}_{j-1} + {}^{I}\mathbf{R}_{j-1}\,{}^{j-1}\boldsymbol{\omega}_j \tag{1} $$

To write ${}^{I}\boldsymbol{\omega}_j$ in the frame I in terms of the base of frame j, we perform the rotation of I to j:

$$ {}^{j}\boldsymbol{\omega}_j = {}^{j}\mathbf{R}_I\,{}^{I}\boldsymbol{\omega}_j \tag{2} $$

This transformation is very useful for the study the dynamics of the manipulator, since it avoids the rotation of the inertia tensor for the inertial frame. Therefore, to obtain the dynamic model of the manipulator when necessary we will apply this transformation to any vector.

With respect to the position, considering any point B belonging to the manipulator, we can write its position vector on the origin of inertial frame as the position vector of the point A, predecessor of B, relative to the inertial frame, more rotation of the vector ${}^{j}\mathbf{r}_{AB}$ between A and B from local frame j to I:

$$ {}^{I}\mathbf{r}_{OB} = {}^{I}\mathbf{r}_{OA} + {}^{I}\mathbf{R}_{j}\, {}^{j}\mathbf{r}_{AB} \tag{3} $$

We write the velocity vector of the point B in the frame I as the derivative of ${}^{I}\mathbf{r}_{OB}$ in relation to time as:

$$ {}^{I}\mathbf{v}_{B} = \frac{d}{dt}\left({}^{I}\mathbf{r}_{OB}\right) = \frac{d}{dt}\left({}^{I}\mathbf{r}_{OA} + {}^{I}\mathbf{R}_{j}\, {}^{j}\mathbf{r}_{AB}\right) \tag{4} $$

The first derivative of ${}^{I}\mathbf{r}_{OB}$ with respect to time yields:

$$ {}^{I}\mathbf{v}_{B} = {}^{I}\mathbf{v}_{A} + {}^{I}\boldsymbol{\omega}_{j} \wedge \left({}^{I}\mathbf{R}_{j}\, {}^{j}\mathbf{r}_{AB}\right) + {}^{I}\mathbf{v}_{REL} \tag{5} $$

where ${}^{I}\mathbf{v}_{A}$ is the velocity vector of the point A in the frame I and ${}^{I}\mathbf{v}_{REL}$ is the relative velocity vector between A and B in frame I.

We write the acceleration vector of B in the frame I as the second derivative of ${}^{I}\mathbf{r}_{OB}$ in relation to time:

$$ {}^{I}\mathbf{a}_{B} = \frac{d^2}{dt^2}\left({}^{I}\mathbf{r}_{OB}\right) = \frac{d^2}{dt^2}\left({}^{I}\mathbf{r}_{OA} + {}^{I}\mathbf{R}_{j}\, {}^{j}\mathbf{r}_{AB}\right) \tag{6} $$

The second derivative of ${}^{I}\mathbf{r}_{OB}$ with respect to time gives:

$$ {}^{I}\mathbf{a}_{B} = {}^{I}\mathbf{a}_{A} + {}^{I}\boldsymbol{\alpha}_{j} \wedge {}^{I}\mathbf{r}_{AB} + {}^{I}\boldsymbol{\omega}_{j} \wedge \left({}^{I}\boldsymbol{\omega}_{j} \wedge {}^{I}\mathbf{r}_{AB}\right) + 2\,{}^{I}\boldsymbol{\omega}_{j} \wedge {}^{I}\mathbf{v}_{REL} + {}^{I}\mathbf{a}_{REL} \tag{7} $$

where ${}^{I}\boldsymbol{\alpha}_{j}$ and ${}^{I}\mathbf{a}_{REL}$ are respectively the angular acceleration of the frame j and the relative acceleration between A and B, both expressed in frame I.

Associating ${}^{I}\mathbf{r}_{OB}$ with the position vector of the center of mass of a body of the manipulator it is possible to get the velocity and acceleration vectors of that center. From here on we only use the notation ${}^{j}(\cdot)$ to terms expressed in the inertial frame I in terms of the base of local frame j, with the exception of inertia tensor ${}^{j}\mathbf{I}_{i}$ of the link i, which in fact we express in the local frame j. This decision aims reduce the size of expressions related to kinematics, and to simplify the notation to obtain the equations of motion to the system.

DYNAMIC MODEL OF THE MANIPULATOR

We model the manipulator of the hydraulic excavator by three different methods: by Lagrange equations, by Gibbs-Appell in the form proposed by Kane and by the principle of virtual works. We considered only the main bodies of the manipulator (base, boom, stick and bucket). We do not consider any dissipative effect.

