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Instability of Binary Subsonic Coaxial Jets

Jhonatan A. A. Manco
Instituto Nacional de Pesquisas Espaciais, CPTEC
jhonatanjaam@gmail.com

Marcio Teixeira de Mendonca
Instituto Nacional de Pesquisas Espaciais, LACP
marcio_tm@yahoo.com
Abstract. *The performance of rocket engines depends strongly on the proper injection and mixing between fuel and oxidizer. The injection of liquid propellants is performed using coaxial shear injectors. The hydrodynamic instabilities formed by the coaxial shear injector allows the mixing between the propellants through vorticity created by the shear layer instabilities. This work investigates the stability characteristics of axisymmetric coaxial jets composed of hydrogen and oxygen using both linear stability theory (LST) and high order simulations (HOS). The LST has shown that when the hydrogen is used in the inner jet the amplification rates of Mode II are larger than in the homogeneous coaxial jet. For Mode I, the binary mixing layer results can not be extrapolated for a coaxial binary jet, since the confinement effect in the inner jet plays an important role. Using High Order Simulations (HOS), the main results of the LST were simulated in order to investigate nonlinear effects and visualize the resulting flow topology.*

Keywords: *Binary coaxial jets, Hydrodynamic stability, Euler equation, High order numerical simulation*

1. INTRODUCTION

The performance of combustion systems in rocket engines depends strongly on the proper injection and mixing between fuel and oxidizer. The injection of propellants in these systems is performed using shear coaxial injectors that define the physical initial conditions for the combustion process and are the most important initial conditions for the ignition behavior and the flame stability. Vehicles such as the ARIANE 5 and 6, Vulcan, and the main engine of the space shuttle use a mix of liquid oxygen and gaseous hydrogen, both in the super critical state in the coaxial injectors. The hydrodynamic instabilities formed by the coaxial shear injector allow the mixing between the propellants through vorticity. Each species that leaves the injector creates a shear interface which is unstable to small perturbation.

Experimental works presented by Schumaker and Driscoll (2007) and Alexander Schumaker and Driscoll (2012) evaluated the overall mixing efficiency of coaxial injectors using the stoichiometric mixing length, which is the distance over the jet axis where two fluids of different species have mixed in a defined concentration. Another experimental investigation was presented by Tani *et al.* (2015), who examined geometric characteristics such as the length, the taper angle and the wall thickness of the shear coaxial injector at super critical pressures.

A linear stability analysis in inviscid and compressible coaxial jets with continuous velocity and temperature profile was performed by Perrault-Joncas and Maslowe (2008), focusing in shear flows with velocities and temperatures similar to the exhaust of aircraft turbofan engines. This study investigated several factors that influence the stability of coaxial jets such as the diameter and velocity radios between the primary and the secondary streams. The study was also interested in the difference in between 2D Bickley jet and axisymmetric coaxial jets.

Recently Gloor *et al.* (2013) published a study on the stability and acoustic characteristics of compressible viscous coaxial jets. The study investigated the parameters that influence the development of hydrodynamic instabilities such as the Reynolds number, Mach number and momentum thickness. As Perrault-Joncas and Maslowe (2008), they studied the effect of velocity and temperature ratios. Gloor *et al.* (2013) highlighted the importance of the acoustic modes for the study of jet noise and the iteration mechanisms between the Kelvin-Helmholtz instability and the acoustic modes.

An investigation on the stability characteristics for coaxial jets using spatial-temporal linear analysis is due to Balestra *et al.* (2015). The work explores the influence of the temperature and the velocity ratio to describe the process of transition between convective and global instability.

Kozusko *et al.* (1996) investigated the stability of binary shear layers in order to understand results from experimental investigations. They found that the species that form the mixing layer have a significant effect on disturbances growth rates and that the density ratio can be more significant than compressibility. Using different species they qualified analytically the effects of density ratio on the stability of mixing layers and compared the results with single gas homogeneous mixing layers. The results show that the neutral modes and the unstable modes are altered by the composition of the mixing layer due to changes in the molecular weight of the mixture. Further, the investigation concludes that when the heavier gas is on the slowest stream the growth rates are greater and evidence the opposite when the heavier gas in the fastest stream that growth rates are smaller.

