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Uncertainty analysis of rotating systems

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Abstract: This work proposes an application of uncertainties analysis in a rotating system. As stochastic modelling is not so common in rotordynamics field, it was chosen a simple model, which can qualitatively represent the main behavior of o rotating machinery. Another advantage of this choice is the processing time of model simulation. As Monte Carlo simulation is used for the stochastic model processing, a lower processing time is desirable. The model takes into account a rotor with two flexible bearing and an outboard disc. It is considered as random parameters the shaft elasticity modulus and the disc mass. In order to analyze the stochastic rotating system response, Campbell diagram is accomplished. In that way, it is possible to verify how the natural frequencies vary with the rotating speed. The rotating speed that is coincident to the natural frequency is called critical speed. Random parameters were adopted and a stochastic Campbell diagram were obtained. Consequently, the histograms of the critical speed were also obtained. Besides, a sensitivity analysis was accomplished, considering the influence of the coefficient of variation on the critical speeds. The is noted an important variation on the critical speeds skewness for high coefficient of variation.

Keywords: Rotordynamics, Jeffcott model, uncertainties

INTRODUCTION

Rotordynamics is important in industry, due to the considerable amount of applications of rotating machineries as in energy generation, power transition and others. Because of that, it is very important to understand the dynamic behavior of these machines. Therefore, mathematical models have been developed, considering the rotor, bearings, and supporting structures. Discrete and continuous models are used for the rotor, while the bearings can be represented by concentrated stiffness and damping coefficients. If hydrodynamic journal bearings are considered, these coefficients are obtained from the Reynolds equation, which models hydrodynamic pressure distribution of the lubricated oil. Due to the complexity of the models, it is not expect to find analytical solutions for the dynamic behavior of rotating machineries. The simplest of rotor model, named as Jeffcott rotor, considers a massless shaft with a rigid disc fixed in the center of the shaft. In this case, the shaft is supported by rigid bearings (Krämer, 1993). There are others formulations based on the Jeffcott rotor that take into account a non-central, or outboard disc. The main advantage of these models is its simplicity and utility to exhibit the main characteristics of the dynamics. Clearly, a more rigorous quantitative analysis demands more elaborated models.

An important aspect, that is not traditionally considered in rotordynamics analysis, is the robustness of the results. Due to the uncertainties of the properties and conditions of operation, stochastic models must be considered to measure the robustness of the results. According to Nordmann et al. (2016), due to the demand of a shorter design cycle and competitiveness in performance and safety criteria in turbomachinery industries, a probabilistic approach is starting to play an important role in turbomachinery design process, allowing the application of robust design methodologies.

However, there are few publications that consider uncertainties in rotor machinery models. Didier et al. (2012) proposed the application of polynomial chaos expansion for uncertainty quantification of rotor model, considering several kind of faults. Later, Sinou and Jacquelin (2015) applied this approach to verify the influence of the polynomial order on the stochastic response. Besides, Sinou et al. (2015) studied the stochastic response of rotors with local non-linearities. Koroishi et al. (2012) also proposed a stochastic modeling of flexible rotors. Gan et al. (2014) considered a Jeffcott rotor with disc offset and proposed a non-parametric modelling for uncertainty analysis. Ritto et al. (2011) proposed an robust optimization of a rotating system, considering the Campbell diagram. Garoli and Castro applied stochastic collocation method to determine the coefficients of a polynomial chaos expansion for linear journal bearing model (2016a) and non-linear model (2016b).

Despite of some advances on the application of uncertainty analysis in rotating systems, this subject still needs a deeper development. Therefore, the objective of this work is to model the critical speeds of a rotating system as stochastic, considering as random variables some of the properties of the rotor. In this paper a simple problem is studied, the analysis of a rotating system with two flexible bearings and an outboard disc. Campbell diagrams are made for this models because, in this case, there are gyroscopic effects and forward and backward natural frequencies are presented in the response.

Since the subject is rather new, the choice of a simple model is justified since our goal is to show the importance of the stochastic aspects. More complex models lead to a significantly higher computational cost. In the analysis, the elasticity modulus and the disc mass are the considered as stochastic. Gamma distribution is assumed to model these parameters. Monte Carlo simulations are done to quantify the stochastic response of the system. As gyroscopic effect is present, the uncertainty of the forward and backward natural frequency is also obtained. A sensitivity analysis is pointed out, in order to verify how parameters dispersion influences the stochastic response. In that way, the second, third and fourth statistical moments of the critical speeds are investigated.

ROTOR MODEL DESCRIPTION

The proposed model (Fig.1) is based on (Krämer, 1993) and takes into account the following properties. The shaft is massless, flexible, and of constant cross-section. Its elasticity modulus is represented by the random variable E and the area moment of inertia I. The disc mass m also is a random variable. The polar and diametral moments of inertia I_p and I_d depends on the disc mass m. The bearings are flexible and the rotor has a constant angular velocity (rotation speed) Ω. An outboard disc is considered, as shown in Fig.1 .The distance between both bearing is represented by the length L and the distance between the bearing B and the disc is given by c. The bearings A and B are flexible, and their stiffness are k_{A1}, k_{A2}, k_{B1} and k_{B2}.

