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Uncertainty Quantification in Volterra Series Analysis of a Nonlinear Beam Considering the Noise Effect

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Abstract: Many mechanical systems can operate with strong nonlinear behavior, making the negligence of such effects a source of errors in the prediction of the system response. A methodology, that has been successfully used, to predict the behavior of such systems is based on the identification of Volterra kernels. However, this technique is subject to uncertainties that are induced by the measurements noise. This work presents a study that assesses the influence of these uncertainties in the Volterra kernels, expanded with Kautz functions, and their propagation through the nonlinear dynamic system. The proposed method is applied to a nonlinear beam. Monte-Carlo simulations are used to compute the propagation of uncertainties in Volterra kernels. The results have been shown that the kernels are greatly influenced by the presence of uncertainties and confidence limits for the system responses can be established.

Keywords: Nonlinear dynamics, Volterra series, Uncertainties quantification.

INTRODUCTION

Many engineering structures have geometries, gaps, bolted connections, clearance, gaps, cracks and materials constitutive relationships that conduces to highly nonlinear effects that make the systems exhibit complex nonlinear responses containing harmonic distortion, jumps, modal interactions, bifurcation, and chaos (Noël and Kerschen, 2016). Thus, it should necessarily be considered in the analysis of the dynamic behavior of such structures, according to many authors (Virgin, 2000; Worden and Tomlinson, 2001). In this sense, the Volterra series expanded in the orthonormal basis, with the Kautz functions, have been used to predict the dynamic responses of system with nonlinear behavior (Shiki et al., 2013a; da Silva, 2011a). A lot of papers as Shiki et al. (2013b), Shiki et al. (2014), Hansen et al. (2014a), Hansen et al. (2014b), Scussel and da Silva (2016) and Shiki et al. (2016) have shown the practical application of this approach in deterministic systems without consider the uncertainties quantification.

However, any real system is uncertain with respect to the project nominal values, due to material imperfections, irregularities on the manufacturing process, noise in the measurements, etc. (Iaccarino, 2009; Oden et al., 2010; Soize, 2013) thus, it is essential for a reliable system identification technique take into account the parameters uncertainties. There are two types of uncertainties in mathematical-mechanical modeling of real systems, the data uncertainties and the model uncertainties (Soize, 2005). In this paper, only the data uncertainties related with the noise effects and changes in the measurements process are considered.

In a previous paper, uncertainties analyses in Volterra series was made considering simulated system that approximates the behavior of a nonlinear beam with variation in the Young's Modulus value (Villani et al., 2015). Now, this paper deals with the uncertainty analysis in the models used in the Volterra kernels identification considering a real system with nonlinear behavior. Hence, it must defined confidence limits for identifying towards a future damage detection in the mechanical system, considering the variabilities in its behavior.

THE STOCHASTIC VOLTERRA SERIES

In this work a parametric probabilistic approach (Soize, 2012, 2013) is employed. In this sense, the model parameters subjected to uncertainties are described as random variables or random processes, defined on the probability space $(\Theta, \Sigma, \mathbb{P})$, where Θ is a sample space, Σ is a σ-algebra over Θ, and $\mathbb{P}$ is a probability measure. It is assumed that any random variable $\theta \in \Theta \mapsto \mathbb{Y}(\theta) \in \mathbb{R}$ in this probabilistic setting, with probability distribution $P_{\mathbb{Y}}(dy)$ on $\mathbb{R}$, admits a probability density function (PDF) $y \mapsto p_{\mathbb{Y}}(y)$ with respect to dy.

The discrete time Volterra series can be used to represents different types of nonlinear systems using the convolution concept (Schetzen, 1980). In a deterministic way, the response $y(k)$ of a nonlinear system can be described by:

$$y(k)=\sum_{\eta=1}^{\infty}\sum_{n_1=0}^{N_1-1}\dots\sum_{n_\eta=0}^{N_\eta-1}\mathcal{H}_\eta(n_1,\dots,n_\eta)\prod_{i=1}^{\eta}u(k-n_i)=y_1(k)+y_2(k)+y_3(k)+\cdots\,, \tag{1}$$

where $u(k)$ is the input signal, $y(k)$ is the system response, $\mathcal{H}_\eta(n_1,\dots,n_\eta)$ is the η-order Volterra kernel, $y_1(k)$ is the linear contribution and $y_2(k)$ e $y_3(k)$ are the quadratic and cubic contributions of $y(k)$.

Now, considering the presence of uncertainties, the system response becomes a random process $(\theta,k)\in\Theta\times\mathbb{R}\mapsto \mathbb{y}(\theta,k)$ that can be described by:

$$\mathbb{y}(\theta,k)=\sum_{\eta=1}^{\infty}\sum_{n_1=0}^{N_1-1}\dots\sum_{n_\eta=0}^{N_\eta-1}\mathbb{H}_\eta(\theta,n_1,\dots,n_\eta)\prod_{i=1}^{\eta}u(k-n_i)=\mathbb{y}_1(\theta,k)+\mathbb{y}_2(\theta,k)+\mathbb{y}_3(\theta,k)+\cdots\,, \tag{2}$$

where the random processes $(\theta,n_1,..,n_\eta)\in\Theta\times\mathbb{R}^\eta\mapsto\mathbb{H}_\eta(\theta,n_1,\dots,n_\eta)$ represents the random version of the η-order Volterra kernel.

