DINAME 2017 - Proceedings of the XVII International Symposium on Dynamic Problems of Mechanics
A. T. Fleury, D. A. Rade, P. R. G. Kurka (Editors), ABCM, São Sebastião, SP, Brazil, March 5-10, 2017

An Algorithm to Improve the Rigidity on the Motion Planning of a Robotic Mechanism

Diego Gabriel Gomes Rosa, José Flávio Silveira Feiteira

Universidade Federal Fluminense – Av. dos Trabalhadores, 420, Vila Santa Cecília – 27255-125, Volta Redonda, RJ, Brazil, diegogabriel@id.uff.br, joseflavio@vm.uff.br

Abstract: This paper deals with trajectory optimization for manipulators. In general robotic applications, some algorithms can be adopted to recognize the minimal joint movements, to avoid points of singularity or to find the minimal power consumption during a trajectory execution. However, in some tasks other strategies might be used, for instance, when it is necessary to place the robot in a joint configuration that permits the execution of more rigid and accurate movements, such as in machining operations. By remembering that a robotic arm can assume various joint positions to complete a task, it is expected that only a few of these configurations will provide higher results. Therefore, the use of a strategy to find these optimized configurations can support on the tasks accomplishing. This way, the main goal of this work is to develop an algorithm that can improve the motion rigidity of a robotic mechanism, by using the dynamics of a robotic arm. This paper is divided in two parts: firstly, a simplified and functional algorithm is developed for a two link robotic arm. From this algorithm, a graphical analysis is proposed, based on the implementation of a performance index and the creation of a surface for torque evaluation. Then, the developed theory is extended to a typical industrial robot and optimal robot postures can be found for real case operations.

Keywords: dynamic rigidity, performance index, robot interaction, trajectory optimization.

INTRODUCTION

Robotic manipulators are known by its elevated flexibility in accomplishing general tasks. However, they are also known by its limited mechanical rigidity and low precision, mainly in operations that demand the application of high forces and moments allied to a high spatial mobility (Qinchuan and Hervé, 2014; Bencsik and Suszter, 1993). With the objective of reducing the impacts of the low rigidity on the robot accuracy, different methods are proposed in literature. Modifications on the robot architecture (Sabourin *et al.*, 2015), adoption of mechanical devices on the robot end effector (Yuan *et al.*, 2014), and use of force control strategies and compensation algorithms (Pan and Zhang, 2008; Rosa *et al.*, 2015; Zaeh and Roesch, 2014) are some procedures that can, in a certain way, improve the rigidity of a robot.

On the other hand, these methods can be costly when it is necessary to change the mechanical structure of the robot, or can promote an elevated waste of time when they are dealing with the robot programming. Moreover, in some cases, a simple adjustment on the robot posture can be sufficient to reach a high advance on the robot accuracy (K'Nevez *et al.*, 2010). In fact, according to some manuals (ABB Product Manual, 1998), if the gravitational forces are the only external forces acting over the mechanism, the load capacity of the robot is higher as the end effector is nearer to the basis of the structure. Nevertheless, what happens when other external forces act over the mechanism, as it occurs in horizontal movements in some machining operations? By considering the lack of information about these particular applications, a novel and simplified solution needs to be found. Here, an algorithm is proposed to solve the problem of selection of the finest robot positioning, in order to improve the robot rigidity and accuracy.

Usually, many parameters are considered to perform this sort of task, such as the moment of inertia of the robot, the acceleration of the robot links or the torque applied on the robot joints. In this work, the torque on the robot joints is used as the main parameter to provide a performance index that should be maximized for the posture selection. In fact, this can be treated as an optimization problem, in which the torque on the joints needs to be minimized. Any mechanism, no matter if it is a robotic manipulator or a human arm, presents a reduction on its rigidity as higher are the torques generated on its joints. If the posture of the body can reduce its moment of inertia, the torque on the joints also can be reduced. Thus, minimizing the torque implies in rising the rigidity of the system and, consequently, the mechanism accuracy.

Conversely, the finding of the best postures in a 6 degree of freedom (DOF) anthropomorphic manipulator can be a laborious task. Thus, on purpose of presenting a detailed study about the optimization algorithm, an elementary case is initially considered. The kinematics and dynamics of a two link planar arm are used to describe the behavior of a typical serial robot. This assumption permits a simplification on the understanding of the problem, and also the possibility of extending the theory in the hereafter. In fact, a brief case study is presented in this work, relating to the robot posture selection for a 6 DOF manipulator in operation involving gravitational, inertial and also contact forces.

