

Response of shock isolators based on nonlinear stiffness and dry friction

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Abstract: The use of shock isolators based on nonlinear stiffness and dry friction is discussed briefly theoretically, where the nonlinear properties are represented using the Bouc-Wen model of hysteresis. Then, the study is focused in the experimental response of cable springs, also known as wire rope isolators. Such isolators present nonlinear stiffness behaviour in different directions, i.e. tension-compression, roll and shear, as well as dry friction damping, and are known for being excellent shock isolators. Different commercially available samples are studied for several configurations and load rates. The experimental study analyzes the cyclic loading, presenting hysteresis loops and discussing the dynamic response and the stiffness and damping behaviour. Then, the shock response is investigated, where the isolators are subjected to pulses of different amplitudes and duration. The advantages of the use of cable isolator over a classic linear system with viscous damping are also discussed in terms of absolute acceleration and relative displacement response. It is found that the shock response is decreased in terms of acceleration, and that the isolation is improved for higher amplitude shocks at the cost of a higher relative motion. The use of the Bouc-Wen model is suggested for further modeling of the shock response of wire rope isolators.

Keywords: shock, dry friction, non linear stiffness, vibration isolation

INTRODUCTION

The protection of sensitive equipment from harsh vibrations and shock is an important task in engineering, which is usually achieved with a resilient element in order to absorb and dissipate mechanical energy. The simplest approach considers a single degree of freedom system with linear elements, used to represent and predict the isolation properties. However, the interest for the nonlinear properties of isolators has recently increased. These approaches are particularly useful for shock vibration due to the high displacements and acceleration involved, where the isolators can behave nonlinearly and these properties can prove useful. A particular type of isolators widely used for shock are cable, or wire rope springs, which are build from steel strands, as presented in Fig. 1. These isolators present nonlinear stiffness properties depending on the direction of deformation, plus dry friction damping due to the relative motion between strands. Although these isolators are regarded as excellent for shock isolation, the dynamic properties that define their shock isolation capabilities are not well understood.



Figure 1 – Examples of different commercially available wire rope isolators.

The Bouc-Wen model of hysteresis, presented by Y. Wen (1976) has been widely used to represent the dynamic

behaviour of wire ropes, or cable isolators, as it can represent a wide variety of hysteresis loops. The Bouc-Wen model of hysteresis is defined by the following set of differential equations, governing the motion of a Single Degree-of-Freedom (SDOF) system.

$$\ddot{x} + 2\zeta\omega_n\dot{x} + (1 - \gamma)\omega^2z = u(t) \quad (1)$$

$$\dot{z} = A\dot{x} - \alpha|\dot{x}|z|z|^{n-1} - \beta\dot{x}|z|^n \quad (2)$$

Where x, \dot{x}, \ddot{x} represent displacement, velocity, and acceleration respectively, z gives the hysteretic restoring force, $u(t)$ is the external excitation, γ is the post yield to pre yield ratio, and the parameters A, α, β regulate the size of the hysteresis loop and n the smoothness.

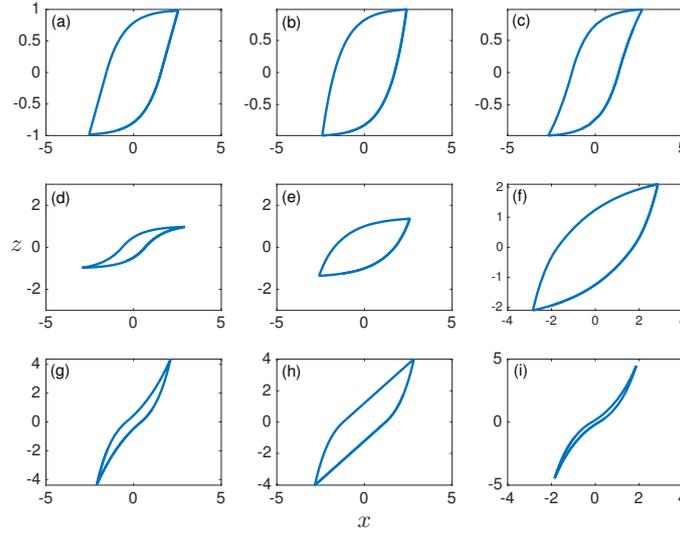


Figure 2 – Hysteresis loops resulting from different parameters of the Bouc-Wen model. (a) $\alpha = 0.5, \beta = 0.5$, (b) $\alpha = 1, \beta = 0$, (c) $\alpha = 0.75, \beta = 0.25$, (d) $\alpha = 0.7, \beta = -0.3$, (e) $\alpha = 0.25, \beta = 0.75$, (f) $\alpha = 0.85, \beta = -0.15$, (g) $\alpha = 0.3, \beta = -0.7$, (h) $\alpha = 0.5, \beta = -0.5$, (i) $\alpha = 0.15, \beta = -0.85$.