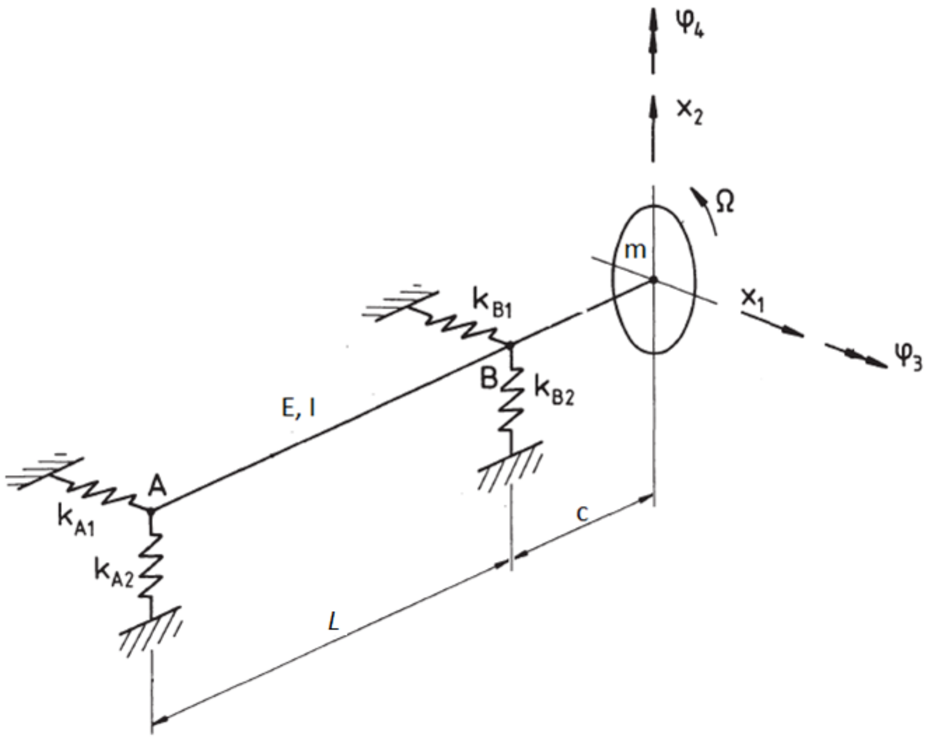

Figure 1 – Rotor model scheme with and outboard disc (based on Krämer, 1993).

The degrees of freedom are represented by the linear displacement in horizontal and vertical direction x_1 and x_2 respectively, and the rotations arounds their axis φ_3 and φ_4. The vector of the degrees of freedom is denoted by $\mathbf{q} = [x_1, x_2, \varphi_3, \varphi_4]^T$.

The equation of motion of the proposed model is shown in Eq.(1):

$$\mathbf{M}\ddot{\mathbf{q}}(t) + (\mathbf{C} + \Omega\mathbf{G})\dot{\mathbf{q}}(t) + \mathbf{K}\mathbf{q}(t) = \mathbf{f}(t) \tag{1}$$

where the mass $\mathbf{M}$, damping $\mathbf{C}$, gyroscopic $\mathbf{G}$ and stiffness $\mathbf{K}$ matrices are given by:

$$\mathbf{M} = \begin{bmatrix} m & 0 & 0 & 0 \\ 0 & m & 0 & 0 \\ 0 & 0 & I_d & 0 \\ 0 & 0 & 0 & I_d \end{bmatrix}, \mathbf{C} = \begin{bmatrix} c_1 & 0 & 0 & 0 \\ 0 & c_1 & 0 & 0 \\ 0 & 0 & c_2 & 0 \\ 0 & 0 & 0 & c_2 \end{bmatrix}, \mathbf{G} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -I_p \\ 0 & 0 & I_p & 0 \end{bmatrix}, \mathbf{K} = \begin{bmatrix} k_{11} & 0 & 0 & k_{14} \\ 0 & k_{22} & k_{23} & 0 \\ 0 & k_{23} & k_{33} & 0 \\ k_{14} & 0 & 0 & k_{44} \end{bmatrix}.$$

The damping matrix $\mathbf{C}$ is characterized by external viscous damping on the disc with coefficients c_1 for displacements and c_2 for rotations.

The equivalent stiffness k_{11}, k_{22}, k_{33}, k_{44}, k_{14} and k_{23} are considered on stiffness matrix $\mathbf{K}$. They take into account the shaft and flexible bearings stiffness. In order to determine each equivalent stiffness, it is necessary to calculate the displacement influence coefficients. The displacement influence coefficients h''_{ik}, for the case where both shaft and bearing are flexible, is given by:

$$h"_{ik} = h'_{ik} + h_{ik} \tag{2}$$

where h_{ik} is the influence coefficient when the shaft is flexible and the bearing is rigid, and h'_{ik} is the influence coefficient for flexible bearing and rigid shaft.

The equivalent stiffness coefficients are obtained by:

$$k_{11} = \frac{h"_{44}}{\Delta_1}, k_{14} = \frac{-h"_{14}}{\Delta_1}, k_{44} = \frac{h"_{11}}{\Delta_1}, k_{22} = \frac{h"_{33}}{\Delta_2}, k_{23} = \frac{-h"_{23}}{\Delta_2} \text{ and } k_{33} = \frac{h"_{22}}{\Delta_2}, \tag{3}$$

where $\Delta_1 = h"_{11}h"_{44} - h"^2_{14}$ and $\Delta_2 = h"_{22}h"_{33} - h"^2_{23}$.

