NONLINEAR DYNAMIC RESPONSES OF LAMINATED COMPOSITE STRUCTURE INVOLVING LARGE DEFLECTION AND MATERIAL UNILATERAL DAMAGE

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Abstract: The aim of this paper is to present the nonlinear dynamic responses of damaged laminated composite structure involving large deflection where a novel unilateral damage model is introduced. Only the matrix cracking is retained as damage form and the investigated structures are made of unidirectional layers of polymer matrix reinforced with long fibers. The damage is described by a single scalar variable representing the degradation of the transverse Young's modulus. The structure behavior depends on the cracks state. Hence, in order to analyze the cracks opening/closure effect, the normal stress component to the fiber direction is chosen as a parameter to control the state of the cracks. If it is positive, the cracks are supposed to be opened and the damage effect is applied. If the sign of the normal stress component is negative, the cracks are closed and the virgin material behavior is recovered. Based on the first-order shear deformation theory and the Von Karman's large deflection plate theory, a Serendib finite element with five degree of freedom incorporating a nonlinear strain displacement relation due to large deflection is developed using the updated Lagrangian approach. Since the damage induces nonlinearity, the resulting nonlinear system of equations, combining the geometric and material nonlinearities, is solved in time domain using Newmark method and Newton-Raphson incremental-iterative method. Numerical examples of composite structures are considered and the influence of geometrical nonlinearity on the dynamic responses and the resulting damage is demonstrated. The stiffness degradation due to impact damage is also investigated. In addition, the proposed model permits the detection and the localization of the damage in laminated structures in large deflection. Thereby, this method can be used in life-time estimations and monitoring strategies of complex composite structures.

Keywords: Unilateral damage, laminated composite, geometric nonlinearity, dynamic analysis.

INTRODUCTION

To reduce the weight of the structures, composite materials are on significant expansion in all industry domains since they are light as well as strong. Since their heterogeneity characteristic, composite structures are subjected to various forms of damage such as matrix cracking, fiber fracture and delamination. They evolve gradually resulting in the mechanical degradation of the structure properties and the modification of its mechanical behavior. Hence, for the security requirements, the knowledge of the damage mechanisms that may occur in composite materials is need. Over the last decades, the behavior of damaged composites has been the subject of several investigations where many models have been proposed. A first meso modelling of damage mechanism is proposed by (Kachanov, 1958) and (Rabotnov, 1969), where the concept of effective stress is introduced permitting to quantify the damage. This meso approach, which is related to the scale of the ply, permits to characterize and represent the damage through a local evolution of the material parameters in damaged regions but it does not provide a geometric representation of the damage.

For laminated composite structures, fiber rupture generally occurs in the final phase of layer rupture whereas matrix cracking is considered as the first damage mechanism and presents a common defect in laminated composites. Using a phenomenological modelling of cracked structures (Boubakar *et al*, 2002), the dynamic behavior of damaged composite structures under an impact load is investigated in works of Mahmoudi *et al* (2016a) where a bilateral damage model is used. The bilateral damage model does not exactly describe the material behavior when the structure is subjected to a harmonic load since the closing of cracks is not taken into account. So, cracks in the matrix can be opened or closed depending on the stress state. To take into account this opening/closure effect, Ladveze and Lemaitre (1984) had introduced the idea of unilateral damage. The unilateral condition of damage is based on the concept of desactivation of the damage procces under a compressive loading.

A large number of structural systems in aerospace engineering are made of thin composite components. Regarding their thiny thickness, these composite structures can often achieve nonlinear vibrations. A reliable design of such structures requires a detailed evelution in the presence of large displacement.

The aim of this paper is to propose the nonlinear dynamic responses of laminated composite structure involving large deflection and a novel unilateral damage model (Mahmoudi *et al*, 2016b). Using a meso-macro approach, the material behavior is only needed at the level of one ply. It represents the degradation of the elastic properties due to the initiation and the growth of cracks in the matrix along the fibers directions. According to Perreux and Oytana (1993), although the damage is anisotropic, it can be described by a single scalar variable representing the degradation of the transverse Young's modulus. Only the matrix cracking is retained as damage form and the investigated structures are made of unidirectional layers of polymer matrix reinforced with long fibers. In order to analyze the cracks opening/closure effect (Trivaudey *et al*, 2009), the normal stress component to the fiber direction is chosen as a parameter to control the state of the cracks. If it is positive, the cracks are supposed to be opened and the damage effect is applied. If the sign of the normal stress component is negative, the cracks are closed and the virgin material behavior should be recovered.

