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Bifurcation analysis of aircraft nose landing gear shimmy

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Abstract: Shimmy is a self-excited nonlinear oscillation that is observed in aircraft landing gears. This phenomenon is influenced by parameters from the landing gear structural dynamics to the tire properties. Structural and tyre nonlinearities lead to complex dynamics of difficult modeling. Moreover, nonlinear behavior of pre- or post-shimmy is typically related to Hopf bifurcation. The objective of this work is to present a shimmy prediction model that considers both structural flexibility and nonlinear tyre representations to the analysis of the respective nonlinear behavior. The model is representative of an aircraft nose landing gear of one wheel. The structure comprises an axle modeled in torsion with smooth nonlinearities on the stiffness values. For the tire model, the straight tangent tyre model is considered. Hopf bifurcations are identified and diagrams of amplitudes versus wheel speed are used to investigate the conditions in which the system is supercritical or subcritical. The bifurcations are also analyzed for parameters range from structure and tyre models.

Keywords: Shimmy, Hopf bifurcation, Nonlinear dynamics, Self-excited systems, Landing gear.

INTRODUCTION

Shimmy is a nonlinear dynamical phenomenon related to large and persistent oscillations of wheels and ground interaction(Besselink, 2000). As a self-excited problem, the shimmy has the same character as the flutter instability on aircraft (Bisplinghoff et al., 1996). The shimmy phenomenon can be observed in any ground vehicles or even in supermarket trolleys, when the wheels present severe oscillations during their rolling on the ground.

Typical reasons for the occurrence of shimmy are associated to inadequate structural torsional stiffness, freeplay in loose mechanism links, unbalanced wheels, etc. Wright and Cooper (2007). On aircraft the shimmy occurs at the main and nose landing gears. Among them the most affected one is the nose landing gear, which is commonly designed as a free wheel supported by a single vertical axis. Such layout is more susceptible to instabilities during its running.

Numerical models to shimmy prediction have to deal with several difficulties, mainly due to the tyre loading in contact with the ground. The contact loading acting upon the tyre is very complex and many models are normally based on empirical or semi-empirical formulations (Pacejka, 2005). If the nonlinear effects from the landing gear structure are added to the shimmy modeling, a higher level of complexity can be reached (Somieski, 1997). For industrial applications, the shimmy predictions practice is normally based on linearized models, therefore leaving behind possible important effects of the phenomena (Coetzee, 2007). However, the nonlinear features of the shimmy may lead the landing gear to manifest undesired behavior and component's failure, Therefore, there is a great interest to include nonlinear effects to the shimmy prediction models.

Landing gear models to predict shimmy have to include a representation of the tyre loading due to ground and may consider the interaction with the elastic reactions from the structure. The tyre interaction with the ground presents a great amount of modeling uncertainties, which have left the approaches to admit semi-empirical formulations (Moreland, 1954; Von Schlippe and Dietrich, 1954; Smiley and Horne, 1958). To reach an adequate model for shimmy analysis the tyre dynamics is essential. A number of efforts towards proper modeling of the tyre interaction with the ground can be observed in the technical literature. Maas (2009) presents comparisons with different tyre dynamical models applied for the shimmy instability analysis. Moreover, Pacejka (2005) developed an empirical approach to the characterization of several tyre configurations. The result of that approach is a formulation that is refereed as the *magic formula* that comprises a set of equations to compute a broad range of tyre behaviors at different operational conditions.

Somieski (1997) has presented a shimmy model for landing gears using a set of nonlinear ordinary differential equations, that allows the analysis of nonlinear behavior such as limit cycle oscillations occurrence. Somieski's model proposes a simplified formulation for the tyre loading and linear structure in torsion, in which the Hopf bifurcation could be assessed. Khapane (2007) has presented a shimmy instability analysis including the effects of brakes vibrations in landing gears and the nonlinear behavior originated from this kind of vibration source. Thota et al. (2008) have studied the Hopf bifurcation in shimmy problems and also characterized the supercritical and subcritical features due to parametric change. Stépán and co-workers (Stépán, 1991, 1998; Takács et al., 2008) shown a series of publications on the analysis of nonlinear shimmy behavior. They have been able to demonstrate the influences on the shimmy dynamics that leads to supercritical to subcritical behavior, as well as to chaotic conditions.