Modeling by the Lagrange method

As shown in (Meirovitch, 2003), we achieve the dynamical model by applying Eq. (8) in the proposed problem, where T is the kinetic energy function of the manipulator, Q_k are the generalized forces related to generalized coordinate k, q_k is the generalized coordinate, with $q_k = \theta_k$ and $k = 1,\dots,n$, where n is the number of generalized coordinates, four in our case.

$$ \frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_k}\right) - \frac{\partial T}{\partial q_k} = Q_k \tag{8} $$

We write the kinetic energy function as the sum of the terms of translational kinetic energy and rotational kinetic energy of the bodies of the manipulator

$$ T = \frac{1}{2}\sum_{i=1}^{N}\left({}^{j}\mathbf{v}_{CGi}{}^{T} m_i\, {}^{j}\mathbf{v}_{CGi} + {}^{j}\boldsymbol{\omega}_{i}{}^{T}\, {}^{j}\mathbf{I}_{i}\, {}^{j}\boldsymbol{\omega}_{i}\right) \tag{9} $$

where m_i is the mass of link i, with $i=j$ and N represents the number of links of the manipulator, ${}^j\mathbf{v}_{CGi}$ is the velocity vector of the center of mass of link i and ${}^j\mathbf{I}_i$ is the inertia tensor of link i. We write the generalized forces of the generalized coordinate k as:

$$Q_k = \sum_{i=1}^{N^*} {}^j\mathbf{F}_i^T \frac{\partial\, {}^j\mathbf{P}_i}{\partial q_k} \tag{10}$$

where ${}^j\mathbf{F}_i$ is the resulting active force vector in a material point, with $i=1,\dots,N^*$ material points, and ${}^j\mathbf{P}_i$ is the position vector of material point i. Developing Q_k to the manipulator results in:

$$Q_k = \sum_{i=1}^{N} {}^j\mathbf{G}_{CGi}{}^T \frac{\partial\, {}^j\mathbf{r}_{CGi}}{\partial \theta_k} + \tau_k \tag{11}$$

where ${}^j\mathbf{G}_{CGi}$ and τ_k are respectively the gravitational force vector of link i, and the torque on the generalized coordinate k.

Modeling by the Gibbs-Appell method

We derive now the model of the manipulator of hydraulic excavator by the method of Gibbs-Appell in the form proposed by Kane, according to Eq. (12) adapted from Baruh (1999):

$$\sum_{i=1}^{N}\left(m_i\, {}^j\mathbf{a}_{CGi}{}^T \frac{\partial\, {}^j\mathbf{v}_{CGi}}{\partial \dot{q}_k} + {}^j\dot{\mathbf{H}}_i^T \frac{\partial\, {}^j\boldsymbol{\omega}_i}{\partial \dot{q}_k}\right) = \sum_{i=1}^{N}\left({}^j\mathbf{F}_i^T \frac{\partial\, {}^j\mathbf{v}_{CGi}}{\partial \dot{q}_k} + {}^j\mathbf{M}_i^T \frac{\partial\, {}^j\boldsymbol{\omega}_i}{\partial \dot{q}_k}\right) \tag{12}$$

where ${}^j\mathbf{a}_{CGi}$ and ${}^j\dot{\mathbf{H}}_i$ are respectively the acceleration vector of the center of mass and variation of the amount of movement vector of link i. The term ${}^j\dot{\mathbf{H}}_i$ is given by:

$${}^j\dot{\mathbf{H}}_i = {}^j\mathbf{I}_i \frac{d}{dt}\left({}^j\boldsymbol{\omega}_i\right) + {}^j\boldsymbol{\omega}_i \wedge \left({}^j\mathbf{I}_i\, {}^j\boldsymbol{\omega}_i\right) \tag{13}$$

The active forces vector ${}^j\mathbf{F}_i$ and the sum of generalized active moments vectors ${}^j\mathbf{M}_i^T\left(\partial\, {}^j\boldsymbol{\omega}_i / \partial \dot{q}_k\right)$ are respectively:

$${}^j\mathbf{F}_i = {}^j\mathbf{G}_{CGi} \tag{14}$$

$$\sum_{i=1}^{N} {}^j\mathbf{M}_i^T \frac{\partial\, {}^j\boldsymbol{\omega}_i}{\partial \dot{q}_k} = \tau_k \tag{15}$$