Based on the first-order shear deformation theory and the Von Karman's large deflection plate theory, a Serendip finite element (Chee, 2000) with five degrees of freedom incorporating a nonlinear strain displacement relation due to large deflection is developed using the updated Lagrangian approach. Since the damage induces nonlinearity, the resulting nonlinear system of equations, combining the geometric and material nonlinearities, is solved in time domain using Newmark method and Newton-Raphson incremental-iterative method.

Numerical examples of composite structures are considered and the influence of geometrical nonlinearity on the dynamic responses and the resulting damage is demonstrated. The stiffness degradation due to impact damage is also investigated. In addition, the proposed model permits the detection and the localization of the damage in laminated structures in large deflection. Thereby, this method can be used in life-time estimations and monitoring strategies of complex composite structures.

UNILATERAL DAMAGE MODEL

For composite materials, the matrix cracking is considered as the first damage mechanism whereas fibre rupture generally occurs in the final phase. These matrix micro-cracks evolve and grow leading mainly to the degradation of the structure stiffness inducing the rupture of the structure. The investigated structures in this study are made of a thin layer of polymer reinforced with long glass fibres oriented with respect to $\vec{e}_1$ direction as depicted in Fig.1 where the micro-cracks are parallel to the fiber direction.
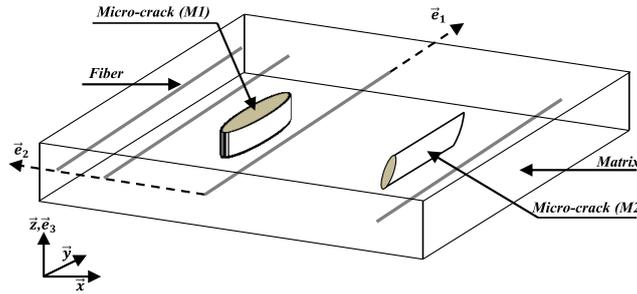

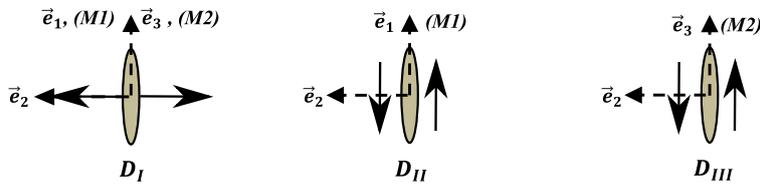**Figure 1 – Orientation of the micro-cracks in the matrix**

Figure 2 – Micro-cracks opening modes

According to experimental tests, the damage in the polymer matrix depends on the micro-cracks opening modes (M1) and (M2) as shown in Fig.2 (Boubakar *et al*, 2002). The three parameters D_I, D_{II} and D_{III} are introduced in order to characterise the reduction of the structure stiffness such as:

$$D_I = 1 - \frac{\tilde{E}_2}{E_2} = 1 - \frac{S_{22}}{\tilde{S}_{22}} \quad ; \quad D_{II} = 1 - \frac{\tilde{G}_{12}}{G_{12}} = 1 - \frac{S_{44}}{\tilde{S}_{44}} \quad ; \quad D_{III} = 1 - \frac{\tilde{G}_{23}}{G_{23}} = 1 - \frac{S_{66}}{\tilde{S}_{66}} \tag{1}$$

Where S_{ii} and $\tilde{S}_{ii}$ are respectively the components of the undamaged and damaged flexibility matrix.

Then, using a self-consistent method (Perreux and Oytana, 1993), the parameters D_{II} and D_{III} can be expressed as a function of D_I. Thereby, the damage is reported through a single scalar variable D representing the relative reduction of

the transverse Young's modulus and a damage matrix $\underline{\underline{H}}$ is defined to express the damaged behavior law as:

$$\underline{\varepsilon}^e = \underline{\underline{\tilde{S}}}\,\underline{\sigma} = \left[\underline{\underline{S}} + \underline{\underline{H}}\right]\underline{\sigma} \quad ; \quad \begin{cases} H_{22} = \dfrac{D}{1-D} S_{22} \\[2mm] H_{44} = \dfrac{D}{\sqrt{1-D}}\sqrt{S_{11}S_{22}} \\[2mm] H_{66} = \dfrac{D}{\sqrt{1-D}} S_{22} \end{cases} \quad ; \quad others\ H_{ij} = 0 \tag{2}$$

