DINAME 2017 - Proceedings of the XVII International Symposium on Dynamic Problems of Mechanics
A. T. Fleury, D. A. Rade, P. R. G. Kurka (Editors), ABCM, São Sebastião, SP, Brazil, March 5-10, 2017

A Model Reduction Method for Aeroviscoelastic Systems

A.G. Cunha-Filho[1], **Y.P.J. Briend**[1], **A.M.G. de Lima**[1], **M.V. Donadon**[2]

[1] Federal University of Uberlandia, School of Mechanical Engineering, Campus Santa Monica, P.O. Box 593, 38400-902, Uberlandia, MG, Brazil, andregc@gmail.com, yvon.briend@gmail.com.

[2] Technological Institute of Aeronautics, Department of Aeronautics, São José dos Campos, SP, 12228-900, Brazil.

Abstract: It is well-known that the flutter boundary prediction through mathematical models for complex aeroelastic panels of industrial interest is not a simple task. In most cases, this is mainly due to the complexity of the resulting aeroelastic models usually composed by a large number of degrees of freedom. Moreover, if the aeroelastic model incorporates a control strategy to suppress the flutter phenomenon, such as the use of viscoelastic materials, the computational cost inflicted to predict the flutter speed of this new aeroviscoelastic model may become impractical. For this situation, in which the flutter boundary must be determined through the solution of a complex eigenvalue problem, the use of a model reduction technique is required. However, when modeling reduction methods based on modal projections for aeroviscoelastic systems with frequency- and temperature-dependency, the construction of the reduction basis becomes a challenge. It is so due to the need of considering simultaneously in the reduction basis the frequency- and temperature-dependency of the viscoelastic properties and the dynamic behavior of the system as a function of the dynamic pressure. Thus, the main contribution of the present paper is to propose a new reduction method strategy for aeroelastic systems incorporating viscoelastic materials based on an iterative enriched Ritz-method with aerodynamic residues iterations. The numerical results for a thin rectangular three-layer sandwich plate under supersonic flow, are presented comparing the exact solution with predictions obtained using the proposed reduction method. The comparisons are presented in terms of the amplitudes of the FRFs, the flutter boundary and the computational efficiency of the proposed new reduction method and through most used methods suggested in the open literature.

Keywords: aeroviscoelastic systems, viscoelastic materials, model reduction, finite element.

INTRODUCTION

Flutter of aeronautical panels is a complex phenomenon that can occur during supersonic flights due to the simultaneous interaction of three forces: elastic, inertial and aerodynamic. Thus, it can significantly affect the fatigue life of existing aeronautical components in flight conditions, leading to catastrophic failure. In the last decades, several works have been proposed as a first attempt to better understand the behavior of composite materials under flutter conditions. Secondly and more recently the application of control strategies to mitigate the undesirable vibrations induced by such phenomenon. In the present paper, a passive control strategy named the constraining viscoelastic layer is investigated. It consists of incorporating a viscoelastic layer sandwiched between a restrained layer and the base-plate forming a moderately thin three-layer sandwich plate. However, viscoelastic material properties are frequency- and temperature-dependent, causing the computation of the flutter boundary to be very costly, sometimes unfeasible. Thus, the proposition of an efficient reduction method to be applied on aeroviscoelastic systems becomes essential.

The constitutive model for the viscoelastic material is based on the complex moduli approach following the original developments made by Drake & Soovere (1984). Thus, the eigenvalue problem obtained from the equations of motion is highly nonlinear due to the simultaneous dependence of temperature and excitation frequency of the viscoelastic material properties. An alternative to deal with such a problem is to linearize it by fixing a value of temperature followed by an iterative process that adjusts the mechanical properties of the viscoelastic material with the excitation frequency. Moreover, with the aim of assessing the stability of the aeroviscoelastic system, the three-layer sandwich panel is submitted to a longitudinal supersonic airflow modeled through the Linear Piston Theory, firstly propose by Lighthill (1953). This kind of formulation involves a certain number of iterations to calculate the properties of the system for each value of the airflow speed, for more detailed information about the iterative process see the work of Cunha-Filho *et al.* (2016). Therefore, a dynamic response evolution of the sandwich plate with respect to the airspeed must be considered in the reduction method.

Among several possibilities available in the open literature, a modal projection basis which is a subspace formed by pseudo-normal modes (nominal projection basis) obtained from the exact solution and enriched by static and dynamic residues of first order is typically used to reduce large finite element (FE) models (Balmès, 1996). It is convenient to use such approach in the present work, since the reduced model contains information about the natural modes of vibration (free, fixed, or loaded boundary conditions) and static responses to a unit load (Balmès, 1997). Clearly, in the context of aeroelastic systems, it means that the load generated by the longitudinal supersonic flow will induce as many static residues as the number of velocities evaluated. It means that a modified iterative projection basis must be constructed.