The displacement coefficients h_{ik} and h'_{ik} are calculated as shown in Eqs.(4) and (5) respectively:

$$h_{11} = h_{22} = \frac{c^2(L+c)}{3EI}, h_{23} = -h_{14} = \frac{c(2L+3c)}{6EI}, h_{33} = h_{44} = \frac{L+3c}{3EI}, \tag{4}$$

$$\begin{aligned} h'_{11} &= \tfrac{(c/L)^2}{k_{A1}} + \tfrac{(1+c/L)^2}{k_{B1}}, h'_{22} = \tfrac{(c/L)^2}{k_{A2}} + \tfrac{(1+c/L)^2}{k_{B2}}, \\ h'_{33} &= \tfrac{1}{k_{A2}L^2} + \tfrac{1}{k_{B2}L^2}, h'_{44} = \tfrac{1}{k_{A1}L^2} + \tfrac{1}{k_{B1}L^2}, \\ h'_{14} &= \tfrac{(c/L)}{k_{A1}L} + \tfrac{1+c/L}{k_{B1}L}, h'_{23} = \tfrac{(c/L)}{k_{A2}L} + \tfrac{1+c/L}{k_{B2}L}. \end{aligned} \tag{5}$$

Free Vibration

To obtain the free vibration response, Eq.(1) is rewritten in space-state:

$$\begin{Bmatrix} \dot{\mathbf{q}} \\ \ddot{\mathbf{q}} \end{Bmatrix} = \begin{bmatrix} \mathbf{0} & \mathbf{I} \\ \mathbf{M}^{-1}\mathbf{K} & \mathbf{M}^{-1}\mathbf{K} \end{bmatrix} \begin{Bmatrix} \mathbf{q} \\ \dot{\mathbf{q}} \end{Bmatrix} + \begin{Bmatrix} \mathbf{0} \\ \mathbf{M}^{-1}\mathbf{f}(t) \end{Bmatrix} \tag{6}$$

Considering only free vibration, and defining $\mathbf{y} = [\mathbf{q}, \dot{\mathbf{q}}]^{\mathrm{T}}$, Eq. (6) is written as:

$$\dot{\mathbf{y}} = \mathbf{A}\mathbf{y} \tag{7}$$

where $\mathbf{A} = \begin{bmatrix} \mathbf{0} & \mathbf{I} \\ \mathbf{M}^{-1}\mathbf{K} & \mathbf{M}^{-1}(\mathbf{C}+\Omega\mathbf{G}) \end{bmatrix}$, depends on the rotating speed Ω.

Assuming a solution of Eq. (7) as following:

$$\mathbf{y}(t) = e^{\lambda t}\psi \tag{8}$$

the problem is stated as an eigenvalue problem:

$$[\mathbf{A} - \lambda\mathbf{I}]\psi = \mathbf{0} \tag{9}$$

where $\lambda = \varsigma \pm i\omega$ is the complex frequency and ψ is the complex mode. The natural frequency of the free motion (whirl frequency) is denoted by the imaginary part ω of the complex frequency λ, while the real part ς is related to the damping. If the real part assumes positive values, the vibration motion becomes unstable (Genta, 2005 and Child, 1993).

A plot of the imaginary part against the rotating speed, Ω, is called a Campbell diagram. Due to the dependence of the gyroscopic matrices on the rotating speed, Ω, this diagram presents an important aspect in rotordynamics analysis, because as the rotating speed increases, the variation on the natural vibration can be observed. This work is not taking into account journal bearing models. In that case, it is also observed an influence of the rotating speed on damping and stiffness coefficients, which also contributes with the effect on the natural frequency variation.

In order to verify how the rotating speed affects the natural frequency in the proposed model, it is taken into account the system parameters as shown in Tab.1.

The polar and diametral moments of inertia I_p and I_d depends on the disc mass m, and its relation is given by:

$$I_p = 1.065m, I_d = 0.5I_p \tag{10}$$

The Campbell diagram for the assumed parameters is shown on Fig. 2.

Table 1 – Rotor model parameters

Parameters	assumed values
Elasticity modulus $E[GPa]$	200
disc mass $m[kg]$	8000
area moment of inertia $I[m^4]$	0.002
bearing stiffness $k_{A1}[N/m]$	$3.33x10^8$
bearing stiffness $k_{A2}[N/m]$	$6.67x10^8$
bearing stiffness $k_{B1}[N/m]$	$0.83x10^8$
bearing stiffness $k_{B2}[N/m]$	$1.67x10^8$
viscous damping $c_1[Ns/m]$	10000
viscous damping $c_2[Ns/rad]$	1000
Length $L[m]$	4
Length $c[m]$	0.8

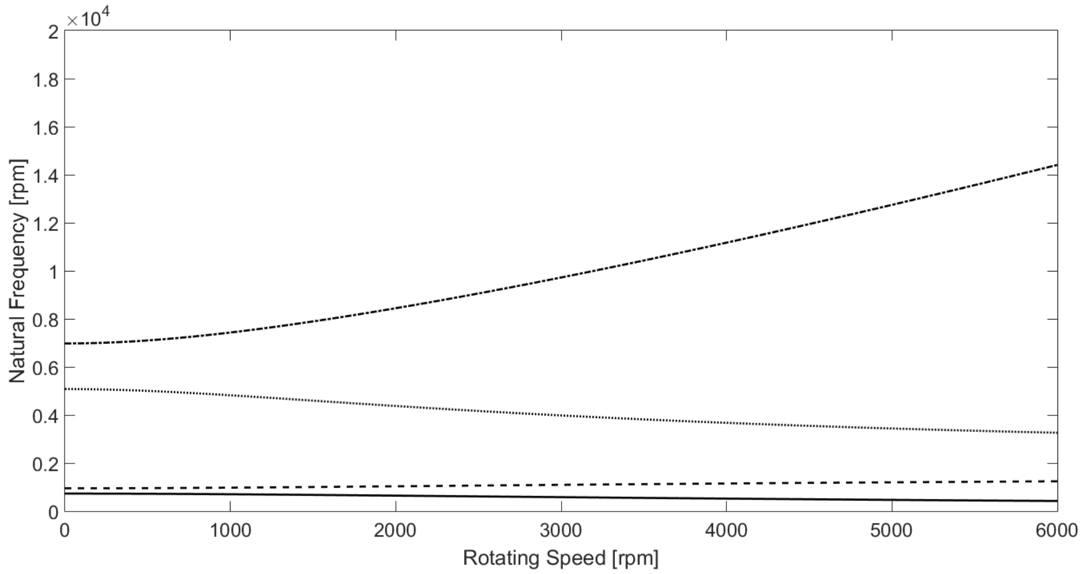

Figure 2 – Campbell Diagram for deterministic parameters.