The Volterra series inconvenient is the difficult in the series convergence when a high number of terms was used $N_1,...,N_\eta$. So, the kernels can be expanded in a base of orthonormal functions to minimize the problem, in this case, with the use of Kautz functions (Kautz (1954); Heuberger et al. (2005); da Silva (2011b)). Now, the Volterra kernels can be defined by:

$$\mathbb{H}_\eta(\theta,n_1,\dots,n_\eta)\approx\sum_{i_1=1}^{J_1}\dots\sum_{i_\eta=1}^{J_\eta}\mathbb{B}_\eta\left(\theta,i_1,\dots,i_\eta\right)\prod_{j=1}^{\eta}\mathbb{\psi}_{i_j}(\theta,n_j)\,, \tag{3}$$

where $J_1,\dots,J_\eta$ are the number of samples in each orthonormal projections of the Volterra kernels, the random processes $(\theta,i_1,\dots,i_\eta)\in\Theta\times\mathbb{R}^\eta\mapsto\mathbb{B}_\eta(\theta,i_1,\dots,i_\eta)$ represents the random Volterra kernels expanded in the orthonormal basis. The definition of Kautz functions is related with the dynamic system response $\mathbb{y}(\theta,k)$ and depend of the system physical parameters (damping ratio and natural frequency). Then, the Kautz functions are assumed as a random process $(\theta,n)\in\Theta\times\mathbb{R}\mapsto\mathbb{\psi}(\theta,n)$. In this context, the Volterra series can be rewrite as the multidimensional convolution between the random orthonormal kernels $\mathbb{B}_\eta(\theta,i_1,\dots,i_\eta)$ and the input signal filtered by the Kautz functions:

$$\mathbb{y}(\theta,k)\approx\sum_{\eta=1}^{\infty}\sum_{i_1=1}^{J_1}\dots\sum_{i_\eta=1}^{J_\eta}\mathbb{B}_\eta\left(\theta,i_1,\dots,i_\eta\right)\prod_{j=1}^{\eta}\mathbb{l}_{i_j}(\theta,k)\,, \tag{4}$$

where the random process $(\theta,k)\in\Theta\times\mathbb{R}\mapsto\mathbb{l}(\theta,k)$ is a simple filtering of input signal $u(k)$ by the Kautz function $\mathbb{\psi}_{i_j}(\theta,n_j)$:

$$\mathbb{l}_{i_j}(\theta,k)=\sum_{n_i=0}^{V-1}\mathbb{\psi}_{i_j}(\theta,n_i)u(k-n_i)\,, \tag{5}$$

where $V=\max\{J_1,\dots,J_\eta\}$.

Then, for each realization θ, the terms of the orthonormal kernels $\mathbb{B}_\eta\left(\theta,i_1,\dots,i_\eta\right)$ can be grouped into uncertain vectors $\widehat{\mathbf{\Phi}}$ and estimated by least squares method:

$$\widehat{\mathbf{\Phi}}=(\widehat{\mathbf{\Gamma}}^T\widehat{\mathbf{\Gamma}})^{-1}\widehat{\mathbf{\Gamma}}^T\widehat{\mathbf{Y}}\,, \tag{6}$$

where the matrix $\widehat{\mathbf{\Gamma}}$ has the input signal filtered $\mathbb{l}_{i_j}(\theta,k)$ and $\widehat{\mathbf{Y}}=[\mathbb{y}(\theta,1)\cdots\mathbb{y}(\theta,k)]$, k is the number of samples used. More information about the method can be found in da Silva et al. (2010) or da Silva (2011a).

Kautz Functions

The Kautz functions perform well in representing the orthonormal kernels to identify the Volterra kernels in oscillatory dynamic models, so they are used in this work (Kautz, 1954).

In a deterministic way, the Kautz functions $\psi(z)$ are defined by the Kautz poles $\beta_{2\eta-1}=\sigma+j\omega$ and $\beta_{2\eta}=\sigma-j\omega$, where $|\beta_{2\eta-1}|,|\beta_{2\eta}|<1$ for a stable system and η represents the number of the kernel, varying in 1, 2 and 3 in this paper. Thus, the Kautz functions of generalized form are given by:

$$\psi_{2j-1}(z)=\frac{\sqrt{1-b^2}\sqrt{1-c^2}}{z^2+b(c-1)z-c}[H_{b,c}(z)]^{j-1}\,, \tag{7}$$

$$\psi_{2j}(z)=\psi_{2j-1}(z)\frac{z-b}{\sqrt{1-b^2}}\,, \tag{8}$$

where $H_{b,c}(z) = \frac{-cz^2+b(c-1)z+1}{z^2+b(c-1)z-c}$ and the values of b and c for the poles $\beta_{2\eta-1}, \beta_{2\eta}$ considered are obtained through relationships:

$$b = \frac{\beta_{2\eta-1} + \beta_{2\eta}}{1 + \beta_{2\eta-1}\beta_{2\eta}}\,, \tag{9}$$

$$c = -\beta_{2\eta-1}\beta_{2\eta}\,, \tag{10}$$

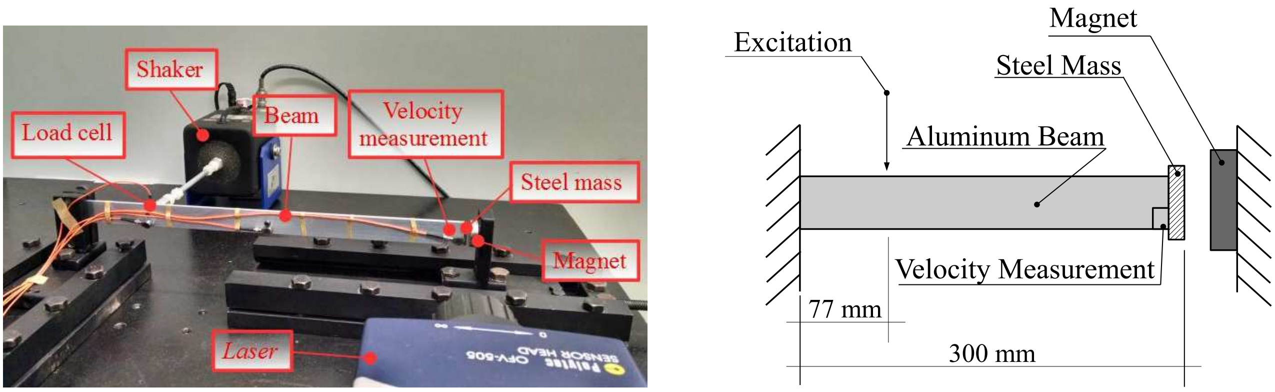
where the definition of Kautz poles are function of ω_η and ξ_η:

$$\beta_\eta = -\xi_\eta\omega_\eta \pm j\omega_\eta\sqrt{1-\xi_\eta^2}\,. \tag{11}$$

A procedure for optimizing the choice of setting parameters of the Kautz functions (β_η) must be used (da Rosa et al., 2007; da Rosa, 2009). So, considering that the system response $\mathbb{y}(\theta, k)$ is a random process, the system parameters ω_η and ξ_η to each kernel are considered as random variables $\theta \in \Theta \mapsto \mathbb{\xi}_\eta(\theta) \in \mathbb{R}$, $\theta \in \Theta \mapsto \mathbb{\omega}_\eta(\theta) \in \mathbb{R}$.