Accuracy and efficiency of the proposed aeroviscoelastic reduction method are compared along with two other already known methodologies. The first one is a method developed by de Lima et al. (2010) named *Robust Enriched Ritz approach*. It is intended to reduce a discrete viscoelastic model through a constant projection basis constructed from residues formed by static displacements associated to external loads and viscoelastic damping forces. Although this method presents an impressive efficiency and accuracy for viscoelastic models, it is not accurate for aeroviscoelastic systems since the projection basis is not updated considering the aeroelastic modifications. The second method is the so-called multi-model approach (MM), initially proposed by Balmès and Plouin (2000). This approach consists in generating a projection basis from pseudo-normal modes obtained from a viscoelastic system. The MM method considers the static response to a prescribed load generated by the imaginary part of the stiffness when exciting a given pseudo-normal mode. The method leads to a good correlation for viscoelastic and aeroviscoelastic systems, but the construction process of the projection basis inflicts high computational costs and may turn out to be prohibitive, depending on the number of dofs to be considered. Finally, an iterative method, firstly proposed by Kergourlay, Balmès and Clouteau (1998) and here modified to attain the chalenge of reducing a aeroviscoeltic system is proposed. This approach has proven to be not only accurate enough to represent the system reliability but also presented feasible time when constructing the projection basis.

Finally, one emphasizes that few works have proposed reduction methods capable of dealing with aeroviscoelastic systems, motivating the study addressed herein. Shin *et al.* (2006) studied the aeroelastic characteristics of cylindrical hybrid composite panels viscoelastically damped using the mechanical properties of the 3M-ISD110 and 3M-ISD112 viscoelastic materials modeled according to the analytic equations of Drake & Soovere (1984). The authors have proposed a reduction method based on a modal approach that consists in generating a simple projection basis formed by a subspace obtained from the equation of motion regardless not only the structural viscoelastic damping but also the aeroelastic effects. This methodology is also compared to the exact model and the results are less satisfactory then the other methods cited above.

BACKGROUND ON AEROVISCOELASTIC MODELING PROCEDURE

This section briefly presents the finite element modeling of a three-layer sandwich panel subjected to a longitudinal supersonic airflow. Further details can be found at Cunha-Filho (2015). One uses a rectangular element containing four nodes and seven DOFs per node, as depicted in Fig. 1. The whole structure is formed by a base plate (1), a viscoelastic core (2) and a constraining layer (3). This set of layers is responsible to introduce shearing stress into the viscoelastic material inducing energy dissipation through hysteretic mechanisms. In-plane displacements in the midplane of the base-plate in directions *x* and *y* are denoted by u_1 and v_1, respectively, and in-plane displacements of the midplane of the constraining layer in directions *x* and *y* are denoted by u_3 and v_3. Transverse displacement is denoted by *w*, which is common for all three layers and the cross-section rotations about *x* and *y*, are denoted by θ_x and θ_y, respectively.

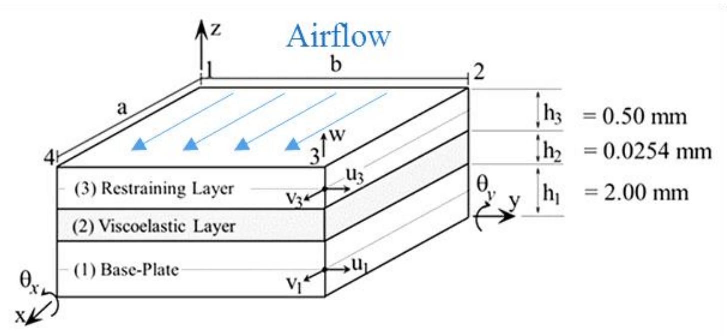

Figure 1 - Illustration of the three-layer sandwich plate element under supersonic airflow

Hypothesis of thin plates made by Kirchhoff are adopted herein for the elastic base-plate and constraining layers. For the viscoelastic core, the Mindlin's theory is considered to account for the transverse shear. Thus, through these assumptions, it is possible to obtain the following equations of motion in the frequency domain:

$$\left[\boldsymbol{K}_e + G(\omega,T)\,\bar{\boldsymbol{K}}_v - \omega^2 \boldsymbol{M}\right]\boldsymbol{Q} = \boldsymbol{L} \tag{1}$$

where $\boldsymbol{M} \in R^{N\times N}$ is the global mass matrix (symmetric, positive-definite), $\boldsymbol{K}_e \in R^{N\times N}$ is the stiffness matrix (symmetric, nonnegative-definite) corresponding to the purely elastic substructure, $\bar{\boldsymbol{K}}_v \in R^{N\times N}$ is relative to the viscoelastic substructure, $\boldsymbol{L}$ is a column vector of the amplitude of the external aerodynamic load and $\boldsymbol{Q}$ is the vector of the responses. The term $G(\omega,T)$ corresponds to the complex modulus function of the viscoelastic material and it contains information about its frequency- and temperature-dependent dynamic behavior. The complex modulus function is expressed by the equations developed by Drake & Soovere (1984) and applied to the ISD112 viscoelastic material.

$$G(\omega_r) = 430700 + \frac{1200\times10^6}{1+3.241\times(i\omega_r/1543000)^{-0.18} + (i\omega_r/1543000)^{-0.6847}} \tag{2}$$

where $\omega_r = \alpha_T \omega$ is the reduced frequency and α_T is the shift factor, which is determined as follows:

$$\log(\alpha_T) = -3758.4\times\left(\frac{1}{T}-\frac{1}{T_0}\right) + 225.06\times\log\left(\frac{T}{T_0}\right) + 0.23273\times(T-T_0) \tag{3}$$