Another research field on the suppression of shimmy is the application of automatic control techniques. Among the most recent works in this field of shimmy control those from Goodwine and Stépán (2000) and Pouly et al. (2008) can be admitted as in the state-of-the-art.

Despite of the need for experimental information to guide the shimmy modeling, a lack of publications can be observed. Historical publications from Moreland (1954) and more recently those from Takács and Stépán (2009) and Takács et al. (2009) represent some few examples of experimental-based shimmy works dedicated to verify numerical models. The results have allowed the assessment of the shimmy stability boundary experimentally. De Falco et al. (2010) have also developed experiments dedicated to shimmy. In this case, the apparatus was designed to study the behavior of motorcycles wheels to determine the damping required to avoid oscillations due to shimmy. Ioi et al. (2015) have presented an experimental analysis of trolley wheel using it in contact with a larger spinning wheel to emulate the ground motion. Nonlinear responses on limit cycle oscillations were measured and used to validate a numerical model.

This paper aims to present a shimmy instability analysis for an arbitrary nose landing gear configuration. The nonlinear behavior as outcome of considering the tyre loading and nonlinear structure in torsion is presented here. Two different tyre-to-ground interaction loading model is used to simulate the shimmy onset velocity, that is, the Hopf bifurcation point. The basic model for shimmy prediction is based on the Somieski's work (Somieski, 1997) and nonlinear tyre model. Another tyre model is the classical von Schlippe's model (Von Schlippe and Dietrich, 1954) that leads to a linear representation for the lateral force and self-aligning moment. Results demonstrate the influence of the modeling on the shimmy prediction and a parametric analysis is also presented.

EQUATIONS OF MOTION

The nonlinear landing gear model consists on the torsional dynamics of its main structure coupled with the contact loading between tyre and ground during rolling. Figure 1 illustrates the elements and respective loading that comprise the landing gear modeling, where the directions correspond to the assumed positive ones.

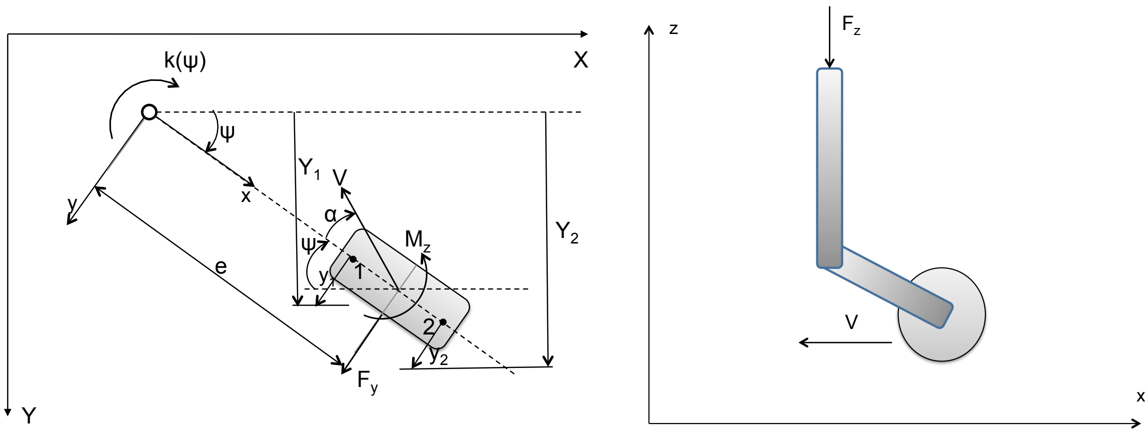


Figure 1 – Landing gear model.

