

Sensibility analysis of the stochastic dynamic response of a rotor

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Abstract: An important element of rotor machines is the journal bearing, which connects the moving part, the rotor, with the static part, the support. To insert the journal bearing into the dynamic model of the rotor system, a nonlinear equation can be considered to evaluate the hydrodynamic supporting forces of fluid in the bearing. However, features and operating conditions of the rotor systems have uncertainties, which can make the simulation results diverge of the collected data of experiments. To include these uncertainties a stochastic model has to be use. Monte Carlo methods are generally utilized to evaluate the stochastic response, due to its simple implementation and convergence. Nevertheless, the Monte Carlo methods take an extended amount of time to compute the simulations. Stochastic collocation methods utilize a deterministic solver, as the Monte Carlo methods, so its implementation is simple. However, a small number of predetermined points is used in the simulation. The stochastic response can be approach by the generalized polynomial chaos expansion. Therefore, only the expansion coefficients need to be determined. The approach by the generalized expansion facilitates the evaluation of the statistical parameters of the response and the sensibility analysis to the input random variables. In this paper the stochastic dynamic response of a rotor system will be evaluated, a nonlinear model will be used to calculate the hydrodynamic supporting forces of the bearing, which will be inserted in the finite element model of the rotor-bearing system as an external force. The stochastic collocation method will be used to evaluate the stochastic response, which will be approach by the generalized polynomial chaos expansion. An input sensibility analysis of the stochastic response will be made. This analysis will provide better understanding of the influence of the random input in the dynamical response of the system.

Keywords: Rotordynamics, Uncertainties propagation, Stochastic collocation, Sensitivity analysis

NOMENCLATURE

Latin symbols

b : boundary condition operator
 c : radial clearance
 d : shaft diameter
 e : eccentricity
 h : oil thickness
 k : normalization factor
 l : shaft length/differential operator
 m : unbalance mass
 p : pressure distribution
 q : nonrandom variables
 r : bearing radius
 s : dimension of the truncated expansion
 t : time
 u : solution of the differential problem
 x : axial coordinate
 y : horizontal coordinate
 \dot{y} : velocity in the horizontal coordinate
 z : vertical coordinate
 \dot{z} : velocity in the vertical coordinate
 f : vector force
 q : vector of non-random variables

Greek symbols

β : proportionality constant
 θ : radial coordinate
 Λ : finite random space
 λ : random variable
 μ : oil viscosity
 ρ : probability density function
 ω : rotating speed
 Φ : set of orthogonal polynomial

Subscripts

h : relative to hydraulic reactions forces
 i, j : counters
 u : relative to unbalance of the rotor system
 PC : relative to polynomial chaos expansion

INTRODUCTION

Due to the wide presence of rotating machines in the industry, it is necessary to understand the behavior and phenomena. The analysis and simulations of rotor systems are made by different approaches, like experimental analysis or mathematical models.

The dynamic response of the rotating system can be used to analyze the fluid-induced instability, which is related to the increase of the cross coupling stiffness coefficient of the fluid force. This instability occurs mainly with journal bearing with fixed geometry and can be extremely harmful. Mathematical models can be considered for real machines simulation with significant confidence.

Researchers continue to develop better models for various rotating machineries. Capone (1986) and Capone (1991) developed a numerical model for a cylindrical hydrodynamic bearing, considering nonlinear hydrodynamic forces. Using this model, the unbalance response in time domain of the shaft inside the bearing can be determined.

The interaction between the rotating inner ring of the bearing and the lubricant oil creates a sub-synchronous vibration, which characterizes the dynamical instability. Newkirk and Taylor (1924 and 1925) identified this behavior, which is known as oil whirl and oil whip. This instability continued to be studied by other researches, like Lund and Saibel (1967), Muszynska (1986 and 1988), Crandall (1990) and Child (1993)

The fluid-induced instability is self-exciting vibration of a rotor-bearing system. The rotor vibrates in a frequency near half the rotating speed, due to the oil whirl. When the rotating speed reaches twice the first natural frequency, the oil whip begins, which is a harmful situation for the rotor-bearing system.

Castro *et al* (2006 and 2008) considered a nonlinear hydrodynamic journal-bearing model to simulate fluid-induced instabilities. The model utilized could numerically represent the dynamic behavior of a rotor-bearing system under the oil whirl and oil whip condition.

Parameters of the rotor-bearing system, as geometry and operational conditions, presents uncertainties. The models mention before are deterministic and a stochastic treatment is needed to obtain better simulations and analysis

To consider these uncertainties, the stochastic collocation method can be used. The method is non-intrusive, so it admits the use of deterministic solvers. A Monte Carlo method could be use as well, but it takes a long processing time, which was noted and is mentioned by Babuska (2007) and Xiu (2007 and 2009). The advantage of the stochastic collocation method is that a previously chosen and smaller sample size of points to simulate is used.