The aeroelasticity can be introduced into the FE model through the so-named Piston Theory based on the original developments made by Lighthill (1953). The simplest form of the Piston Theory is called Quasi-Steady Model of Ackeret (Almeida *et al.* 2012), which is a linear approximation of the aerodynamic loading regardless the aerodynamic damping. This later can be neglected since it has a small influence over the prediction of the critical dynamic pressure (Bismarck-Nasr, 1999; Dowell, 1975; Pegado, 2003; Kuo, 2011). According to this theory, the variation of pressure for a panel can be expressed as follows:

$$\Delta P = P - P_\infty = \frac{2q}{\beta}\left(\frac{\partial w}{\partial x}\right) \tag{4}$$

The energy involved in the interaction between the supersonic airflow and the plate can be described through the work done by the aerodynamic load as:

$$W = -\int_A \Delta P w dA = -\frac{2q}{\beta}\int_A \frac{\partial \boldsymbol{w}}{\partial x} w dA = -\frac{2q}{\beta}\boldsymbol{\delta}^T \boldsymbol{A}\boldsymbol{\delta} \tag{5}$$

where the vector $\boldsymbol{\delta}$ represents the DOFs of the sandwich plate, $\boldsymbol{A} \in R^{NxN}$ designates the aerodynamic stiffness matrix, $q = 0.5\rho_{air}V^2$ is the dynamic pressure over the plate generated by the supersonic airflow, and $\beta = \sqrt{M^2-1}$. The parameter ρ_{air} is the air density at a specific altitude and V is the speed of the airflow. Thus, the aerodynamic stiffness matrix, $\boldsymbol{A}$, can be introduced into the equations of motion (1):

$$\begin{gathered}\left[\left(\boldsymbol{K}_e + G(\omega,T)\bar{\boldsymbol{K}}_v + 2\frac{q}{\beta}\boldsymbol{A}\right) - \omega^2\boldsymbol{M}\right]\boldsymbol{Q}(\omega,T) = \boldsymbol{F}(\omega) \\ \boldsymbol{F}(\omega) = \boldsymbol{bu}(\omega); \qquad \boldsymbol{y}(\omega,\mathrm{T}) = \boldsymbol{cQ}(\omega,T)\end{gathered} \tag{6}$$

where $\boldsymbol{F}(\omega) \in R^N$ and $\boldsymbol{y}(\omega,T) \in R^C$ are, respectively, the vectors corresponding to the applied loading and complex responses. The vectors $\boldsymbol{b} \in R^{Nxf}$ and $\boldsymbol{c} \in R^{cxN}$ are Boolean matrices used to select, among the DOFs, those which the responses are computed and the excitation forces are applied, respectively.

Equation (6) can be interpreted as if the viscoelastic system had inherent aeroelastic properties and a constant projection basis containing such information should be enough to approximate the solution through a linear transformation of the form $\boldsymbol{T}^{N\times N_R}$, where $N_R << N$ is the number of reduced vectors that forms the base. However, if the dynamic pressure increases, it also changes the aeroelastic properties of the system, modifying the modal shapes of the panel under supersonic flow condition. Such information is not considered in a constant projection basis $\boldsymbol{T}$. This fact suggests that an iterative reduction method, such as the strategy proposed by Kergourlay *et al.* (1998), could be used to consider the mode shape evolution into the projection basis.

Next, one shows a brief description of the standard procedure used to obtain a nominal projection basis and how such basis can be improved according to each reduction method implemented here.

REVIEW OF SOME MODEL REDUCTION TECHNIQUES

Clearly, it is expected that the temperature- and frequency dependent behavior of viscoelastic materials imposes some difficulty when dealing with model reduction techniques applied to aeroviscoelastic systems. By performing a modal analysis with the aim to define the exact properties of the viscoelastic material in consonance with the excitation frequency it is convenient to implement an iterative process to ensure that the eigenvalues and the global FE matrices converge toward a unique value. More detailed information about this procedure is available at the reference (Cunha-Filho *et al.*, 2016). Thus, the application of model reduction methods in the iterative process to obtain the flutter boundary can make the computation of flutter speed faster, since it avoids the use of the exact FE model. However, it will be shown that different reduction techniques lead to different results and some of them are not accurate enough to deal with aeroviscoelastic systems, requiring improvements to fulfill expectations. Starting from the reduction methods used to deal with viscoelastically damped structures as discussed in the introduction section, it will be shown firstly their advantages and disadvantages, before introducing the new model reduction technique proposed herein.

Typical reduction methods applied to viscoelastic systems

Condensation methods are used to reduce the model dimension and to accelerate the computation burden while keeping a reasonable predictive result capacity. Such an impressive achievement is possible under the assumption that an exact solution, taken as an example Eq. (6), which can be approximated by projections on a reduced vector basis as:

$$\boldsymbol{Q}(\omega,T)=\boldsymbol{T}\hat{\boldsymbol{Q}}(\omega,T) \tag{7}$$

where $\boldsymbol{T}\in C^{N\times N_R}$ is a linear transformation matrix spanned by a vector basis obtained arbitrarily from the exact coordinates $\boldsymbol{Q}(\omega,T)$, the term $\hat{\boldsymbol{Q}}(\omega,T)\in C^{N_R}$ is the generalized coordinates.