STOCHASTIC MODEL

The methodology for stochastic modelling follows the scheme shown on Fig. 3. In this process, the first step is the probabilistic modelling of the random variables. This part consists in determining the most indicated probability distribution, based on the Maximum Entropy Principle, (MEP), (Shannon 1948, Jaynes 1957a,b), which means that the probability distribution is the one that best represents the current state of knowledge of the random variable. Considering a hypothetic random variable x, the MEP states that the uncertainty, measured by the Shannon entropy, should be maximized.

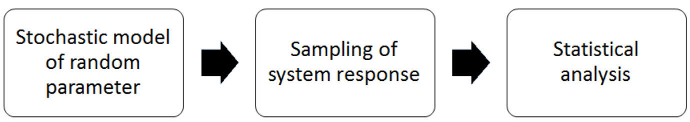

Figure 3 – Stochastic Modelling process

The mathematical formulation proposed by Shannon (1948) to quantify the entropy considering continuous random function is:

$$S(f_X) = \int_C f_X(x) \ln(f_X(x))\, dx, \tag{11}$$

where C is the region (support) where probability density function $f_X(x)$ is defined.

The entropy maximization problem consists in determining the distribution $f_X(x)$ that maximizes the entropy, taking into account the appropriated constraints related to the problem addressed. The Lagrange multipliers method is then applied to the maximization of $S(f_X)$ as proposed by Jaynes (1957 a, b).

The random parameters considered in this work are the elasticity model E and the disc mass m. The Gamma distribution was chosen to model such variable due to the MEP . This distribution is completely defined by its mean and standard deviation. The distribution support is the interval between 0 and ∞. The probability density function *pdf* in term of mean μ and standard deviation σ is presented on Eq. (12):

$$f(x,\mu,\sigma) = \mathbb{1}_{[0\infty)}(x)\,\frac{1}{\mu\Gamma(\frac{\mu^2}{\sigma^2})}\left(\frac{\sigma^2}{\mu}\right)^{\frac{\mu^2}{\sigma^2}} x^{\frac{\mu^2}{\sigma^2}-1}\exp\left(-x\frac{\sigma^2}{\mu}\right), \tag{12}$$

where x is a general random variable.

The second step of this process is focused on the generation of samples of the system response. So, deterministic model is solved several time in a Monte Carlo simulation process (Cursi and Sampaio, 2015, and Sampaio and Lima, 2012, and Rubinstein, 2008).

In order to obtain the stochastic responses of the system, Monte Carlo simulation were accomplished. The means of the elaticity modulus μ_E and the disc mass μ_m assume the values proposed in Tab. 1. In a first simulation the coefficients of variation ($\delta = \sigma/\mu$) is set to 5% for both random variable.

Figure 4 presents the Campbell diagram for stochastic parameters.

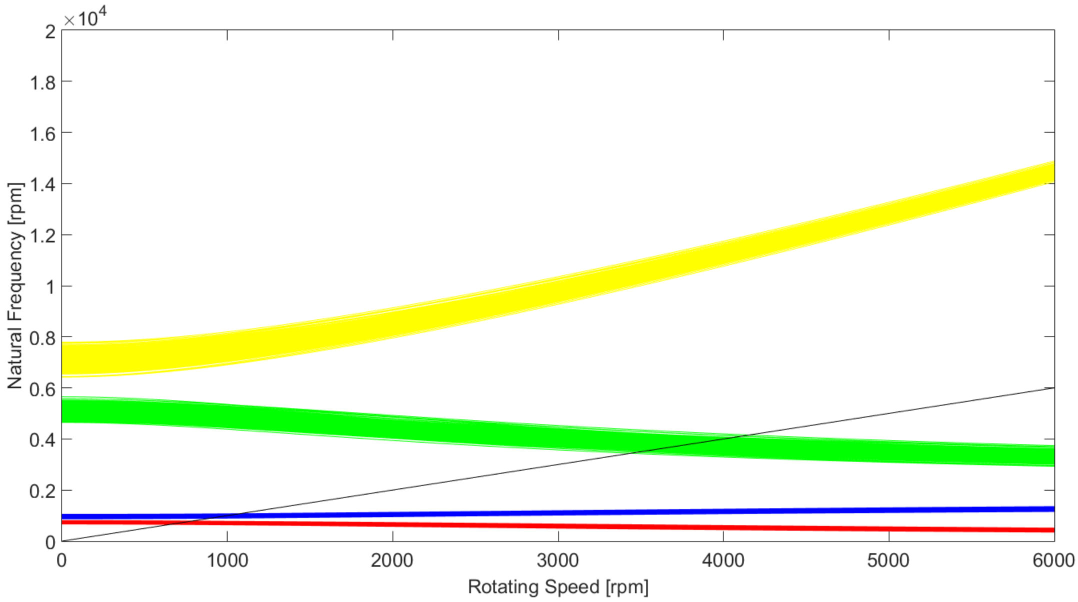

Figure 4 – Campbell diagram for stochastic parameters.

The black line crossing the natural frequencies represent the first harmonic excitation. It corresponds to an excitation in frequency coincident with the frequency spin (rotating speed), which is a common excitation frequency in rotordynamics. The most common source of vibration in rotating machinery is the mass unbalance that produces a vibration in a frequency equal to rotating speed (or synchronous excitation). When the harmonic line crosses a natural frequency, the system enters in a resonance motion. The rotating speed in this situation is called critical speed.