EXPERIMENTAL SETUP

The experimental setup used in the tests is composed by an aluminum beam with dimensions of $300 \times 18 \times 3$ [mm] with a steel mass connected in the free end of the beam to cause an magnetic interaction between the beam and the magnet. A schematic representation of this system with the experimental apparatus is illustrated in Fig. 1.



(a) Photo (b) Scheme

Figure 1: Experimental setup.

Note that, a MODAL SHOP shaker (Model Number: K2004E01) is attached 77 mm from the clamped and used to excite the structure considering different levels of voltage amplitude 0.01 V (low level), 0.10 V (medium level) and 0.15 V (high level). A vibrometer *laser Polytec* (Model: OFV-525/-5000-S) and a *Dytran* load cell (Model: 1022V) is used to measure the velocity in the free end of the beam and the force excitation, respectively. All signals are measured considering a sampling rate of 1024 Hz and 4096 samples saved using a m+p *Vib Pilot* data acquisition system. It is important to note that, the input signal considered is the voltage applied in the shaker, as done in Tang et al. (2015), and the study considers the beam-shaker system. Through this strategy, the applied voltage in the shaker is kept constant, while the beam applied force varies over a range of frequencies.

The magnet interaction with the beam generates a nonlinear hardening behavior (Kovacic and Brennan, 2011). This hardening effect is illustrated in the beam extreme velocity curves obtained during the stepped sine test shown in Fig. 2. When the input signal is high, the beam presents a jump phenomenon that is represented by a sudden drop in the amplitude of the response. This result clearly shows the nonlinear behavior of the mechanical system one wants to identify.

Fig. 3 shows the spectrogram of input and output signals for two levels of excitation (Low - 0.01 V and High - 0.15 V) when a linear chirp signal is applied in the beam-shaker system with a rate of 10 Hz/s, between 10 and 50 Hz. The nonlinear effects can be also seen when the high level of input signal is applied, with the arise of multiple harmonics in the system response. With low level of input signal applied (Fig. 3a and 3b) it can be seen that the system responds with the excitation frequency and the natural frequency (linear behavior), so with high level of input signal applied (Fig. 3c and 3d) the system responds with the excitation frequency, the natural frequency and two and three times the natural frequency when the first mode is excited (nonlinear behavior). The quadratic and cubic effects are related with the shaker and magnet presence, respectively.

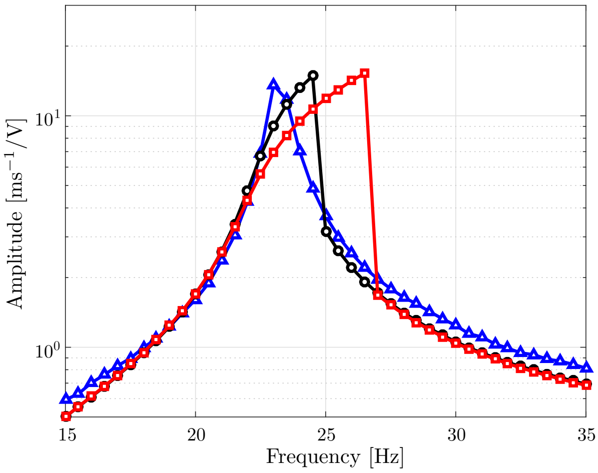
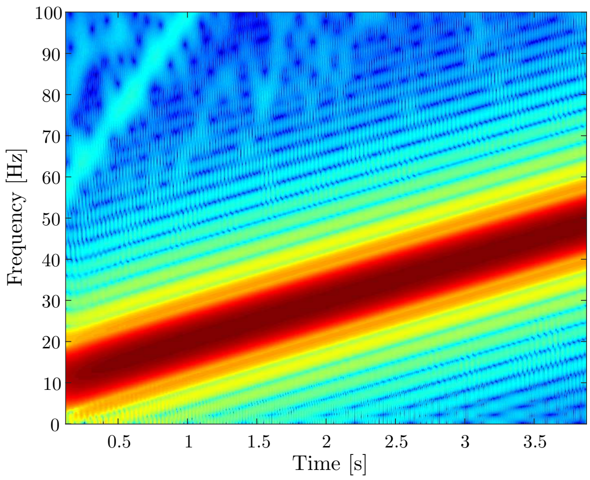
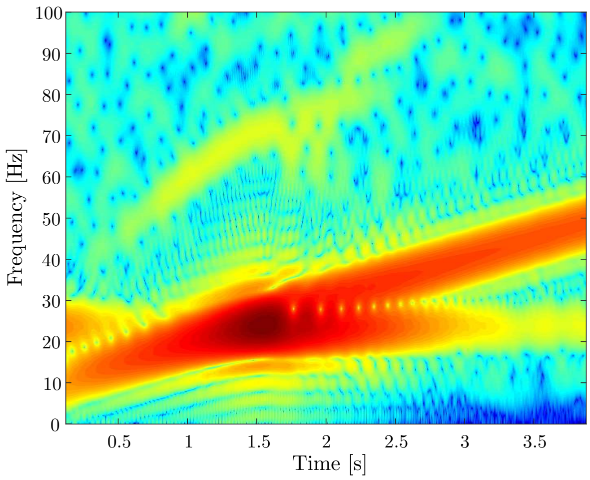


