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DYNAMIC BEHAVIOR OF TIMBER FOOTBRIDGES WITH UNCERTAIN MECHANICAL PROPERTIES AND STOCHASTIC WALKING LOADS

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Abstract: A dynamic study of timber footbridges with uncertain mechanical properties under the action of stochastic walking loads is presented in this paper. These structural systems made of timber are increasingly employed due to the high relation stiffness/weight that wood exhibits in relation to others structural materials. More, the development and implementation of laminated beams permits larger spans. These features can lead to lightweight structural systems in which the acceleration levels can exceed the human comfort limits. The sources of uncertainty of this structural model are the timber mechanical and physical properties, Modulus of Elasticity (MOE) and mass density. Also, the geometrical design of the boards that compose the laminated timber beams supporting the floor involves variability in the distances between finger joints. Probability Density Functions (PDFs) of the timber properties are formulated from the Principle of Maximum Entropy (PME). The finger joints distance generates the lengthwise variability of the MOE and mass density functions in each board of the laminated beams. The influence of these stochastic variables in the structural response on a forced vibration problem that includes a stochastic model of the load induced by the human walking is assessed. Pedestrians arrive to the footbridge under a Poisson distribution. The arrival velocity is such that a medium/low transit density is achieved in accordance with the footbridge dimension. The PDFs of the natural frequencies of the structure, the mode shapes and the structural response are numerically obtained through the Finite Element Method (FEM) and Monte Carlo Simulations (MCS). Values of peak accelerations produced at the mean span of the footbridge are evaluated in relation to the footbridge occupancy at each instant. The present stochastic model contributes to obtain a more realistic description of the response of this type of structures.

Keywords: Dynamics, Timber footbridge, Uncertain properties, Stochastic walking load, Poisson process.

INTRODUCTION

Pedestrian footbridges are one of the most common timber structures mainly due to the high relation stiffness/weight that this material presents in comparison to other construction materials and the possibility of covering long spans owing to the development and implementation of laminated beams. This type of beams has become greatly employed, and has allowed the construction of slender timber structures. The footbridges are composed of a deck and longitudinal laminated beams. In this work, the complete structure is made of Argentinian *Eucalyptus grandis*, one of the most important renewable species cultivated in Argentina. A simple method for visually strength grading sawn timber of these species has been developed by Piter (2003). As reported in this paper, the presence of pith and knots are considered the most important visual characteristic for strength grading this material by the Argentinian standard IRAM 9662-2 (2006). Experimental studies related with the bending strength and stiffness in *Eucalyptus grandis* laminated beams have been presented by Piter *et al*. (2007) and Saviana *et al*. (2009).

The stochastic approaches employed for the modelling of timber mechanical properties are derived from the probabilistic theories of random variables and processes. They allow simulating the timber mechanical properties within a structural analysis. Köhler *et al*. (2007) presented a probabilistic model of timber structures where the MOE is represented as a random variable with a lognormal PDF and the mass density through a random variable with normal distribution, both assuming a homogeneous value within a structural element. Recently, Fink and Köhler (2015) present a probabilistic approach for modelling the tensile strength and stiffness properties of timber boards and finger joint connections.

Timber footbridges must satisfy both strength and serviceability requirements. Generally, due to its lower weight, the serviceability requirements in terms of peak accelerations constitute the most restrictive condition in the design. The control of the structural system maximum acceleration can just be made by increasing the structural damping or mass.

The first natural frequency is also significant because the harmonic force intensity generally decreases with increasing harmonics. Actually, the design criteria for floor and footbridge vibration analysis due to people walking are based on these principles. In Živanović *et al*. (2005), an extensive literature review and state of the art report of the dynamic behaviour of these structures is presented. Among other topics, the loads models, standards requirements and studies of the walk of people and crowns are reported. In Segundinho (2010), a study of the vibrations induced by pedestrian in timber footbridges carried out in a parametric way is presented. Numerical results are compared with experimental ones obtained from a scaled model.

The sources of uncertainty in the structural model herein presented are assumed in the timber mechanical and physical properties, and in the geometrical design of the laminates that compose the laminated timber beams that support the floor of the footbridge. This geometric uncertainty involves the distances between finger joints which were obtained from visual survey of structural size *Eucalyptus grandis* laminated beams. Then, the Probability Mass Function of the distance between finger joints was constructed. Between the pieces of timber, defined by the distance among fingers joints, the properties vary stochastically and in a non correlated way. According to the standard IRAM 9662-2 (2006), each board of the laminated beams comes from a specific strength class. Within this quality class, the properties vary stochastically. The propagation of these sources of uncertainty in the first natural frequency of the footbridge that constitute one of the evaluation parameters of the serviceability performance is studied. To accomplish this, the PDF of the first natural frequencies was found via Monte Carlo simulations (Rubinstein, 1981). The PDFs of the MOE and the mass density are obtained by means of the Principle of Maximum Entropy (Shannon, 1948), and its parameters by means of the Maximum Likelihood Method (MLM) applied to MOE values that were obtained experimentally. In order to measure the fit between the experimental and theoretical PDFs of the MOE and the mass density, the Kolmogorov-Smirnov (K-S) test of fit is used.