Through proper selection of the parameters, a wide variety of loops can be represented, as well as energy dissipation characteristics, as depicted in Fig. 2, which presents different loops for hardening and softening behaviour. For instance, when $\alpha = 0.75, \beta = 0.25$ the behaviour is softening, as depicted by the loop in Fig. 2(c). On the other hand, when $\alpha = 0.3, \beta = -0.7$ the system present hardening characteristics as can be seen in Fig. 2(g).

This paper presents an experimental approach to characterise and quantify the isolation properties of a commercially available isolator. Static and dynamic properties are measured, focusing on the shock response and the effect of the input amplitude in the response of the isolator. The use of the Bouc-Wen model of hysteresis is suggested for further modelling of the response.

BACKGROUND

One of the first applications of dry friction in the isolation of shocks was conducted by Mercer and Rees (1971), who designed an optimal isolator based on the principle of adjustable friction, obtaining better results than previous studies where polymers were studied, as Snowdon (1961) considered previously. Molyneux (1956) studied the behaviour of different arrangements with low stiffness springs and limited displacement ranges for mechanical vibration isolation in aeronautical applications. Eshleman and Rao (1969) studied the response of different configurations of impact insulators as coil springs, fluids, pneumatics, ring springs and friction dampers demonstrating their high capacity in terms of energy dissipation. Cutchins et al (1987) published studies on nonlinear stiffness and damping, in which an analytical model for the description of the hysteresis loop, that is common in systems with nonlinear stiffness, was derived. The later study focused on the wire springs, the authors noted that the spring wire strands tend to separate when compressing, while when tensioned a greater number of contact points exist and the resistance to relative motion is increased, forcing the force displacement function to behave differently in tension than in compression. Subsequently, Cutchins and Tinker (1992) continued their investigations in which they sought to develop a semi-empirical analytical model, which could fully describe the impact isolator behaviour arranged by wire coil springs under axial loads. Demetriades et al (1993) also investigated the response of cable isolators under earthquakes excitation and derived an analytical model that was calibrated using experimental results. Ni, et al (1999) developed dynamic test in wire rope isolators under shear, roll and tension-compression loads, finding a symmetric soft-hardening stiffness loops for shear and roll, and assymetric behaviour otherwise. Popp (2000) performed a theoretical investigation, citing relevant studies to the discontinuous nature of both

phenomena: impact and friction, highlighting everyday examples where they are seen and was considered the importance of their study. Other applications of the wire spring are in the civil engineering, Georges and Vickery (2003) designed and tested experimentally a tuned mass damper using wire springs. Hoge and Bausic (2005), and also Foss (2005) performed experimental tests on cable isolators subjected to axial excitation, obtaining data about dynamic stiffness and damping under harmonic loads. As mentioned before, the Bouc-Wen model is commonly used to model wire rope isolators. The parameters that represent the Bouc-Wen restoring force can be obtained from experimental data in forced or displacement controlled experiments by system identification procedures, such as Kalman filters, Differential evolution, Genetic algorithms, Particle Swarm Optimization, (Charalampakis, 2008). The model has been extensively studied, for instance, the nonlinear characteristics, bifurcation and chaos were analysed by Li, et al (2007) for a variety of parameters giving typical loops for hardening and softening systems, finding that chaotic behaviour is possible. Di Massa, et al (Di Massa, 2013) developed a prototype of wire rope isolator restraining the vertical motion with a ball transfer unit, allowing only horizontal motion. Based on experimental results, the authors identified the restoring force by means of the Bouc-Wen model, simulating the nonlinear response of the system. Wang, et al (2015) analysed a cable isolator for shear, roll, and compression-tension, fitting the experimental data using a modified Bouc-Wen model, finding that the system under compression presents highly nonlinear behaviour, whilst the shear and roll motion can be approximated as linear. Despite the recent interest on cable isolators, most of the published works are related to parameter identification and modeling, with few works related to vibration isolation properties. A recent paper by Guzman-Nieto, et al (2015) explored damping properties and briefly presented the shock response of several cable isolator samples.