The equation of motion is given by,

$$I_z\ddot{\psi} + c\dot{\psi} + k(\psi) = \mathscr{M}(y, \psi, \dot{\psi}) \quad , \tag{1}$$

where I_z and c are the landing gear mass inertia moment and structural damping coefficient, respectively, $k(\psi)$ is the nonlinear stiffness function, and $\mathscr{M}(\cdot)$ denotes the total moments acting through the tyre contact with the ground as function of the tyre lateral displacement y, landing gear yaw angle deflection ψ, and yaw angle rate $\dot{\psi}$.

The structural nonlinearity in the landing gear torsional stiffness is assumed as a hardening effect. The smooth function based on cubic polynomial approximation for the hardening effect in torsion is given by,

$$k(\psi) = b_3\psi^3 + b_2\psi^2 + b_1\psi + b_0 \quad (Nm/rad) \quad , \tag{2}$$

where b_i for $i = 0$ to 3 represent the real-valued polynomial coefficients.

The total moments $\mathscr{M}(\cdot)$ have been modeled using different assumptions, but typically leading to nonlinear formulations of the tyre-to-ground behavior coupled with the structure dynamics (Pacejka, 2005). To capture the self-excited phenomenon of shimmy from the present model, two approaches for tyre-to-ground behavior are considered.

Somieski (1997) proposed the moments $\mathscr{M}(\cdot)$ given by two terms, *i.e.*, one is a tyre moment due to lateral deformations on side slip and yaw rate (self-aligning moment) and another is a damping moment term. Those moments terms are given by,

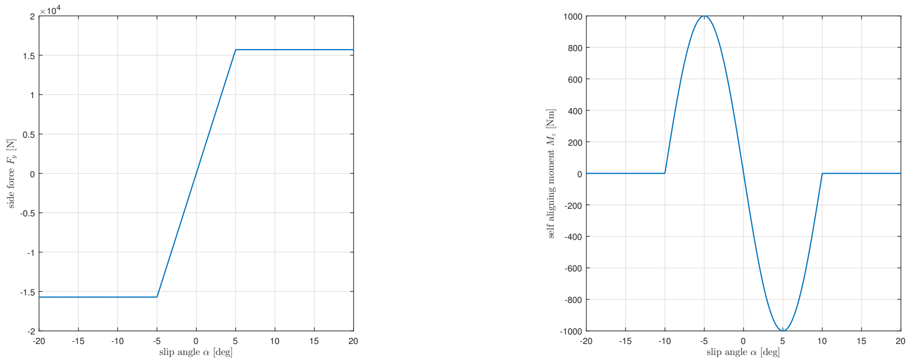
$$\mathscr{M}(y, \psi, \dot{\psi}) = M_s + M_d \quad . \tag{3}$$

The moment due to tyre deformation, Somieski (1997) admits,

$$M_s = M_z - eF_y \quad , \tag{4}$$

where M_z is the self-aligning moment, e is the caster length, and F_y is the side force.

The nonlinearity from tyre-to-ground interaction is modeled in the side force F_y and in the self aligning moment M_z. For the side force and the self aligning moment of the tyre, the loading is supposed to vary proportional to a vertical loading F_z acting through the landing gear structure, and as a function of the slip angle α as a result of tyre deformation (Somieski, 1997). Figure 2 depicts the side force and aligning moment functions of slip angle as proposed in Somieski (1997) in terms of piecewise functions approximations of slip angle.