The solution of the stochastic problem can be approached by the generalized polynomial chaos expansion. Wiener (1938) proposed the approach of the solution through a polynomial series, which use the “homogeneous chaos” to span Hermite polynomials of a Gaussian distribution. Ghanem and Spanos (1991) described the construction of the polynomial chaos expansion and Xiu and Karniadakis (2002) utilize the Askey scheme to determine the generalized polynomial chaos, which can expand other distributions.

Garoli and Castro (2016) validated the usage of the stochastic collocation method with generalized polynomial chaos expansion to evaluate the stochastic dynamical response of a rotor. The method was compared with the results of a Monte Carlo simulation and satisfying results was obtained, which are good convergence, like the Monte Carlo method, and smaller computing time.

A sensitivity analysis can be made to evaluate which input parameters has more influence in the output. Sudret (2008) made a correlation between the Sobol decomposition with the polynomial chaos expansion. From the expansion coefficients, the author evaluated the Sobol indices, which is the influence of the input random variable on the output.

In this work, the stochastic dynamical response of a rotating machine is evaluated. The solution is approached by the generalized polynomial chaos expansion and evaluated by the stochastic collocation method. Then, the Sobol indices are calculated and a sensitivity analysis is made.

Nonlinear journal bearing model

The mathematical model of the rotor-bearing system is based on the model developed by Castro *et al* (2008), which divide the system in two parts: the finite element model of the shaft and the rigid disc and the nonlinear model of the cylindrical journal bearing.

The nonlinear hydrodynamic supporting forces of the journal bearing is modeled by the Reynolds equation for short bearings, which is denoted by the Eq. (1). A short bearing scheme and the cylindrical coordinate system are shown in Fig. 1

$$\frac{\partial}{\partial \theta} \left(h^3 \frac{\partial p}{\partial \theta} \right) + \left(\frac{r}{l} \right)^2 \left(\frac{\partial}{\partial x} \left(\frac{h^3}{\mu} \frac{\partial p}{\partial x} \right) \right) = \frac{\partial h}{\partial \theta} + 2 \frac{dh}{d(\Omega t)} \quad (1)$$

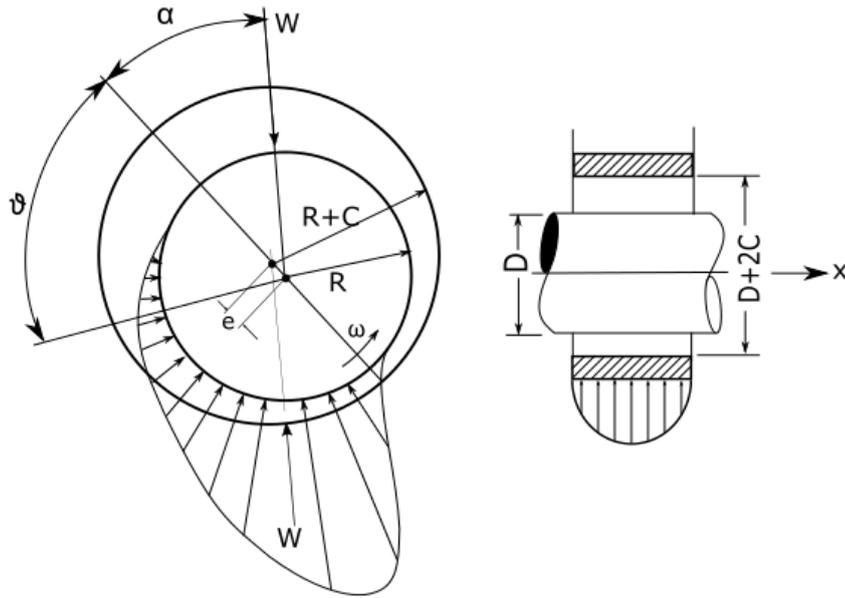


Figure 1 – Journal bearing scheme

In the short journal bearing model, the angular pressure distribution is neglected, and the solution of the axial pressure distribution is:

$$p(\theta, x) = \frac{1}{2} \left(\frac{l}{d} \right)^2 \left[\frac{((y - 2\dot{z}) \sin(\theta) - (z - 2\dot{y}) \cos(\theta))}{(1 - y \cos(\theta) - z \sin(\theta))^3} \right] (4x^2 - 1) \quad (2)$$