Shock isolation

Mechanical shock is defined as a short, i.e. transient excitation, with a duration usually less than twice the natural period of the system. The standard approach for mathematical modelling and predicting the shock response is to consider a pulse function as external excitation, such as a half sine, rectangular, triangular, or another pulse function. Fig. 3(a) shows a half sine pulse of different durations, one of the most common pulses used in the study of shock response. The choice of pulse function depends upon the particular situation and it can represent a variety of scenarios i.e. a base displacement, base acceleration, force input, etc. In order to present the shock response in a condensed format, a useful tool is the Shock Response Spectra (SRS) which is a plot of the normalised shock response, either maximum, residual, or relative, as a function of the ratio of the duration of the shock τ and the natural period of the system T .

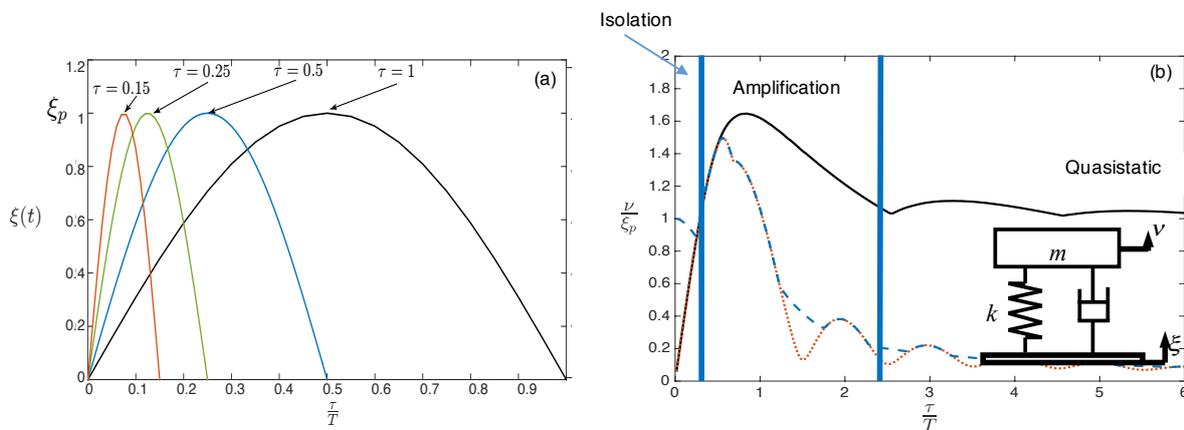


Figure 3 – (a) Representation of a half sine pulse as shock excitation, considering different values of the duration τ of the pulse, (b) Shock Response Spectra of a lightly damped SDOF system. v and ξ represent the response and the input pulse respectively, whilst ξ_p represents the maximum pulse amplitude. (– Maximax absolute, - - Relative response, ... Residual response)

The response parameter can represent absolute acceleration, displacement, or relative displacement depending upon the nature of the input function. There are three typical zones in the SRS depending on the relative duration of the pulse. When the pulse is short compared to the natural period, it is said to be impulsive, and the response is smaller than the input amplitude, resulting in isolation from the impact. Pulses of duration similar to the natural period result in an amplified response. Finally, for longer pulses, the excitation is no longer transient but quasistatic, and it follows closely the input. These three scenarios can be observed in Figure 3(b), which gives a SRS corresponding to a lightly damped SDOF system under a half sine pulse excitation. The continuous line represent the maximum response at any time, sometimes called Maximax. The relative and residual responses are given by the broken line, and the dotted line, respectively.

DESCRIPTION OF EXPERIMENTAL PROCEDURE

For this study, a commercially available wire rope isolator was selected. Due to limitations in length of this work, only one sample of isolator is considered in axial, i.e. compression-tension loading, however, different models have been tested in the laboratory, resulting in similar results. The general setup used in the experiments is shown in Fig. 4.