Where $\underline{\varepsilon}^e$ is the elastic strain tensor, $\underline{\underline{\tilde{S}}}$ is the damaged flexibility matrix and $\underline{\sigma}$ the stresses tensor. To define the damage evolution law, The thermodynamic of irreversible processes is used. Then, considering only the elastic domain and at constant temperature, the free specific energy Ψ can be written as:

$$\Psi(\underline{\varepsilon}^e, D) = \frac{1}{2\rho}{}^T\underline{\varepsilon}^e\,\underline{\underline{\tilde{S}}}^{-1}\underline{\varepsilon}^e = \frac{1}{2\rho}{}^T\underline{\varepsilon}^e\left[\underline{\underline{S}} + \underline{\underline{H}}\right]^{-1}\underline{\varepsilon}^e \tag{3}$$

The Clausius-Duhem inequality gives the state law and the thermodynamic force, Y, associated to the damage:

$$\sigma = \rho\frac{\partial\Psi}{\partial\underline{\varepsilon}^e} = \underline{\underline{\tilde{S}}}^{-1} : \underline{\dot{\varepsilon}}^e \quad ; \quad Y = \frac{1}{2}{}^T\underline{\sigma}\,\underline{\underline{H}}'\,\underline{\sigma} \tag{4}$$

where ρ is the mass density and $\underline{\underline{H}}'$ is the derivative of $\underline{\underline{H}}$ with respect to D

According to the definitions of the damage matrix and the thermodynamic force of damage, it can be deduced that only the σ_{22}, σ_{12} and σ_{23} are able to activate damage increase through $\underline{\underline{H}}'$. Then, using the principle of maximum dissipation (Boubakar *et al*, 2002), a threshold function $\bar{Y}$ is defined in order to record the damage level and adapt the threshold of damage activation to the damage level. Thereby, the following yield function which controls the triggering or not of the damage evolution is obtained:

$$f_d = Y - \bar{Y} = \frac{1}{2}\underline{\sigma}^T\underline{\underline{H}}'\,\underline{\sigma} - (Y_c + qD^p) \tag{5}$$

where Y_c, q and p are damage material parameters.

Since the mechanical behavior depends on the crack state, a criterion is defined in order to decide when a crack is opened or closed. Hence, if the transverse stress component σ_{22} is positive or null, the cracks are assumed to be opened, and they are considered closed when σ_{22} is negative.

Open cracks

When the cracks are opened, (i.e: $\sigma_{22} \geq 0$), the damage is taken into account and the structure elastic behavior is modified by adding the matrix $\underline{\underline{H}}$ to the virgin flexibility matrix $\underline{\underline{S}}$. The damage evolution is controlled by the yield function f_d. If $f_d < 0$, it means that there is no damage trigerring and the previous damage level is kept in memory. But, if f_d tends to become positive, a damage increment occurs in order to neutralize the yield function f_d. The damage level is obtained by $\partial f_d/\partial D = 0$.

Closing cracks

When the transverse stress σ_{22} decrease until becoming null, the behavior is modified. As a result, a new flexibility matrix, $\underline{\underline{\tilde{S}}}^*$ is defined from the virgin flexibility in order to verify the strain continuity at the cracks closure as follows:

$$(\underline{\underline{\tilde{S}}}^* - \underline{\underline{\tilde{S}}}) : \underline{\sigma} = 0 \tag{6}$$

The virgin behavior in recovered only in the transverse direction but it still elastic damaged in the two other directions:

$$\tilde{S}_{22}^* = S_{22} \quad ; \quad \tilde{S}_{44}^* = S_{44} + H_{44}(D_c) \quad ; \quad \tilde{S}_{66}^* = S_{66} + H_{66}(D_c) \tag{7}$$

Where D_c is the damage value when cracks close.

Closed cracks

When the transverse stress decrease until becoming negative ($\sigma_{22} < 0$), the cracks are closed. The virgin behavior in transverse direction by keeping $\tilde{S}_{22}^* = S_{22}$, from the cracks closure. In the shear directions, the behavior depends on the contact between the crack lips which can have sticking or sliding behavior. For simplicity, the crack lips are assumed to be sticking and the virgin behavior is recovered in all direction in the case of closed cracks.

Opening cracks

As soon as the cracks open ($\sigma_{22} > 0$), the damage behavior is recovered in all directions. The flexibility matrix is now equal to $\left[\underline{\underline{S}} + \underline{\underline{H}}(D_c)\right]$ where D_c is the damage level, kept in memory, at the cracks closure.