After the sampling of the system response, a statistical analysis should be accomplished. Histograms are a graphical option for this analysis. Another possibility is the calculation of statistical moments. In this case, information about the distribution can be summarized in one number. The n^{th} moment of the probability density function of a random variable x is defined as:

$$\mu_n = \mathbb{E}[x^n] = \int_{-\infty}^{\infty} x^n f_X(x)dx. \tag{13}$$

The central moment can also be considered for higher order. In this case, the moment is defined arround the mean, which is the moment of first order:

$$\bar{\mu}_n = \mathbb{E}[(x-\mu)^n] = \int_{-\infty}^{\infty} (x-\mu)^n f_X(x)dx. \tag{14}$$

Besides the mean (moment of first order), there are other parameters defined throuth statistical moments. The variance, which is a measure of the sample dispersion, is the second central moment. The standard deviation σ is the square root of the variance.

A normalized moment is also defined:

$$\bar{\bar{\mu}}_n = \frac{\mathbb{E}[(x-\mu)^n]}{\sigma^n}. \tag{15}$$

The skewness η is a measure of asymmetry of the probability density function and it is the third normalized moment. For simmetric distribution, the skewness is zero. If the skewness is negative, the samples are concentrated in the higher values of the distribution. In the case of positive skewness, the concentration of samples is in the lower values.

Another parameter that describe the distribution is the kurtosis κ, which is defined as the fourth normalized moment, and it is a measure of heaviness of the tail of the distribution.

Figures 5 to 7 show the three critical speeds histograms of the system.

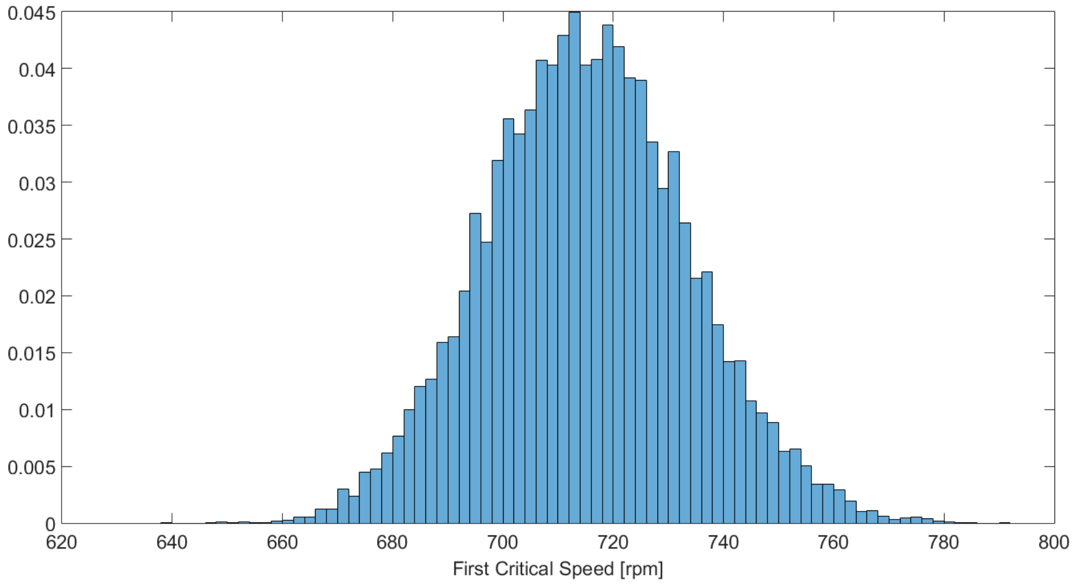

Figure 5 – First critical speed histogram.

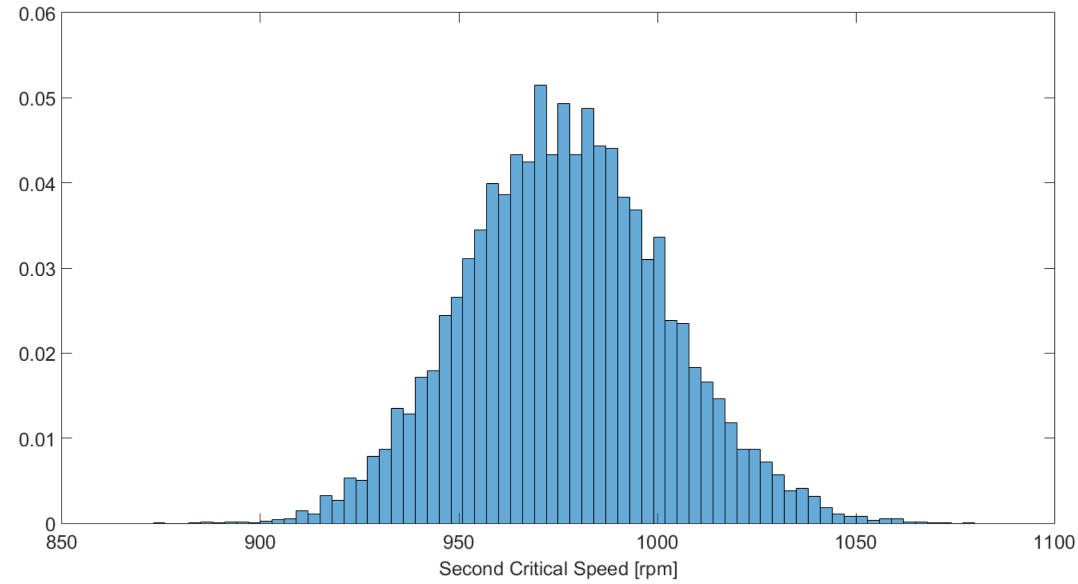

Figure 6 – Second critical speed histogram.