DYNAMICAL MODELLING

The algorithm here developed is based on the inverse dynamic model for a n-link manipulator, *i.e.*, given the vectors of acceleration ($\ddot{\boldsymbol{q}}$), velocity ($\dot{\boldsymbol{q}}$) and position ($\boldsymbol{q}$) on the joint space, besides the vector of forces applied on the end effector ($\boldsymbol{h}$), the actuating torque on each joint ($\boldsymbol{\tau}$) can be obtained. Equation (1) is found by the Lagrangian formulation. It shows the joint space dynamical model of a typical open chain manipulator (Corke, 2011). The term $\mathbf{B}$ is the inertia matrix, the matrix $\mathbf{C}$ represents the Coriolis and centripetal terms, $\boldsymbol{g}$ is the gravitational acceleration and $\mathbf{J}$ is the Jacobian matrix. Effects of friction and backlash on the joints are not considered.

$$\mathbf{B}(\boldsymbol{q})\ddot{\boldsymbol{q}} + \mathbf{C}(\boldsymbol{q},\dot{\boldsymbol{q}})\dot{\boldsymbol{q}} + \boldsymbol{g}(\boldsymbol{q}) = \boldsymbol{\tau} - \mathbf{J}^{\mathbf{T}}(\boldsymbol{q})\boldsymbol{h} \tag{1}$$

Simulations based on the two link planar arm

As the matrices and vectors on the Eq. (1) are n-dimensional, the use of a simplified model is essential for the development of a consistent theory. A two dimensional structure is chosen on this work. Figure 1, adapted from Sciavicco and Siciliano (2005), displays the two link planar arm used in this paper. Table 1 shows the Denavit-Hartenberg parameters for the same structure.

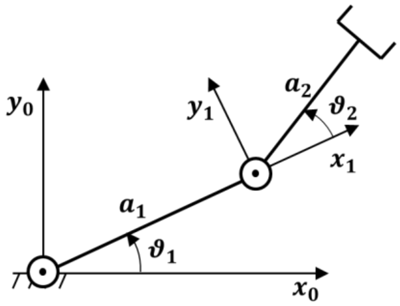


Figure 1 – The two link planar arm.

Table 1 – Denavit-Hartenberg parameters for the two link planar arm.

Link	a_i	α_i	d_i	ϑ_i
1	a_1	0	0	ϑ_1
2	a_2	0	0	ϑ_2

The terms a_1 and a_2 represent the lengths of the robot links, while the angles ϑ_1 and ϑ_2 represent the orientation of the link frames. The mass of each link, m_{l1} and m_{l2}, the moment of inertia of the links, I_{l1} and I_{l2}, the mass of the motors, m_{m1} and m_{m2}, the moment of inertia of the motors, I_{m1} and I_{m2}, and the gear reduction ratios, k_1 and k_2, are also considered on the obtaining of the dynamical model presented on Eq. (2) and (3). These equations, developed from Eq. (1), are extensions from the model implemented by Sciavicco and Siciliano (2005). The elements f_1 and f_2 are the terms of force already multiplied by the transposed Jacobian.

$$\tau_1 = (I_{l1} + 0.25\, m_{l1} a_1^2 + k_1^2 I_{m1} + I_{l2} + m_{l2}\left(a_1^2 + 0.25 a_2^2 + a_1 a_2 \cos(\vartheta_2)\right) + I_{m2} + m_{m2} a_1^2)\ddot{q}_1 + (I_{l2} + 0.25 m_{l2}(a_2^2 + 4a_1 a_2 \cos(\vartheta_2)) + k_2 I_{m2})\ddot{q}_2 - m_{l2} a_1 a_2 \sin(\vartheta_2)\dot{q}_1\dot{q}_2 - 0.5 m_{l2} a_1 a_2 \sin(\vartheta_2)\dot{q}_2^2 + a_1 g(0.5 m_{l1} + m_{m2} + m_{l2})cos(\vartheta_1) + 0.5 m_{l2} a_2\, g cos(\vartheta_1 + \vartheta_2) + f_1; \tag{2}$$

$$\tau_2 = (I_{l2} + 0.25 m_{l2}(a_2^2 + 2a_1 a_2 \cos(\vartheta_2))k_2 I_{m2})\ddot{q}_1 + (I_{l2} + 0.25 m_{l2} a_2^2 + k_2^2 I_{m2})\ddot{q}_2 + 0.5 m_{l2} a_1 a_2 \sin(\vartheta_2)\dot{q}_1^2 + 0.5 m_{l2} a_2\, g cos(\vartheta_1 + \vartheta_2) + f_2 \tag{3}$$

The simulations were done by using the data presented on Tab. 2. The total time of simulation was fixed in 8.0 seconds. This is the necessary time to accomplish a programmed path of 40 mm and with a constant velocity of 5 mm/s, as experimentally performed by Rosa *et al* (2015) in machining operations with robots. The frequency of data writing and acquisition was fixed in 1 kHz. The initial postures varied according to the angles given on Tab. 3. The dislocations on the operational space were made only in the positive x_o (horizontal) direction, according to Fig. 1, and with 0.4 m of magnitude. The applied forces were constant in y_o direction, at 100 N of intensity – that represents the robot tool – but two different values of force were tested in x_o direction, 100 N and 1000 N. In usual robotic applications these loads in horizontal direction are negligible. However, they can appear in operations in which the robot interacts with its ambiance. Moreover, other considerations were made for the calculations: the orientation of the end effector was disregarded, the mass of each link was considered to be applied as concentrated on the medium point of the link length and the mass of the motors on the center of each joint axis. The trajectory was programmed as a point-to-point movement and a polynomial of third order was used to represent it. The initial and final velocities were set to zero.