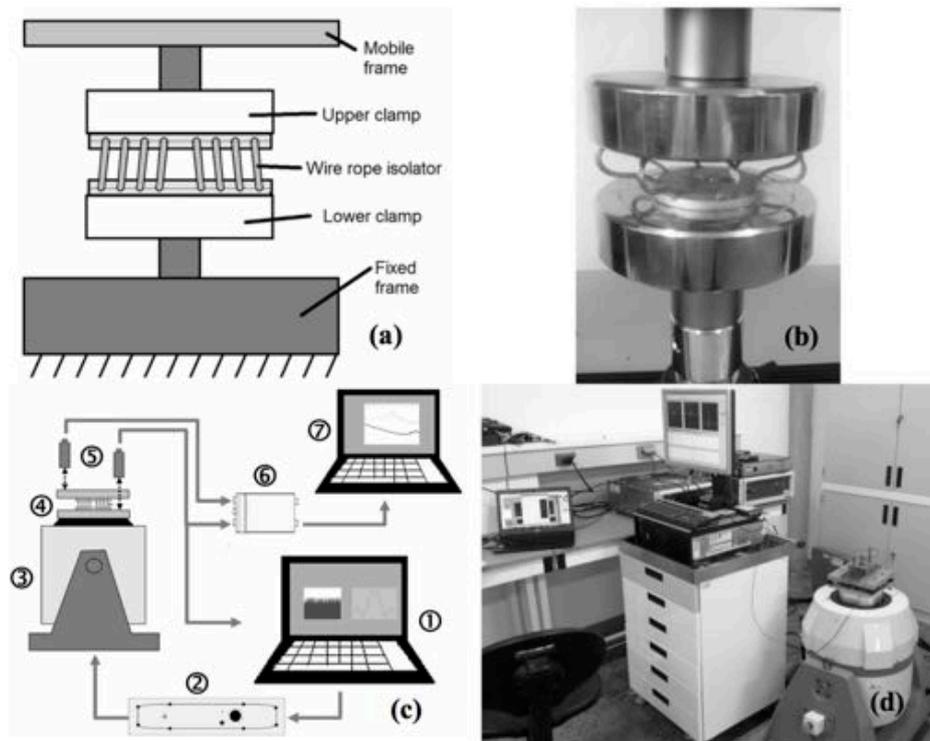


Figure 4 – Description of the experimental procedure. (a) Schematics for the static tests, (b) picture of one sample of isolator during compressive testing, (c) experiment diagram of the setup used during the dynamic testing, (d) photograph of the actual experimental setup of the dynamic testing.

In order to acquire a stable system under mass loading, a symmetrical arrangement of two identical isolators was devised. Firstly, a static test was performed in the two isolator arrangement in an universal machine Shimadzu model AGS-X 10. Load was applied slowly in order to get the force deflection curve. For the dynamic tests, the isolators were arranged with a reference plate on top, for a quasi-unloaded situation, i.e. no significant static deformation registered, and then, a mass load of 4.7 kg was placed on the plate in order to attain an equilibrium position about the region with the lower stiffness in the force-deformation plot. The isolator arrangement was attached to an electrodynamic shaker LDS model V721 (Figure3(a)). Hysteresis loops for the isolators were measured by applying a sinusoidal excitation on the base of fixed frequency of 5 Hz, and recording the restoring force on the isolator, for different values of the input amplitude. Then, the response to shock excitation for a half sine acceleration pulse applied from the shaker base was evaluated. The input acceleration profile was programmed in the shaker control system. The pulse has a constant value of acceleration amplitude of 1g, and different pulses of duration ranging from 15ms to 40ms with 5ms intervals were considered. Then, the shock amplitude was increased for a fixed duration of 15 ms. For both the sinusoidal and the shock tests, the response at the top of the isolator and excitation at the base of the shaker were measured with piezoelectric accelerometers PCB 352B10 and an impedance sensor 288D01, then acquired and processed with a Dynamic Analyser Dataphysics QUATTRO. Figure 4 shows schematics and pictures of the experimental procedure.

RESULTS AND DISCUSSION

Fig. 5(a) presents the force vs static deflection plot obtained in the universal machine. The isolator is characterised for a nonlinear hardening stiffness behaviour for large deformations, and a region of low stiffness for most of the displacement range. This is useful to determine the optimum loading when low stiffness is preferable for shock and vibration isolation. As previous studies have suggested, the nonlinear stiffness of wire rope springs can be modelled after a cubic force-deflection relationship. The result of the cubic fit evaluated with MATLAB is depicted by the blue line. Then, the expected stiffness behaviour can be evaluated by the derivative of the force displacement relationship with respect to the displacement, which is presented in Fig. 5(b). This confirms the hardening effect and the low stiffness region. In this particular case, the cubic curve fit was assessed with a R squared of 0.998. However, in this case the cubic fit might not be the best option, as the behaviour can be simplified as a piecewise linear function, distinguishing three linear regions in the

force displacement plots. These regions are for very small deformations, then there is a large region of low stiffness, and for very large deflections the system turns hardening. Previous studies of the individual characteristics of these isolators have shown a better compatibility with a cubic behaviour, but in this case the linear approximation can be useful as the displacement ranges are known and within the linear limits. This is also a benefit as the best force-deflection range is known, in terms of low stiffness, hence better isolation.