(a) Side force (b) Self aligning moment

Figure 2 – Sideslip features for the Somieski's model (*cf.* Eqs. (5) and (6))

So, from Somieski (1997), for $\alpha_F = 5°$ e $\alpha_M = 10°$, F_y and M_z take the form,

$$F_y = \begin{cases} c_{F_\alpha} F_z \alpha & if\ \alpha \leq \alpha_F \ , \\ c_{F_\alpha} F_z \ \text{sign}(\alpha)\alpha_F & if\ \alpha \geq \alpha_F \ , \end{cases} \tag{5}$$

$$M_z = \begin{cases} c_{M_\alpha} F_z \sin\left(\frac{180°}{\alpha_M}\alpha\right)\left(\frac{\alpha_M}{180°}\right) & if\ |\alpha| \leq \alpha_M \ , \\ 0 & if\ |\alpha| \geq \alpha_M \ . \end{cases} \tag{6}$$

The slip angle is assumed as a function of the tyre lateral displacement on the front contact point with the ground (y_1), which is described by the elastic string model. This model considers the tire to be a massless string of infinite length supported elastically in the lateral direction, as shown in Fig. 3.

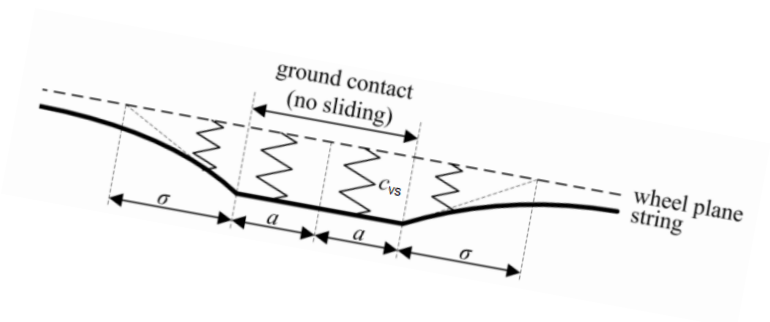


Figure 3 – Elastic string analogy for tyre-to-ground contact model.

In order to derive an equation that describes the lateral displacement a simplified case is admitted. The contact path between the tire and the ground is assumed to be a point with the caster length e equals to zero, as well as an initial angle ψ is applied and let the tire rotate freely. The yaw angle at a generic moment is the derivative of the deflection of the tire (y) with respect to its position. This results in the inicial angle ψ minus the restoring effect of the tire, which is assumed to be linear with respect to the deflection, that is:

$$\frac{dy}{ds} = \psi - \left(\frac{1}{\sigma}\right) y \quad , \tag{7}$$

where s is the distance travelled by the tire, and σ is a relaxation length.

When the contact path has reached $2a$ (*cf.* Fig. 3), the caster length becomes significant. Consequently, the applied ψ angle is variable (rather than an initial condition) and an additional term is included. Moreover, the equation must now be related to the first contact point, here denoted as y_1, therefore,

$$\frac{dy_1}{ds} = \psi - \left(\frac{1}{\sigma}\right) y_1 + (e-a)\frac{d\psi}{ds} \quad . \tag{8}$$

Finally, given that the travelled distance is Vt:

$$\frac{dy_1}{dt} = V\psi - \left(\frac{V}{\sigma}\right) y_1 + (e-a)\frac{d\psi}{dt} \quad , \tag{9}$$

being V the tire constant speed.

Therefore, a final approximation for the slip angle is assumed as,

$$\tan\alpha = \frac{y_1}{\sigma} \;\Rightarrow\; \alpha \approx \frac{y_1}{\sigma} \quad . \tag{10}$$

The tyre damping moment due the contact with the ground during the motion, is given by,

$$M_d = -(0.15/V)a^2 c_{F_\alpha} F_z \dot{\psi} \;\; . \tag{11}$$

The Somieski's model governing equations are summarized by Eqs. (1), (3), and (9). A state-space representation for the Somieski's model can be achieved by adopting,

$$x_1 = \psi \;, \qquad x_2 = \dot{\psi} \;, \qquad x_3 = y \;,$$

which leads to the following set of first-order nonlinear differential equations,

$$\begin{array}{rcl} \dot{x}_1 &=& x_2 \\ \dot{x}_2 &=& (1/I_z)\left[-k(x_1) - cx_2 - (0.15/V)a^2 c_{F_\alpha} F_z\, x_2 + M_s(x_3/\sigma)\right] \\ \dot{x}_3 &=& Vx_1 + (e-a)x_2 - (V/\sigma)x_3 \end{array} \quad . \tag{12}$$