FINITE ELEMENT MODELLING

Consider a laminated structure consists of n thin transversely isotropic layers, oriented arbitrarily (*i.e* θ) and having a total thickness h. The global coordinate system $(\vec{x}, \vec{y}, \vec{z})$ is chosen such that the plane $(\vec{x}, \vec{y})$ is in the central plane of the structure and $\vec{z}$ axis is oriented in the thickness direction. The kth layer has a thickness $h_k = z_{k+1} - z_k$, where z_{k+1} and z_k are the coordinates of the upper and lower surfaces of the layer as shown in Fig 3. The first-order shear deformation theory (FSDT) and the Von Karman's large deflection plate theory are retained to establish the finite element model and the governing equation of motion. The displacement field $\{\underline{u}\}$ can be expressed as:
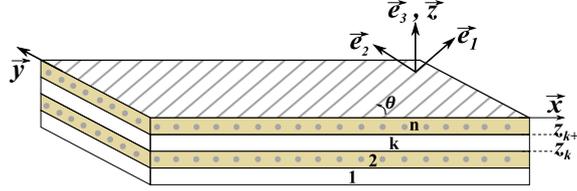

Figure 3 – *Coordinate system of a layered composite plate*

$$\{\underline{u}(x,y,z,t)\} = \left\{ \begin{array}{c} u(x,y,z,t) \\ v(x,y,z,t) \\ w(x,y,z,t) \end{array} \right\} + z \left\{ \begin{array}{c} \varphi_y(x,y,t) \\ -\varphi_x(x,y,t) \\ 0 \end{array} \right\} \tag{8}$$

where (u, v, w) are the displacements in the median plane $(\vec{x}, \vec{y})$ and (φ_x, φ_y) are respectively the rotation of the normal to the median plane with respect to $\vec{x}$ and $\vec{y}$. The problem is discretized by finite elements method where the quadrangular element Serendip $Q8$ with eight nodes is used to mesh the 2D model. The $Q8$ element is characterized by its high performance in the finite element modelling of thin and thick composite structures. The global displacements $\{\underline{u}\}$ can be expressed in local coordinates as:

$$\{\underline{u}(\xi, \eta, z, t)\} = [A(z)][N(\xi, \eta)]\{\underline{u}_e\} \tag{9}$$

Where $[A(z)]$ is the matrix of z-coordinates along the thickness direction, $[N(\xi, \eta)]$ is the matrix of the shape functions and $\{\underline{u}_e\}$ is the elementary nodal displacements.

In the updated lagrangian formulation, all variables are referred to the last calculated configuration at time t. The virtual work principle expressing the equilibrium for a single element can be expressed as:

$$\delta W_e = \delta W_i^e \tag{10}$$

where δW_e is the virtual work done by external forces and δW_i is the virtual work done by internal forces. The internal virtual work is given by:

$$\delta W_i^e = \int_{t_{V^e}} \delta \underline{E}^T \underline{S}^* \, dV^e \tag{11}$$

where ${}^t V^e$ is the volume of the element at time t, $\underline{E}$ is the vector of incremental Green-Lagrange strain, and $\underline{S}^*$ is the vector of second Piola-Kirchhoff stress. Using the derivative of shape function matrix, the Green-Lagrange strain and its variation can be expressed as:

$$\underline{E} = [B_l + \frac{1}{2}B_{nl}]\{\underline{u}_e\} \quad ; \quad \delta\underline{E} = [B_l + B_{nl}]\delta\{\underline{u}_e\} \tag{12}$$

According to Mohan and Kapania (1998), The Piola-Kirchhoff stress can be decomposed as:

$$\underline{S}^* = \underline{\sigma} + \Delta\underline{S}^* \tag{13}$$

where $\underline{\sigma}$ is the vector of Cauchy stresses at time t and $\Delta\underline{S}^*$ is the vector of incremental Piola-Kirchhoff stresses. For small strains, the incremental Piola-Kirchhoff stresses can be expressed as (Fafard *et al*, 1989):

$$\Delta\underline{S}^* = {}^t\underline{C}\Delta\underline{E} \tag{14}$$

where ${}^t\underline{C}$ is the matrix of elastic constants at time t and $\Delta\underline{E}$ is the incremental Green-Lagrange tensor obtained from t to $t + \Delta t$. Hence, the internal force becomes $q = \delta W_i^e = \int_{t_{V^e}} [B_l + B_{nl}]^T \underline{S}^* \, dV^e$. The residue r can be written as $r = q - f$, where f is the element external force vector. Assuming that the equilibrium is statisfied at time t, the equilibrium equation at the current time step $t + \Delta t$ can be linearized as

$$r_{t+\Delta t} = q_t + \frac{\delta q}{\delta\{\underline{u}_e\}}\delta\{\underline{u}_e\} - f_{t+\Delta t} = q_t + [K_{tg}^e]\delta\{\underline{u}_e\} - f_{t+\Delta t} \tag{15}$$