Modeling by the principle of virtual work

The sum of the virtual work done by all the forces of the system, including the forces of actuators, inertia forces and gravitational forces are equal to zero (Meirovitch, 2003). Therefore, we write the resulting dynamic model using the virtual work as follows:

$$\delta W = \sum_{i=1}^{N} m_i \mathbf{g}^T \delta\, {}^j\mathbf{r}_{CGi} - \sum_{i=1}^{N} m_i\, {}^j\mathbf{a}_{CGi}{}^T \delta\, {}^j\mathbf{r}_{CGi} + \boldsymbol{\tau}^T \delta\mathbf{q} - \sum_{i=1}^{N}\left[{}^j\mathbf{I}_i \frac{d}{dt}\left({}^j\boldsymbol{\omega}_i\right) + {}^j\boldsymbol{\omega}_i \wedge \left({}^j\mathbf{I}_i\, {}^j\boldsymbol{\omega}_i\right)\right]^T \delta\, {}^j\boldsymbol{\Theta}_i = 0 \tag{16}$$

where $\boldsymbol{\tau} = [\tau_1 \;\; \dots \;\; \tau_n]^T$ is the torque vector, $\mathbf{q} = [\theta_1 \;\; \dots \;\; \theta_n]^T$ is the generalized coordinates vector, and $\mathbf{g} = [0 \;\; -g \;\; 0]^T$ is the gravity acceleration vector.

The following expressions give the virtual linear displacement vector $\delta\, {}^j\mathbf{r}_{CGi}$ and the virtual angular displacement vector $\delta\, {}^j\boldsymbol{\Theta}_i$:

$$\delta\, {}^j\mathbf{r}_{CGi} = \sum_{k=1}^{n} \frac{\partial\, {}^j\mathbf{r}_{CGi}}{\partial q_k} \delta q_k \tag{17}$$

$$\delta\, {}^{j}\boldsymbol{\Theta}_i = \sum_{k=1}^{n} \frac{\partial\, {}^{j}\boldsymbol{\Theta}_i}{\partial q_k} \delta q_k \tag{18}$$

The sum of the virtual work is equal to zero for any virtual displacement (Almeida, 2013). To obtain the equations of motion of the manipulator, the sum of terms which multiply each of the virtual displacements ($\delta\theta_1, \delta\theta_2, \delta\theta_3, \delta\theta_4$) must be zero.

FORCES OF THE HYDRAULIC ACTUATORS

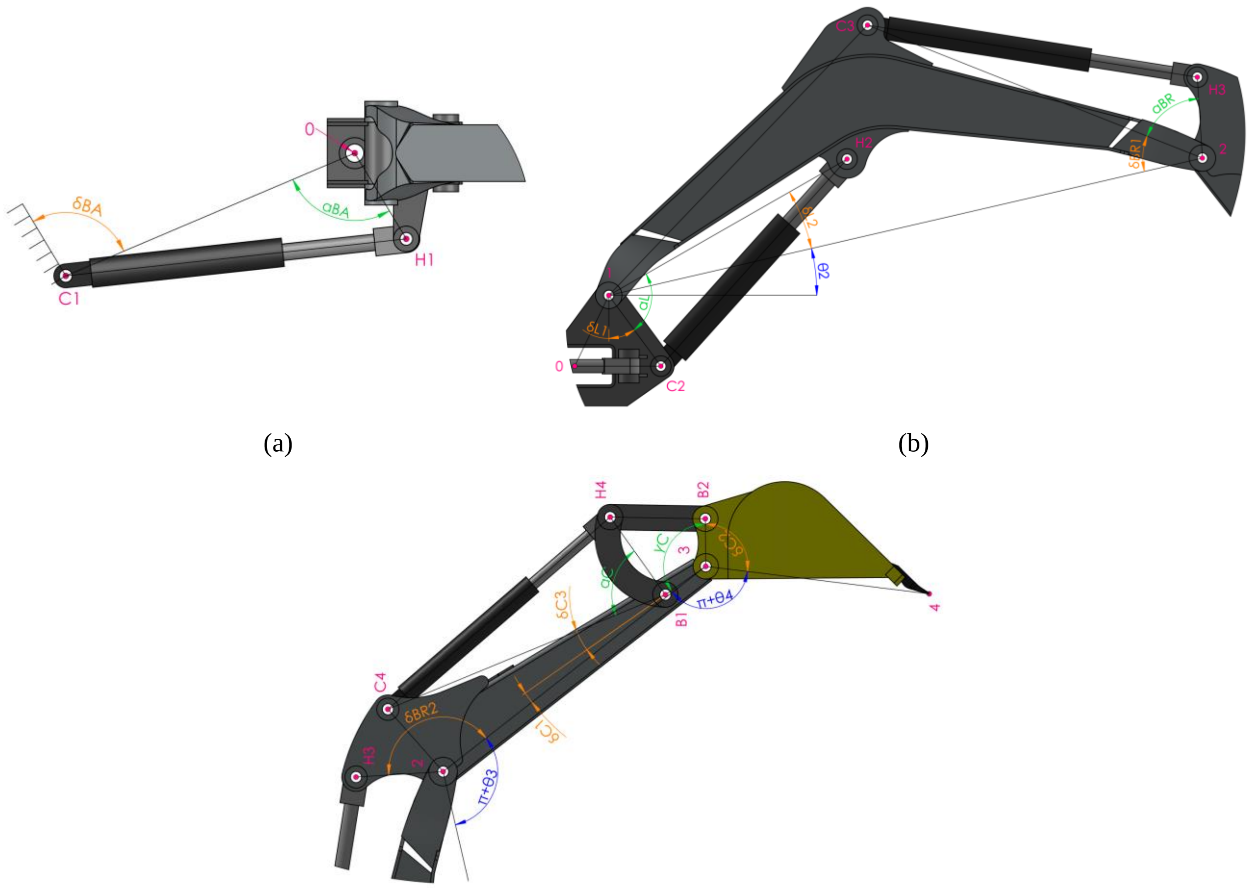
The motor torque τ_k can be related to the force F_{hk} of the hydraulic actuator k by the principle of the virtual works (Šalinić *et al.*, 2014), resulting in:

$$\tau_k = F_{hk} \frac{\partial l_{CkHk}}{\partial q_k} \tag{19}$$

where l_{CkHk} is the length of the actuator k , with $k = 1, \ldots, n$ generalized coordinates.

Lengths of actuators

We determine the lengths of actuators in function of the generalized coordinates of the manipulator by inverse kinematics analysis of actuators, through the use of geometry (Frankel, 2004), using Fig. 2 as reference.



(a) (b) (c)

Figure 2 – Kinematic of the hydraulic actuators.
(a): kinematic of actuator 1; (b): kinematic of actuator 2 and 3; (c): kinematic of actuator 4.

Based on Fig. 2 (a) we have for actuator 1:

$$\alpha_{BA} = \theta_1 + \delta_{BA} \tag{20}$$

$$l_{C1H1} = \sqrt{{l_{0C1}}^2 + {l_{0H1}}^2 - 2 l_{0C1} l_{0H1} cos\alpha_{BA}} \tag{21}$$

From Fig. 2 (b) and (c) we have for actuators 2 and 3, respectively:

$$\alpha_L = \frac{\pi}{2} + \theta_2 - \left(\delta_{L1} - \delta_{L2}\right) \tag{22}$$

$$l_{C2H2} = \sqrt{{l_{1H2}}^2 + {l_{1C2}}^2 - 2 l_{1H2} l_{1C2} cos\alpha_L} \tag{23}$$

$$\alpha_{BR} = \pi - \theta_3 - \left(\delta_{BR1} + \delta_{BR2}\right) \tag{24}$$

$$l_{C3H3} = \sqrt{{l_{2C3}}^2 + {l_{2H3}}^2 - 2 l_{2C3} l_{2H3} \, cos\alpha_{BR}} \tag{25}$$

For the actuator 4 in Fig. 2 (c) follows the relationships:

$$\gamma_C = \pi - \theta_4 - \left(\delta_{C1} + \delta_{C2}\right) \tag{26}$$

$$l_{B1B2} = \sqrt{{l_{3B2}}^2 + {l_{3B1}}^2 - 2 l_{3B2} l_{3B1} cos\gamma_C} \tag{27}$$