The determination of the reaction forces of the film of oil is done by the Eq. (3):

$$\mathbf{f}_h = \begin{Bmatrix} f_{h_y} \\ f_{h_z} \end{Bmatrix} = -\mu\omega \frac{(r \cdot l)^3}{(c \cdot d)^2} \frac{[(y - 2\dot{z})^2 + (z + 2\dot{y})^2]^{\frac{1}{2}}}{(1 - y^2 - z^2)} \cdot \begin{Bmatrix} 3y \cdot v(y, z, \alpha) - \sin(\alpha) g(y, z, \alpha) - 2 \cos(\alpha) b(y, z, \alpha) \\ 3z \cdot v(y, z, \alpha) - \cos(\alpha) g(y, z, \alpha) - 2 \sin(\alpha) b(y, z, \alpha) \end{Bmatrix} \quad (3)$$

where:

$$v(y, z, \alpha) = \frac{2 + (z \cos(\alpha) - y \sin(\alpha)) \cdot g(y, z, \alpha)}{(1 - y^2 - z^2)}, b(y, z, \alpha) = \frac{y \cos(\alpha) + z \sin(\alpha)}{(1 - y^2 - z^2)}$$

$$g(y, z, \alpha) = \frac{\pi}{\sqrt{1 - y^2 - z^2}} - \frac{2}{\sqrt{1 - y^2 - z^2}} \tan^{-1} \left(\frac{z \cos(\alpha) - y \sin(\alpha)}{\sqrt{1 - y^2 - z^2}} \right)$$

$$\alpha = \tan^{-1} \left(\frac{z + 2\dot{y}}{y - 2\dot{z}} \right) - \frac{\pi}{2} \text{sign} \left(\frac{z + 2\dot{y}}{y - 2\dot{z}} \right) - \frac{\pi}{2} \text{sign}(z + 2\dot{y})$$

Equation of motion of the system

The equation of motion of the rotor-bearing system is given by the Eq. (4):

$$\mathbf{M} \cdot \ddot{\mathbf{q}} + \mathbf{C} \cdot \dot{\mathbf{q}} + \mathbf{K} \cdot \mathbf{q} = \mathbf{f}_u + \mathbf{f}_h - \mathbf{w} \quad (4)$$

A finite elements method is implemented to obtain the mass, gyroscopic and stiffness matrix. The damping matrix considers the gyroscopic matrix times de rotating speed and structural damping proportional to the stiffness: $\mathbf{C} = \beta \cdot \mathbf{K} + \omega \cdot \mathbf{G}$.

The system excitation force is due to the unbalanced mass m and its eccentricity e , expressed in Eq. (5):

$$\mathbf{f}_u = m \cdot e^2 \begin{Bmatrix} \vdots \\ \omega^2 \cos(\omega \cdot t) \\ \omega^2 \sin(\omega \cdot t) \\ \vdots \end{Bmatrix} \quad (5)$$

The solution of the equation of motion is numerically obtained. The Newmark integration method was chosen, because it is a robust algorithm to solve nonlinear equations in the time domain, which is the case of this research.

The output of Eq. (4) is a vector of displacements in both directions, horizontal and vertical. These displacements form an ellipse, presented at Fig. (2). To simplify the outputs which will be analyzed, the parameters of this ellipse will be evaluated by the method utilized by Castro *et al.* (2008). The characterization of the ellipse will be done by the SDI, the dimension of the major axis and its inclination with respect to the horizontal axis.

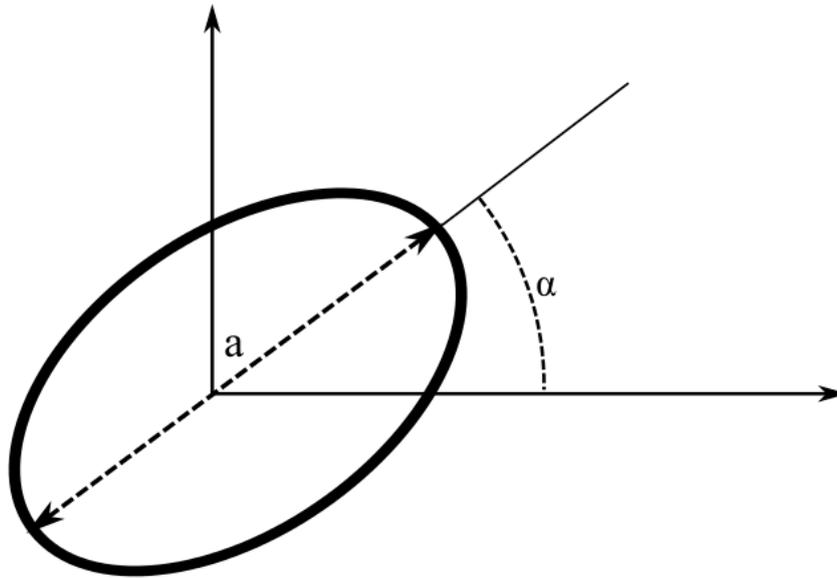


Figure 2 – Elliptical orbit parameters

The displacement in y e z directions will be described by Eq. (6) and (7), respectively, where y_0 , y_s , y_c , z_0 , z_s and z_c are the first order Fourier coefficients.