$[K_{tg}^e]$ is the element tangent stiffnes matrix which can be decomposed on a linear and nonlinear stiffness matrix $[K^e]$ and a geometric nonlinear matrix $[K_\sigma^e]$:

$$[K_{tg}^e] = [K^e] + [K_\sigma^e] = \int_{{}^tV^e} [B_l + B_{nl}]^T \, {}^t\underline{\underline{C}} [B_l + B_{nl}] \, dV^e + [K_\sigma^e] \tag{16}$$

Assembling the element matrix and adding the dynamic components leads to the equation of motion

$$[M]\{\underline{\ddot{u}}(t+\Delta t)\} + [B]\{\underline{\dot{u}}(t+\Delta t)\} + F_{int} = F(t+\Delta t) \tag{17}$$

The problem is solved using the Newton-Raphson method by determining the incremental displacement $\{\Delta u\}$:

$$\left([M]\frac{1}{\beta\Delta t^2} + [B]\frac{\gamma}{\beta\Delta t} + [K_{tg}]\right)\{\Delta u\} = -(F_{int} - F(t+\Delta t)) \tag{18}$$

where γ and β are the Newmark constant, $[M]$ and $[B]$ are the mass and damping matrices.

NUMERICAL RESULTS

According to the mesoscopic approach, the damage is characterized through a decreasing of stiffness. Under an applied load, the damage evolution is related to the stress state. Several numerical simulations have been performed in order to highlight the mutual influence between damage and dynamic response. The investigated structure consists of two laminated beams where each one is composed of four layers oriented as $(90°/0°/0°/90°)$. The mechanical properties of the considered laminated structure are given in Tab 1. The investigated structure is clamped at $x = \pm L_1/2$ and $z = -L_2$ and

Table 1 – Mechanical properties of the laminated structure

Elastic modulus E_1 ($\vec{e}_1$) (MPa)	45680
Elastic modulus E_2 ($\vec{e}_2$) (MPa)	16470
Shear modulus G_{12} (MPa)	6760
Poisson ratio ν_{12}	0.34
Poisson ratio ν_{23}	0.34
Y_c (MPa)	0.0027
q (MPa)	1.246
p (parameter in Eq.5)	0.816

its geometric dimensions are $(L_1 = L_2 = 0.1, b = 0.01)\,m$. Two thickness h_1 and h_2 are attributed to the two components of the structure as shown in Fig 4. It is subjected to a distributed harmonic load in $\vec{z}$ direction and along the side of junction between the two beams with $F = 2.2\,10^5 sin(wt)$ where $w = 9173.5\,rad/s$. During the applied load, the damage level is computed in Gauss points and the modification of the material properties induced by the damage is introduced via the elementary updated stiffness computed in these points.
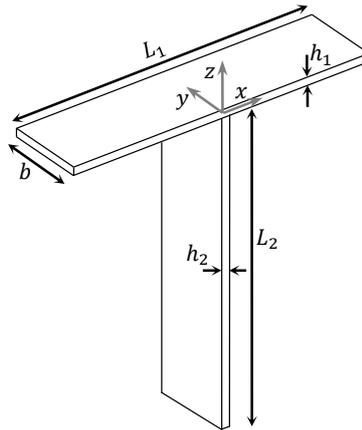

Figure 4 – Assembled composite structure

Two cases are investigated in this paper. For the first one, $h_1 = 4 \times h_2 = 4.8\,10^{-3}\,m$, and as depicted in Fig 5, when the damage is considered in the behavior law, the displacement amplitude is greater than the undamaged one. In addition, The dynamic behavior of the structure is modified during the harmonic load and the nonlinear transient response is dominated by the damage effect.