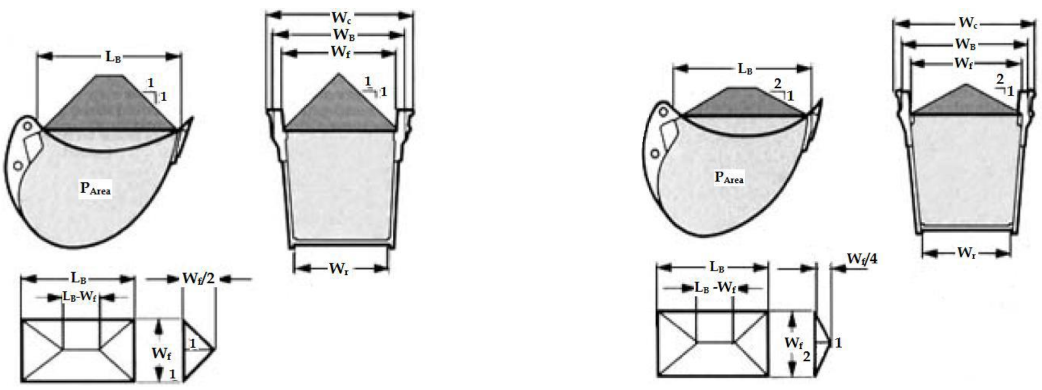
$$\alpha_C = acos\left(\frac{{l_{B2H4}}^2 - {l_{B1B2}}^2 - {l_{B1H4}}^2}{2 l_{B1B2} l_{B1H4}}\right) - asin\left(\frac{l_{3B2} sin\gamma_C}{l_{B1B2}}\right) - \delta_{C3} \tag{28}$$

$$l_{C4H4} = \sqrt{{l_{C4B1}}^2 + {l_{B1H4}}^2 - 2 l_{C4B1} l_{B1H4} cos\alpha_C} \tag{29}$$

The analysis of the kinematics of the fourth actuator is more intricate because of the presence of the six bars mechanism, formed by the actuator 4 with the two-bar linkage, stick and bucket. With the lengths of the actuators in function of the generalized coordinates, it is possible to determine the forces carried out by hydraulic actuators.

LOAD ON THE BUCKET

We estimate the maximum volumetric capacity of a bucket of a small hydraulic excavator by the following standard norms (Patel, 2012): (i) SAE J296 "Mini excavator and backhoe bucket volumetric rating" (ii) CECE (Committee of European Construction Equipment). The norms show how the calculation of maximum material volume in a bucket, considering the volume in excess. The difference between these procedures is in the calculation of the excess volume. We calculate the excess volume at a 1:1 angle of repose for SAE J296 and at 1:2 angle of repose for CECE. Figure 3 shows the dimensions specified by SAE J296 and CECE for the calculation of the maximum volume.



(a) (b)

Figure 3 – Dimensions specified for the calculation of the maximum volumetric capacity of a bucket (Patel, 2012). (a): Dimensions specified by SAE J296; (b) Dimensions specified by CECE.

We show the procedure as follows, where V_s is the useful volume of the bucket.

$$V_s = P_{área}\left(\frac{W_f + W_r}{2}\right) \tag{30}$$

The excess volume V_e is given by:

$$(V_e)_{\text{SAE J296}} = \frac{L_b W_f^{\,2}}{4} - \frac{W_f^{\,3}}{12}\,;\ (V_e)_{\text{CECE}} = \frac{L_b W_f^{\,2}}{8} - \frac{W_f^{\,3}}{24} \tag{31}$$

We estimate the total volume of material or heap volume in the bucket as $V_h = V_s + V_e$, and the mass of material in the bucket is given by $m_C = \rho V_h$, where ρ is the density of the material, in the case, soil.

For inclusion of load on the bucket in the equations of motion, it is necessary to add τ_{Ck} equations in k equations of motion, with $k = 1,\ldots,n$ generalized coordinates. We assume that the load on the bucket has no inertia tensor and its center of mass coincides with the center of mass of the bucket, therefore:

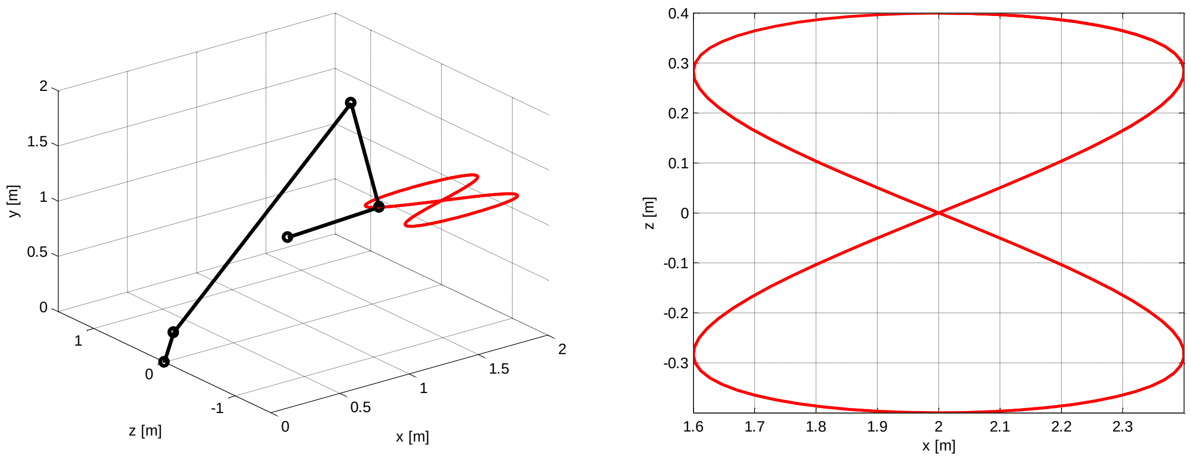
$$\tau_{Ck} = m_C\, {}^4\mathbf{a}_{CG4}{}^T \frac{\partial\, {}^4\mathbf{v}_{CG4}}{\partial \dot{\theta}_k} - {}^4\mathbf{G}_{C4}{}^T \frac{\partial\, {}^4\mathbf{v}_{CG4}}{\partial \dot{\theta}_k} \tag{32}$$