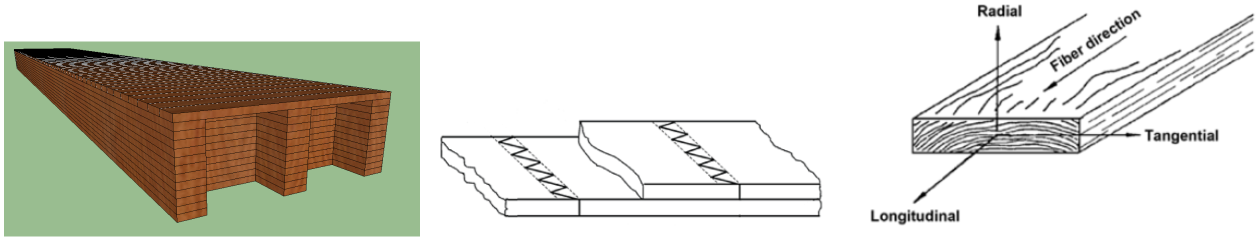
The second important parameter in the study of the serviceability performance is the acceleration. In addition to the work presented in García *et al*. (2016), in the current study, the footbridge was excited by a stochastic walking load model as the one presented in Živanović *et al*. (2007). Also, the transit of multiple pedestrians is considered through an arrival process simulated by a Poisson random variable. Two values of damping ratio are used. The stochastic model introduces the step frequency by a normal random variable, the probability of synchronization between the fundamental frequency of the footbridge and a multiple of the step frequency is low. This fact is frequently employed in the deterministic load models. This load model constitutes a more real representation of the variability in the human walking and in combination with the pedestrians arrival process conduces to more real serviceability conditions. A sensitivity study of the beams stiffness and the arrival velocity shows the changes in the conditions in which the maximum accelerations were produced.

PROBLEM STATEMENT

The study of the vertical dynamical behavior of a timber footbridge with stochastic mechanical properties under a walking load is presented. The structure is composed of three longitudinal laminated beams with a length of 13.2 m and a separation of 0.6 m in the transversal direction of the structure, five transversal laminated beams with a separation o 3.3 m in the longitudinal direction of the structure and a deck of timber boards. The width of the laminated beams and the timber boards of the deck was fixed in 0.15 m, and the height of each lamina of the beams and of the timber boards in 0.0375 m. The total height of the beams was dimensioned for resistance according to the Argentinean standard CIRSOC 601 (2013) and for serviceability requirements. The latter constitutes the main factor of the structural design analyzed later in this work. The number of laminates of the beams was considered between 12 and 16, laminated beams with 0.45 m and 0.6 m of height respectively, Fig. 1a. Laminated beams are composed of several laminates formed by the union of boards with different mechanical properties. Upper and lower faces of the boars are glued to the superior and inferior continuous board. Previously, the boards of each lamina are assembled by finger joints union. The influence of the finger joints configuration in the natural frequencies will be studied assuming randomness. Distances between finger joint obtained from a visual survey of laminated beams (Torrán, 2013) were employed in order to simulate the different boards that conform a laminated beam. With the results of the survey, the Probability Mass Function (PMF) of the distance between fingers joints was constructed. The mean value and standard deviation of the distance between joints are 0.865 m and 0.247 m, respectively (Fig. 1b).

TIMBER MATERIAL

The material model derived from the general assumption of orthotrophy in which each board has a longitudinal direction parallel to the main material fibres direction that coincides with the x axis, $L = x$, a tangential direction with respect to the growth rings of the transversal section that coincides with the y axis, $T = y$ and finally a radial direction with respect to the growth rings of transversal section that coincides with the z axis, $R = z$. Figure 1c shows the orthotrophy assumption.



(a) Transversal section of the timber footbridge. (b) Distance between finger joints. (c) Orthotrophy of a wood board.

Figure 1 – Structural model assumptions.

$$\begin{Bmatrix} \varepsilon_L \\ \varepsilon_R \\ \varepsilon_T \\ \gamma_{LR} \\ \gamma_{LT} \\ \gamma_{RT} \end{Bmatrix} = \begin{bmatrix} \frac{1}{E_L} & -\frac{\nu_{RL}}{E_R} & -\frac{\nu_{TL}}{E_T} & 0 & 0 & 0 \\ -\frac{\nu_{LR}}{E_L} & \frac{1}{E_R} & -\frac{\nu_{TR}}{E_T} & 0 & 0 & 0 \\ -\frac{\nu_{LT}}{E_L} & -\frac{\nu_{RT}}{E_R} & \frac{1}{E_T} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{G_{LR}} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{G_{LT}} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{G_{RT}} \end{bmatrix} \begin{Bmatrix} \sigma_L \\ \sigma_R \\ \sigma_T \\ \tau_{LR} \\ \tau_{LT} \\ \tau_{RT} \end{Bmatrix} \tag{1}$$

In this work, the orthotropic model is reduced to the transversal isotropic model with two main directions the longitudinal also named parallel to the main fibres direction $E_L = E_x$, and the perpendicular direction that includes the radial and tangential material direction $E_R = E_T = E_{yz}$. The basic stochastic properties proposed in this work are the longitudinal MOE and the mass density. In what follows, these stochastic properties will be represented by $\mathbb{E}_x$ and ρ. For a transversally isotropic material the elastic and shear modulus are defined as $\mathbb{E}_{zy} = \frac{\mathbb{E}_x}{15}$, $\mathbb{G}_{xy} = \mathbb{G}_{xz} = \frac{\mathbb{E}_x}{16}$ (UNE-EN 408, 1995). Meanwhile, in a general form the poison coefficients for hardwood are $\nu_{RT} = 0.67, \nu_{LT} = 0.46$ and $\nu_{LR} = 0.39$ (Argüelles Álvarez *et al*. 2000). For a transversally isotropic formulation $\nu_{zy} = \nu_{RT} = 0.67$ and $\nu_{xzy} = \frac{\nu_{LT}+\nu_{LR}}{2} = 0.425$ and $\mathbb{G}_{zy} = \frac{\mathbb{E}_{zy}}{2\left(1+\nu_{zy}\right)}$.