$$y(t) = y_0 + y_c \cos(\omega \cdot t) + y_s \sin(\omega \cdot t) \quad (6)$$

$$z(t) = z_0 + z_c \cos(\omega \cdot t) + z_s \sin(\omega \cdot t) \quad (7)$$

The inclination of the major axis α and the major radius a are given by:

$$\alpha = 0.5 \tan^{-1} \left(\frac{2(y_c \cdot z_c + y_s \cdot z_s)}{y_c^2 + y_s^2 - z_c^2 - z_s^2} \right), \quad (8)$$

$$a = \sqrt{\frac{2(y_s z_c - y_c z_s)^2}{y_c^2 + y_s^2 + z_c^2 + z_s^2 - \sqrt{(z_c^2 + z_s^2 - y_c^2 - y_s^2) + 4(y_s z_s + y_c z_c)^2}}}}. \quad (9)$$

The Shape and Directivity Index (SDI) describe the degree of ellipticity:

$$-1 \leq sdi = \frac{|r^f| - |r^b|}{|r^f| + |r^b|} \leq 1, \quad (10)$$

where r^f and r^b are the forward and backward coefficients of the complex harmonic signal $p(t)$ of frequency ω .

$$p(t) = y(t) + j \cdot z(t) = r^f \cdot e^{j\omega t} + r^b \cdot e^{-j\omega t} \quad (11)$$

Generalized polynomial chaos expansion

The operating conditions of the rotor-bearing system have uncertainties. The insertion of these uncertainties in the model will be made by the generalized polynomial chaos expansion. A brief explanation of the generalized polynomial chaos expansion is presented below, which is an abridgment of the work of Xiu and Karniadakis (2002).

Lets the orthogonality condition of a set of orthogonal polynomial $\Phi_i(\lambda)$ be:

$$\int_{\Lambda} \Phi_i(\lambda) \Phi_j(\lambda) \cdot \rho(\lambda) d\lambda = k_i^2 \cdot \delta_{ij} \quad (12)$$

where

$$k_i^2 = \int_{\Lambda} \Phi_i^2(\lambda) d\lambda$$

and δ_{mn} is the Kronecker delta.

Then, ρ in Eq. (12) is the integration weight, which determines the type of the orthogonal polynomial set. For Gamma distributed random variable, which will be used in this work, its probability density function defines Laguerre polynomials.

To define a truncated basis of orthogonal polynomial with maximum degree p in a n -variable random space the Eq. (13) is use:

$$\Phi_n^p = \otimes_{|d| \leq p} \Phi^{i,d_i}, s = \dim(\Phi_n^p) = \binom{n+p}{n} \quad (13)$$

The generalized polynomial chaos expansion will be use with the stochastic collocation to solve the stochastic problem.

Stochastic collocation

Xiu (2007) presents a stochastic collocation for parametric uncertainty analysis with a finite number of random variable, an abridgment of this work is presented below.

Consider a partial differential equations:

$$\begin{aligned} l(q, u; \lambda) &= 0 \\ b(q, u; \lambda) &= 0 \end{aligned} \quad (14)$$

where q are the nonrandom variables and λ are the random variable. l is the differential operator and b is de boundary conditions operator. The solution of the Eq. (8) will be approach by:

$$u \cong u_n^p(q, \lambda) = \sum_{i=1}^s \hat{u}_i(q) \cdot \Phi_i(\lambda) \quad (15)$$

The expansion coefficients $\hat{u}_i(q)$ are determinate through the least square method:

$$\begin{Bmatrix} u(q, \lambda^1) \\ \vdots \\ u(q, \lambda^a) \end{Bmatrix} = \begin{bmatrix} \Phi_0(\lambda^1) & \cdots & \Phi_s(\lambda^1) \\ \vdots & \ddots & \vdots \\ \Phi_0(\lambda^a) & \cdots & \Phi_s(\lambda^a) \end{bmatrix} \cdot \begin{Bmatrix} \hat{u}_1 \\ \vdots \\ \hat{u}_a \end{Bmatrix} \quad (16)$$

where $u(q, \lambda^i)$ is the solution of Eq. (8) and $\Phi_i(\lambda^i)$ is the polynomial i , both calculated at the node λ^i . The choice of the nodes is made by the sparse grid.

The sparse grid is constructed by the Smolyak algorithm, which is a linear combination of product formulas, additionally the linear combination is chosen in such a way that an interpolation property for $N=1$ is preserved for $N>1$. Only products with a relatively small number of points are used and the resulted nodal set has significantly less number of node compared to the tensor product rule. This method is interesting to the presented paper, due to the considerable time used by the deterministic solver. A Chenshaw-Curtis quadrature rule to determine the points for a unidimensional interpolation and the Smolyak algorithm does the combination to construct the nodes for the stochastic collocation method.