Temporal linear stability analysis in three dimensional compressible binary shear layers, as well as a direct numerical simulations were performed by Fedioun and Lardjane (2005). Density radios from 1 to 32 associated with different species at different temperatures are evaluated at high velocities, with convective Mach number M_c up to 2. The extreme cases evaluated were $\mathrm{O_2 - N_2}$ and $\mathrm{O_2 - H_2}$ for $M_c > 0.6$, with nearly the same maximum amplification factor but different most amplified wavelengths and phase velocities. This does not happen in lower convective Mach numbers, when the system $\mathrm{O_2 - N_2}$ presents the largest amplification factor, decreasing around 25% to 30% for the M_c range at the angle of propagation $\theta = 0$. Further an empirical model was developed to predict the stability properties within 5% error starting with the incompressible solution as recommended by Kozusko *et al.* (1996).

Using linear stability theory and direct numerical simulations Manco (2014); Manco *et al.* (2014) studied binary mixing layers stability modified by jets and wakes. The vortical structures development by the resulting flow due to the interaction of mixing layers, jets and wakes is also considered in that study.

This paper extends the study on coaxial jets of Perrault-Joncas and Maslowe (2008) in different aspects, i) the base flow is now composed of two different chemical species in the inner and outer jets and ii) a direct numerical simulation is used to view the nonlinear vortical structures developed by the instabilities.

2. METHODOLOGY

The stability analysis of binary coaxial jets was performed using the Euler equations. Being more specific the Euler equations for compressible, thermally perfect gas with no heat addition and without externals forces were used to model a coaxial binary jet. In non dimensional form the Euler equations are:

$$\frac{D\rho}{Dt} + \rho\nabla\cdot\mathbf{u} = 0, \tag{1}$$

$$\rho\frac{DY_i}{Dt} = 0, \tag{2}$$

$$\rho\frac{D\mathbf{u}}{Dt} = -\frac{\gamma_0}{M_0^2}\nabla p, \tag{3}$$

$$\frac{Dp}{Dt} + \gamma_0\gamma p\nabla\cdot\mathbf{u} = 0, \tag{4}$$

where γ is the ratio between the specific heats c_p and c_v, γ_0 is the reference specific heat ratio and a_0 is the reference speed of sound

The non-dimensional state equation for the gas mixture and for each species are

$$\bar{p}\gamma_0 = \bar{\rho}\bar{R}\bar{T}. \qquad p_i\gamma_0 = \rho_i R_i T, \tag{5}$$

respectively, where $p = \sum p_i$, using the Dalton's law of partial pressures.

In order to arrive at a simple form of the systems of equations 2 to 4 and derive the liner stability equations, the hypothesis that the instantaneous flow can be decomposed in a perturbation flow and a base flow is used

$$\mathbf{u}(t, r, \theta, z) = \bar{\mathbf{u}}(t, r, \theta, z) + \epsilon\mathbf{u}'(t, r, \theta, z), \tag{6}$$

where $\mathbf{u}$ represents the variables present in the Euler equations. The base flow of the coaxial jets is considered parallel, $i.e.$, the properties do not depend of the axial z and azimuthal θ direction,

$$\bar{\mathbf{u}} \equiv [0, 0, \bar{w}(r)], \tag{7}$$

where the component of the velocity in the axial direction $\bar{w}$ is function only the radial coordinate.