Basically, all the approaches addressed here have the same initial idea in which the construction of the projection basis $\boldsymbol{T}$ is performed on the conservative associated viscoelastic system without any aerodynamic contribution. This initial subspace is called nominal basis, $\boldsymbol{\Phi}_0$, which is determined by performing the following eigenvalue problem:

$$\begin{gathered}\left[\boldsymbol{K}_e+G_0\bar{\boldsymbol{K}}_v-\boldsymbol{\Lambda}_0\boldsymbol{M}\right]\boldsymbol{\Phi}_0=\boldsymbol{0}\\ \boldsymbol{\Phi}_0=\left[\boldsymbol{\phi}_1^* \quad \boldsymbol{\phi}_2^* \quad \ldots \quad \boldsymbol{\phi}_{N_R}^*\right],\quad \boldsymbol{\Lambda}_0=diag\left(\Lambda_1^*,\ldots,\Lambda_N^*\right)\end{gathered} \tag{8}$$

If the nominal base, $\boldsymbol{\Phi}_0$, is taken as the reduction basis, it could be stated that a first reduction method is defined. Next, a reduced system can be determined by performing the following transformation applied on the complete system (6):

$$\boldsymbol{T}^T\left[\boldsymbol{K}_e+G(\omega,T)\bar{\boldsymbol{K}}_v+2\frac{q}{\beta}\boldsymbol{A}-\omega^2\boldsymbol{M}\right]\boldsymbol{T}\hat{\boldsymbol{Q}}(\omega,T)=\boldsymbol{T}^T\boldsymbol{F}(\omega) \tag{9}$$

In the open literature, Shin *et al.* (2006) used such procedure to reduce an aeroviscoelastic system formed by curved composite sandwich panels under supersonic flow. However, no comparison between the exact solutions and the predictions obtained by the reduced model were presented. In order to certificate the efficiency and precision of such approach, the resulting sysem (9) is compared to the exact solution (6). To do so, three different systems are evaluated through FRFs curves: the exact viscoelastic system, the exact aeroviscoelastic system and the reduced model through the nominal projection basis, $\boldsymbol{\Phi}_0$, for an airspeed value of 1500m/s (right before the flutter condition).

The idea is to notice not only the influence of the aeroelastic properties over the dynamic response of the sandwich panel, but also to show how important those information are for the construction of the projection basis. Figure 2 shows how inacurate reducing the system using only the nominal basis $\boldsymbol{\Phi}_0$ can be and the influence of the aerodynamic loading over the plate. Figure 2 (b) shows that aerodynamic informations must be inserted into the projection basis, othewise it becomes impossible to the reduced model predict the coalescense. Nevertheless, the nominal basis, $\boldsymbol{\Phi}_0$, is an important step to be considered in the construction of the projection basis as it will be seen in the sequel.

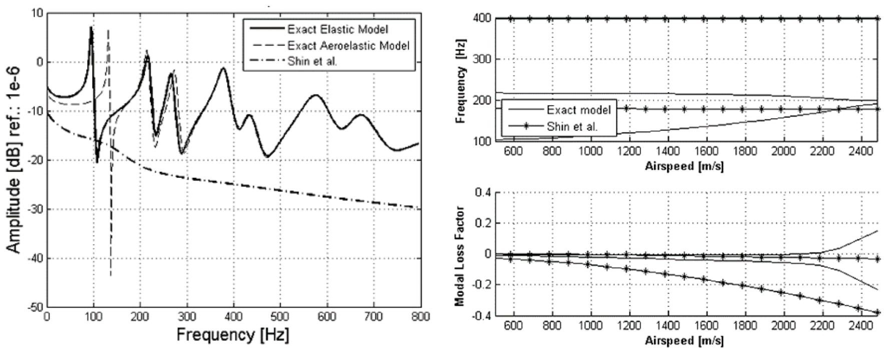

Figure 2 – Comparison between the amplitudes of the FRFs for the exact and reduced systems by considering the projection basis Φ_0 (left); V-g diagram for the solutions by considering the projection basis Φ_0

Enriched Ritz Method (ERM)

A considerable improvement of the projection basis, $\boldsymbol{T}$, can be achieved by enriching it with static first order residual vectors associated to the viscoelastic damping forces, as implemented by de Lima *et al.* (2010):

$$\boldsymbol{R}_v^0 = \boldsymbol{K}_0^{-1}\bar{\boldsymbol{K}}_v\boldsymbol{\Phi}_0 \tag{10}$$

where $\boldsymbol{K}_0 = \boldsymbol{K}_e + G_0\bar{\boldsymbol{K}}_v$ and G_0 is the property of the viscoelastic material for $\omega = 0$.