The first critical speed (Fig. 5) mean value is close to 715 rpm and it varies from 640 to 790 rpm. The histogram seems to be simmetric. A similar appearance can be observed for the second critical speed (Fig. 6), whose mean is about 980 rpm and the bounds are 870 and 1080 rpm. This results justify the stochastic analysis in rotordynamics. If the operational rotating speed is determined based on a deterministic response and set between the first and second critical speed, which

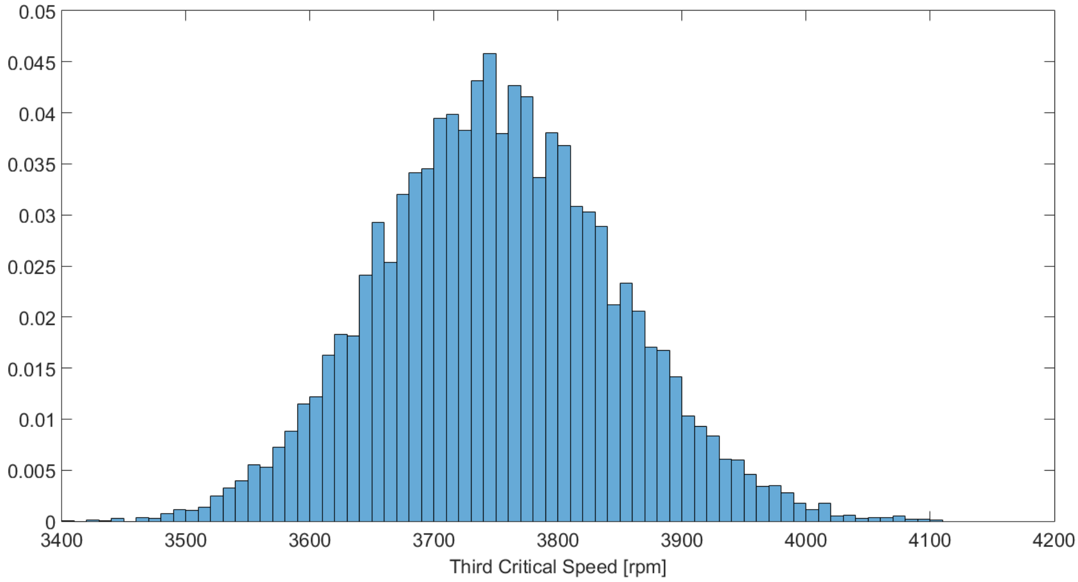

Figure 7 – Third critical speed histogram.

is a common practice in rotating machinery design, there is a considerable probability of a resonance motion. Besides, the third critical speed (Fig. 7) is near 3750 rpm and it can assume values between 3400 to 4100 rpm.

It is an important result for rotordynamics analysis, because an operational rotating speed can be set in a probabilistic bound of a critical speed, which increases the possibility of a resonance motion. For example, it is common to set the operational rotating speed between the first and the second critical speed. For the considered uncertainty level, the first critical speed can vary from 640 to 790 rpm, while the second critical speed can assume values between 870 and 1080 rpm. There is a tiny interval between the both histograms. If the level of uncertainty is higher, it is possible to decrease significantly this interval, and the operational rotating speed can be coincident to one of these two critical speeds, as shown in Fig. 8, where the disc mass coefficient of variation is set to 20%.

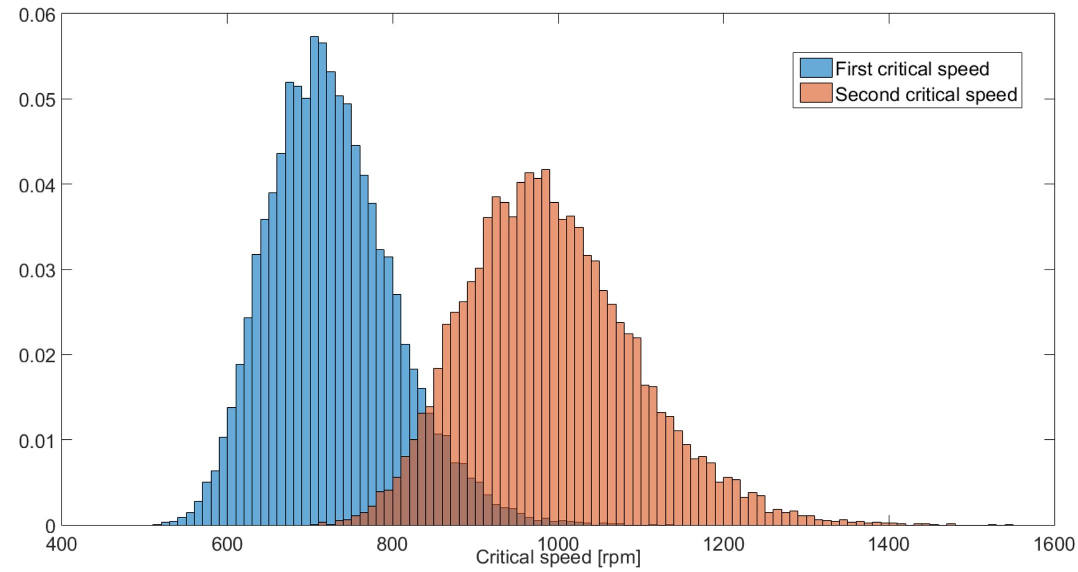

Figure 8 – First and Second critical speed histograms for δ_E = 5% and δ_m = 20%.

Another possible investigation is calculation of the stochastic response statistical moments, as mentioned before. It is accomplished in the next section, where a sensitivity analysis is carried out.

Sensitivity analysis

A sensitivity analysis is an important procedure to verify how the variation of random parameters changes the stochastic system response. Therefore, it is proposed a complete factorial test with two variables (mass and elasticity modulus coefficients of variation, δ_E and δ_m) and three levels (5%, 10% and 20%). So, it should be considered as presented on

Tab. 2.