Table 2 – Data used for the simulations.

a_1 (m)	a_2 (m)	m_{l1} (Kg)	m_{l2} (Kg)	I_{l1} (Kgm²)	I_{l2} (Kgm²)
0.84	0.75	50	40	10	5

m_{m1} (Kg)	m_{m2} (Kg)	I_{m1} (Kgm²)	I_{m2} (Kgm²)	k_1	k_2
5	5	0.010	0.005	100	100

Table 3 – Initial postures tested for the two link planar arm movement.

Angles	Posture A	Posture B	Posture C
ϑ_1	60º	45º	30º
ϑ_2	-120º	-105º	-90º

Figure 2 displays the joint position (a), velocity (b), acceleration (c) and applied torque (d) for a movement starting from the posture A and with a total applied force of 100 N in horizontal direction. The third order polynomial trajectory originates a parabolic velocity and a linear acceleration curve. The torque curve does not have a specific layout.

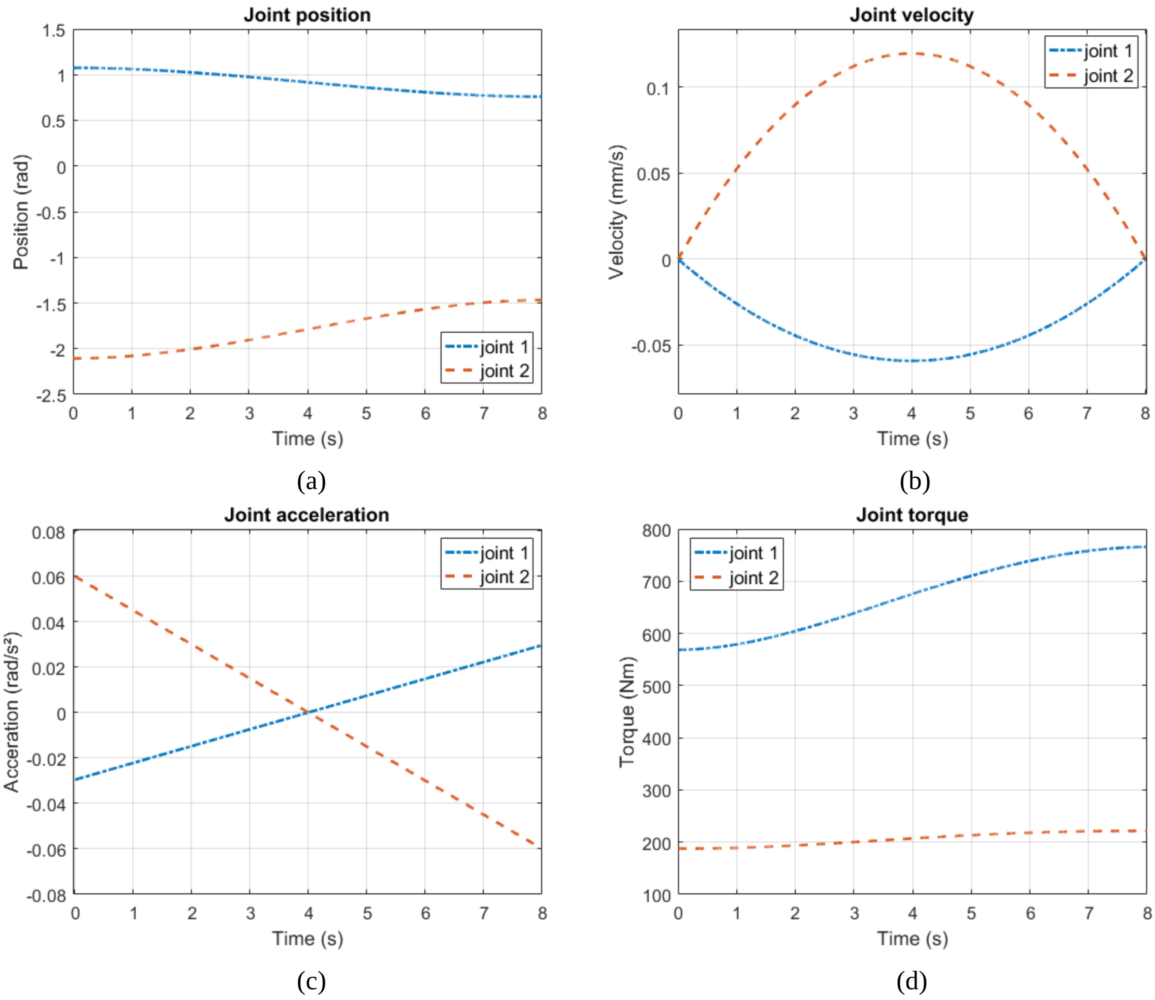


Figure 2 – Joint parameters for a path starting from posture A and with a horizontal applied load of 100 N. (a) position; (b) velocity; (c) acceleration; (d) torque.

On postures B and C, the position, velocity, acceleration and torque curve layouts were quite similar to the represented in Fig. 2. However, the main change was on the intensity of the applied torque on each joint. As it is possible to see in Fig. 3 (a), the initial and final torques on joint 1 increased more than 100 Nm, if compared to the torques in Fig. 2 (d). Similarly, the final torque on joint 1 increased in Fig. 3 (b), if compared to the final torque presented in Fig. 3 (a).