The residues obtained from Eq. (10) can be interpreted as columns of the flexibility matrix associated to the undamped system (Balmès and Germes, 2002). For this system, three types of forces are involved: the applied external excitations, the damping forces and the aerodynamic forces. The damping forces can be interpreted by examining the Eq.(8). By moving the term involving the viscoelastic properties to the right-hand side of the equation, it plays a role of additional forces applied on the associated conservative structure. The aeroelastic residue, generated by the aerodynamic loading is obtained by the following expression:

$$\boldsymbol{R}_A^0 = \boldsymbol{K}_0^{-1}\boldsymbol{A}\boldsymbol{\Phi}_0 \tag{11}$$

Thus, the enriched projection basis is determined as follows:

$$\boldsymbol{T}_{R_{v+A}} = \left[\boldsymbol{\Phi}_0 \quad \boldsymbol{R}_v^0 \quad \boldsymbol{R}_A^0\right] \tag{12}$$

From Eq.(12), the following reduced aeroviscoelastic model can be written,

$$\boldsymbol{T}_{R_{v+A}}^T \left[\boldsymbol{K}_e + \boldsymbol{K}_v\left(\omega,T\right) + 2\frac{q}{\beta}\boldsymbol{A} - \omega^2\boldsymbol{M}\right]\boldsymbol{T}_{R_{v+A}}\hat{\boldsymbol{Q}}\left(\omega,T\right) = \boldsymbol{0} \tag{13}$$

The Multi-Model Approach (MM)

The Multi-model approach is based on the knowledge relative to the associated conservative system and dynamic responses obtained from the higher values of the frequency band of interest. It consists in considering the sub-spaces obtained from Eq.(6), but admitting responses from different values of frequencies: $\boldsymbol{\Phi}_{(\omega_{min})}$ and $\boldsymbol{\Phi}_{(\omega_{max})}$, as developed by Balmès and Plouin (2000). Those subspaces can be determined according to a number of pseudo-normal modes as follow:

$$\begin{aligned} \boldsymbol{\Phi}\left(\omega_{\min}\right) &= \left[\phi_1\left(\omega_{\min}\right) \quad \phi_2\left(\omega_{\min}\right) \quad \dots \quad \phi_k\left(\omega_{\min}\right)\right] \\ \boldsymbol{\Phi}\left(\omega_{\max}\right) &= \left[\phi_1\left(\omega_{\max}\right) \quad \phi_2\left(\omega_{\max}\right) \quad \dots \quad \phi_k\left(\omega_{\max}\right)\right] \end{aligned} \tag{14}$$

where, in the present study, $k = 1,...,10$.

From Eq. (14), the projection basis of the MM approach, $\boldsymbol{T}_{MM}$, is obtained as:

$$\boldsymbol{T}_{MM} = \left[\boldsymbol{\Phi}\left(\omega_{\min}\right) \quad \boldsymbol{\Phi}\left(\omega_{\max}\right)\right] \tag{15}$$

In order to adapt the MM method to the aeroelastic case, instead of solving the eigenvalue problem only for the associated conservative viscoelastic system of Eq.(8), one solves it for the eigenvalue problem of the fully aeroviscoelastic system represented by Eq.(6), for both frequencies of interest determined in Eq.(14). This operation will introduce the required aeroelastic information into the reduction basis. Clearly, the factor, $2q/\beta$, that multiplies the aerodynamic matrix, has a strong influence on the result in terms of the flutter point prediction. An optimal value would be the one in which the flutter point occurs itself, which is unknown by definition. However, one could add the aerodynamic residue of Eq.(11) into the Multi-model projection basis as shown in Eq.(16), which has proved to be more efficient.

$$\boldsymbol{T}_{MM} = \left[\boldsymbol{R}_A^0 \quad \boldsymbol{\Phi}(\omega_{\min}) \quad \boldsymbol{\Phi}(\omega_{\max}) \right] \tag{16}$$

Finally, the reduced model through the projection basis, $\boldsymbol{T}_{MM}$, assumes the following form:

$$\boldsymbol{T}_{MM}^T \left[\boldsymbol{K}_e + \boldsymbol{K}_v(\omega,T) + 2\frac{q}{\beta}\boldsymbol{A} - \omega^2 \boldsymbol{M} \right] \boldsymbol{T}_{MM} \hat{\boldsymbol{Q}}(\omega,T) = \boldsymbol{0} \tag{17}$$

Iterative Ritz Method (IRM)

This approach is inspired by the technique proposed by Bobillot and Balmès (2002), whose purpose was to reduce FE models incorporating damping and coupled with compressible fluids. Clearly, the resulting systems contain complex stiffness matrices, whose eigenvalue computations play an important role in the quest of achieving a time efficient model. Because of the frequency-dependent behavior of the damped systems, the authors proposed a projection basis that contains information of the system for a certain number of natural frequencies associated to the conservative structure. Thus, by applying an efficient iterative process, the eigenvectors for non-zeros frequencies are introduced into the projection basis complying with a strain energy criterion. After the construction process, the new enriched basis remains constant and it does not need to be updated during the computation process, allowing an important reduction of time processing.