PDF of the MOE and mass density

In order to determine the marginal PDF of the MOE and the mass density, the Principle of Maximum Entropy (PME) (Shannon, 1948) for a continuous random variable is applied, Eq. (2). The measure of uncertainties of the continuous random variable X is defined by the following expression:

$$S(f_X) = -\int_D f_X(x) ln(f_X(x)) dx \tag{2}$$

in which f_X stands for the PDF of the random variable X and D is its domain. It is possible to demonstrate that the application of the principle under the constraints of positiveness and bounded second moment, leads to a gamma PDF. The PME conduces to this PDF due to the fact that the domain of the MOE and the mass density is real and positive.

To find the parameters of the marginal PDF of the MOE and mass density, experimental data presented by Piter (2003) are employed. These values were obtained by means of two point load bending tests, performed with 349 sawn beams of Argentinean *Eucalyptus grandis* with structural dimensions and density measurement. Bending tests and density measurement were carried out according to the standard UNE-EN 408 (1995). The parameters of the gamma marginal PDF of the MOE are estimated with the help of the Maximum Likelihood Method (MLM). Then, the Kolmogorov-Smirnov (K-S) test of fit is used, (*e.g.* Benjamin and Cornell, 1970). The PDF that best fits with the experimental values is the gamma, in accordance with the PME result. The test of fit was also carried out with the lognormal and normal PDFs, the first one since Köhler *et al*. (2007) proposed it to model the MOE and, the second PDF as is often employed to represent mechanical properties. The normal PDF fits the experimental data best. However, the use of this PDF in the model would occasionally lead to negative values of the MOE which are physically unacceptable. Thus, the gamma and lognormal PDF seem to be more suitable. Köhler *et al*. (2007) represented the variability of the mass density between structural timber beams with a normal PDF for European Softwood species. The parameters of the PDF of the mass density are estimated with the help of the MLM like in the previous case. Finally, the K-S test of fit is used to choose the PDF that fits best. As before, assuming α=0.05, the critical value for the K-S test of fit is equal to c=1.36. The results of the K-S test are the following: lognormal 0.65, gamma 0.69 and normal 0.75. As can be seen, the three PDFs fulfill the critical value, but the lognormal and gamma fit best with respect to the experimental values. Here, following the PME and due to the difference found among the lognormal and gamma is small, we adopted the last PDF in order to introduce in the stochastic model the mass density uncertainty. The gamma marginal PDF and CDF of the MOE and mass density are:

$$f(x \mid a,b) = \frac{1}{b^a\Gamma(a)} x^{a-1} e^{-\frac{x}{b}} \qquad F(x \mid a,b) = \frac{1}{b^a\Gamma(a)} \int_0^x t^{a-1} e^{-\frac{t}{b}} dt \tag{3}$$

where a and b, denote the shape and scale parameters, respectively. For the MOE, the parameters are a = 34.582 and b= 0.402 with a mean value of the MOE equal to 13.902 GPa and a standard deviation of 1.498 GPa. In the case of the mass density, a = 72.179 and b = 7.659, the mean value of the mass density equal to 552.819 kg/m^3 and a standard deviation of 65.069 kg/m^3.

LOAD MODEL

Stochastic load models are the most real representation of the forces induced by the pedestrian walking (Racic *et al.*, 2009). Deterministic models are not capable to model adequately the variability introducing by the human walking and result in an over representation of the forces induced by the pedestrian. The stochastic walking load model employed in this work was presented in Živanović *et al.* (2007). Parameters of the stochastic model that describe the variability in the forces induced by different individuals are:

- Step frequency and length:
  Step frequency f_s and step length l_s are considered independent parameters, the time employed by one pedestrian to cross the footbridge of length L is defined as $T_c = L/f_s l_s$ in which f_s and l_s are normal variables with mean value μ_{f_s}= 1.87 Hz and standard deviation σ_{f_s}= 0.186 Hz, for the first; and μ_{l_s}= 0.71 m and σ_{l_s}= 0.071 m, for the second.
- Pedestrian load magnitude:
  The total force is composed by the harmonics and sub-harmonics contribution, then is necessary to define the amplitudes of each one of them. These amplitudes are given in terms of Dynamic Load Factors (DLFs).
  - DLFs for harmonics:
    This load model consider five harmonics components. For the first one, the following expression presented by Kerr (1998) is employed $\mu_{DLF1} = -0.2649 f_s^3 + 1.3206 f_s^2 - 1.7597 f_s + 0.7613$. The probability distribution of $DLF1$ is obtained when its mean value is multiplied by a normal random variable MF with μ_{MF}= 1 and σ_{MF}= 0.16. The DLFs adopted for the second to fourth harmonics were presented in Kerr (1998) and for the five harmonic in Brownjohn *et al.* (2004). They are normal random variables independents of the step frequency f_s. Its mean values are 0.07, 0.05, 0.05 and 0.03; and its standard deviations are 0.03, 0.02, 0.02 and 0.015, respectively for the second to the fifth harmonic.
  - DLFs for sub-harmonics:
    The DLFs for the five sub-harmonics are given in function of $DLF1$ and were obtained by a lineal fit:

$$DLF_1^S = 0.026 DLF_1 + 0.0031 \qquad DLF_4^S = 0.013 DLF_1 + 0.0093$$
$$DLF_2^S = 0.074 DLF_1 + 0.0100 \qquad DLF_5^S = 0.015 DLF_1 + 0.0072$$
$$DLF_3^S = 0.012 DLF_1 + 0.0160$$

- Pedestrian weight:
  In this model, the individual weight has a unique mean value equal to 750 N.