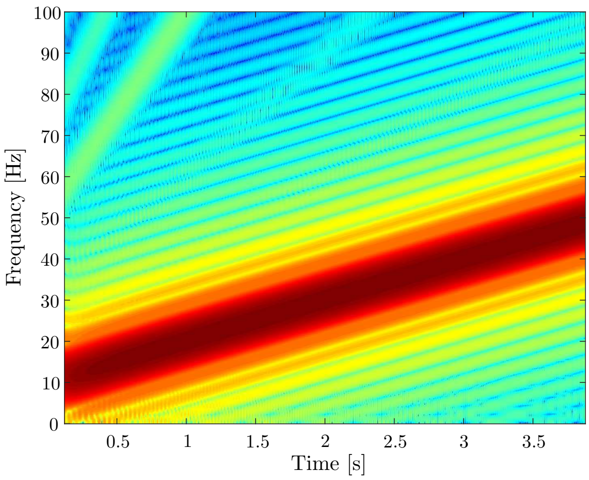
Figure 2: Beam extreme velocity during a stepped sine test, where △ represents the low level of input signal (0.01 V), ○ the medium level (0.10 V) and □ the high level (0.15 V).



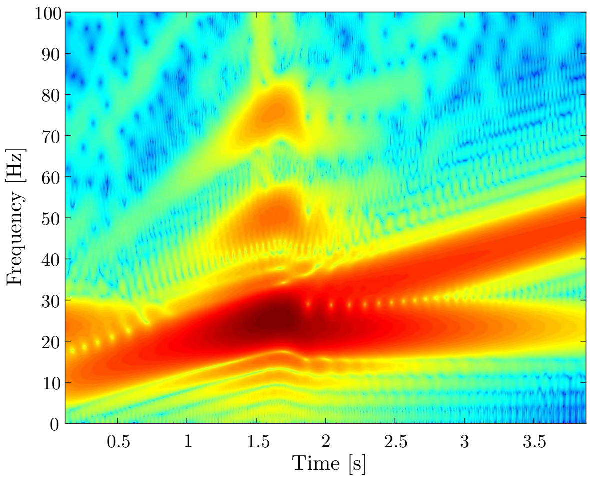
(a) Input low level (0.01 V).



(b) Output low level (0.01 V).



(c) Input high level (0.15 V).



(d) Output high level (0.15 V).

Figure 3: Specgram of the inputs and outputs considering differents levels.

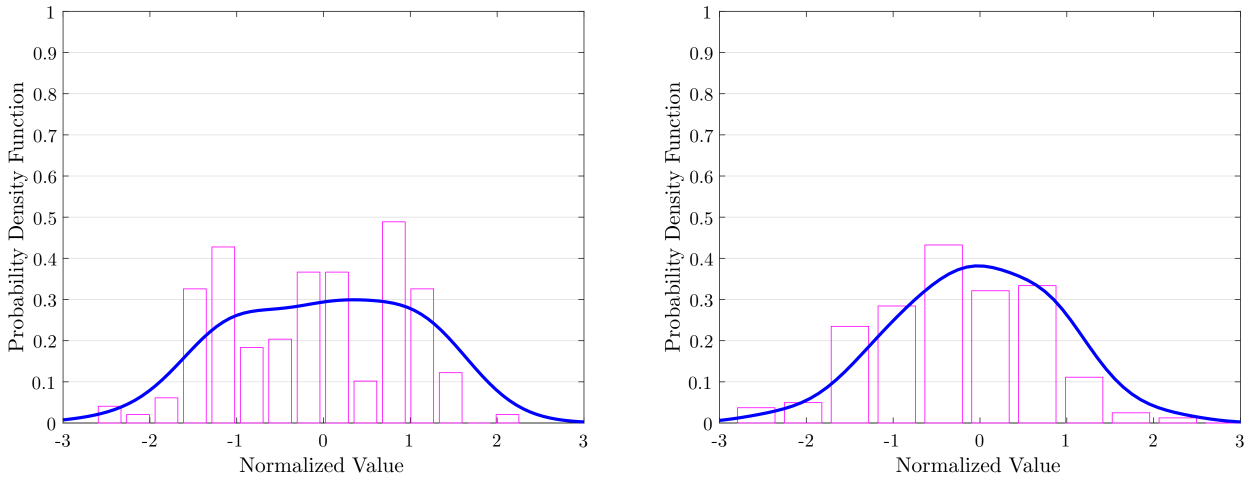
RESULTS AND DISCUSSIONS

In all simulations were used sampling frequency of 1024 Hz and 4096 samples. The structure was excited considering two different levels of voltage amplitude 0.01 V (low level) and 0.15 V (high level) with chirp signal varying the frequency between 10 to 50 Hz with rate of 10 Hz/s and repeated 150 times in different days between April and May of 2015 to study the system variations.

The identification of modal parameters damping ratio (ζ) and natural frequency (ω_n) uses the underlying linear dynamics of the beam, obtained when input has low level of amplitude (0.01 V) through the system FRFs and the line-fit method (Kouroussis et al., 2012). The modal parameters are important and used in the poles parameters optimization process. After identifying several realizations of the modal parameters (150 in fact), the PDFs (Probability Density Function) of damping ratio ζ, natural frequency ω_n are nonparametrically estimated, through histograms and kernel smoothed density curves (Ridgeway, 2007). Fig. 4 shows the normalized PDFs of damping ratio and natural frequency, where the normalization is performed according to the following expression:

$$A^{(n)}_{norm} = \frac{A^{(n)} - \mu_A}{\sigma_A}\,, \tag{12}$$

where $A^{(n)}_{norm}$ is the normalized value of the parameter A to each realization n, μ_A is the mean and σ_A the standard deviation.