With the parallel assumption for the base flow and taking the first order approximation of the Euler equations 2 to 4, a linear system for the perturbation variables can be combined in a single equation for the perturbation pressure

$$\frac{\bar{D}}{\bar{D}t}\left(\frac{\gamma_0}{M_0^2}\nabla^2 - \frac{\bar{\rho}}{\gamma_0\gamma\bar{p}}\left(\frac{\bar{D}}{\bar{D}t}\right)^2\right)p' - \left(\frac{1}{\bar{\rho}}\frac{\partial\bar{\rho}}{\partial r}\frac{\bar{D}}{\bar{D}t} + 2\frac{\partial\bar{w}}{\partial r}\frac{\partial}{\partial z}\right)\frac{\partial p'}{\partial r} = 0 \tag{8}$$

Where ∇^2 is the Laplace operator.

The pressure perturbation can be approximated using normal modes ansatz because of the linearity and the single dependence of the coefficient on the radial direction r in the above equation. Then, to transform the partial pressure disturbances equation in a ordinary differential equation a wave solution of the form

$$p'(r,\theta,z,t) = \hat{p}(r)e^{i(kz+n\theta-\omega t)} + \hat{p}^*(r)e^{i(k^*z+n^*\theta-\omega^*t)} \tag{9}$$

is used, where "*" represent the complex conjugate.

Using the ansatz in the pressure disturbance equation 8 and rearranging, results

$$\frac{\partial^2 p'}{\partial r^2} + \left(\frac{1}{r} - \frac{1}{\bar{\rho}}\frac{\partial\bar{\rho}}{\partial r} + \frac{2k}{\Omega}\frac{\partial\bar{w}}{\partial r}\right)\frac{\partial p'}{\partial r} + \left(\frac{M_0^2}{\gamma_0}\frac{1}{a^2}\Omega^2 - \left(\frac{m^2}{r^2} + k^2\right)\right)\hat{p}+ = 0, \tag{10}$$

where a is the local speed of sound.

2.1 Solution Method

The stability equations are transformed in a generalized eigenvalue problem of the kind

$$\mathbf{A}\mathbf{x} = \mathbf{B}\lambda\mathbf{x}, \tag{11}$$

where $\mathbf{A}$ and $\mathbf{B}$ are matrices, $\mathbf{x}$ is the generalized eigenvector and λ is an generalized eigenvalue of $\mathbf{A}$ and $\mathbf{B}$. If $\mathbf{B}$ is a non-singular matrix this problem turns to a classical eigenvalue problem.

$$\mathbf{C}\mathbf{x} = \lambda\mathbf{x} \tag{12}$$

with $\mathbf{C} = \mathbf{B}^{-1}\mathbf{A}$.

To solve the generalized eigenvalue problem the spectral collocation method was used.

A High order numerical code was used to solve the Euler equations for binary coaxial jets using low dissipation, low dispersion, high order numerical methods, to represent appropriately the waves characteristics. The spatial discretization uses a 4th order dispersion relation preserving finite difference (DRP) proposed by Tam and Webb (1993). For the temporal advance a 6 steps, a low storage Runge Kutta specialized in non-linear operator implemented by Berland *et al.* (2006) was used. Non-reflecting boundary conditions (NRBC) were implemented to avoid reflections of outgoing waves at the boundaries of the computational domain. More specifically a buffer zone was used. This NRBC shows the most efficients results for hydrodynamics stability problems Manco and de Mendonca (2019). A 4th order 13 points low dispersive and low dissipative explicit selective filter was implemented, following the work of Bogey and Bailly (2004), to avoid the grid-to-grid oscillations caused by the use of central finite difference schemes. This filter removes the short waves without affecting the instability long waves, that in this case are the Kelvin-Helmholtz.

In the master thesis of Manco (2014) the complete development of the numerical tools is presented along with the reasons to choose these numerical schemes and types of non-reflecting boundary conditions recommended in the numerical simulation of unstable mixing layers.

2.2 The mean flow profiles

To represent the mixing process of coaxial injector which is performed by the hydrodynamics stability formed by two shear layers. The first shear layer is formed between the inner and outer jet and the respectively hydrodynamic mode is knows as Mode I. The second shear layer is formed by the outer jet and the ambient and its hydrodynamics mode is known as Mode II. The shear layers and its hydrodynamics modes are shown in Fig. 1.