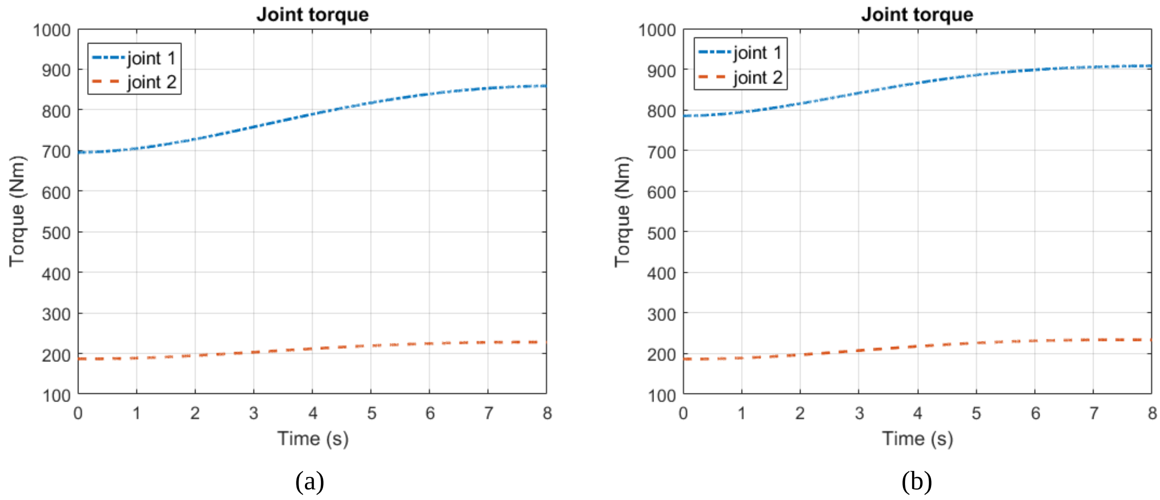


Figure 3 – Torque on the joints for a path starting from different postures and with an applied load of 100 N. (a) posture B; (b) posture C.

These results were expected. If the robot end effector is close to the robot basis, as it happens on posture A, the torque is minor than the required torque on the other postures. Consequently, it is possible to assume that the robot rigidity is higher on posture A. This proceeding is actually presented in technical manuals (ABB Product Manual, 1998), and also used in several industrial applications, in which robots normally do not need to execute forces in the horizontal direction or these forces are minimal.

Nevertheless, when 1000 N were applied in the horizontal direction, different and interesting results were found. Figure 4 shows the total applied torque on the joints for postures A, B and C in this new situation. In this figure, it is possible to observe that the torques on the joint 1 decrease from posture A to B and from posture B to C in the end of the movements. It is the opposite of the previous case. At the same time interval, joint 2 has an irregular behavior, even the variations on the torque are small.

Truthfully, as shown on Eq. (2) and (3), the torque on the joints has a very non-linear design. Consequently, it is not possible analyze each applied torque in an easy manner. Thus, an option is to develop an algorithm that can summarize the contributions of the torques relative to each joint during the execution of the movement. The algorithm is supposed to realize the torque analysis and solve the problem of the best posture selection for operations that involve contact between the robot and its ambiance.

The optimization algorithm

Trajectory optimization is a challenge that has been investigated on the last decades and still needs new enhancements. In fact, this technique is actually used in many procedures, for obstacle avoidance and minimization of travelling distance, time interval or energy consumption. However, to combine the optimization with the kinematics and dynamics of a robot is a problematic task, mainly because of the torque constraints of the manipulators (Ata, 2007). Therefore, a torque optimization criterion can be helpful not only for operations dealing with robot interaction, but also for general robotic tasks.

Several methods can be used for the establishment of a torque optimization criterion. B-spline interpolations (Feddema, 1996), finite element models (Schoenwald *et al.*, 1991) and inverse dynamics based algorithms (Vincent, 1995) are some techniques present on literature. Garg and Kumar (2002) also proposed a procedure, based on genetic algorithms, to minimize the joint torque in collaborative robotic systems. Here, an adaptation of this method is proposed, to assist the robot posture selection and, consequently, to minimize the torque for generalized force vectors acting on the robot end effector.

The algorithm here proposed is based on integration of the joint torque vector, as shown in Eq. (4). The total torque has to be minimized to promote the increasing on the robot rigidity. The term PI is the performance index and its value is inversely proportional to the summation of torques on the robot joints. The term t_f expresses the final time for a complete trajectory. Notice that this formulation does not permit a zero value for the total torque, situation that might promote an infinite performance index. However, the torque will never be null, as the masses and moments of inertia are continually acting on the robot joints. Moreover, the presented method is prescribed to be used offline, but an online application can be proposed for future applications.