The walking force is not a perfectly periodic process. The factors that represent the variability in the walking of one individual are taking into account through the force representation in the frequency domain. This representation is obtained through the normalized amplitude spectrum established for each one of the five harmonics and sub-harmonics. The normalized spectrum amplitude for each harmonic is represented by the following equation:

$$\overline{DLF_i}(\overline{f}_j) = a_{i,1} \exp^{-\left(\frac{\overline{f}_j - b_{i,1}}{c_{i,1}}\right)^2} + a_{i,2} \exp^{-\left(\frac{\overline{f}_j - b_{i,2}}{c_{i,2}}\right)^2} + a_{i,3} \exp^{-\left(\frac{\overline{f}_j - b_{i,3}}{c_{i,3}}\right)^2} \tag{4}$$

in which $a_{i,k}$, $b_{i,k}$ y $c_{i,k}$ ($k = 1,2,3$) are fit parameters (Živanović *et al.*, 2007), $\overline{f}_j$ is the ratio between the considered frequency and the step frequency f_s and belong to the interval $[i-0.25, i+0.25]$ with a frequency step of $1/80$.

The normalized spectrum amplitude for each sub-harmonic is represented by the following equation:

$$\overline{DLF_i}^S(\overline{f}_j^S) = a_{i,1}^S \exp^{-\left(\frac{\overline{f}_j^S - b_{i,1}^S}{c_{i,1}^S}\right)^2} + a_{i,2}^S \exp^{-\left(\frac{\overline{f}_j^S - b_{i,2}^S}{c_{i,2}^S}\right)^2} \tag{5}$$

in which $a_{i,k}$ y $b_{i,k}$ ($k = 1,2$) are fit parameters (Živanović *et al.*, 2007), $\overline{f}_j^S$ is the ratio between the considered frequency and the step frequency f_s and belong to the interval $[i-0.75, i-0.25]$ with a frequency step of $1/80$.

The variation of the force in the time domain is obtained through the Spectral Representation Method (Shinozuka and Deodatis, 1991). The force induced by the harmonic component i with a frequency if_s has the following time representation:

$$F_i(t) = P \times DLF_i \times \sum_{\overline{f}_j = i-0.25}^{\overline{f}_j = i+0.25} \overline{DLF_i}(\overline{f}_j) \cos\left(2\pi \overline{f}_j f_s t + \theta\left(\overline{f}_j\right)\right) \tag{6}$$

and for the sub-harmonic i, the force in the time domain has the following form:

$$F_i^S(t) = P \times DLF_i^S \times \sum_{\overline{f}_j^S = i-0.75}^{\overline{f}_j^S = i-0.25} \overline{DLF}_i^S(\overline{f}_j^S) \cos\left(2\pi \overline{f}_j^S f_s t + \theta\left(\overline{f}_j^S\right)\right) \quad (7)$$

in which i is the considered harmonic or sub-harmonic component, $\overline{f}_j f_s$ is the spectrum sub-division and $\theta\left(\overline{f}_j\right)$ is the phase angle of each sub-division. The random phase angle has a uniform distribution between $[-\pi, \pi]$. The total fluctuating force is obtained as the summation of the five harmonics and sub-harmonics contributions.

$$F(t) = \sum_{i=1}^{5} F_i(t) + \sum_{i=1}^{5} F_i^S(t) \quad (8)$$

In order to simulate the transit of multiple pedestrians the arrival time is simulated through a Poisson process. The inter-arrival times have an exponential distribution of parameter λ and they are simulated as $A_i = -(1/\lambda)\ln U_i$ in which U_i are random values from an uniform distribution between $[0,1]$ (Rubinstein, 1981). Then, the arrival times $T_i = A_1 + \cdots + A_i$ are simulated within the analysis time $[0,T]$ in which T is equal to 30 s and the mean arrival velocity λ is equal to 0.2 pedestrians/s. The mean arrival velocity λ was adopted in order to ensure a low pedestrians density according to the dimensions and the use of the footbridge presented in this work. A limit of low occupancy density is considered equal to 0.6 pedestrians /m^2 (Schlaich, 2005). For this limit or lower values, the individuals are capable to walk freely keeping their transit characteristics (step frequency and length). In the case of multiple pedestrians, the weight is assigned through a random variable with uniform distribution between 650 and 850 N.

FINITE ELEMENT DISCRETIZATION

The application of the Hamilton's principle and the discretization of the system lead to the following matricial expression:

$$\mathbf{M}\ddot{\mathbf{x}}(t) + \mathbf{C}\dot{\mathbf{x}}(t) + \mathbf{K}\mathbf{x}(t) = \mathbf{f}(t) \quad (9)$$

in which $\mathbf{M}$, $\mathbf{C}$ and $\mathbf{K}$ are the global matrices of mass, damping and stiffness, respectively; $\mathbf{f}(t,x)$ is the global nodal forces vector and $\ddot{\mathbf{x}}(t)$, $\dot{\mathbf{x}}(t)$ and $\mathbf{x}(t)$ are the global vectors of nodal accelerations, velocities and displacements, respectively. The equation of motion was discretized for laminated beams using Timoshenko beam elements with two nodes and three degrees of freedoms per node. Cubic and quadratic shape functions for the translational and rotational degrees of freedom and a lineal shape function for the torsional one, were employed (Reddy, 1993). The torsional stiffness was obtained from Swanson (1998). Rotational, translational and torsional inertial terms were considered. The equation of motion was discretized for the deck of the footbridge by rectangular bilinear plate elements with four nodes and three degrees of freedom per node (Reddy, 1993). The damping matrix in Eq (9) was considered proportional to the stiffness matrix, considering damping ratios equal to 5 % and 7 % in the first natural vibration mode. These values have been suggested by Chopra (1995) for wood structures with nailed or bolted joints. Experimental studies carried out in real footbridges found similar values (Segundinho, 2010). Natural frequencies and mode shapes were obtained through the following equation:

$$[\mathbf{K} - \Omega_n^2 \mathbf{M}]\Phi_n = 0 \quad (10)$$

in which Ω_n is the n^{th} natural circular frequency and Φ_n is its associated mode shape. Then and in order to obtain the dynamic response, the Modal Superposition Method is applied (Chopra, 1995). The nodal displacements vector is expressed as the product between the mode shape matrix and the vector of modal amplitudes $\mathbf{x}(t) = \Phi\mathbf{y}(t)$. Then, replacing the nodal displacements vector in Eq. (9):

$$\mathbf{M}\Phi\ddot{\mathbf{y}}(t) + \mathbf{C}\Phi\dot{\mathbf{y}}(t) + \mathbf{K}\Phi\mathbf{y}(t) = \mathbf{f}(t) \quad (11)$$

in which the components of nodal forces vector are defined as:

$$f_i(t) = \begin{cases} F(t) & \text{for} \quad t_0 \le t \le t_0 + t_d \\ 0 & \text{for} \quad t < t_0 \quad \text{y} \quad t > t_0 + t_d \end{cases}$$

where $F(t)$ is equal to the load function in the time domain, t_0 is the arrival time to the node i, $t_d = Le/v_p$ is the step time between nodes, v_p is the pedestrian velocity and L_e is the distance between nodes. Replacing $\mathbf{C} = a\mathbf{K}$ and multiplying the Eq. (12) for the transpose of the modal shape vector Φ_n and due to the orthogonality property of the mode shapes, we can obtain the equation of motion in generalized coordinates for each mode n:

$$M_n\ddot{y}_n(t) + aK_n\dot{y}_n(t) + K_n y_n(t) = f_n(t) \quad (12)$$

in which $f_n(t) = \Phi_n(v_p t)F(t)$ for the time between $t_0 \le t \le t_0 + t_d$. For the adopted damping model, the damping relationship for each one of the considered vibration modes is expressed as $\zeta_n = (a/2)\Omega_n$ where a is obtained assuming

ζ_n equal to 5-7 % for the first vibration mode. Applying the principle of effects superposition, the total mid-span (MS) accelerations induced for the total number of pedestrians (NP) considering N vibration modes is obtained from:

$$\ddot{x}_{MS}(t) = \sum_{i=1}^{NP} \left(\sum_{n=1}^{N} \Phi_{MS,n} \ddot{x}_n(t) \right) \tag{13}$$

NUMERICAL RESULTS

A convergence study of the first natural frequency was carried out due to the fact that the step harmonic, the step frequency and the step distance of the load model are considered in this work as functions of the first natural frequency of the footbridge. A number of 3000 independent Monte Carlo Simulations (MCS) was adopted for the stochastic study. The PDF of F_1, $f(F_1)$ is shown in Fig. 2 for laminated beams with and without finger joints union. As can be observed the mean value remain equal in both cases, $\mu(F_1) = 6.78$ Hz while the standard deviation decreases in the first case, $\sigma(F_1) = 0.06$ Hz and $\sigma(F_1) = 0.12$ Hz respectively. This effect is based in the variation of the effective stiffness and mass properties along the beams, introduced by the stochastic model with finger joints union. In Fig. 3, the first five

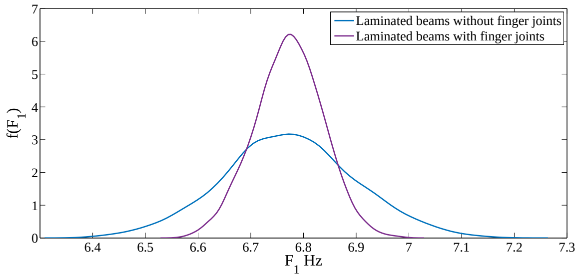


Figure 2 – PDFs of the first natural frequencies. Laminated beams with and without finger joints.

mode shapes and natural vibration frequencies of the mean model are shown. The study of the modes shapes and the frequency content of the forces allows determine the number of modes employed in the mode superposition method for the resolution of the equation of motion. For the load models applied at the center of the deck, the bending vibration modes have the higher contribution in the response.

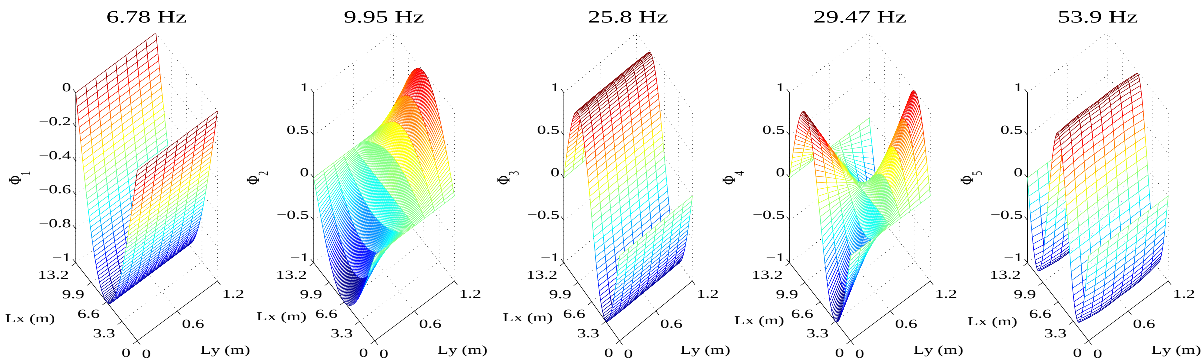


Figure 3 – Mode shapes and natural frequencies of the mean model.