Another proposal for tyre-to-ground interaction model to assess moment representation in the equation of motion given by Eq. (1) is the von Schlippe's tyre model (Von Schlippe and Dietrich, 1954). The von Schlippe's model allows the calculation of the lateral force and self-aligning torque by using the values of lateral displacement of the tire at the front (y_1) and rear (y_2) points with the contact path (*cf.* Fig. 3). The position y_1 is computed from Eq. (9). To obtain the position y_2, the total displacements Y_1 and Y_2 with respect to a landing gear reference system are considered. The von Schlippe's model results in linear representation of the loads with respect to the tyre deformation.

Given that we have y_1, it is easy to see geometrically for small ψ angles that:

$$Y_1 = y_1 + \psi(e-a) \quad . \tag{13}$$

To relate Y_1 and Y_2, Von Schlippe's model admits another assumption, that is: the contact rear point follows the movement of the front point, but with $2a$ delay in distance (corresponding to a $(2a)/V$ time delay). Therefore:

$$Y_2 = Y_1(t - \frac{2a}{V}) \quad . \tag{14}$$

Finally, the geometrical relations lead to:

$$y_2 = Y_2 - \psi(e+a) \quad . \tag{15}$$

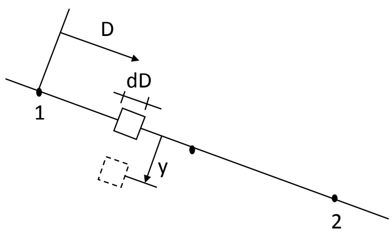


Figure 4 – Infinitesimal string element.

Considering an infinitesimal element of the tyre string, as depicted in Fig. 4, the differentials of force and torque can be calculated using the concept of representative elastic foundation, given,

$$\begin{array}{rcl} dF_y &=& c_{vs} y dD \\ dM_z &=& (a-D)c_{vs} y dD \end{array} \quad , \tag{16}$$

where c_{vs} is the stiffness of the elastic foundation, y is the displacement in a generic point of the string, and D is the distance from the front contact point to the generic point under consideration.

The next step is to integrate the differential loads given by Eq. (16) from the beginning to the end of the tyre. Since the von Schlippe's model consideres an infinite elastic string, integration interval results $-\infty$ to ∞. The proposed behaviour of y prior and after contact (from $D = -\infty$ to 0 and $2a$ to ∞) is assumed as exponent functions of D. Within the contact path is adopted as linear for the Von Schlippe's model. The final form for the linear tyre-to-ground contact loads is,

$$\begin{array}{r} F_y = (y_1 + y_2)(a + \sigma)c_{vs} \\ M_z = (y_1 - y_2)[\frac{a^2}{3} + \sigma(a + \sigma)]c_{vs} \end{array} . \quad (17)$$

Substituting the set of Eqs. (17) in Eq. (4), and subsequently in Eq. (3), the resulting state-space equations for the von Schlippe's tyre model become,

$$\begin{array}{rcl} \dot{x}_1 & = & x_2 \\ \dot{x}_2 & = & (1/I_z)\left[-k(x_1) - cx_2 + [(y_1 - y_2)(\frac{a^2}{3} + \sigma(a + \sigma))c_{vs} - e(y_1 + y_2)(a + \sigma)c_{vs}](x_3/\sigma) + M_d(x_2)\right] \\ \dot{x}_3 & = & Vx_1 + (e - a)x_2 - (V/\sigma)x_3 \end{array} . \quad (18)$$

