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Asynchronous modes of vibration in a heavy-chain model

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Abstract: Asynchronous modes of vibration, characterized by localized free oscillations, are unexpected findings in linear structural systems. Yet, among other examples, they may be found in non-conservative systems, such as those with follower forces, as well as in conservative systems, such as simple portal frames, provided the system parameters are duly calibrated. In the present paper, a three-degree-of-freedom heavy-chain model is also shown to display such a behaviour, depending on the hanging force, the dead weight, the system stiffness, mass and geometric characteristics. Two types of asynchronous modes are seen to exist: the so-called isolated mode, with a single non-null physical coordinate, and the so-called non-isolated mode, with more than one non-null physical coordinate, although still localized. It is curious that in this simple model the companion modes of isolated modes are non-isolated ones, all of them being of course asynchronous. At this stage, a linearized formulation of modal analysis is carried out, leaving to a later moment the discussion of non-linear effects upon the asynchronicity phenomenon. The heavy-chain model addressed here bears similarities with offshore risers, cylindrical reservoirs and cylindrical pressure vessels under pre-stressing. Understanding the asynchronicity phenomenon is the first step towards using it in technological applications, such as vibration control, energy harvesting and micro-sensors.

Keywords: heavy chain, asynchronous modes, localization

INTRODUCTION

Classical modes are characterized by undamped free-vibration motions in which all physical coordinates attain their maximum/minimum values simultaneously or synchronously.

Yet, some systems for special tuning of control parameters may exhibit an altogether different behaviour, more specifically, while parts of it vibrate in unison, other ones are at rest. Hence, localized free vibration is the distinguishing feature of the asynchronous modes.

When initial conditions lead to a combination of an asynchronous mode with other modes, which may be synchronous or not, the frequency content of the response is clearly different in different parts of the system.

Such a behaviour has already been captured in non-conservative (Mazzilli and Lenci, 2016; Lenci and Mazzilli, 2017a) and conservative (Lenci and Mazzilli, 2017b) systems alike.

The present study explores still another case of these latter ones, namely the heavy-chain model that bears similarities with offshore risers, cylindrical reservoirs and cylindrical pressure vessels under pre-stressing.

Understanding asynchronous modes seems an essential step towards exploring their properties in the design of vibration controllers, energy-harvesting devices and micro-sensors.

There are not many references in the literature on the asynchronicity phenomenon, as we understand it. Some of the few references that associate asynchronicity and localization are Chao et al (1997) and Issa et al (2015).

In fact, different meanings for "asynchronous oscillations" appear in a literature survey. For example, referring to machine vibrations, any (forced) vibration that is not a multiple of a natural frequency is called a "non-synchronous vibration". Still with different meanings, "asynchronous motions" have been reported in rotor dynamics (Smith and Wachel, 1983; Scalzo et al, 1986; Ehrich, 2007; Wu et al, 2010; Clark, 2013; Wang et al, 2015). Yet, this has nothing to do with the asynchronous free vibrations, in general, or the asynchronous modes of vibration, in particular, to which we refer here.

THE MODEL

A three-degree-of-freedom model of a heavy chain is addressed here. A generalization for a larger number of degrees of freedom is out of the present scope.

The model is shown in Fig. 1, in which hinged-hinged rigid bars of length ℓ are interconnected and constrained by transversal elastic springs of stiffness k. Lumped masses m are positioned at the hinges. A tension T is applied upwards at the top, whereas dead weight forces P are applied downwards at the hinges. Physical (also generalized) coordinates are indicated in Fig.1.

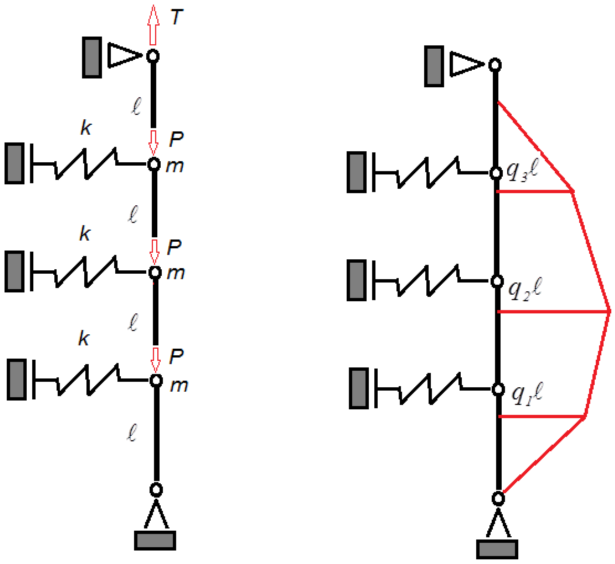

Figure 1 – heavy-chain model

Having in mind a linearized modal analysis, the following expressions apply for the total potential and kinetic energies, respectively:

$$E_p = \frac{1}{2}k\ell^2\left(q_1^2+q_2^2+q_3^2\right)+\frac{1}{2}T\ell\left[q_1^2+\left(q_2-q_1\right)^2+\left(q_3-q_2\right)^2+q_3^2\right] \tag{1}$$

$$-\frac{1}{2}P\ell\left[3q_1^2+2\left(q_2-q_1\right)^2+\left(q_3-q_2\right)^2\right]$$

$$E_k = \frac{1}{2}m\ell^2\left(\dot{q}_1^2+\dot{q}_2^2+\dot{q}_3^2\right) \tag{2}$$

where over dots mean differentiation with respect to time. The free-vibration equations of motion can be obtained from Euler-Lagrange's equation:

$$\frac{d}{dt}\left(\frac{\partial E_k}{\partial \dot{q}_i}\right)+\frac{\partial E_p}{\partial q_i}=0 \quad i=1,2,3 \tag{3}$$

or, in matrix form:

$$\mathbf{M}\ddot{\mathbf{q}}+\mathbf{K}\mathbf{q}=\mathbf{0} \tag{4}$$

with

$$\mathbf{K}=k\ell^2\begin{bmatrix}1+2\theta-5\sigma & -\theta+2\sigma & 0\\ -\theta+2\sigma & 1+2\theta-3\sigma & -\theta+\sigma\\ 0 & -\theta+\sigma & 1+2\theta-\sigma\end{bmatrix} \tag{5}$$

$$\mathbf{M} = m\ell^2 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{6}$$

$$\mathbf{q} = \begin{Bmatrix} q_1 \\ q_2 \\ q_3 \end{Bmatrix} \tag{7}$$

where $\theta = \dfrac{T}{k\ell}$ and $\sigma = \dfrac{P}{k\ell}$.

The associated eigenvalue problem that allows for determination of vibration modes $\mathbf{q}$ and natural frequencies ω can be written as:

$$\left(\mathbf{K} - \omega^2\mathbf{M}\right)\mathbf{q} = k\ell^2 \begin{bmatrix} 1+2\theta-5\sigma-\lambda & -\theta+2\sigma & 0 \\ -\theta+2\sigma & 1+2\theta-3\sigma-\lambda & -\theta+\sigma \\ 0 & -\theta+\sigma & 1+2\theta-\sigma-\lambda \end{bmatrix} \begin{Bmatrix} q_1 \\ q_2 \\ q_3 \end{Bmatrix} = \mathbf{0} \tag{8}$$

with $\lambda = \dfrac{m\omega^2}{k}$.

Assuming that the generalized coordinates are also physical coordinates (asynchronicity is a phenomenon that must be perceived in the physical space!), localization implies that at least one of the entries of vector $\mathbf{q}$ should be zero.

For this three-degree-of-freedom system, if two entries are null (and hence just one is non-null), then the asynchronous mode is said to be *isolated*. If only one entry is null (and hence two are non-null), the asynchronous mode is said to be *non-isolated*.

ASYNCHRONOUS MODES OF VIBRATION

It can be shown that stable modes $(\lambda > 0)$ of the isolated types: (a) $\mathbf{q} = \begin{pmatrix} 1 & 0 & 0 \end{pmatrix}^T$, (b) $\mathbf{q} = \begin{pmatrix} 0 & 1 & 0 \end{pmatrix}^T$ and (c) $\mathbf{q} = \begin{pmatrix} 0 & 0 & 1 \end{pmatrix}^T$; or non-isolated types: (d) $\mathbf{q} = \begin{pmatrix} 1 & x & 0 \end{pmatrix}^T$ and (e) $\mathbf{q} = \begin{pmatrix} 0 & 1 & x \end{pmatrix}^T$, with $x \neq 0$, are possible, provided some parameter requirements are met:

Case (a): $0 \le \sigma < 1$, $\theta = 2\sigma$; the frequency of this isolated asynchronous mode is $\lambda = 1 - \sigma$.

Case (b): $\theta = \sigma = 0$; the frequency of this isolated asynchronous mode is $\lambda = 1$.

Case (c): $\theta = \sigma \ge 0$; the frequency of this isolated asynchronous mode is $\lambda = 1 + \sigma$.