$$PI = \left(\int_0^{t_f} \|\boldsymbol{\tau}(t)\| \, dt \right)^{-1} \tag{4}$$

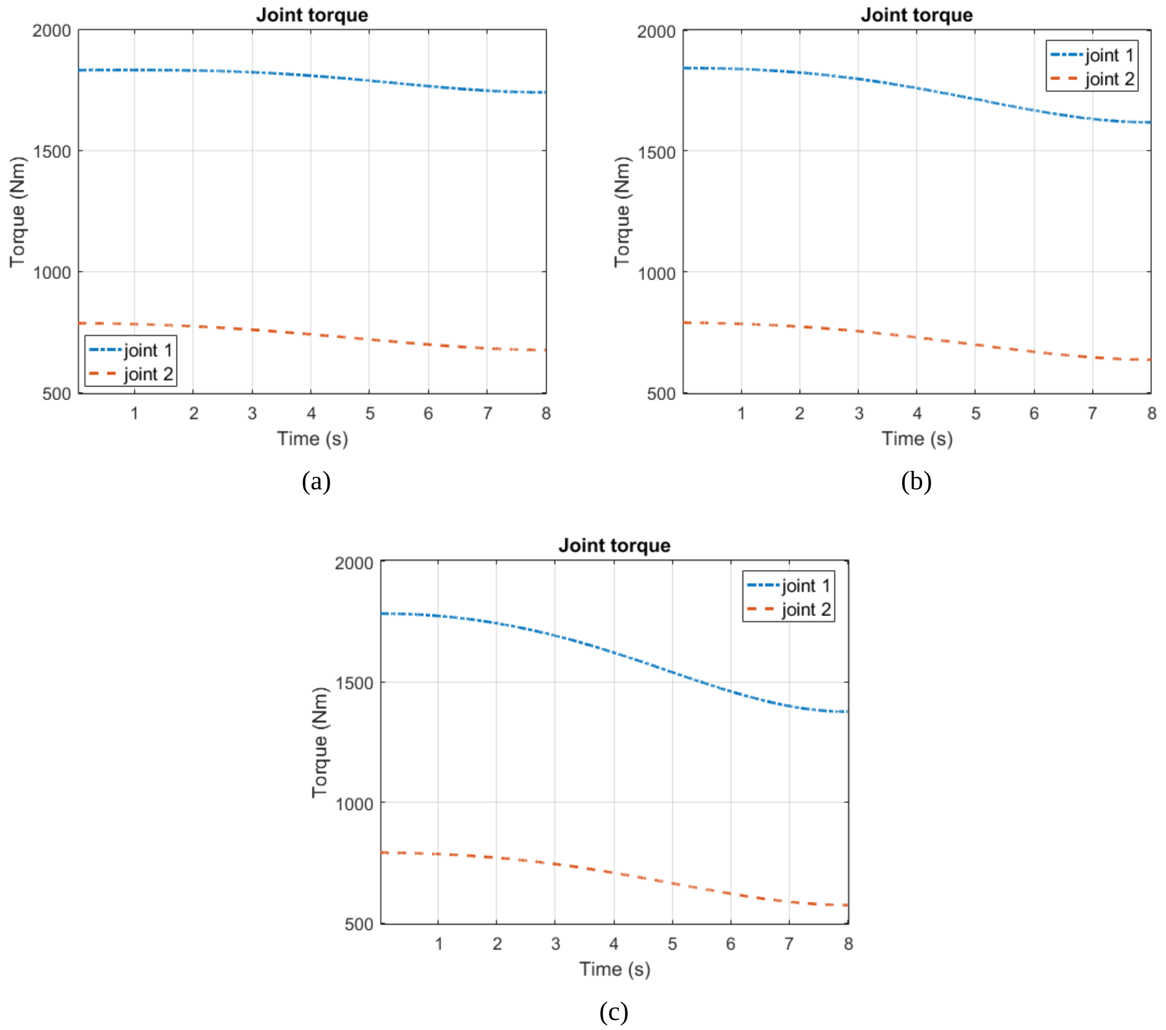


Figure 4 – Torque on the joints for a path starting from different postures and with an applied load of 1000 N. (a) posture A; (b) posture B; (c) posture C.

A first examination of the algorithm can be performed by using the data processed on anterior section of this work. When the horizontal applied force on the end effector was set to 100 N, it was verified that the best joint positioning was related to posture A. When the force was raised to 1000 N, the posture C presents the smallest joint torques. On Tab. 4 it is possible to observe that the higher performance indexes were found as expected, *i.e.*, posture A gave the finest positioning with an applied load of 100 N and posture C with 1000 N. In addition, it is possible to observe that the calculated performance indexes also permit a comparison between gauges related to different applied loads. In fact, the values confirm that the rigidity of the robot is lower when higher forces are applied on the robot end effector. Furthermore, it should be noted that even if the value of the indexes are nearby, a small difference means a high reduction on the active torque on the joints, as a result of the inversion of the integral in Eq. (4).

Table 4 – Performance index (Nm^{-1} x 10^{-7}).

Force (N)	Posture A	Posture B	Posture C
100 N	13.8	12.0	11.0
1000 N	5.13	5.26	5.68

However, the presented indexes were calculated only for three postures and for a specific movement, while a real robot can assume thousands of possible poses and make different movements. Thus, according to Chen and Dong (2012), it would be interesting to develop a mapping method to find postures that minimize the applied torque or to find regions on the joint space in which adjacent postures should be avoided.