Differently from the methods described previously, the construction of the IRM projection basis is based not only on the nominal basis, but also in the dynamic stiffness matrix, $\boldsymbol{Z}(\omega,T)$. The method proposed consists firstly in introducing the aerodynamic matrix defined in Eq.(5) into the dynamic stiffness matrix. Then, it is considered the aerodynamic evolution of the system by the insertion of several values of the aerodynamic factor, $2q/\beta$, simultaneously to the frequency iterative process. The values of $2q/\beta$ are inferred from an interval of airspeed that may respect the aerodynamic theory $\left(\sqrt{2} < M < 5.5\right)$ and certainly contains the critical airspeed. It makes this method more general than the previous one. Such procedure allows updating the projection basis with respect to the modification of the aerodynamic loads.

The construction of the projection basis $\boldsymbol{T}_{IRM}$ can be performed, firstly defining the dynamic stiffness

$$\begin{aligned} \boldsymbol{Z}(\omega,T)\boldsymbol{Q}(\omega,T) &= \boldsymbol{bu}(\omega,\mathrm{T}) \\ \boldsymbol{y}(\omega,T) &= \boldsymbol{cQ}(\omega,T) \end{aligned} \tag{18}$$

where $\boldsymbol{Z} = \left[-\omega^2 \boldsymbol{M} + \boldsymbol{K}_e + \boldsymbol{K}_v(\omega,T) + 2\frac{q}{\beta}\boldsymbol{A} \right]$. Then, it is assumed that the exact dynamic response given by Eq. (18) can be approximated by projection basis formed by the nominal basis, $\boldsymbol{T} = \boldsymbol{\Phi}_0$, leading to, $\boldsymbol{Q} = \boldsymbol{T}\hat{\boldsymbol{Q}}$, where $\hat{\boldsymbol{Q}}$ is the reduced vector of generalized DOF. After, assuming that the equilibrium equations are projected onto the basis $\boldsymbol{T}$, the following reduced model is obtained:

$$\begin{aligned} \boldsymbol{T}^T \boldsymbol{Z}\boldsymbol{T}\hat{\boldsymbol{Q}} &= \boldsymbol{T}^T \boldsymbol{bu}(\omega) \\ \boldsymbol{y}(\omega,T) &= \boldsymbol{cT}\hat{\boldsymbol{Q}} \end{aligned} \tag{19}$$

The solution of Eq. (19) gives an approximation of the dynamic response of the system excited by $\boldsymbol{bu}(\omega,\mathrm{T})$. Then one can compute the load and displacement residues according to the following equations:

$$\boldsymbol{R}_L(\omega,T) = \boldsymbol{Z}(\omega,T)\boldsymbol{T}\hat{\boldsymbol{Q}} - \boldsymbol{bu}(\omega,T) \tag{20}$$

$$\begin{aligned} & \boldsymbol{R_D}(\omega,T) = \boldsymbol{K}_0^{-1}\boldsymbol{R_L}(\omega,T) \\ & \boldsymbol{T}_{IRM} = \boldsymbol{R_D} \end{aligned} \tag{21}$$

where $\boldsymbol{R_L}$ is the load residual and $\boldsymbol{R_D}$ is the displacement residual. Once the displacement residual has been determined, one can compute the associated error using a criteria based on the strain energy:

$$E = \frac{\boldsymbol{R_D}(\omega,T)^T \boldsymbol{K}(0,T)\boldsymbol{R_D}(\omega,T)}{\hat{\boldsymbol{Q}}^T(\omega,T)\boldsymbol{T}^T\boldsymbol{K}(0,T)\boldsymbol{T}\hat{\boldsymbol{Q}}(\omega,T)} \tag{22}$$

If the error *E* is bigger than a defined tolerance, then the ultimate displacement residual $\boldsymbol{R_D}$ is added to the previous basis $\boldsymbol{T}$, and the process from Eq.(19) to (22) is repeated until the condition is satisfied. Once it is done, one can go to next iteration *k+1*, that is, to equation (18) with a new value of ω and $2\frac{q}{\beta}$ corresponding to ω_{k+1} and $2\frac{q_{k+1}}{\beta_{k+1}}$. Thus, one can understand that, for *N* iterations, the size of projection basis $\boldsymbol{T}$ can increase from more than *N* vectors, given that for one iteration, several displacement residuals can be added. Moreover, to avoid some possible vector collinearities, it is prudent to perform an orthogonalization of the projection basis at each loop, which generally reduces its order.

NUMERICAL RESULTS

The numerical results are presented in three parts: first, it is compared the quality of the FRFs obtained from each reduction method proposed hereby; next, the interest is to evaluate the capability of flutter prediction of the reduced models through a V-g diagram; finally, one shows the comparison between the dynamic characteristics of the results obtained by the projection bases and the corresponding computational efficiency.

The physical system considered herein is formed by a three-layer rectangular sandwich panel subjected to supersonic airspeed, where the base plate and constraint layer are made of aluminum ($E = 70Gpa$, $\upsilon = 0.34$, $\rho = 2700 kg/m^3$), while the viscoelastic core is made of ISD112 3M® viscoelastic material ($\upsilon = 0.49$, $\rho = 950 kg/m^3$). The length of the plate in the *x* and *y* directions are 0.39m and 0.33m, respectively. The characteristics of the air considered in the present study are $\rho_{air} = 1.25 kg/m^3$, $V_{sound} = 340 m/s$, and the reference temperature of 15°C.