Statistical Parameters

Considering the approach of the solution, Eq. (9), the mean and the variance can be easily calculated by the Equations (11) and (12), respectively.

$$E[u(q, \lambda)] \approx \int_{\Lambda} \left(\sum_{i=1}^s \hat{u}_i(q) \cdot \Phi_i(\lambda) \right) \rho(\lambda) d\lambda = \hat{u}_1 \quad (17)$$

$$Var(u(q, \lambda)) = E[u(q, \lambda)^2] - E[u(q, \lambda)]^2 = \sum_{i=2}^s \hat{u}_i^2(q) \quad (18)$$

Sensitivity analysis

The quantification of the respective effects of the random variables onto the stochastic response of the model can be made by a global sensitivity analysis. The Sobol' indices have received much attention since they provide accurate information for most models. Based on the developments of Sudret (2008) the evaluation of the Sobol' indices trough a generalized polynomial chaos expansion is presented.

Let y be a scalar output of the function $f(\mathbf{x})$, where \mathbf{x} is a vector of the n random variables. The Sobol' decomposition of $f(\mathbf{x})$ into summands of increasing dimension is:

$$y = f(x_1, \dots, x_n) = f_0 + \sum_{i=1}^n f_i(x_i) + \sum_{1 \leq i < j \leq n} f_{ij}(x_i, x_j) + \dots + f_{1,2,\dots,n}(x_1, \dots, x_n) \quad (19)$$

Where f_0 is the mean value of the function and the integral of each summand $f_{i_1, i_2, \dots, i_s} = (x_{i_1}, x_{i_2}, \dots, x_{i_s})$ over any of its independent variables is zero, i.e.:

$$\int f_{i_1, i_2, \dots, i_s} = (x_{i_1}, x_{i_2}, \dots, x_{i_s}) dx_{i_k} = 0 \text{ for } 1 \leq k \leq s \quad (20)$$

Consider that the input random variables are independent. Therefore, the model response is a random variable, whose variance σ^2 reads:

$$\sigma^2 = Var[f(\mathbf{x})] = \int f^2(\mathbf{x}) d\mathbf{x} - f_0^2 \quad (21)$$

By integrating the square of Eq. (19) and due to Eq. (20), it is possible to decompose the total variance (21) as follows:

$$\sigma^2 = \sum_{i=1}^n \sigma_i^2 + \sum_{1 \leq i < j \leq n} \sigma_{ij}^2 + \dots + \sigma_{1,2,\dots,n}^2 \quad (22)$$

where the partial variances appearing in the above expansion is:

$$\sigma_{i_1, \dots, i_s}^2 = \int f_{i_1, \dots, i_s}^2(x_{i_1}, \dots, x_{i_s}) dx_{i_1}, \dots, dx_{i_s}, \quad 1 \leq i_1 < \dots < i_s \leq n, \quad s = 1, 2, \dots, n \quad (23)$$

The Sobol indices (s_i) are defined as follow:

$$s_{i_1, \dots, i_s} = \frac{\sigma_{i_1, \dots, i_s}^2}{\sigma^2} \quad (24)$$

By definition, according to (22), they satisfy

$$\sum_{i=1}^n s_i + \sum_{1 \leq i < j \leq n} s_{ij} + \dots + s_{1,2,\dots,n} = 1 \quad (25)$$

Thus, each index s_{i_1, \dots, i_s} is a sensitivity measure describing which amount of the total variance is due to the uncertainties in the set of input parameters $\{i_1, \dots, i_s\}$. The first order indices s_i give the influence of each parameter taken alone whereas the higher order indices account for possible mixed influence of various parameters.

Sobol decomposition of the polynomial chaos expansion

Considering that, the expansion coefficients of Eq. (15) were evaluated and the variance of the solution was calculated. The expansion can be rearranged according to the random parameters:

$$u_n^p(q, \lambda) = \hat{u}_0 + \sum_{i=1}^n \sum_{\mathbf{j} \in \xi_i} \hat{u}_{\mathbf{j}} \Phi_{\mathbf{j}}(\lambda_i) + \sum_{1 \leq i_1 < i_2 \leq n} \sum_{\mathbf{j} \in \xi_{i_1, i_2}} \hat{u}_{\mathbf{j}} \Phi_{\mathbf{j}}(\lambda_{i_1}, \lambda_{i_2}) + \dots + \sum_{\mathbf{j} \in \xi_{1,2,\dots,n}} \hat{u}_{\mathbf{j}} \Phi_{\mathbf{j}}(\lambda_1, \dots, \lambda_n) \quad (26)$$

where

$$\xi_{i_1, \dots, i_s} = \left\{ \mathbf{j} : \begin{array}{l} j_k > 0, \forall k = 1, \dots, n, k \in (i_1, \dots, i_s) \\ j_k = 0, \forall k = 1, \dots, n, k \notin (i_1, \dots, i_s) \end{array} \right\} \quad (27)$$

Putting into words ξ_{i_1, \dots, i_s} corresponds to the polynomials depending only on parameter x_i .