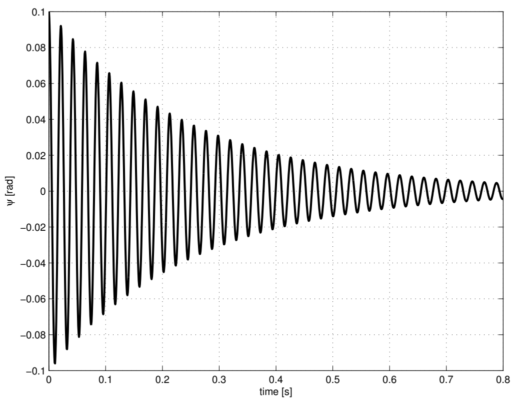
RESULTS

Simulations of Eq. (12) were performed for cases based on parameters given in Somieski (1997). Table 1 presents the parameters used to simulate the shimmy problem with Somieski's and von Schlippe's models, based on values from Somieski (1997), for $\sigma = 3a$. The nonlinear torsional stiffness parameters used in all cases admit the hardening effect by taken $b_0 = 0.0$, $b_1 = 8.5994 \times 10^4$, $b_2 = 7.0$, and $b_3 = 8.8986 \times 10^4$ in Eq. (2).

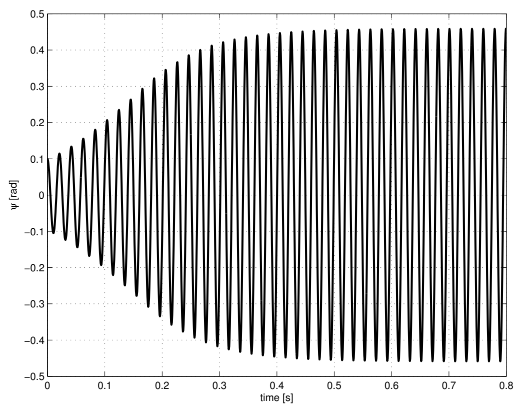
Table 1 – Simulations parameters

parameter	value
a	0.1 m
k	10^5 Nm/rad
c	10 $Nm/rad/s$
c_{F_α}	20 rad^{-1}
c_{M_α}	-2 m/rad
e	0.1 m
F_z	9000 N
I_z	1.0 kgm^2
c_{vs}	1.2411×10^6 $N/m/m$

The shimmy phenomenon is observed when the landing gear dynamics leave stable damped responses to limit cycle oscillations. By setting the initial conditions $[\psi_0, \dot{\psi}_0, y_0]$ and keeping other structural parameters constants, Eqs. (12) and (18) are integrated in time at different arbitrary velocities. Selected time histories of the yaw angle for two different velocities are presented in Fig. 5 for the case of Somieski's model simulation. Here, limit cycle oscillations are present at $30m/s$, thereby demonstrating the shimmy occurrence. Stable damped yaw angle response is observed at $15m/s$ establishing a range in which the shimmy onset is present. For the von Schlippe's model, the yaw angle time histories prior and after the shimmy onset velocity is presented in Fig. 6. One can observe that both model seem to follow the same landing gear dynamics given reasonable prediction of the shimmy phenomenon.