The damage distributions at the end of the numerical simulation are depicted per ply in Fig 6. The percentage $D[\%]$ represents the relative reduction of the transverse Young's modulus. The damage state depends on the fibre orientations since the stress state is highly depends on the fibre orientations. Hence, the layers oriented as $90°$ present a high damage level since the fibers are perpendicular to the direction of the local transverse stress σ_{22}.
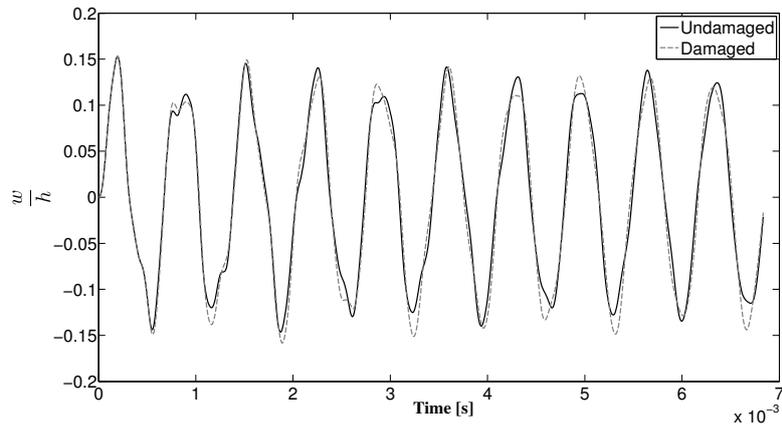
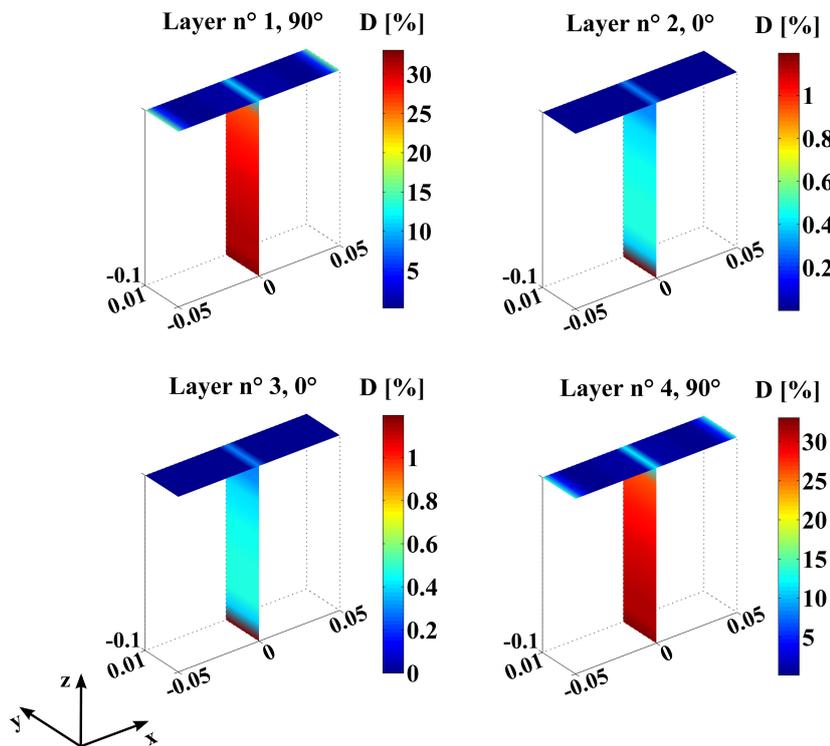**Figure 5 – Normalized displacement of the nodes in the junction side**

Figure 6 – Damage state per layer of the assembled composite structure, $(h_1 = 4 \times h_2 = 4.8\,10^{-3}\,m)$

For the second case, $h_1 = h_2$. Consequently, the excitation frequency becomes closed to the fourth natural frequency of the structure. Since it is thiny, the horizontal component has a bending behavior with large deflection and the nonlinear transient responses are depicted in Fig 7. The difference between the undamaged and the damaged responses is more important when the structure has a vibrational behavior with large displacements. The Damage state is presented in Fig 8 where the damage level is higher comparing to the case of linear vibration in Fig 6 since the membrane-bending effect is more pronounced in the case of large displacement.

The strain-stress curve in the point located $(x = 0, y = 0.005, z = -0.01)$ is depicted in Fig 9. The damage increases when stress increases leading to a nonlinear curve, and a damaged slope is observed when the stress decreases as long as the cracks are opened. When the crack are closed, the virgin slope is recovered but the damage state is kept in memory as show in Fig 10 where the damage can be only grow or remain constant. The damaged slope is recovered when the cracks are opened and a damaged elastic behavior is again observed until the stress state leads to a damage increase.

The damage induces a stiffness degradation, the eigenfrequencies of the structure are changed comparing to their initial values. A ratio of decrease, as show in Fig 11, is defined as the per cent of the shift between f_D and f_0 respectively eigenfrequencies of the damaged and undamaged structure, *i.e ratio of decrease* $R_d = 100\|\frac{f_0 - f_D}{f_0}\|$.