FRFs comparison

In order to evaluate the quality of each method in predicting the dynamic response of the aeroviscoelastic system, firstly the FRFs have been investigated. The amplitudes of the FRFs are evaluated as they represent a valuable source of information to determine the frequency bands where reduction methods are efficient. For the proposed structure, the most important natural frequencies are the first and second modes, since they are the first ones to coalesce. Thus, the exact aeroviscoelastic model, the enriched Ritz Method (ERM), the Multi-Model Approach (MM) and the Iterative Ritz Method (IRM) are presented in Figure 3.

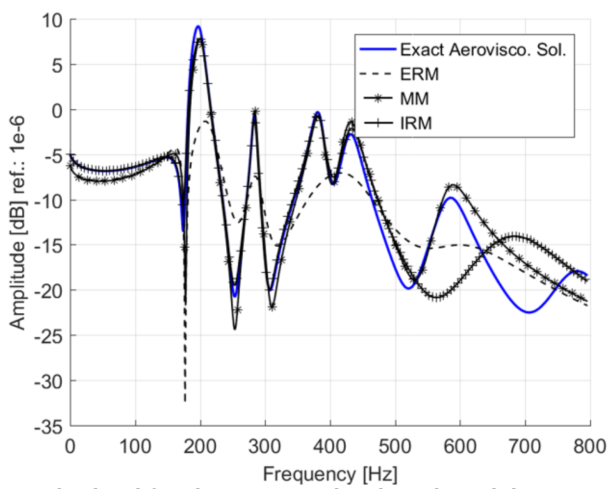

Figure 3 – FRFs obtained for the exact and reduced models: ERM, MM and IRM.

The arbitrary airspeed of study is defined as $v = 1500 m/s$ (before flutter) in order to assess the difference between the reduction methods that are influenced by the aerodynamic effect. Analysing the results shown in Figure 3 it is evident that the IRM presents the best correlation until 400 Hz for both resonance and anti-resonance regions. A zoom from 100 to 300 Hz, depicted in Fig. 5, enables to conclude that the IRM method overlaps the exact solution, whereas the MM and ERM methods are out of the exact one.

Figure 4 shows that the best approximation is obtained by the IRM projection basis, while the MM and ERM diverge at the anti-resonance, not only for the frequency values but also for the amplitude values. Moreover, despite the ERM predicts correctly the value of the first natural frequency, the amplitude is largely underesstimated. Such good correlation of the IRM can be explained by the fact that the displacement residues are more accurate and reliable than the other reduction methods thanks to the iterative construction process conforming to the strain energy criterion.

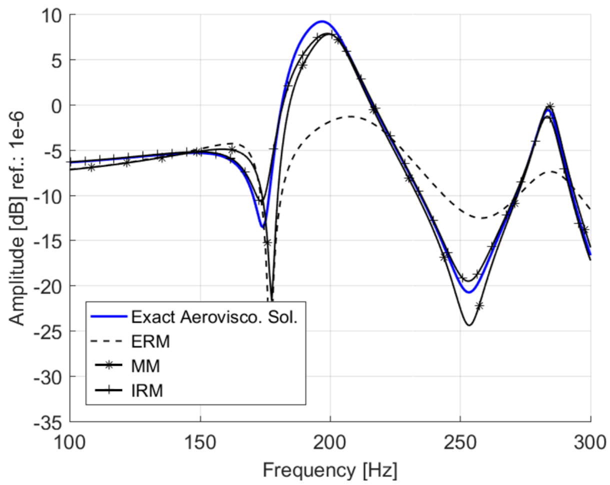

Figure 4 – A zoom from 100 to 300Hz to visualize the accuracy of each method

Flutter prediction through V-g diagrams

To evaluate the flutter boundaries of the exact and reduced models, the V-g diagram is under consideration, in Figure 5. More details about this methodology can be found in the references (Hodges and Pierce, 2002) and (Singha and Ganapathi, 2005).

The V-g diagram is used to evaluate the mode frequencies of the plate along the increasing airflow speed over the plate. Flutter can be assessed by the analysis of the imaginary part of the eigenvalue, which is composed by the sum of the viscoelastic damping and the imaginary part generated by non-symmetric aerodynamic matrix. Flutter occurs when the imaginary part of one of the eigenvalues turns greater than zero. Frequencies are normalized according to the next formulation:

$$\tilde{\omega} = \omega \frac{A^2}{h} \sqrt{\frac{\rho_{eq}}{E_{eq}}} \tag{23}$$

where A is the length of the plate in the x direction, h is the total width of the plate, $\rho_{eq} = \frac{h_1}{h}\rho_1 + \frac{h_2}{h}\rho_2 + \frac{h_3}{h}\rho_3$ the equivalent plate density and $E_{eq} = \frac{h_1}{h}E_1 + \frac{h_2}{h}E_2 + \frac{h_3}{h}E_3$ the equivalent plate Young's modulus.