Therefore, it is now easy to calculate the Sobol indices from the polynomial chaos expansion (s_{PC}), which is described by:

$$s_{PC_i} = \sum_{\mathbf{j} \in \xi_{i_1, \dots, i_s}} \frac{\hat{u}_{\mathbf{j}}^2}{\sigma^2} \quad (28)$$

Results

For the simulations, a Laval rotor is use, which contains 13 nodes; the model is presented in Fig 3. The journal bearings was in a symmetric position to the rigid disc at the center of the shaft. The uncertainties in the journal bearing is considered in the radial clearance and the lubricant viscosity.

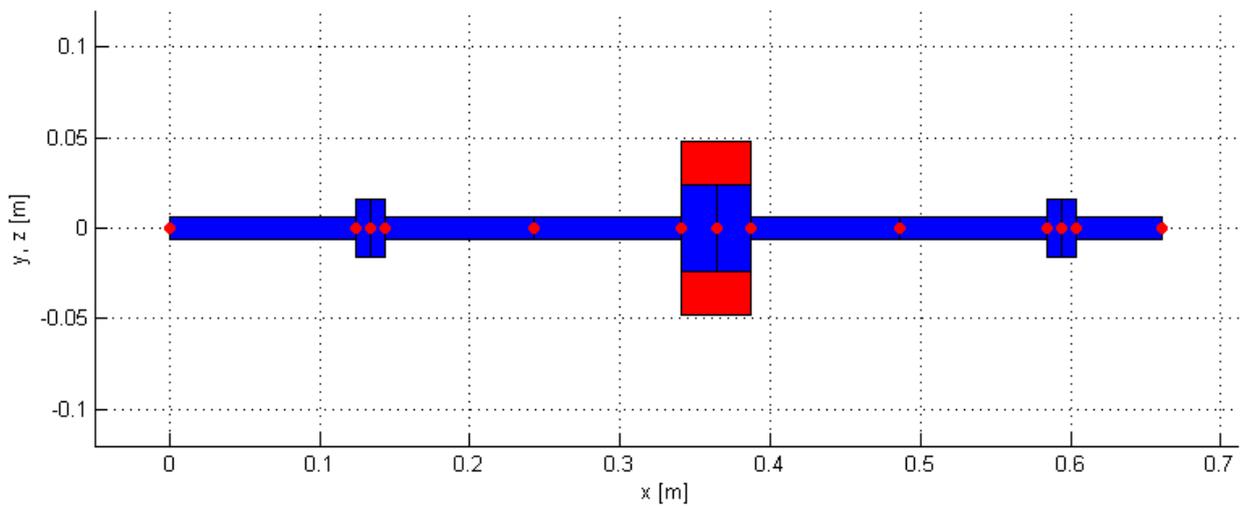


Figure 3 - Rotor system finite elements model

A Gamma distribution is considered for both of them, the shape parameter and the scale parameter of each one is presented in the Table 1. Due to the Gamma distribution, Laguerre polynomial is chosen for the generalized polynomial chaos expansion.

Table 1 – Shape and scale parameters of each random variable

Variable	Shape parameter	Scale parameter
Radial clearance	1296	$6.94 \cdot 10^{-8}$ (10^{-6} m)
Lubricant viscosity	156.25	$4.46 \cdot 10^{-4}$ (Pa·s)

The Laguerre polynomials are solutions of the Laguerre equation:

$$xy'' + (1-x)y' + ny = 0 \quad (29)$$

The closed form of the polynomials is:

$$L_n(x) = \sum_{k=0}^n \binom{n}{k} \frac{(-1)^k}{k!} x^k \quad (30)$$

The collocation method was applied with a sparse grid of level 4. Then the degree of the generalized polynomial chaos expansion was evaluated, which the max polynomial degree used was 3, therefore a total of 10 expansion coefficients was evaluated. The dynamical response was calculated at rotating speed of 35, 40 and 45 Hz, before, during and after resonance, respectively.

For each time instance and each coordinate position, a generalized polynomial chaos expansion is evaluated, then, for each expansion, the equations (17) and (18) are applied to evaluate the mean position of the orbit of the rotor and its standard deviation.

The mean orbit, which consist of the mean positions of the rotor orbit, and the mean orbit plus two standard deviation and minus two standard deviation are presented in Figure 3. The area between the two dashed lines is not the region that covers the 95%; this is only true if and only if a Gaussian distribution can model the stochastic solution, which is not known. However, the Figure 4 show part of the range of values that the stochastic orbit of the rotating system can achieve.