(a) ω_n ($\mu_{\omega_n} = 23.3$ Hz and $\sigma_{\omega_n} = 0.1$ Hz). (b) ξ ($\mu_{\xi} = 0.0175$ and $\sigma_{\xi} = 0.0009$).

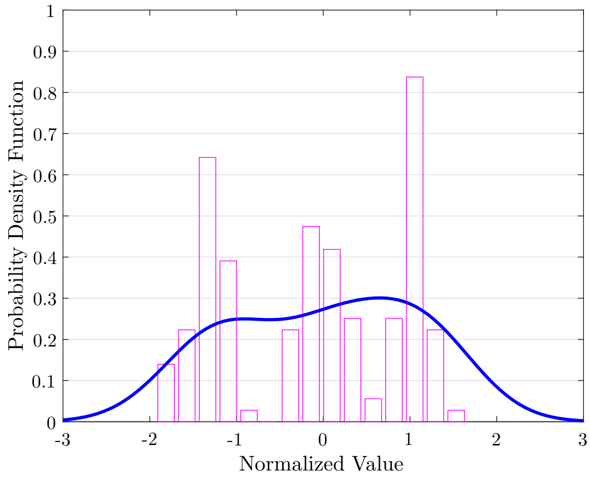
Figure 4: PDFs of the natural frequency and damping ratio of the equivalent linear system.

First, the number of Kautz functions should be selected. For the linear kernel, it is usual to select an order $J_1 = 2$ functions since this value maintains the second-order nature of linear vibrating systems. The orders of J_2 and J_3 was chosen as 2 and 6 functions because the nonlinearity of the system has cubic behavior what makes the number of functions of third kernel be higher. The parameters ω_n and ζ were used to defined the optimization process limits to find the optimal Kautz poles values to each realization. Tab. 1 shows the limits used to each kernel that are function of the modal parameters. The optimization process is more complicated in systems with data uncertainties as opposed to deterministic systems, then, more studies should be performed in the future to observe if the optimization is really needed or if only the stochastic identification of Volterra kernels would be able to describe the variability of such systems. Fig. 5 shows the distribution of Kautz parameters, estimated by the optimization process using a genetic algorithm, to each response realization. This parameters are used to estimated the Kautz functions that are used to filter the input signal. It can be seen that the PDFs of the parameters related with the first kernel (ω_1 and ξ_1) are very similar with the PDFs of modal parameters (ω_n and ζ), because the first kernel describes the linear behavior of the system. The dispersion of the second kernel parameters (ω_2 and ξ_2) are larger, because of the fact that the second kernel has lower contribution in the system response. Finally, the third kernel parameters (ω_3 and ξ_3) has multi-modal behavior with means close to the system modal parameters that influences the higher contribution of the third kernel in the response.

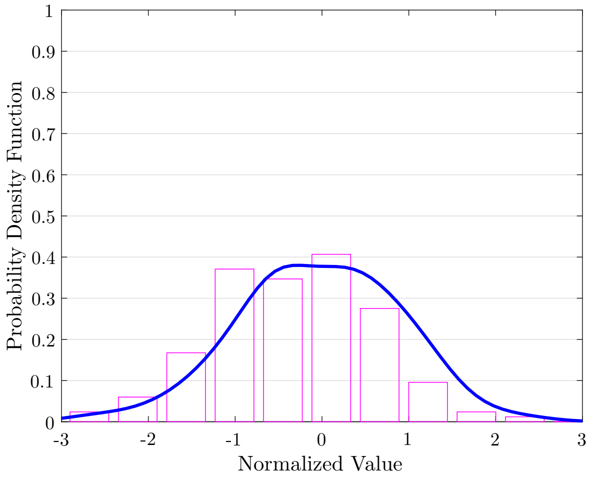
Table 1: Kautz parameters limits for each realization.

Parameter	$\omega_1(\theta)$	$\xi_1(\theta)$	$\omega_2(\theta)$	$\xi_2(\theta)$	$\omega_3(\theta)$	$\xi_3(\theta)$
Inferior Limit	$0.99 \times \omega_n(\theta)$	$0.99 \times \zeta(\theta)$	$0.80 \times \omega_n(\theta)$	$0.020 \times \zeta(\theta)$	$0.9 \times \omega_n(\theta)$	$0.90 \times \zeta(\theta)$
Superior Limit	$1.01 \times \omega_n(\theta)$	$1.01 \times \zeta(\theta)$	$1.20 \times \omega_n(\theta)$	$5.00 \times \zeta(\theta)$	$1.10 \times \omega_n(\theta)$	$3.00 \times \zeta(\theta)$

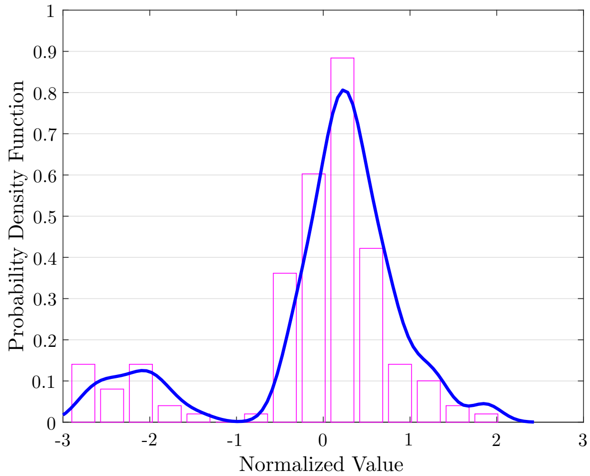
Fig. 6 shows the dispersion of the Volterra kernels in time domain considering 98% of confidence. It can be seen that the dispersion of first kernel is lower than the other two, because the linear parameters and the linear system responses vary lower than the nonlinear effects. The second kernel has large dispersion and low amplitude, because it has low contribution (system response is symmetric). Finally, the third kernel has large dispersion related with the difficult in estimated of Kautz parameters related with it and the nonlinear behavior that changes a lot in the data measured. The kernels variations permit to the prediction model describe changes in the system response over the following days. This is very important in the sense of to describe the systems behavior through a prediction model by separating the uncertainties to damage.