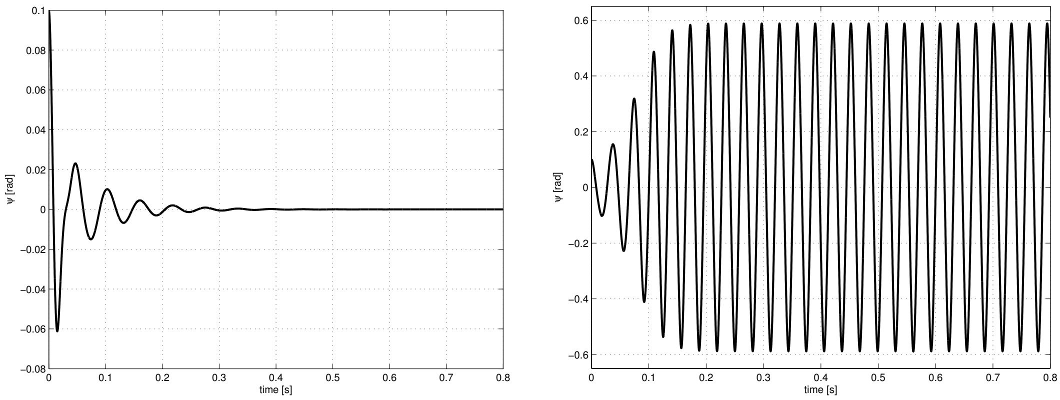


(a) $15m/s$



(b) $30m/s$

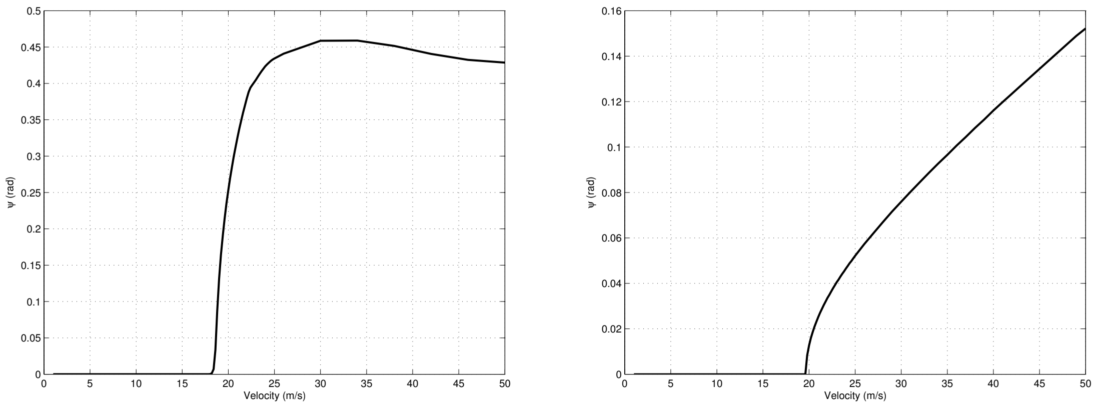
Figure 5 – Yaw angle time histories for the Somieski's model.



(a) $15m/s$ (b) $30m/s$

Figure 6 – Yaw angle time histories for the von Schlippe's model.

With the existence of limit cycle oscillations, Hopf bifurcation behaviour drives the landing gear dynamics and characterizes the shimmy phenomenon. Figures 7(a) and 7(b) show the Hopf bifurcation diagram extracted from the signals for ψ and y variables from respective equations of motion. It can be observed that the shimmy onset for the models present different predicted velocities. For the case of Somiesky's model the shimmy onset velocity, that is, the Hopf bifurcation point, is $17.8m/s$. For the von Schlippe's model the shimmy velocity is predicted at $19.6m/s$. Regarding the fact that the landing gear dynamics for the two approaches only differs from the tyre-to-ground interaction model, one may conclude on the importance of the tyre model. However, because structural nonlinear behavior is all included in the landing gear torsion, bifurcation can be observed.



(a) Somieski's model (b) von Schlippe's model

Figure 7 – Hopf bifurcation diagrams.