CONCLUSION

In this paper, the large deflection of damaged composite structure is investigated where a novel unilateral damage model is proposed. The proposed model is able to describe, locally,the material behavior with respect to the cracks state
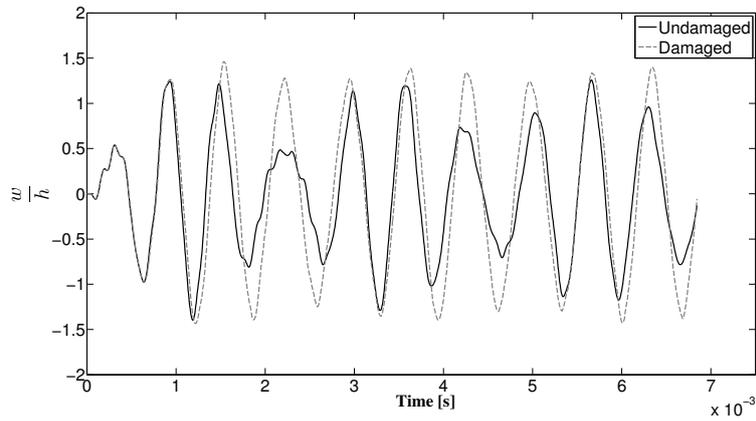
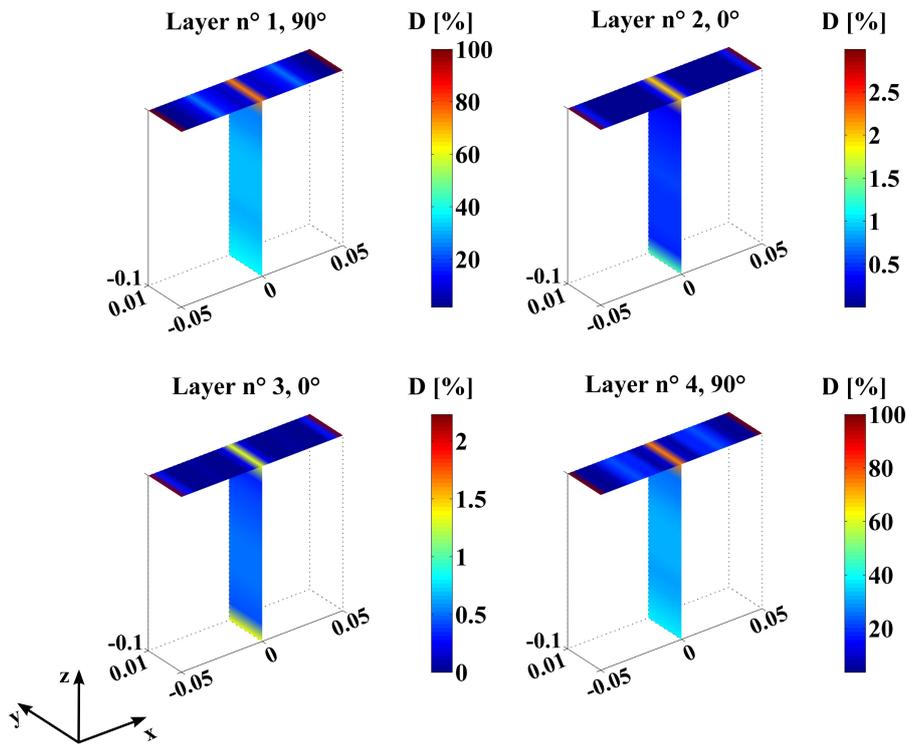**Figure 7 – Normalized maximum deflection in the point** $(x = \pm 0.025, y, z)$

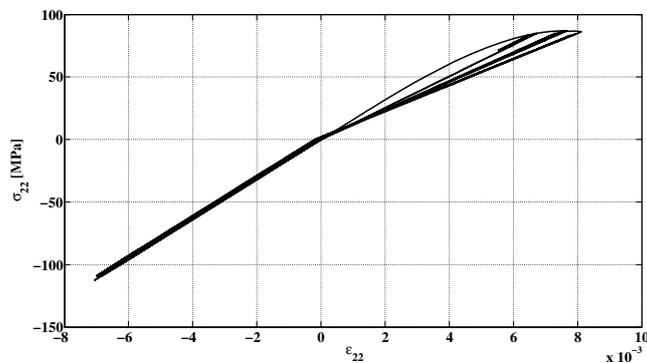**Figure 8 – Damage state per layer of the assembled composite structure,** $(h_1 = h_2)$