Garoli and Castro (2016) validated the stochastic collocation with the generalized polynomial chaos expansion to evaluate the stochastic dynamical response of rotor systems. Therefore, the validation of the method won't be made on this paper.

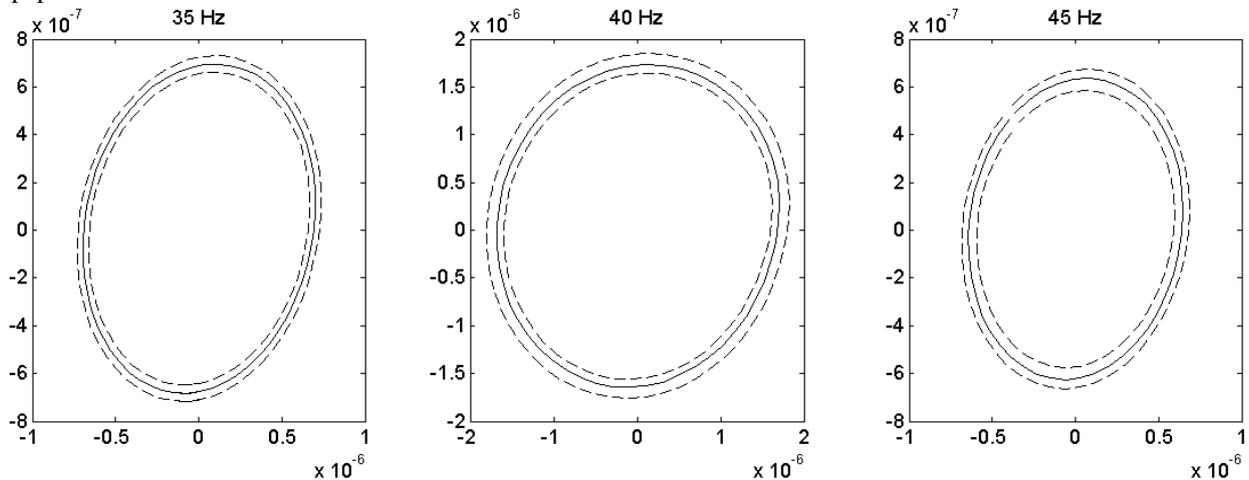


Figure 4 – Mean orbit and the mean orbit \pm two standard deviation on each rotating speed (Full line: mean orbit; Dashed lines: mean orbit \pm two standard deviation)

Afterward, the Sobol indices was evaluated. Table 2, 3 and 4 presents the indices for each variable and both combine on the SDI, the dimension of the major axis and its inclination with respect to the horizontal axis, respectively.

Table 2 – Sobol indices of the input parameters at 35 Hz

Sobol index relative to	SDI	Dimension of the major axis	Inclination
Radial clearance	0.0000	0.3100	0.0097
Lubricant viscosity	1.0000	0.3250	0.9869
Both of them	0.0000	0.3650	0.0033

Table 3 – Sobol indices of the input parameters at 40 Hz

Sobol index relative to	SDI	Dimension of the major axis	Inclination
Radial clearance	0.0042	0.3215	0.0001
Lubricant viscosity	0.9938	0.3249	0.9999
Both of them	0.0020	0.3536	0.0000

Table 4 – Sobol indices of the input parameters at 45 Hz

Sobol index relative to	SDI	Dimension of the major axis	Inclination
Radial clearance	0.0091	0.3009	0.0000
Lubricant viscosity	0.9879	0.3045	1.0000
Both of them	0.0030	0.3928	0.0000

The Sobol indices is a sensitivity measure of the influence of the input parameters on the total variance of the solution. It is important to note that it estimates the sensitivity relative to the modeled random variables. Therefore, due to the small number of input parameters that has its uncertainties modeled, the SDI has a strong influence by the lubricant viscosity and only by it. The same can be conclude to the inclination, the viscosity has affects the output more than the others input parameters. The influence of the viscosity on these two outputs parameters was steady, even with the change of the rotating speed.

For the dimension of the major axis, which is relative to the vibration of the system, both input parameters has, nearly, the same influence and both combined has a considerable effects in the solution. Therefore, to evaluate the dynamical response of a rotor system, these parameters are essential. For the sake of the vibration levels of a rotor system, consider the radial clearance and lubricant viscosity uncertainties an assumption that should be made. The variation of the rotating speed do not make a significant change on the indices.