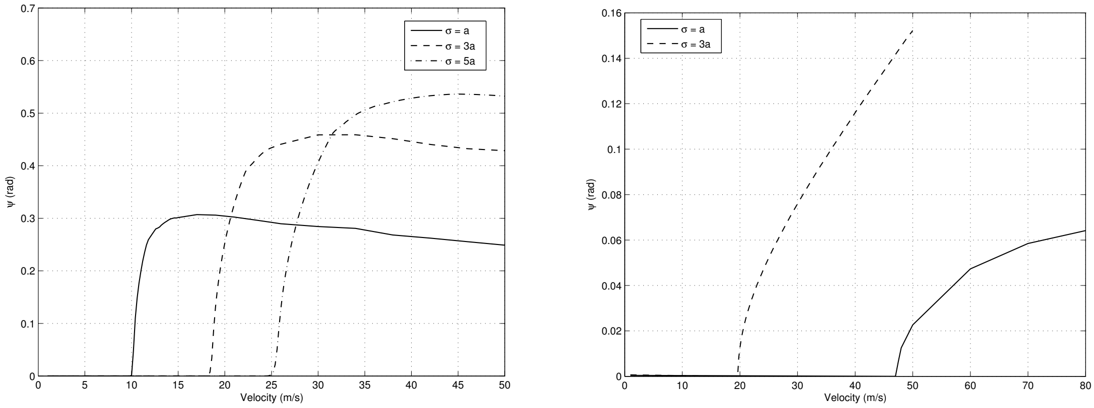
Loading pattern allows to conclude that differences in the models are part related to the features of the tyre-to-ground interaction approaches. To verify how the tyre models influence the bifurcation occurrence, the Somieski's and von Schlippe's models are simulated admitting linear structural dynamics. For both cases, the linear structural stiffness is considered as $10^5 Nm/rad$. The result on bifurcation diagram can only be seen with the Somieski's model, since the von Schlippe's model is based on linear tyre-to-ground loading.

Finally, a parametric analysis of the shimmy problem with respect to the models under consideration is presented. For the Somieski's model the Hopf bifurcation diagram is achieved for different values of σ values, as illustrated in Fig. 8(a). Here the σ represents the relaxation length, that is, the length of deformed tyre in contact with the ground. Therefore, if the tyre contact length increases the shimmy onset is delayed to higher velocities. The von Schlippe's model parametric study shows that the σ variation produces opposite influence on the Hopf bifurcation onset. That condition is illustrated in Fig. 8(b).

The von Schlippe's model is conceived using a elastic foundation stiffness distribution that can be related to the tyre pressure. In this case, the parameter c_{vs} for different values is used to verify its influence on shimmy phenomenon. Figure 9 shows how c_{vs} influences the bifurcation onset for three different values that correspond to different tyre pressure values. The relation between tyre pressure and c_{vs} is given by $0.9P$, where P is the tyre pressure.

CONCLUDING REMARKS

The present paper presents an investigation on the landing gear shimmy prediction and parameter influence depending on the tyre-to-ground interaction model assumptions. The landing gear structure is assumed with nonlinear stiffness,



(a) Somieski's model (b) von Schlippe's model

Figure 8 – Influence of σ on the shimmy onset velocity.

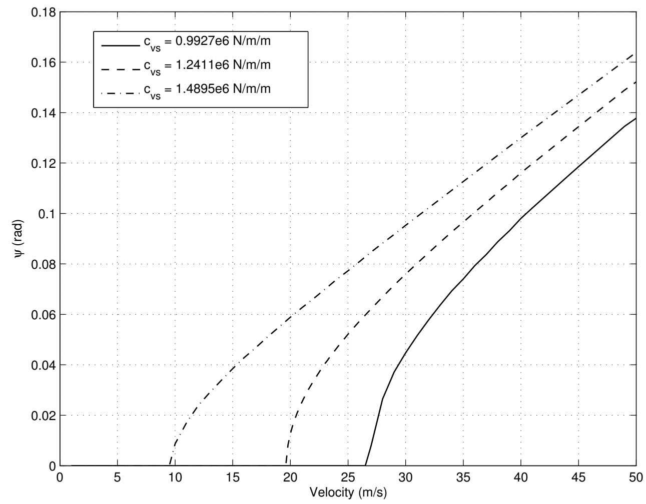


Figure 9 – Influence of c_{vs} (tyre pressure related parameter) on the shimmy onset velocity.

thereby ensuring complex interaction with the tyre loads. Somieski's nonlinear and von Schlippe's linear tyre models are used for this analysis. Results show that shimmy prediction can be fairly obtained with both tyre models for minor adjustments on the models parameters. The importance of considering nonlinear structural behaviour in stiffness for shimmy prediction is also observed. Parameters change also show that tyre-related variables have important role on instability prediction. Future investigation will considere other tyre-to-ground models and different stiffness behavior of the landing gear structure.

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