This mapping is shown in Fig. 5 and 6. A performance evaluation surface was created based on the industrial robot ABB IRB 2400. The joints 1 and 2 in the figure correspond to the second and third joints of the real robot, respectively. Figure 5 shows the performance indexes for a horizontal load of 100 N on the robot end effector, while Fig. 6 corresponds to an applied load of 1000 N. The angles are set in radians. In both cases, only the initial posture of the programmed trajectory is presented, while the calculated index considers the whole movement contributions.

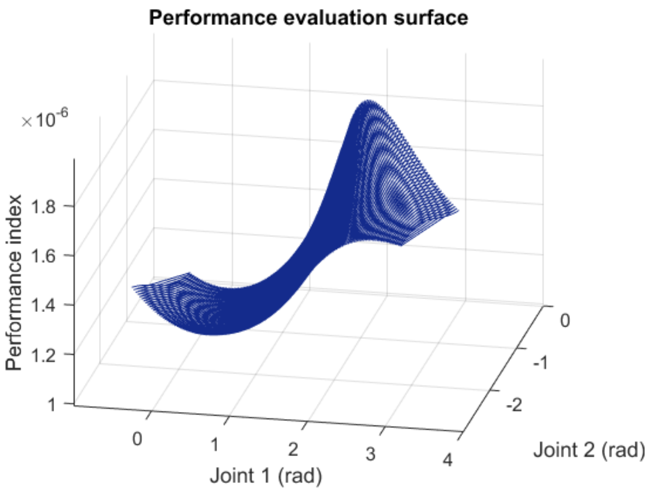


Figure 5 – Surface related to a horizontal load of 100 N on the robot end effector.

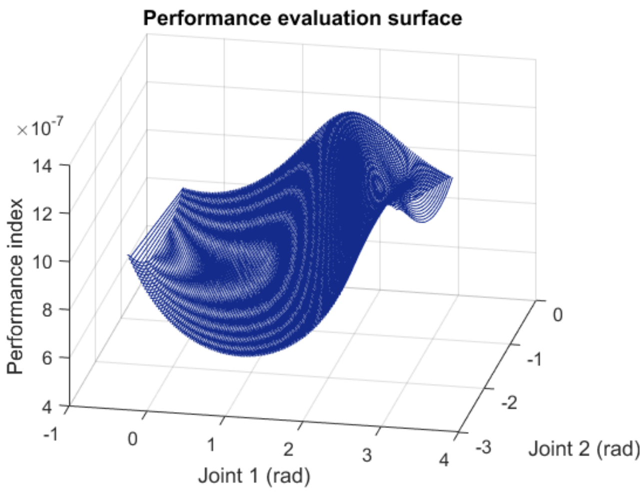


Figure 6 – Surface related to a horizontal load of 1000 N on the robot end effector.

Based on the Fig. 5, it is possible to determine minimum and maximum performance index locations. Besides, by analyzing the same figure, it becomes easy to perceive that the highest performance indexes are recurrently related to the higher positions of joint 1. It is also notable that, from the 1.9 rad angle onwards, the performance begins to fall quickly. On the other hand, joint 2 also influences the surface feature, even though the effects of this influence are smaller.

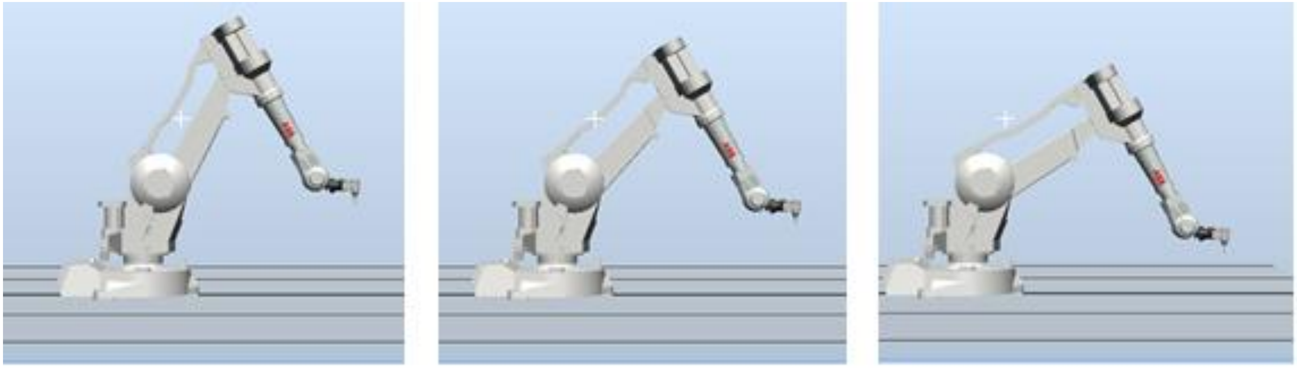
Figure 6 displays a more interesting behavior for the performance evaluation surface. Joint 2, in this case, realizes high influence on the performance, as joint 1. Moreover, in some postures, a small changing on the joint space might result in high variations on the total applied torque. It requires a superior attention on the trajectory planning, on purpose of maintaining the robot as far as possible from these postures. Indeed, by this analysis it is possible to perceive that the usual postures for robotic operations, represented by configurations similar to the ones presented on postures A, B and C are not appropriate when high horizontal forces are applied on the robot end effector.