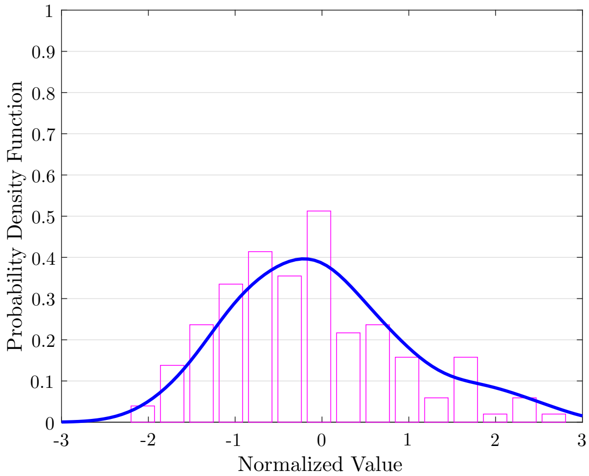
(a) ω_1 (μ_{ω_1} = 23.26 Hz and σ_{ω_1} = 0.04 Hz).



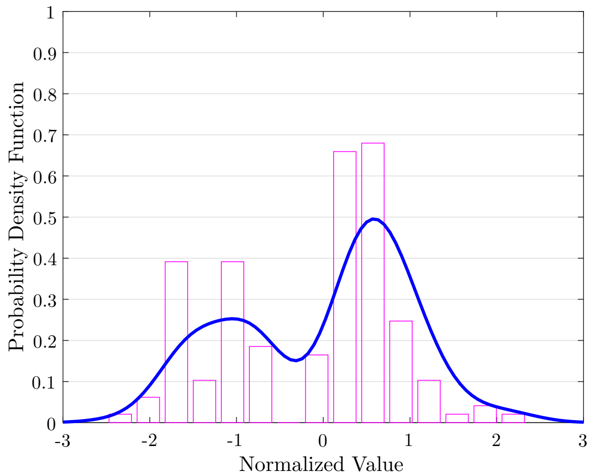
(b) ξ_1 (μ_{ξ_1} = 0.0177 and σ_{ξ_1} = 0.0006).



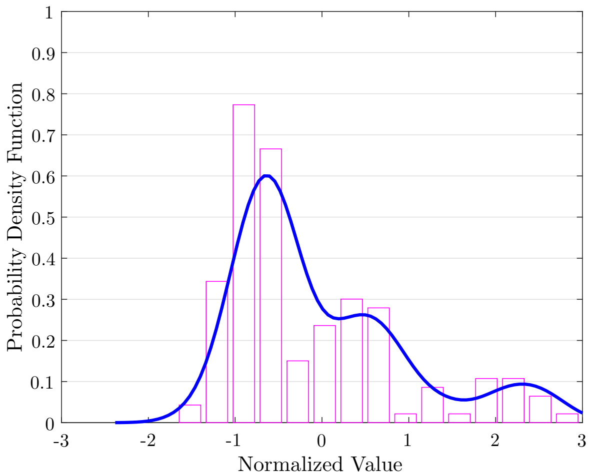
(c) ω_2 (μ_{ω_2} = 23.92 Hz and σ_{ω_2} = 1.83 Hz).



(d) ξ_2 (μ_{ξ_2} = 0.0391 and σ_{ξ_2} = 0.0187).



(e) ω_3 (μ_{ω_3} = 23.52 Hz and σ_{ω_3} = 0.24 Hz).

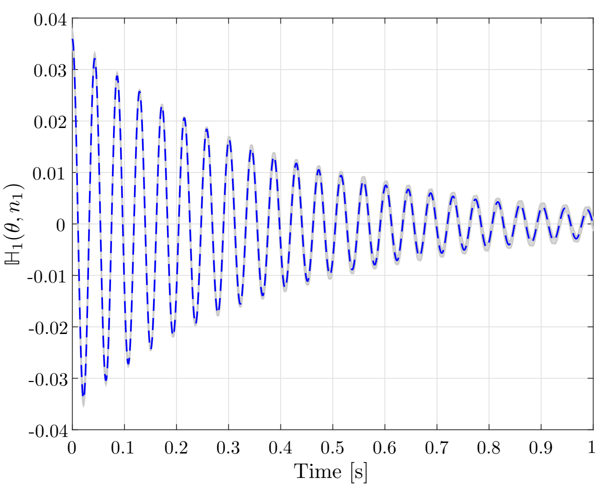


(f) ξ_3 (μ_{ξ_3} = 0.0197 and σ_{ξ_3} = 0.0027).

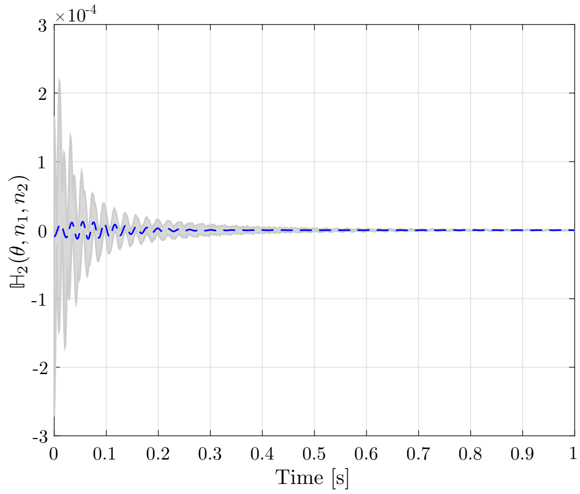
Figure 5: PDFs of the Kautz parameters.

Then, it can be seen in Fig. 7 the comparison between the system responses obtained through the Volterra model considering two levels of input (Low - 0.01 V and High - 0.15 V) and their limits with 98% of confidence and the experimental data obtained in different days. In both cases (linear and nonlinear behavior) the model prediction is good and, despite the variation of the data, the model is able to describes the systems behavior.