Table 2 – Combination of the complete factorial test

Test number	δ_E	δ_m
1	5%	5%
2	5%	10%
3	5%	20%
4	10%	5%
5	10%	10%
6	10%	20%
7	20%	5%
8	20%	10%
9	20%	20%

In Fig. 9, the influence of both coefficients of variation in the first critical speed is shown. It is possible to observe that the mass coefficient of variation causes a significant increase in the first critical speed standard deviation. Its effect on the skewness is similar, but not so evident. The kurtosis does not vary due to the mass coefficient of variation. The coefficient of variation of the elasticity modulus does not have significant influence on the three considered moment.

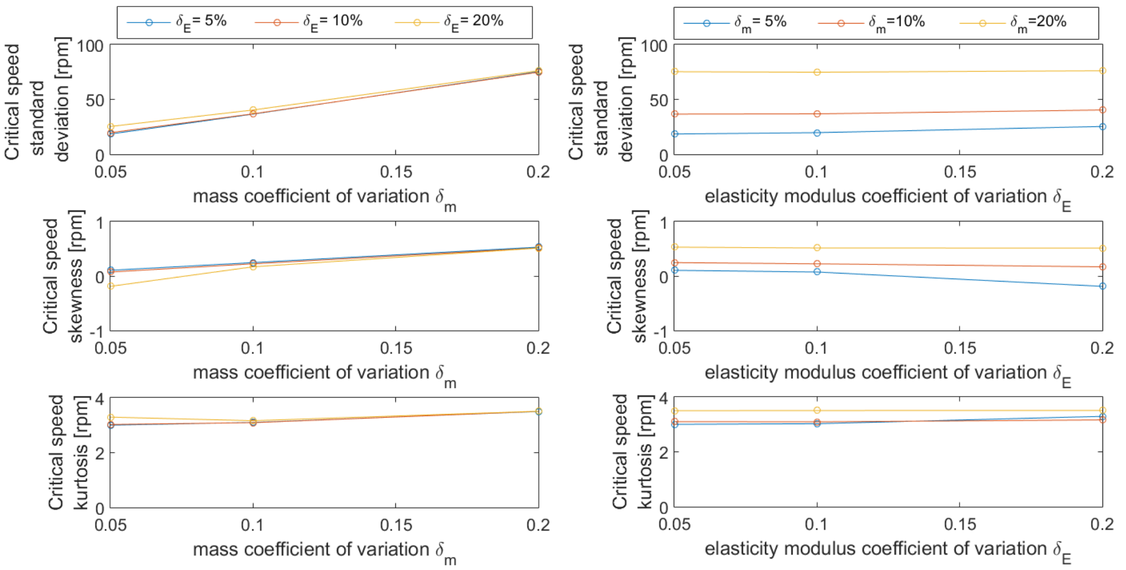

Figure 9 – First critical speed statistic moments variation

The same analysis can be applied to the second critical speed, according to Fig. 10. The increase of the standard deviation with the disc mass coefficient of variation is more intense.

However, the effects in the third critical speed is quite different, see Fig. 10. The skewness and the kurtosis significantly increase for a high elasticity modulus coefficient of variation (20%). In order to better characterize this variation, Figure 12 presents the histogram for the third critical speed with elasticity modulus and disc mass coefficient of variation of 20% and 5%, respectively (test 7). As the previous cases, the standard deviation of the critical speed increases with the disc mass coefficient of variation.

CONCLUSIONS

An uncertainty analysis in rotordynamics was proposed in this work. A simple rotating system model is considered, taken into account an outboard disc and two flexible bearings. In order to verify the system natural frequencies, a Campbell Diagram is obtained for deterministic and stochastic response. The disc mass and elasticity modulus were set as random parameters

The critical speeds are estimated, when a harmonic excitation crosses the natural frequencies of the Campbell Diagram. For stochastic response, histograms were carried out for the three critical speed of the system. This is an important result, because it allows a more suitable analysis to determine or verify the operational rotating speed.

Besides, the effect of the coefficient of variation for each random parameter are also accomplished. So a complete factorial test is applied for three levels of coefficient of variation of each random parameter (5%, 10% and 20%). It is noted a direct influence of the disc mass coefficient of variation on the standard deviation of the three critical speed. The same effect, but not so evident, can be observed on the skewness of the three critical speed for the cases where the elasticity modulus coefficient of variation is lower than 20%. The kurtosis has a weak influence of the mass disc variation