Case (d): $\dfrac{2 \pm \sqrt{2}}{2} > \theta = \sigma \ge 0$; the non-isolated asynchronous modes are characterized by $x = 1 \pm \sqrt{2}$ and their corresponding frequencies are $\lambda = 1 + \left(-2 \pm \sqrt{2}\right)\sigma$.

Case (e): $0 \le \sigma < 1$, $\theta = 2\sigma$; the non-isolated asynchronous modes are characterized by $x = -\left(1 \pm \sqrt{2}\right)$ and their corresponding frequencies are $\lambda = 1 + \left(2 \pm \sqrt{2}\right)\sigma$.

Figure 2 summarizes the above results, in two sets, left and right, depicting the normal force diagram along the heavy chain, the isolated asynchronous mode and the two companion modes, which happen to be asynchronous non-isolated modes. By companion modes it is meant that they come out together as solutions of the same eigenvalue/eigenvector problem.

It is readily seen that (a) and (e) are companion modes (Fig. 2 left), as well as (c) and (d) (Fig. 2 right).

Note that Fig. 2 left and right display specular modes, although the associated frequencies are different, as a result of anti-symmetric normal force diagrams.

Also, when $\theta = \sigma = 0$ any $\mathbf{q}$ is an eigenvector, that characterizes either synchronous or asynchronous (isolated or not) modes. In fact, in such a case, the system behaves as independent one-degree-of-freedom oscillators, for which any combination of initial conditions will correspond to a mode; in particular, a localized and therefore asynchronous one.

It is also seen that in the heavy-chain model, asynchronous modes require that part of the structural system be subjected to compression.

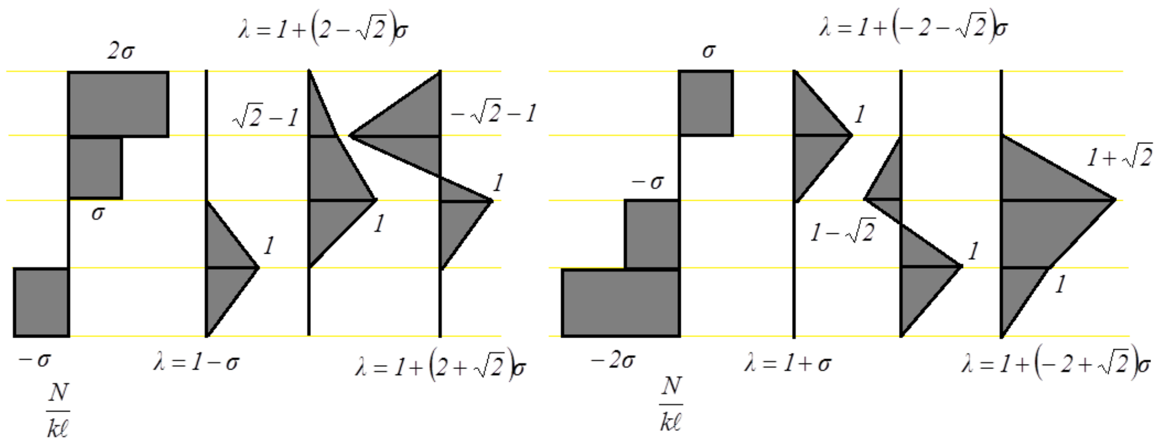

Figure 2 – left, companion modes (a) and (e); right, companion modes (c) and (d).

CONCLUSIONS

The paper addressed the asynchronous modes of a three-degree-of-freedom model of a heavy chain, indicating the particular “tuning” between the system parameters (namely the hanging and dead weight forces, stiffness, length and mass of the chain modules) that are necessary for the phenomenon to show up.

It is seen that to isolated asynchronous modes correspond companion modes that are also asynchronous, but of the non-isolated type. A full discussion on the asynchronicity phenomenon is beyond the scope of this paper, but it has been the subject of Lenci and Mazzilli (2017a).

The heavy-chain model anticipates the possibility of asynchronous modes in vertical cylindrical shells subjected to the action of pre-stressing and dead weight, situation that may be found in pressure vessels, pipes and, possibly, in risers.

In cylindrical shells, using the classical analogy with a beam on elastic foundation (Timoshenko, 2002), it is immediate to associate the transversal spring stiffness per unit arc length to $\frac{Ee}{R^2}\ell$, where E is Young’s modulus, e is the shell thickness and R is the shell medium radius, as far as axi-symmetrical modes are concerned.

The effect of the longitudinal flexural stiffness upon the results obtained in this study is examined in (Mazzilli and Lenci, 2017).

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