Although, the main limitation of this method is related to the high computational cost on its implementation. If only few forces and movements are necessary to accomplish a process, the analysis can be executed in an easy way. In fact, only one surface is necessary for each part of a trajectory, if the applied load do not vary. However, if other considerations are necessary, the simulation time only can be reduced by some adjustments on the computational codes used for the algorithm implementation.

GENERALIZATION: THE 6 DOF MANIPULATOR

On purpose of applying the subjects discussed in this work, a case study based on an industrial robot was executed. The manipulator chosen is the ABB IRB 2400, which has a 6 DOF structure. A simplification was made, by considering that the posture variations on the first and the last three joints of the robot do not make high influences on the robot rigidity. Hence, the robot became similar to a two link planar arm.

Figures 7 (a), (b) and (c) display the cited robot arm on postures A, B and C, respectively. By analyzing the figures, it is possible to realize that, on exclusive action of gravitational forces, the minor joint torques are related to posture A, on which the moments of inertia of the links are minor. Nonetheless, by considering the action of horizontal forces, as it was previously discussed in this work, this sense analysis cannot be used and it should be changed to a dynamical analysis.

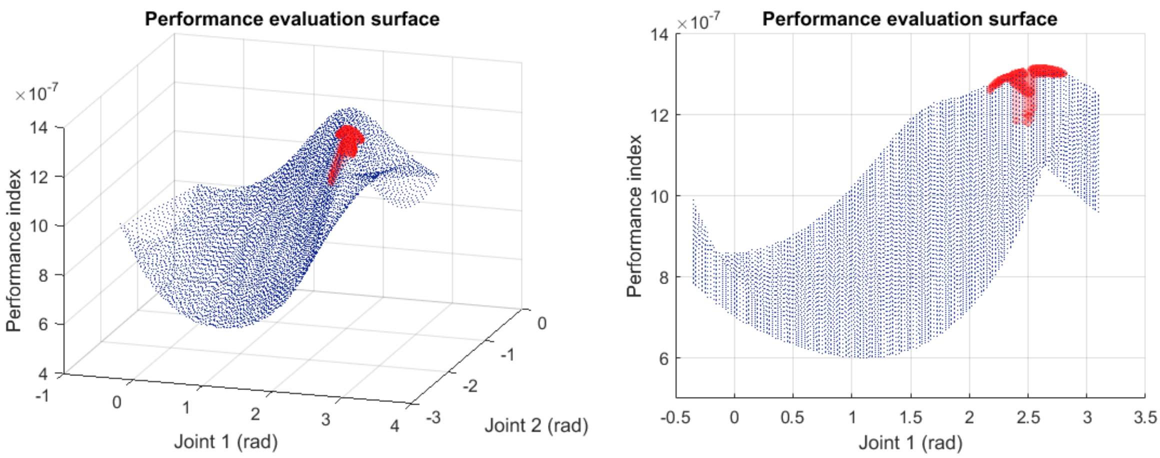


(a) (b) (c)

Figure 7 – 6 DOF manipulator in three different postures.
(a) posture A; (b) posture B; (c) posture C.

According to Fig. 6, the angles related to postures A, B and C are not ideal for a robot with horizontal loads applied on its end effector. Consequently, new and different configurations should be established by the use of the developed performance index. A first possibility is to use the global or some local maximum point extracted from the Fig. 6. However, these points cannot be on the surface boundaries, as it typically coincides with points of singularity of the robot. Hence, an option is to use, instead of the maximums of the function, some regions on the surface close to them.

Figure 8 (a) shows one of those regions, randomly selected, for the applied load of 1000 N. Figure 8 (b) shows a planar projection extracted from Fig. 8 (a), on which it is possible to observe more accurately the brushed region as the region with higher performance indexes. In Fig. 8 (b) it is also possible to observe the values of angles for the joint 1 that provides better performance indexes.



(a) (b)

Figure 8 – Analysis to an applied load of 1000 N.
(a) Posture region found from Performance Index; (b) Bidimensional projection.

Figure 9 shows the posture for the industrial robot on which the trajectory starts from the mean point of the brushed region in Fig. 8 (a). The robot posture described in Fig. 9 is not commonly used. However, by the dynamical and numerical calculations, it is known that, for the quoted case, this posture provides the lowest torques and guarantees better rigidity and accuracy for the robotic system. Moreover, by remembering that if the angle of the joint 1 is higher than 1.9 rad the performance index starts to decrease, it is possible to consider the angle ϑ_1 provided in Fig. 9 as a limiter for a movement accomplishing.

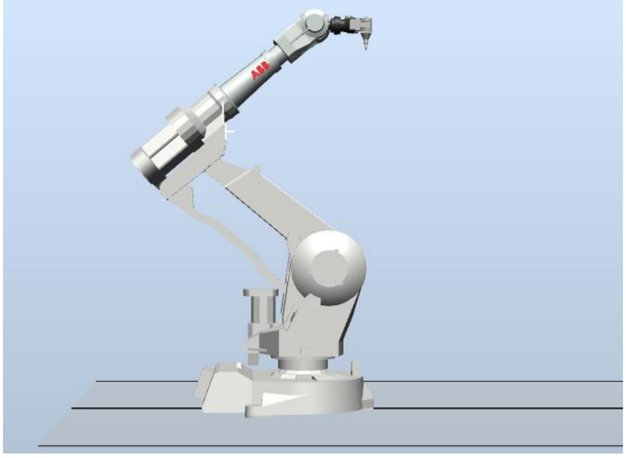


Figure 9 – Industrial robot on posture for an applied load of 1000 N.