In this work, we present the calculation only for the SAE J296 with the following dimensions for the bucket geometry: $P_{área} = 0.11\ \text{m}^2$, $W_f = 0.47\ \text{m}$, $W_r = 0.46\ \text{m}$ and $L_b = 0.54\ \text{m}$. We consider a sandy-loam for the soil with: $\rho = 1200\ \text{kg/m}^3$ (Quang, 2000). The geometrical parameters of the bucket results in $V_h = 0.072\ \text{m}^3$ therefore the mass of load in the bucket is $m_C = 86.78\ \text{kg}$.

PATH FOR THE MOVE OPERATION

We construct the path executed by the manipulator with periodic functions. Figure 4 shows the path executed by the manipulator in space xyz and in plane xz. The equation in meters for this trajectory with respect to point 3 is:

$${}^I\mathbf{r}_3 = \begin{bmatrix} x_3 & y_3 & z_3 \end{bmatrix}^T = \left[0.40\,sin\left(\frac{4\pi}{3}t\right) + 2 \quad 0.76 \quad 0.40\,cos\left(\frac{2\pi}{3}t\right)\right]^T \tag{33}$$



(a) (b)

Figure 4 – Path executed by the manipulator in the move operation.
(a): Path for the move operation in xyz space; (b): Path for the move operation in xz plane.

This path was set for the joint center of the bucket or point 3 to simplify the analysis of inverse kinematics. We need this assumption to determine the joints displacements for a desired path; therefore, we consider the orientation of the bucket, relative to plane xz, constant during the operation. To start solving the problem of inverse kinematics, first we calculate the distance from point 1 to point 3 of the manipulator in the plan xz, which is:

$$d = \sqrt{\left(x_3 - l_{01}\cos\delta_{L3}\cos\theta_1\right)^2 + \left(-z_3 - l_{01}\cos\delta_{L3}\sin\theta_1\right)^2} \tag{34}$$

Later we write the resultant distance from point 1 to point 3 as follows:

$$r = \sqrt{d^2 + \left(y_3 - l_{01}\sin\delta_{L3}\right)^2} \tag{35}$$

At the end, we calculate the angular displacements of the joints by:

$$\theta_1 = -atan\left(\frac{z_3}{y_3}\right) \tag{36}$$

$$\theta_2 = acos\left(\frac{{l_{12}}^2 + r^2 - {l_{23}}^2}{2 l_{12} r}\right) + asin\left(\frac{y_3 - l_{01}\sin\delta_{L3}}{r}\right) \tag{37}$$

$$\theta_3 = acos\left(\frac{{l_{12}}^2 + {l_{23}}^2 - r^2}{2 l_{12} l_{23}}\right) - \pi \tag{38}$$

$$\theta_4 = -\pi - \theta_3 - \theta_2 \tag{39}$$

RESULTS AND DISCUSSIONS

In this section, we present the results of the simulation of the manipulator for the move operation under load. We show the dimensions required for the calculation of the kinematic greatness in Fig. 5. Tables 1 and 2 gather the dimensions and the proprieties of mass and inertia of the manipulator.