Figure 9 – Stress-strain curve in the point $(x = 0, y = 0.005, z = -0.01)$

and reporting the opening/closure effect of the cracks. In addition, the damage state is kept in memory even during compression, and the damage level is recovered as soon as tensile occurs. In addition, the influence of geometrical nonlinearity on the dynamic damaged responses is considered. The stiffness degradation due to damage effect is also investigated. The proposed model permits the detection and the localization of the damage in laminated structures in large deflection. Thereby, this method can be used in life time estimations and monitoring strategies of complex composite
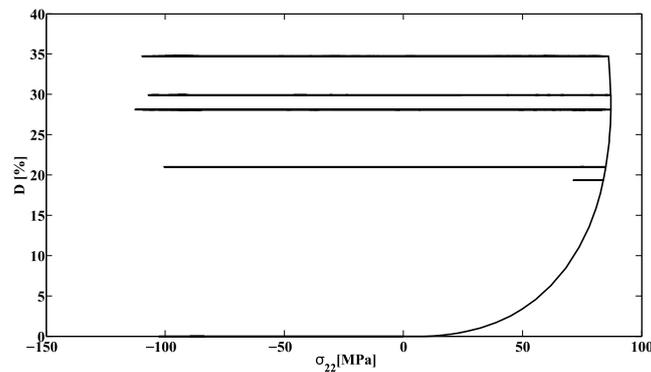
Figure 10 – Damage evolution with respect to the local transverse stress σ_{22} in the point $(x = 0, y = 0.005, z = -0.01)$
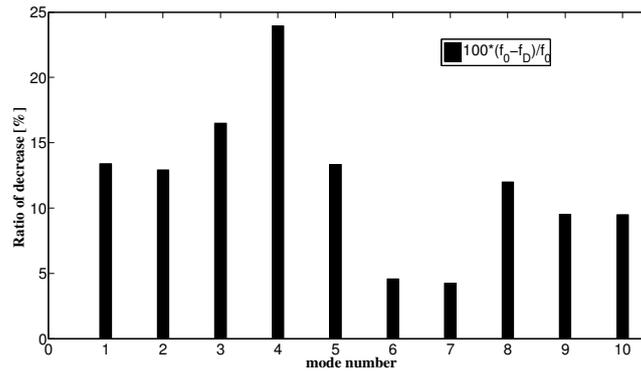

Figure 11 – Ratio of decrease between the eigenfrequencies, f_D and f_0 of damaged and undamaged cases, respectively

structures.

REFERENCES

Boubakar, M. L., Trivaudey, F., Perreux, D., Vang, L., 2002, "A meso-macro finite element modelling of laminate structures: Part I": time-independent behavior, Composite structures, Vol. 58, No. 2, pp. 271-286.

Chee, C . Y . K ., 2000, "Static shape control of laminated composite plate smart structure using piezoelectric actuators", Ph.D. thesis.

Fafard, M., Dhatt, G., Batoz, J. L., 1989, "A new discrete Kirchhoff plate/shell element with updated procedures", Computers and Structures, vol. 31, No. 4, pp. 591-606.

Kachanov, L. M., 1958, "Time of the rupture process under creep conditions", Izv. Akad. Nauk. S.S.R. Otd. Tech. Nauk, Vol. 8 pp. 26-31.

Ladveze, P., Lemaitre, J., 1984, "damage effective stress in quasi unilateral conditions", 16th International Congress of Theoretical and Applied Mechanics, Lyngby, Denmark.

Mahmoudi, S. , Trivaudey, F., Bouhaddi, N., 2016 (a), "Benefits of metamodel-reduction for nonlinear dynamic response analysis of damaged composite structures", Finite element in analysis and design, Vol. 119, pp. 1-14.

Mahmoudi, S. , Trivaudey, F., Bouhaddi, N., Boubakar, M. L., 2016 (b), "Model for prediction of dynamic behaviour of damaged composite structures with large displacements", submitted to journal of composite structures

Mohan, P . and Kapania, R ., 1998, "Updated Lagrangian Formulation of a Flat Triangular Element for Thin Laminated Shells", AIAA Journal, vol. 36, No. 2, pp. 273-281.

Perreux, D., Oytana, C., 1993, "Continuum damage mechanics for micro cracked composites", Composites Engineering, vol. 3, pp. 115–122.

Rabotnov, Y. N., 1969, "Creep rupture", in: Applied Mechanics, International Union of Theoretical and Applied Mechanics, Springer Berlin Heidelberg, pp. 342-349.

Trivaudey, F., Guicheret-Retel, V. , Boubakar, M. L. , 2009, "Unilateral damage model for laminate analysis", 6th International Congress of Croatian Society of Mechanics, Dubrovnik, Croatia.

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