The comparisons between the PDFs and CDFs of the maximum acceleration a_{max} for one and multiple pedestrians considering f_s random and $if_s = F_1$ are presented in Fig. 4. As is expected, the highest a_{max} values were obtained for the last case. But, comparing the PDF of f_s and the PDF of F_1, the real probability of $if_s = F_1$ are less than 0.159 and 0.096 for the fourth harmonic and sub-harmonic, respectively. These harmonics components are the most influential in the a_{max} for the footbridge considered. Then, the random f_s seems to be the most real option in structures in which the slenderness and dynamic behavior can not modified the human walking characteristic and in conditions of low occupancy density.

Then, the case of a footbridge with laminated beams composed for 10, 12, 14 and 16 laminates is considered. When the number of laminates is modified, the natural frequency of the structure changes and in consequence, the level of the load. In Fig. 5, the CDFs of the a_{max} for 5 % damping are depicted. For this damping value, the obtained mean values and standard deviations $[\mu(a_{max}), \sigma(a_{max})]$ are: [0.586, 0.168], [0.698, 0.202], [0.762, 0.207] and [1.074, 0.321] m/s^2 for 16:2:10 laminates, respectively. For 7 % damping the obtained mean values and standard deviations $[\mu(a_{max}), \sigma(a_{max})]$ are: [0.517, 0.135], [0.603, 0.158], [0.671, 0.170] and [0.927, 0.257] m/s^2 for 16:2:10 laminates, respectively. As can be observed, the highest difference with respect to the other configurations was obtained for the case with 10 laminates. This behavior is explained due to the change in the relation between F_1 and f_s that, in this case, leads to the highest load levels for the second harmonic of the force model. In the other cases, the footbridge is excited for the highest harmonics which

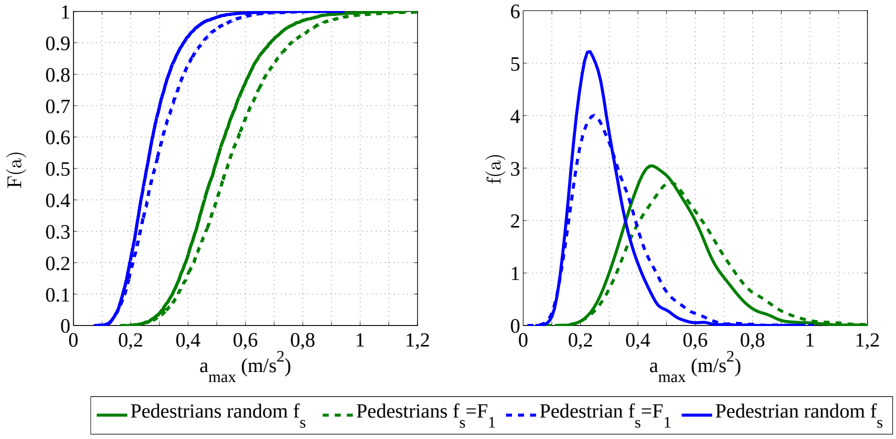


Figure 4 – Influence of the synchronization $if_s = F_1$, individual pedestrian (blue) and multiple pedestrians (green).

present the lower values of DLFs.

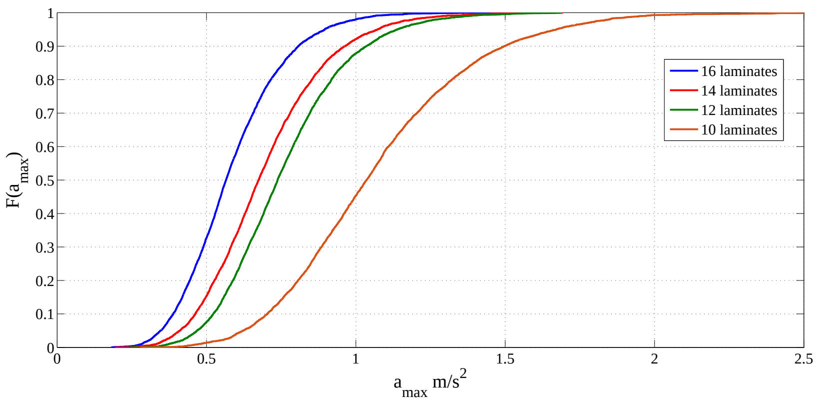


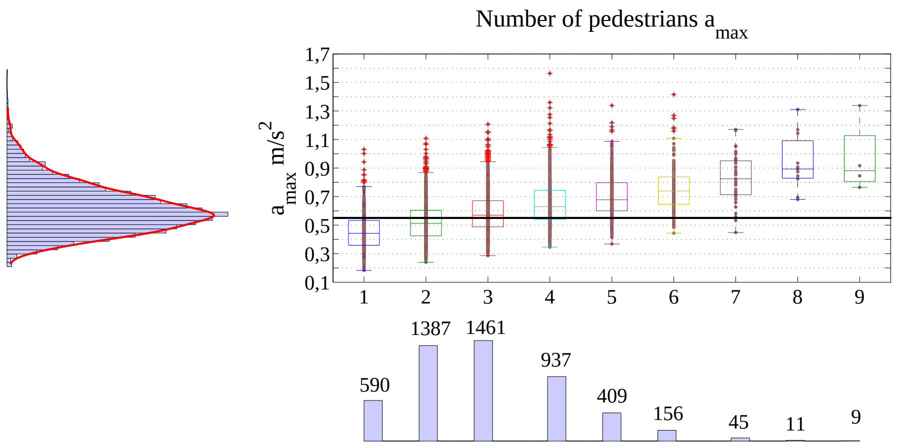
Figure 5 – Influence of the laminated beams stiffness. Number of laminates variation.