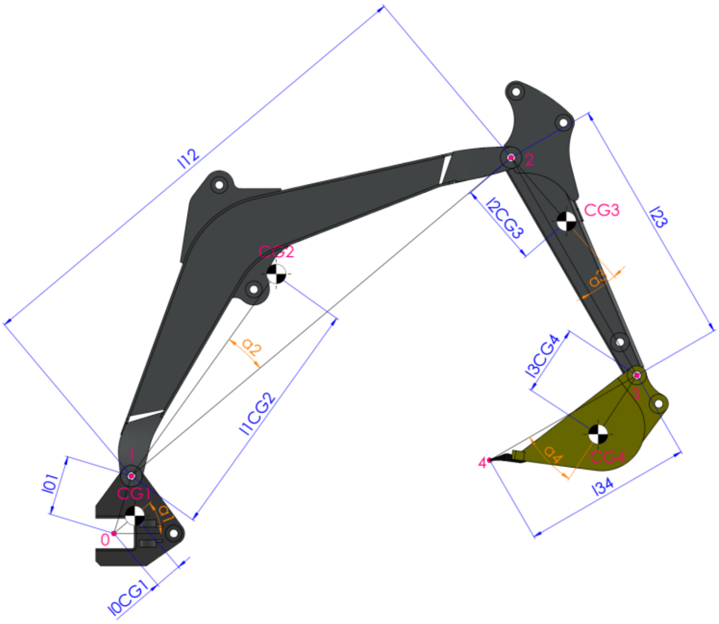


Figure 5 – Dimensions of the manipulator for kinematic calculations.

Table 1 – Dimensions of the manipulator.

VECTORS OF SYMBOLS	VALUES
$[\delta_{B4}, \delta_{L1}, \delta_{L2}, \delta_{L3}, \delta_{BR1}, \delta_{BR2}]$ (°)	[98.09, 36.40, 16.73, 73.12, 34.62, 145.59]
$[\delta_{C1}, \delta_{C2}, \delta_{C3}, \alpha_1, \alpha_2, \alpha_3, \alpha_4]$ (°)	[2.95, 97.49, 12.62, 34.24, 14.53, 11.53, 26.66]
$[l_{1C2}, l_{1H2}, l_{2C3}, l_{2H3}, l_{0C1}, l_{0H1}, l_{C4B1}, l_{3B1}, l_{B1H4}]$ (m)	[0.30, 0.93, 1.22, 0.28, 0.86, 0.27, 0.95, 0.16, 0.30]
$[l_{B2H4}, l_{3B2}, l_{01}, l_{12}, l_{23}, l_{34}, l_{0CG1}, l_{1CG2}, l_{2CG3}, l_{3CG4}]$ (m)	[0.30, 0.15, 0.25, 2.07, 1.05, 0.71, 0.12, 1.03, 0.35, 0.29]

Table 2 – Mass and inertia propriety's of the manipulator.

VECTORS OF SYMBOLS	VALUES
$[m_1, m_2, m_3, m_4]$ (kg)	[31.87, 175.48, 50.54, 42.56]
$[I_{xx1}, I_{yy1}, I_{zz1}, I_{xy1}, I_{xz1}, I_{yz1}]$ (kg·m²)	[0.66, 0.47, 0.69, -0.05, 0.02, -0.04]
$[I_{xx2}, I_{yy2}, I_{zz2}, I_{xy2}, I_{xz2}, I_{yz2}]$ (kg·m²)	[6.81, 76.55, 82.00, -0.95, 0, 0]
$[I_{xx3}, I_{yy3}, I_{zz3}, I_{xy3}, I_{xz3}, I_{yz3}]$ (kg·m²)	[0.54, 9.01, 9.36, -1.16, 0, 0]
$[I_{xx4}, I_{yy4}, I_{zz4}, I_{xy4}, I_{xz4}, I_{yz4}]$ (kg·m²)	[1.46, 3.03, 2.30, -0.08, 0, 0]

Figure 6 shows results for the angular displacement for the joints with the specified path, which we obtained with the study of inverse kinematics of the manipulator.

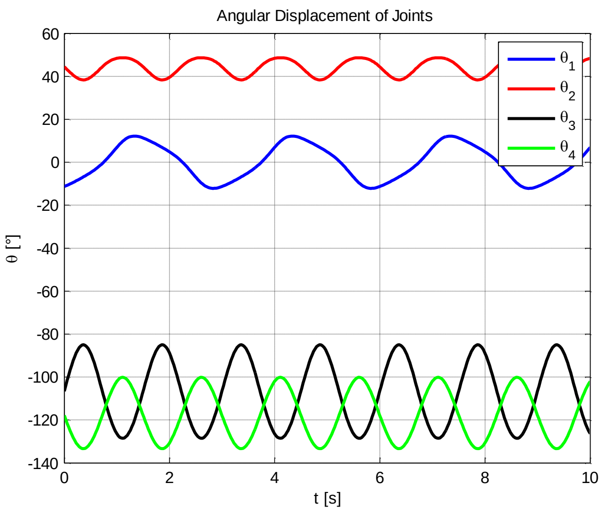


Figure 6 – Angular displacements of the joints for the desire path.