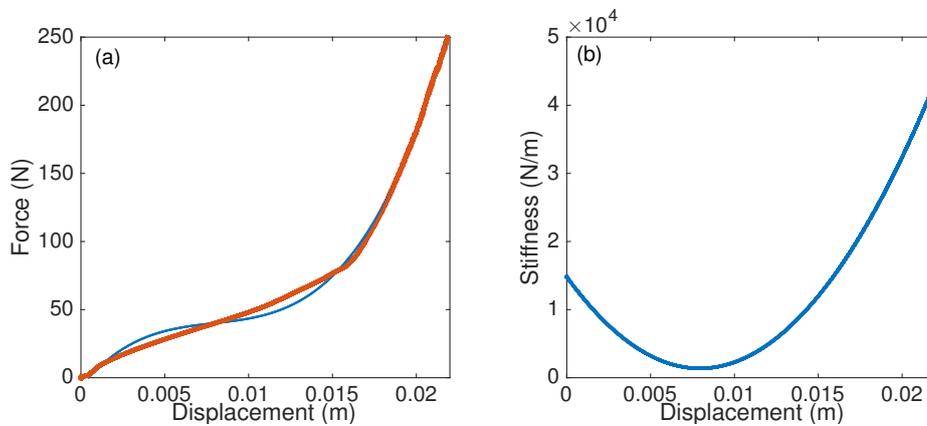


Figure 5 – Static tests results for the studied isolator. (a) Experimental force deflection curve, with cubic fit included, (b) Plot of stiffness as a function of the displacement, calculated from the cubic fit.

Fig. 6 shows the hysteresis loops measured for cyclic loading. In this case, the test was displacement controlled, and the acceleration time histories recorded were filtered and integrated twice to obtain displacement. The loops reflect the nonlinear properties of the isolator, due to the stiffness properties and the dry friction damping. Moreover, it can be seen that the loops are asymmetrical, which has been reported previously for these isolators when working on axial loading (Ni, et al, 1999). The Bouc-Wen model has been used previously to model the nonlinear characteristics of these isolators, where several parameter identification techniques can be applied to obtain the model parameters. As it was explained before, the Bouc-Wen model can represent a huge variety of hysteresis loops for different nonlinear scenarios. However, the use of this model has not been applied for studying the shock response. In a first attempt, the work presented in this paper provides the experimental background needed for further modelling considering this approach.

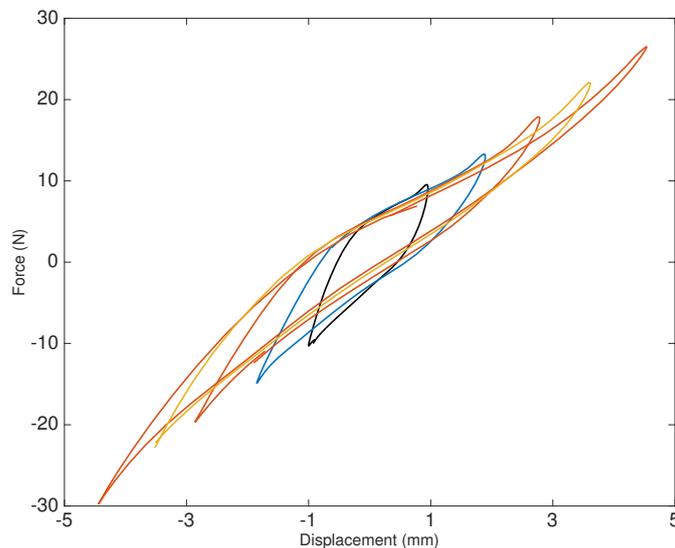


Figure 6 – Example of hysteresis loops measured during cyclic loading. Each loop corresponds to a value of the input amplitude, from 1mm to 5mm