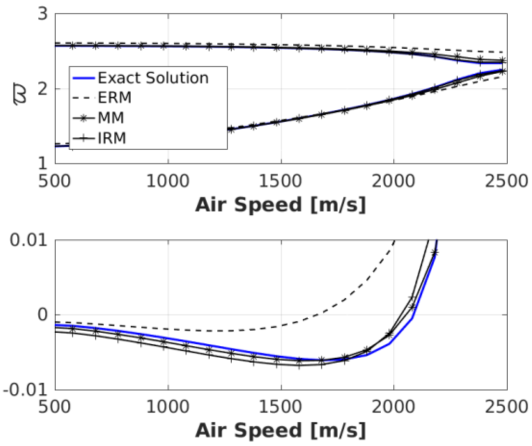

Figure 5 – V-g diagram comparing the exact solution and the ERM, MM and IRM reduced models

In Figure 5, one shows that the Enriched Ritz Method (ERM) presents the poorest result with a critical airspeed of $v_c = 1663 m/s$, while the exact solution is at $v_c = 2087 m/s$. It could be explained by the fact that the aerodynamic information in the projection basis comes from a constant matrix without any aerodynamic information. The MM approach is performed assuming an arbitrary value of the factor $2q/\beta$. The result presents considerable improvement when compared to the ERM method, with $v_c = 2055 m/s$. The IRM method, also obtained a good approximation, $v_c = 2033 m/s$, using a general interval of factor $2q/\beta$. As seen in the approximation of the FRF, the aerodynamic information inserted in the projection basis during the iterative process is crucial to refine the approximation.

Projection basis characteristics

Finally, the efficiency of each method is evaluated by the estimation of the time taken to perform the iterative process. Such process will depend directly on the size of each projection basis, which is determined by the mesh refinement and the reduction method. For the analysis, three mashes are evaluated: 6x6, 12x12 and 24x24 elements within an interval of airspeed varying from $380 m/s$ to $2500 m/s$ stepped by $100 m/s$, totalizing 21 iterations. The efficiency of each one is measured taking the IRM calculation time as reference, once it is the proposal of this work. All the conditions evaluated were performed in computer with the following hardware caracteristics: Intel Core i7-6700K CPU @ 4.00GHz 4,00 GHz, 16GB RAM, 64 Bits. The results are available in Table 1.

Table 1 – Characteristics of each projection basis

Mesh Size	Method	Basis Size	Construction Time [s]	Iterative Sol. [s]
6x6	Exact	-	-	84.8
	ERM	319x24	0.016	0.224
	MM	319x37	0.555	0.428
	IRM	319x53	0.3764	0.8192
12x12	Exact	-	-	5277
	ERM	1135x24	0.2000	0.5750
	MM	1135x37	53.8750	1.0970
	IRM	1135x57	4.7129	1.4480
24x24	Exact	-	-	Prohibitive
	ERM	4279x24	6.8740	1.5000
	MM	4279x37	4476.407	6.3550
	IRM	4279x58	150.835	12.9925

Among the three conditions evaluated, the IRM method has proven to be most efficient if pondering time and quality of the approximation. The IRM method allows to assess much larger systems containing a number of d.o.f that would be impossible to evaluate without a reduction approach.

Although the MM approach has shown excelent results, the time taken to construct the projection basis drastically increases with the size of the system.

CONCLUDING REMARKS

For aeroviscoelastic problems represented by panels under aerodynamics loads modeled through the linearized Piston Theory, several reductions methods can be adapted in order to reduce computation time, keeping a relatively good prediction of flutter occurrence. It has been seen through FRF studies that simple methods only composed of conservative information are not good enough to represent the dynamic behavior of the system as the analyzed frequency band increases. The conservative basis can be enriched by residuals that allow the solution of reduced model to approximate to the exact one. However, depending on the construction process of these residuals, accuracy will not be the same. The ERM enables the prediction of the evolution of the first mode, but the prediction of the imaginary part of the eigenvalue is not accurate enough resulting in a flutter point accurence considerably different from the exact one. This problem can be related to the lack of dynamic information at higher airspeed values and consequently higher frequencies.

Improved methods like MM get around it by adding mode shapes at non-zeros frequencies and non-zeros airspeed to the projection basis. This approach could be understood as a frequency and aerodynamic update. It has been shown that the accuracy of the MM is then better in terms of FRF as well as for flutter prediction, however a particular value of aerodynamic pressure has to be chosen, what makes this method not as general as the IRM.

The IRM method presented relatively good results, due to a more efficient iterative process of construction of the displacement residual, that enriches the projection basis for a large number of frequencies and airspeed values. This method was able to reduce time computation to 98% giving an excellent correlation to the exact model.

ACKNOWLEDGMENTS

The authors are grateful to the Minas Gerais State Agency FAPEMIG for the financial support to their research activities and the Brazilian Research Council – CNPq for the continued support to their research work, especially through research projects 303020/2013-0 and 480994/2013-7 (A.M.G. de Lima), and Grant 300990/2013-8 (M.V. Donadon). The authors express their acknowledgements to the National Institute of Science and Technology of Smart Structures in Engineering, jointly funded by CNPq, CAPES and FAPEMIG.

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