Conclusions

The stochastic dynamical response of a rotor system was evaluated. The stochastic collocation method was implemented and the solution was approached by a generalized polynomial chaos expansion. The sparse grid with the Clenshaw-Curtis quadrature rule was used to pre-determine the nodes to be calculated. The polynomial expansion facilitates the post processing of the stochastic solution, such as the statistical parameters, the mean and standard deviation, and the sensitivity of the solution to its random variables inputs.

The deterministic model was a Laval rotor and the uncertainties of the viscosity of the lubricant oil and the radial clearance between the shaft and the bearing was considered, it was evaluated the SDI, the dimension of the major axis and the its inclination with respect to the horizontal axis. After the evaluation of the expansion coefficients, a sensitivity analysis was made.

The sensitivity analysis presented the two considered input random variables has similar influence on the major axis. However, only the viscosity appears to influence the SDI and the inclination of the major axis. More random variables has to be considered to evaluate if inputs has effects on these last two parameters and on the major axis, due to this last one is an estimation of the vibration levels of the system.

Nevertheless, the Sobol indices is an easy to implement together the stochastic collocation method with the generalized polynomial expansion and it is a straightforward method to evaluate the sensitivity of the stochastic solution.

For future works, an embracing sensitivity analysis will be made to evaluate which inputs interferes on the dynamical response of the rotor system.

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REFERENCES

- Babuska, I, Nobile, F, Tempone, R, 2007, A Stochastic Collocation Method for Elliptic Partial Differential Equations with Random Input Data, *SIAM Review*, Vol. 52, No. 2, pp. 317-355
- Capone, G., 1986, Orbital motions of rigid symmetric rotor supported on journal bearings, *La Meccanica Italiana*, Vol. 199, pp. 37-46
- Capone, G., 1991, Descrizione analitica del campo di forze fluidodinamico nei cuscinetti cilindrici lubrificati, *L.Energia Elettrica*, Vol. 3, pp. 105-110
- Castro, H F, Cavalca, K L, Nordmann R., 2006, Rotor-bearing system instabilities considering a non-linear hydrodynamic model, 7th IFToMM - Conference on Rotor Dynamics, Vienna, Austria, paper ID-136, pp. 1-10
- Castro, H F, Cavalca, K L, 2008, Analysis of Fluid-induced instability using dynamic stiffness and nonlinear journal bearing model, *IMEchE, Tribology group*
- Castro, H F, Cavalca, K L, Camargo, L. W. F., 2008, Multi-Objective Genetic Algorithm Application in Unbalance Identification for Rotating Machinery, *IMEchE, Tribology group*
- Childs, D., 1993, *Turbomachinery Rotordynamics*. Wiley-Intersciences, New York
- Crandall, S., 1990, Form Whirl to Whip in Rotordynamics, *Transactions of IFToMM 3rd International Conference on Rotordynamics*, Lyon, France, pp. 19-26
- Garoli, G. Y., Castro, H. F., 2016, Uncertainty quantification of a nonlinear rotor-bearing model with stochastic

- collocation, *Vibrations in Rotating Machinery 11.*, Manchester, England, pp. 397-406
- Ghanem, R.G., Spanos, P., 1991. "Stochastic Finite Elements: a Spectral Approach". Springer-Verlag
- Lund, J. W., Saibel, E., 1967, Oil whip whirl orbits of a rotor in sleeve bearings, *ASME Journal of Engineering for Industry.* n. 89, pp. 813-823
- Muszynska, A., 1986, Whirl and Whip – Rotor Bearing Stability Problems, *Journal of Sound and Vibration*, Vol. 110 n. 3, pp. 443-462
- Muszynska, A., 1988, Stability of Whirl and Whip in Rotor Bearing System, *Journal of Sound and Vibration.* Vol. 127 n. 1, pp. 49-64
- Newkirk, B. L., and Taylor, H. D., 1924, Shaft Whipping, *General Electric Review*, Vol. 27, n. 3, pp. 169-178
- Newkirk, B. L., and Taylor, H. D., 1925, Shaft Whipping due to Oil Action in Journal Bearings, *General Electric Review*, Vol. 28, n. 8, pp. 559-568
- Sudret, B. 2008, Global sensitivity analysis using polynomial chaos expansions, *Reliability Engineering and System Safety* 93, pp. 964-979
- Wiener, N., 1938. "The homogeneous chaos", *American Journal of Mathematics*, Vol. 60, pp. 897-936
- Xiu, D., 2007, Efficient Collocational Approach for Parametric Uncertainty Analysis, *Communications in Computational Physics*, Vol. 2, No. 2, pp. 293-309
- Xiu, D., 2009. "Fast numerical methods for stochastic computations: A review", *Communications in Computational Physics*, Vol. 5, pp. 242-272
- Xiu, D., Karniadakis, G., 2002. "The Wiener–Askey polynomial chaos for stochastic differential equations", *SIAM Journal on Scientific Computing*, Vol. 24, pp. 619-644

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