The shock response results are presented in Fig. 7. In this case, each sub plot represents a response to a particular pulse duration as stated before, with a peak amplitude constant of 1 g. The solid blue line depicts the excitation pulse, showing the pre and post pulses generated by the control system in order to suppress the dynamic response of the shaker after the pulse. The red dotted line represents the acceleration response of the payload mass. The first effect to notice in these plots, is the progressive increase in response as the pulse duration increases. The reason behind this is the successive progression from the impulsive region for very short pulses, where the response is always smaller than the pulse amplitude,

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to the amplification region, when the pulse duration is longer and it approaches the natural period of the system. For this example, when the duration is longer than 40 ms it surpasses the input amplitude. For the scenario considered, the system does not reach the maximum amplification, since the estimated natural frequency of the system with the payload mass considered is measured at approximately 5 Hz, giving a natural period of 0.2 s. This means, the longest pulse considered is below the natural period.

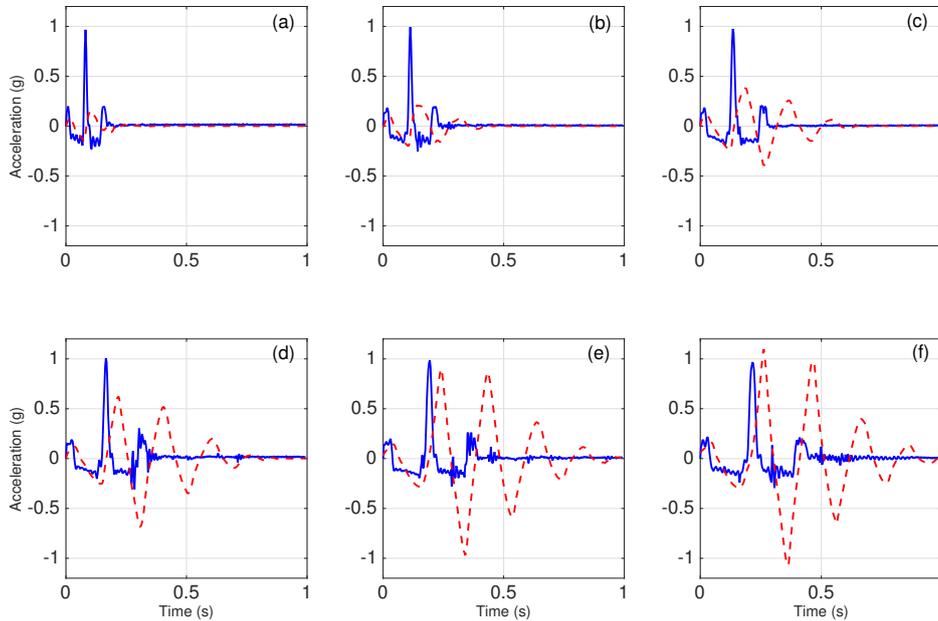


Figure 7 – Experimental shock response of the isolator, for a pulse of constant amplitude of 1 g and different pulse durations. (a) 15ms, (b) 20ms, (c) 25ms, (d) 30 ms, (e) 35 ms, (f) 40 ms (– Pulse input, - - Response)

The information from the time histories presented previously are condensed in Fig. 8, where the maximum value of acceleration response for each pulse is plotted as a function of its respective duration. The blue line with square markers represents absolute acceleration response, whilst the red line with cross markers depicts relative motion. It can be noted how the relative displacement is large for short duration inputs, where the absolute acceleration response is significantly smaller. This effect can be explained due to the increased deformation in the elastic element as it needs to absorb the energy from the impact thus providing good isolation. This compromise between relative i.e. space, and absolute responses is well known in the theory of linear shock isolation, and it is valid in these wire rope isolators. As the pulse becomes longer, the absolute acceleration increases with the subsequent reduction in relative motion.