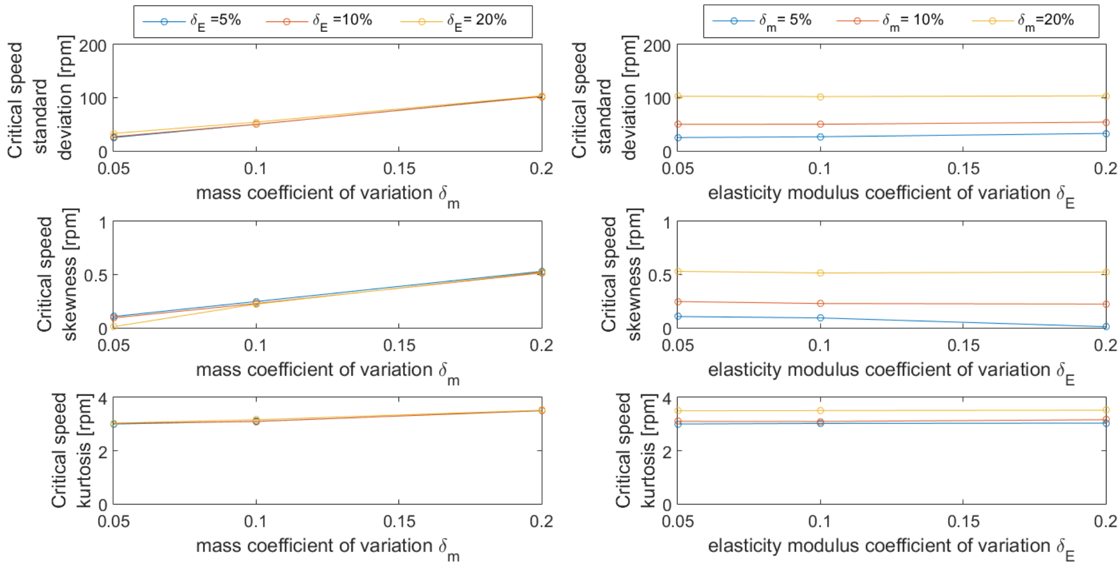

Figure 10 – Second critical speed statistic moments variation

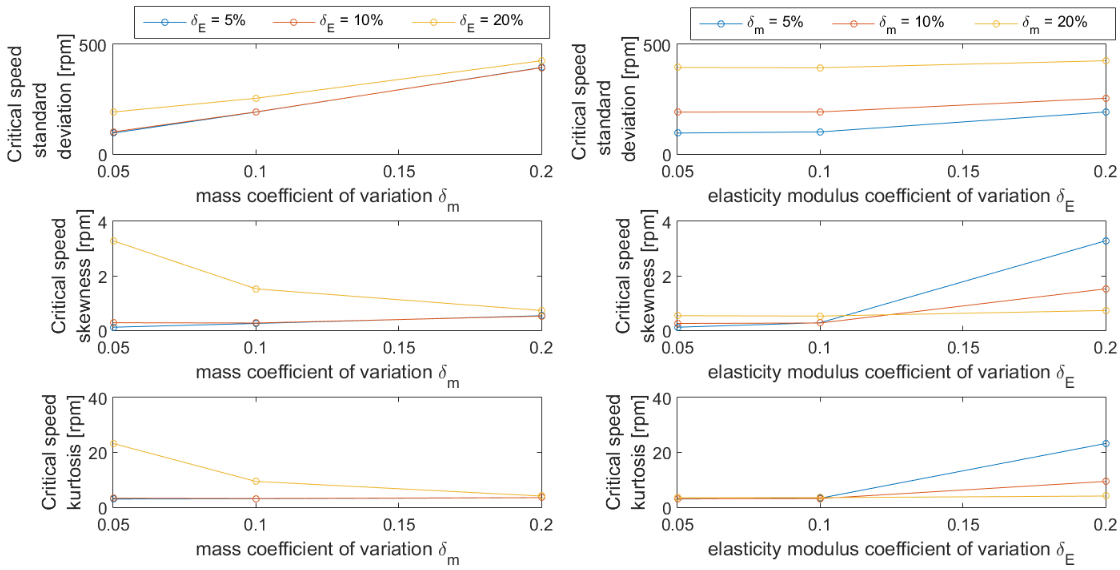

Figure 11 – Third critical speed statistic moments variation

when the elasticity modulus coefficient of variation is lower than 20%. However, for the case that the elasticity modulus coefficient of variation is 20%, skewness and kurtosis are strongly affected, and it inverts the relation with the disc mass coefficient of variation. When this coefficient increases, the skewness and kurtosis decrease.

For future works, it is intended to extend this analysis. A first step is to considerer geometric parameters as random variable, like disc position or shaft diameter. Besides, the effect of the variation of flexible bearing stiffness can also be considered. Other models can also take into account, for example the case where the disc is located between bearing, or more complex system solved by finite element method. One more option, is the inclusion of journal bearing model and an analysis of its parameter effect on the stochastic response of the system

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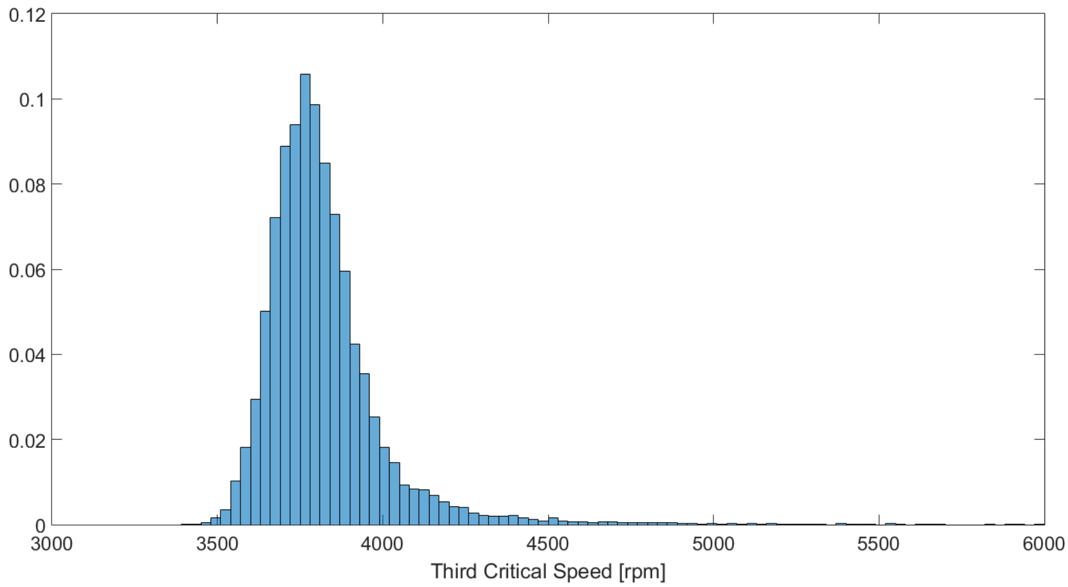

Figure 12 – Third critical speed histogram for δ_E = 20% and δ_m = 5%.

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