Figure 7 shows the forces of the hydraulic actuators for the move operation, with the load in the bucket. We also verify that the result obtained is the same for the three methods.

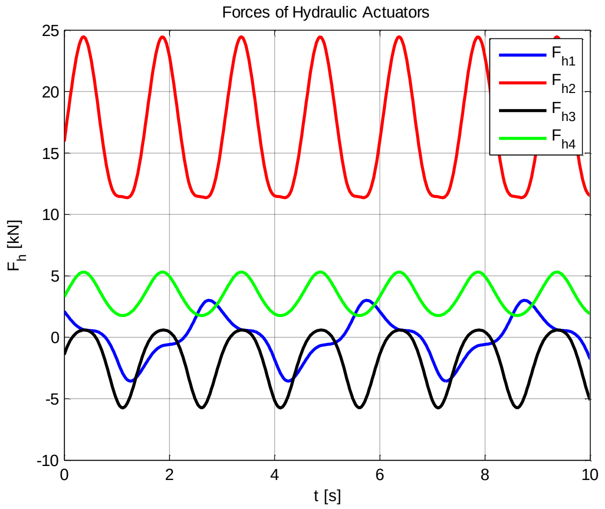
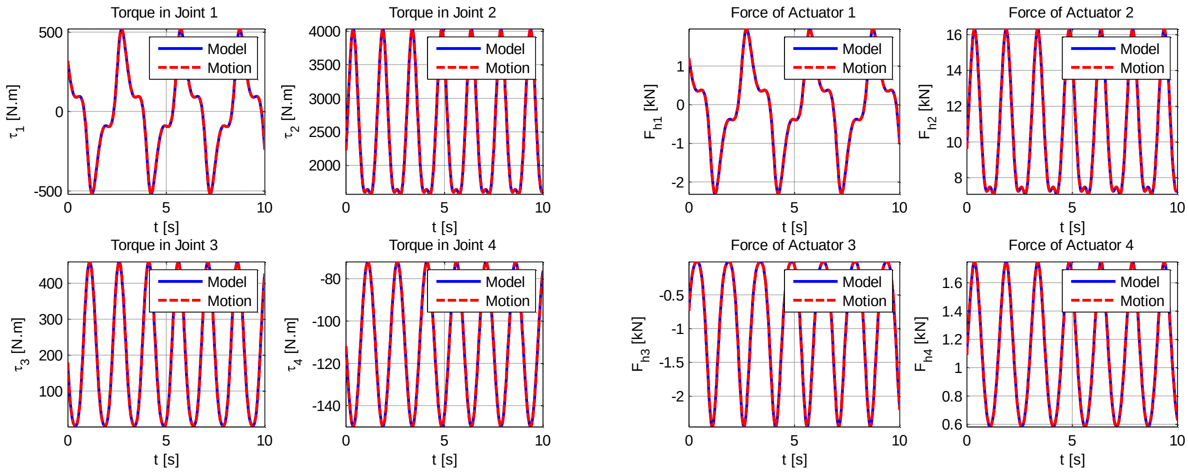


Figure 7 – Forces in the hydraulic actuators in the move operation with load.

We validate the dynamic model presented in this work by comparison with a computational model based on CAD geometry of the manipulator. We create a validation model using the dynamic analysis tool (Motion) of the SOLIDWORKS software 2016. Figure 8 show the comparison of motor efforts for the move operation, without load in the bucket, where we observe no significant difference.



(a) (b)

Figure 8 – Comparison of the efforts by the propose model and the model based on CAD geometry. (a): Caparison of torques in joints; (b): Comparison of forces in hydraulic actuators.

CONCLUSION

Using three different methods, we developed the dynamic model of the manipulator of a small hydraulic mini-excavator in the move operation under load, with four degrees of freedom and four links. The results of the different models are identical for all three methods. Regarding the computational efficiency methods, the principle of virtual work and the method of Gibbs-Appell in the form proposed by Kane were superior to the Lagrange method. This is because it is not necessary to derive scalar functions to obtain the equations of motion. To conclude, we perform a comparison of efforts obtained analytically, with those provided by a commercial software of dynamic analysis. The analytical model showed outstanding results. For future works, we intend to couple the models of mechanical and hydraulic systems and perform the control.

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