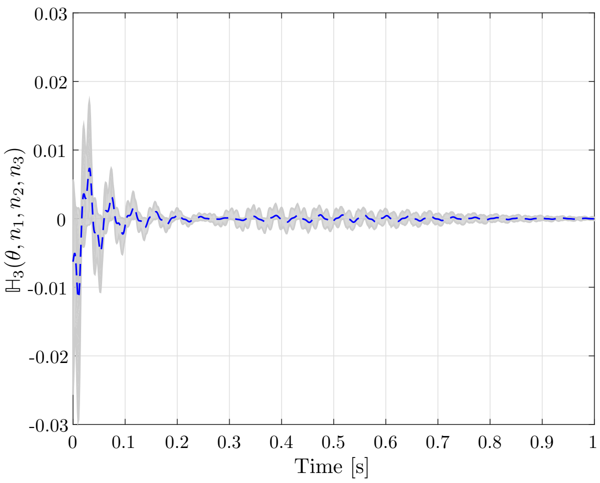
The great advantage of using Volterra series is to become easier the separation of linear and nonlinear contributions of the total response obtained through the model. In this sense, Fig. 8 shows the contributions of first, second and third kernels with 98% of confidence, considering low and high input. When the input has low level of amplitude (0.01 V), the system has linear behavior and the contribution in the response is only of the first kernel. Now, when the input has high level of amplitude (0.15 V), the total response is composed by the



(a) First kernel.

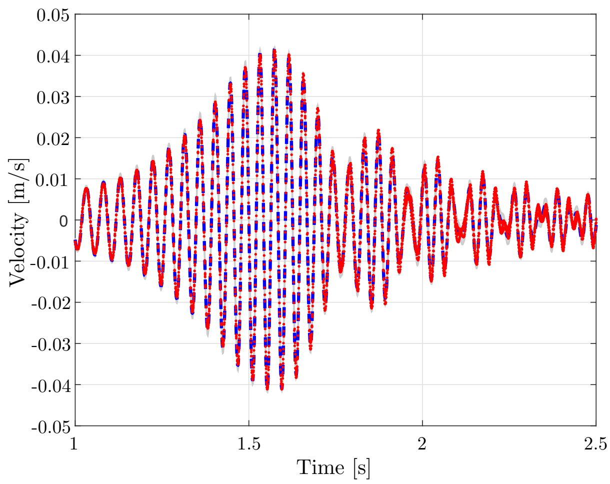


(b) Main diagonal of second kernel.

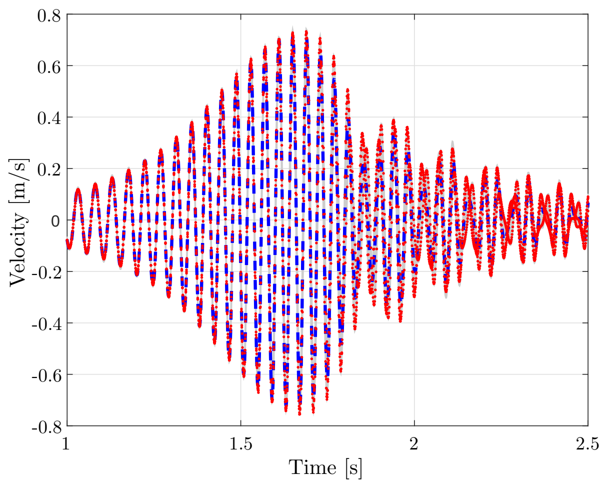


(c) Main diagonal of third kernel.

Figure 6: Volterra kernels limits with 98 % of confidence, the blue line represents the mean value.



(a) Low input.

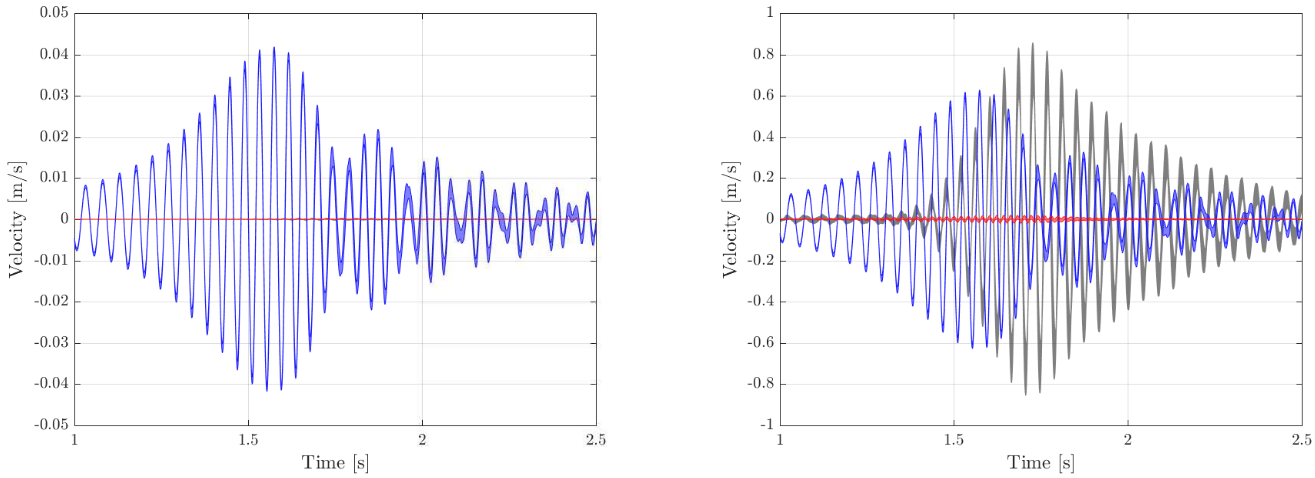


(b) High input.