CONCLUSIONS

Based on the dynamical model for a two link planar arm, it was possible to develop a functional algorithm for the torque optimization on the trajectory planning. Indeed, some tests were made for different robot postures, by using the developed algorithm, and the performance indexes obtained functioned as expected. Besides the easy implementation, the algorithm also permits the development of a surface for posture evaluation. The methodology can be used for movements in different directions and with diverse force requirements.

The same technique can be extended to industrial manipulators with open kinematic chains, assisted by commercial software used in robot programming and simulations. From the evaluation surface, it is possible to categorize regions for the joint space in which the robot can develop a better performance. With the lower torques on the joints more rigid and accurate movements can be executed, guaranteeing quality in a performed task. Moreover, this concept can be developed and used in online robot programming, the same way as in control theory of manipulators, equilibrium of humanoid robots or robotic prosthesis development and motion.

REFERENCES

ABB Product Manual, 1998, Article number: 3HAC 5644-1, Issue: M98A, ABB Robotics Products, Västerås, Sweden.

Ata, A.A., 2007, Optimal Trajectory Planning Of Manipulators: A Review, Journal of Engineering Science and Technology, Vol. 2, No. 1, pp. 32-54.

Bencsik, D.A. and Suszter, G., 1993, Investigation of Robot Arm Rigidity, Mechatronics, Vol. 3, No. 2, pp. 185-203.

Chen, Y. and Dong, F., 2012, Robot Machining: Recent Development and Future Research Issues, The International Journal of Advanced Manufacturing Technology, Vol. 66, pp. 1489–1497.

Corke, P., 2011, Robotics, Vision and Control – Fundamentals Algorithms in MATLAB®, STAR (Springer Tracts in Advanced Robotics), Berlin and Heidelberg.

Feddema, J.T., 1996, Kinematically Optimal Robot Placement for Minimum Time Coordinated Motion, SPIE, Vol. 2596, pp. 22-31.

Garg, D.P. and Kumar, M., 2002, Optimization Techniques Applied to Multiple Manipulators for Path Planning and Torque Minimization, Engineering Applications of Artificial Intelligence, Vol. 15, pp. 241-252.

K'Nevez, J, Cherif, M., Zapciu, M. and Gerard, A., 2010, Experimental Characterization of Robot Arm Rigidity in order to be used in Machining Operation, Proceedings in Manufacturing Systems, Vol. 5, No. 1, pp. 1-10.

Pan, Z. and Zhang, H., 2008, Robotic Machining from Programming to Process Control: a Complete Solution by Force Control, Industrial Robot: An International Journal, Vol. 35, No. 5, pp. 400-409.

Qinchuan, L. and Hervé, J.M., 2014, Type Synthesis of 3-DOF RPR-Equivalent Parallel Mechanisms, IEEE Transactions on Robotics, Vol. 30, No. 6, pp. 1333-1343.

Rosa, D.G.G., Feiteira, J.F.S., Abreu, P.A.F. and Lopes, A.M., 2015, Robotic System with Force Control for Drilling Operations, Proceedings of the 23rd ABCM International Congress of Mechanical Engineering, Rio de Janeiro, Brazil.

Sabourin, L., Subrin, K., Cousturier, R. and Gogu, G., 2015, Redundancy-Based Optimization Approach to Optimize Robotic Cell Behaviour: Application to Robotic Machining, Industrial Robot: An International Journal, Vol. 42, No. 2, pp. 167-178.

Schoenwald, D.A., Feddema, J.T. and Segalman, D.J., 1991, Minimum-Time Trajectory Control of a Two-Link Flexible Robot Arm, Proceedings of the IEEE International Conference on Robotics and Automation, Sacramento, Vol. 3, pp. 2114-2122.

Sciavicco, L. and Siciliano, B., 2005, Modelling and Control of Robot Manipulators, Springer, London, 6th Edition.

Vincent, Y., 1995, Inverse Dynamic-Based Programming Method for Optimal Point-to-Point Trajectory Planning for Robotic Manipulators, International Journal of Systems Science, Vol. 26, No. 2, pp. 181-195.
Yuan, P.J., Wang, Q.S., Shi, Z.Y., Wang, T.M., Wang, C.K., Chen, D.D. and Shen, L.H., 2014, A Micro-Adjusting Attitude Mechanism for Autonomous Drilling Robot End-Effector, Science China - Information Sciences, Vol. 57, 120203:1-120203:12.
Zaeh, M.F. and Roesch, O., 2014, Improvement of the Machining Accuracy of Milling Robots, Production Engineering Research and Development, Vol. 8, pp. 737-744.

RESPONSIBILITY NOTICE

The authors are the only responsible for the material included in this paper.