In what follows, the number of pedestrians and the f_s at the moment in which the values of a_{max} were produced for each register are analyzed. The results obtained for 16 laminates and 5 % of damping discretized in relation to the number of pedestrians at the footbridge at the moment in which a_{max} was registered for each one of the 5000 simulations are presented in Fig. 6. As can be observed in Fig. 6(a), the most frequent situation is that in which 3 pedestrians are at the footbridge at the moment of a_{max}, and this situation conduces to the nearest mean value in relation to the global mean value of a_{max}. The highest a_{max} was obtained with 4 pedestrians over the footbridge although situations with 8 and 9 individuals were reported. This result indicates that the a_{max} values occur for determinate configurations of pedestrian weight and step frequency f_s. However, the general tendency of increasing values of a_{max} with the increment of the number of pedestrians over the footbridge can be observed through the box plot. In Fig. 6(b), the relation between the pedestrians in a_{max} and the total pedestrians that transit the footbridge in each simulation is presented. The most frequent situations is that in which 6 to 8 pedestrians cross the footbridge during the total time of analysis and at the moment of a_{max}, 3 of them are present over the structure. In the comparison of the histograms, similitudes among them are apparent. A number of 3 pedestrians at the moment of a_{max} is coincident with the values of higher frequency in the Poisson arrival process which has a mean value equal to 0.2 pedestrians/s $\times$ 30 s = 6 pedestrians. This situation is observed in most of the simulations.

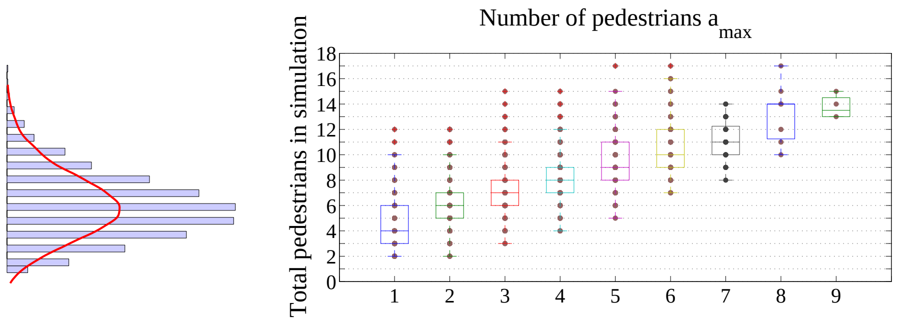
The number of pedestrians at the footbridge at the moment in which a_{max} were registered in function of the laminates of the laminated beams and the damping (5 % left plot and 7 % right plot) is presented in Fig. 7. Due to the fact that the acceleration levels are function of the relation between frequencies and the individuals weight, the number of pedestrians involved in a_{max} does not vary significantly with the laminates and change slightly with the damping. This behavior is observed due to the fact that the stochastic process of the load is the same and only its levels were modified with the change of the natural frequency of the footbridge and its relation with the step frequency f_s.

In Fig. 8, the variation of the CDFs of a_{max} in function of the arrival velocity is shown. Laminated beams with 16 laminates and 7 % of structural damping were employed. The arrival velocity was considered to range between 0.1 and 0.3 pedestrians/s. As can be observed, a_{max} increases with the arrival velocity due to the increment in the number of pedestrians in the analysis time. The obtained mean values and standard deviations $[\mu(a_{max}), \sigma(a_{max})]$ are: [0.419, 0.121], [0.472, 0.132], [0.517, 0.135], [0.563, 0.139] and [0.601, 0.147] m/s^2 for 0.1:0.05:0.3 pedestrians/s, respectively.

The results obtained for the arrival velocities 0.1 and 0.3 pedestrians/s are presented in Figure 9. As can be observed,



(a) Relationship between a_{max} and the number of pedestrians at the footbridge at the moment in which a_{max} was registered.



(b) Relationship between the total pedestrians in each simulation and the number of pedestrians at the footbridge at the moment in which a_{max} was registered.

Figure 6 – Study of the conditions in which a_{max} were produced.

the shape of the histograms of a_{max} were modified in the same form that the number of pedestrians involved in a_{max}. When the arrival velocity increases, the Poisson process tends to the normal one and a more symmetric distribution of the number of pedestrians at the footbridge at the moment in which a_{max} was registered is observed.

CONCLUSIONS

A stochastic study of the structural behavior of a timber footbridge was presented. The material properties and the assumptions of the model were presented. The stochastic analysis allows us to extend the range of the response starting from a model with mean properties. In comparison with footbridges structures with similar natural frequencies and different materials, the obtained values of accelerations are significant and the displacements small. This fact can be explained due to the high relation stiffness/weight of the timber material.

also, the influence of the finger joints distances in the natural frequencies was presented. From a visual survey carried out in structural size laminated beams, the PMF of the distance between finger joints was constructed. The variation in the effective stiffness and mass along the elements producing by the union of boards with different properties produces a lower standard deviation in the natural frequencies than when the finger joints union are not considered. The mean values of the natural frequency remains equal and are not influenced by the laminated beam model.

A stochastic load model that represents in a more real form the human walking has been employed. Employing this model, the maximum accelerations producing by the transit of individual and multiple pedestrians were studied. The pedestrian arrival was simulated through a Poisson process. The introduction of a step frequency random variable and the comparison with the distribution of the fundamental frequency of the footbridge conduces to a lower probability of synchronization between the frequencies. This fact is very common in the deterministic design of this type of structures. Through the simulation of multiple pedestrians, the conditions in which the maximum accelerations happen has been studied. First, the influence of the variation in the number of laminates of the laminated beams and then, the influence of the variation in the arrival velocity were studied. The influence of the step frequencies and weights of the pedestrians is evident. With the variation of the beam stiffness, the maximum accelerations were controlled by the relationship between the step frequencies of the pedestrians and the fundamental frequency of the footbridge. When the arrival velocity increases, the maximum accelerations are controlled only by the variation in the pedestrian quantities.