Figure 7: Confidence bands for the system responses obtained through the Volterra model with 98 % of confidence. The mean is represented by the blue line and the experimental data are represented by the red dots.

linear component (first kernel) and the cubic component (third kernel) and the sum of the contributions allows the model to describe the nonlinear behavior of the system. It can be seen in Fig. 8b that the cubic component has more dispersion that the linear, because the estimation of the third kernel is more uncertain.

The model validation was made considering a simple sine input in the region of the first mode (≈ 20 Hz).



(a) Low input.

(b) High input.

Figure 8: Confidence bands for the contributions to the system responses obtained through the Volterra model with 98 % of confidence. represents the linear contribution, represents the quadratic contribution and represents the cubic contribution.

First, a low input signal (0.01 V) is applied, Figs. 9a and 9b shows the response in time and frequency domain with 98% of confidence. It can be seen that the experimental data are within the confidences limits obtained by the Volterra models. Then, high input signal (0.15 V) is applied to excite the nonlinearities of the system, Figs. 9c and 9d shows the response in time and frequency domain with 98% of confidence. In this case also, the model can predict the system response but now the dispersion in the frequency domain is higher. It can be seen the multiples harmonics in the system response when we consider high level of input (nonlinear behavior).

FINAL REMARKS

The results have showed that there is variation in system response over the days that must be considered in the modeling of the system. The Volterra kernels expanded through the Kautz functions are able to predict such variations when they are considered in system identification. The uncertainties in the measurements cause large changes in the Kautz parameters, especially the parameters related with high order kernels, because their values are optimized and this procedure is susceptible to variations in the system responses. The confidence limits established to the system response can be used in the future to study damage detection approaches in nonlinear uncertain systems. More studies should be conducted to observe the real need to optimize the Kautz parameters, because this process is very uncertain and causes great variability in the system response. Possibly, only the stochastic identification of Volterra kernels is able to describe the uncertainties in a simpler way.

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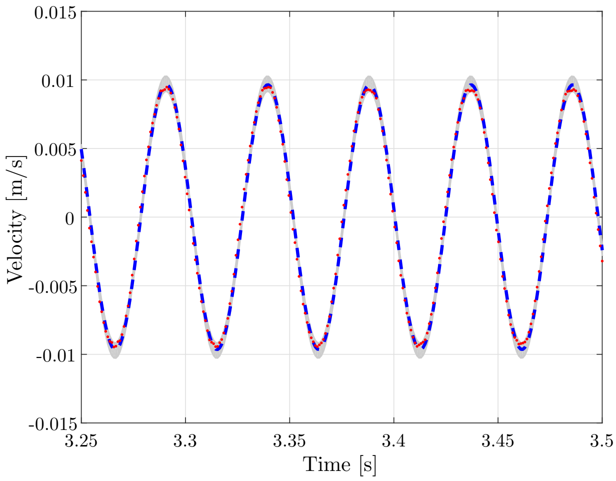
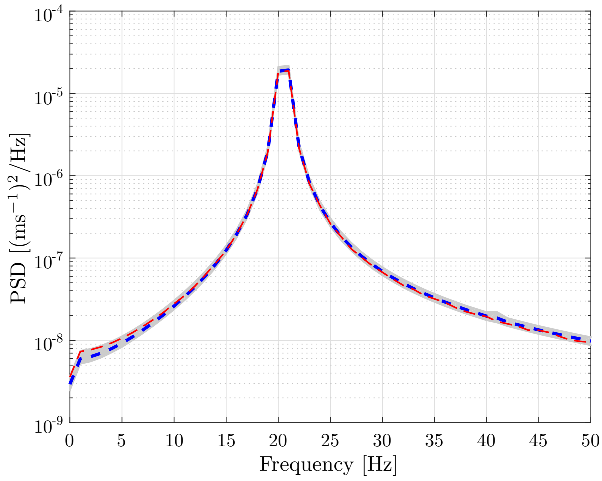
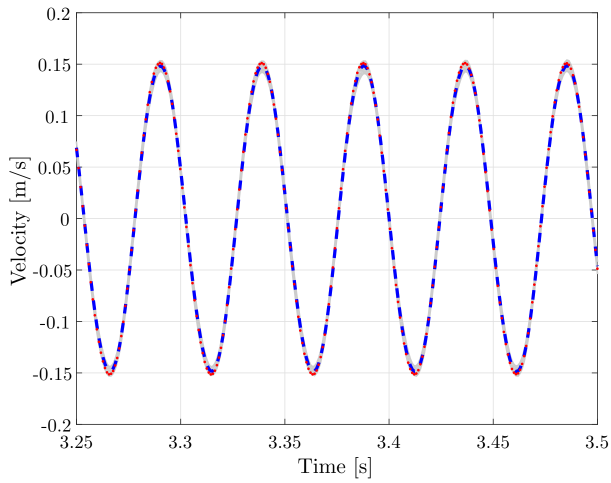


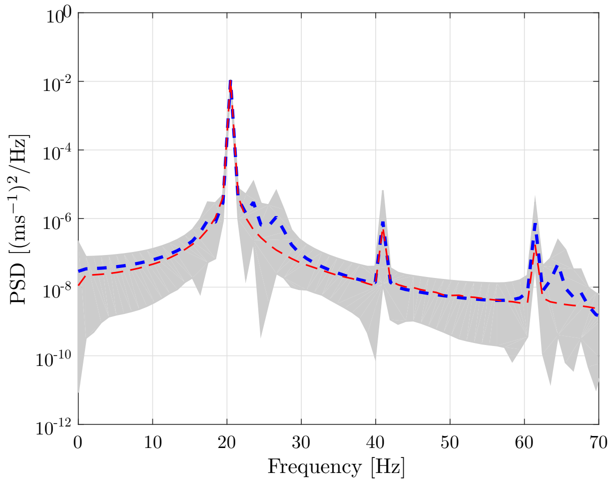
(a) Response low input.



(b) PSD low input.



(c) Response high input.



(d) PSD high input.

Figure 9: Confidence bands for the system responses obtained through the Volterra model with 98 % of confidence. The mean is represented by the blue line and the experimental data are represented by the red line.

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