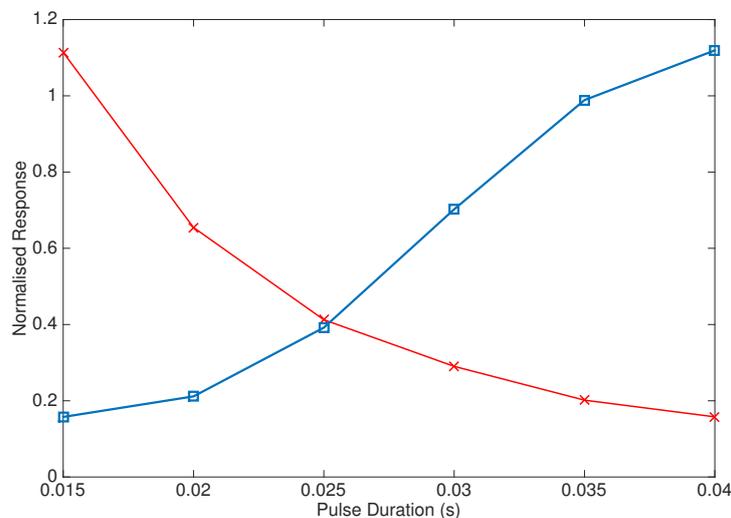


Figure 8 – Normalised Shock Response as a function of the pulse duration. Square markers depict absolute acceleration response, whilst cross markers represent relative motion.

The effect of the input amplitude can be observed in Fig. 9. In this case, the pulse duration was fixed at 15 ms, due to restraints in the displacement limits of the testing system, this duration gave the widest range of amplitudes in order to properly observe the effect on the response. The amplitude was increased in 1 g steps, from 1 g to 10 g. The curves depict the time responses for each input amplitude. This figure presents the maximum normalised absolute and relative responses, as a function of the input amplitude. The nonlinear effect on the input output relationship as the shock amplitude increases can be appreciated in Fig. 9, i.e. doubling the input amplitude does not increase the response in the same ratio, as expected in a linear system. In fact, the normalised response decreases for higher amplitude inputs, meaning that the system presents better isolation for stronger impacts, probably because of the lower dynamic stiffness for larger deformations.

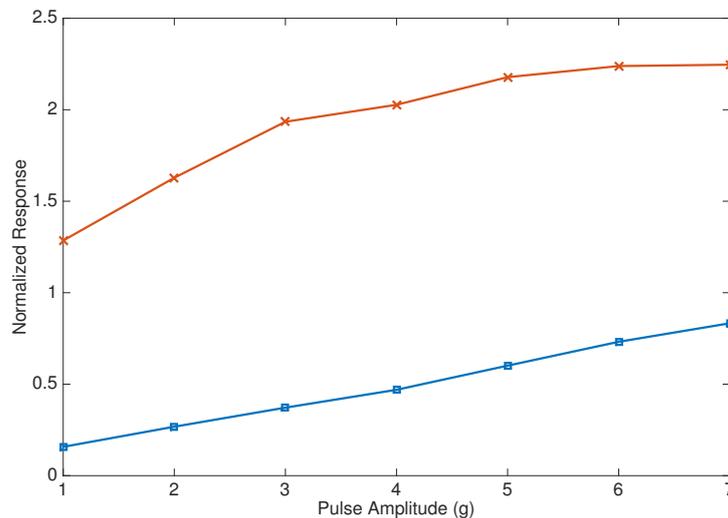


Figure 9 – Maximum Shock Response as a function of the input pulse amplitude. Square markers depict absolute acceleration response, whilst cross markers represent relative motion. The pulse has a duration of 15 ms.

CONCLUSIONS

An insight of the constructions and characteristics of the wire ropes was presented, as well as a brief literature review of the applications and studies related to these isolators. Transient vibration and shock isolation were briefly introduced, where the shock response spectra are used to assess shock isolation. The selection of the wire rope isolator, and the experiments developed during this study were explained, with the objective to get the shock response and isolation properties considering an optimal load to achieve low dynamic stiffness. The static deflection curves were obtained, demonstrating the nonlinear properties for large deformations, however, a piecewise linear approximation can be useful for simplifying the modelling of this particular wire rope isolator. The hysteresis loops were also presented, for which the isolator was subjected to a low frequency sinusoidal displacement and the restoring force was measured. The loops obtained are non symmetrical, and reflect the hardening stiffness of the isolators for large deformations. Then, shock responses were obtained for pulses of different duration and amplitude, demonstrating the isolation and amplification regions of the response. However, when considering relative displacement as the response parameter, the response is very similar to the pulse amplitude for short duration pulses, the response is high, due to the wire rope isolator considerably deforming during impact, thus reducing the energy transmission. As a result, it was demonstrated how cable isolators present excellent shock isolation capabilities, but more research is suggested in order to test different isolator models in other situations, and the developing of a mathematical model that allows to predict the response, by using the Bouc-Wen model of hysteresis.

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