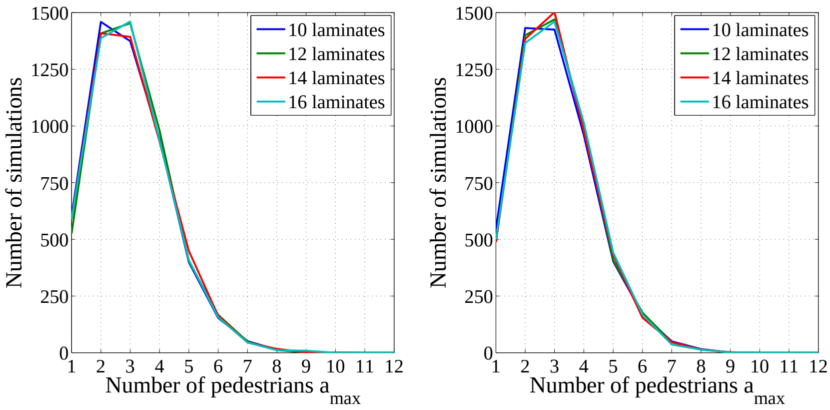


Figure 7 – Relationship between the number of pedestrians at the footbridge at the moment in which a_{max} was registered and the beams stiffness. Damping 5% left and 7 % right.

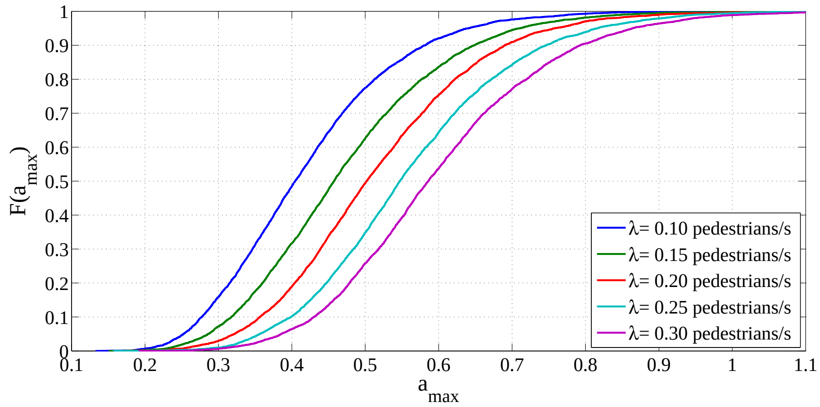


Figure 8 – Influence of variation in the arrival velocity of the Poisson process.

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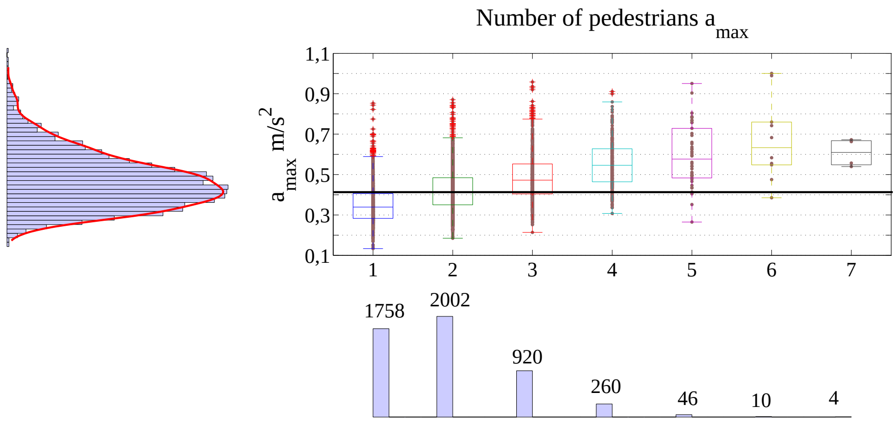
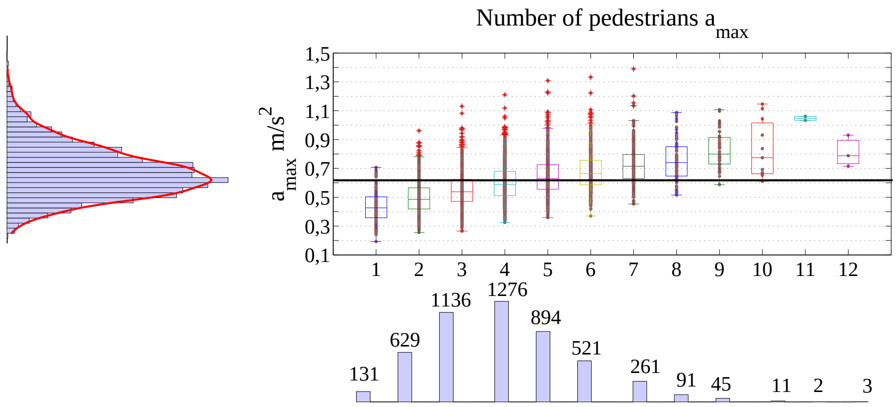


(a) λ=0.1 pedestrians/s.



(b) λ=0.3 pedestrians/s.

Figure 9 – Relationship between a_{max} and the number of pedestrians at the footbridge at the moment in which a_{max} was registered.

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