To represent the base flow which will be perturbed to form the two hydrodynamics modes of the coaxial injector the mean flow proposed by Perrault-Joncas and Maslowe (2008) was used both for the linear stability analysis and the High

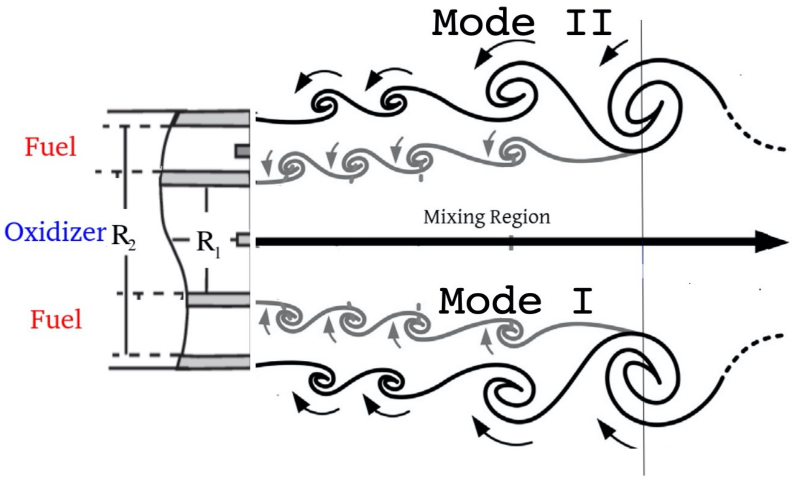


Figure 1. Mixing process between oxidizer and fuel in a shear coaxial injector produced by the Kelvin-Helmholtz instability.

Order numerical simulations. To this base flow, the velocity and density profiles are defined by canonical profiles. For velocity

$$\bar{w} = (1-h)w_1 + hw_2, \tag{13}$$

where

$$w_n = \frac{1}{2}\left\{1 + tanh\left[b_n\left(\frac{R_n}{r} - \frac{r}{R_n}\right)\right]\right\} M \quad n = 1, 2. \tag{14}$$

Where, h parameter is the velocity ratio between the primary stream, defined by the value of the velocity at radii R_1, and the velocity of the secondary stream, defined by the velocity value in the radii R_2 of the coaxial jet. M represents the Mach number of the inner jet in relation to oxygen speed of sound at the temperature at the center of the coaxial jet, $r = 0$. In the evaluated cases $M = 0.6558$ was selected because as shown by Joncas and Maslowe, above $M > 0.8281$ radiating modes exist which have different behavior than the well known instability modes, the Kelvin-Helmholtz mode. The other parameters, b_1 and b_2 are related to the momentum thickness of the shear layer of the different streams θ_1 and θ_2, and are defined by the relation $b_n = R_n/4\theta_n$, where θ_n is the momentum thickness. Another important parameter that defined the base flow configuration is the radii relation $\Gamma = R_2/R_1$.

As was reported by Perrault-Joncas and Maslowe (2008) the geometric parameter Γ and the velocity ratio h of the coaxial jet, control the two instability modes, Mode I and Mode II, respectively. Density gradients are due only to the choice of species, since the flow is considered isothermal. A typical base flow density distribution is presented in Fig. 2. In these base flow $h = 0.7$ and $\Gamma = 2.0$.

The reference properties, as can be see in the Fig. 2, were chosen always with respect to the same chemical species, the oxygen. This fact corresponds to keep the Mach number in reference to the oxygen, as it is more similar to air than in relation to the hydrogen, where the speed of sound is about 4 times greater.

The first species that appears in the nomenclature in the label indicates the species used in the inner jet and the second represent the species used in the outer jet and the ambient. The homogeneous case, without density gradients, formed by a single species in both the inner and outer jet, oxygen or hydrogen, has $\bar{\rho} = 1$.

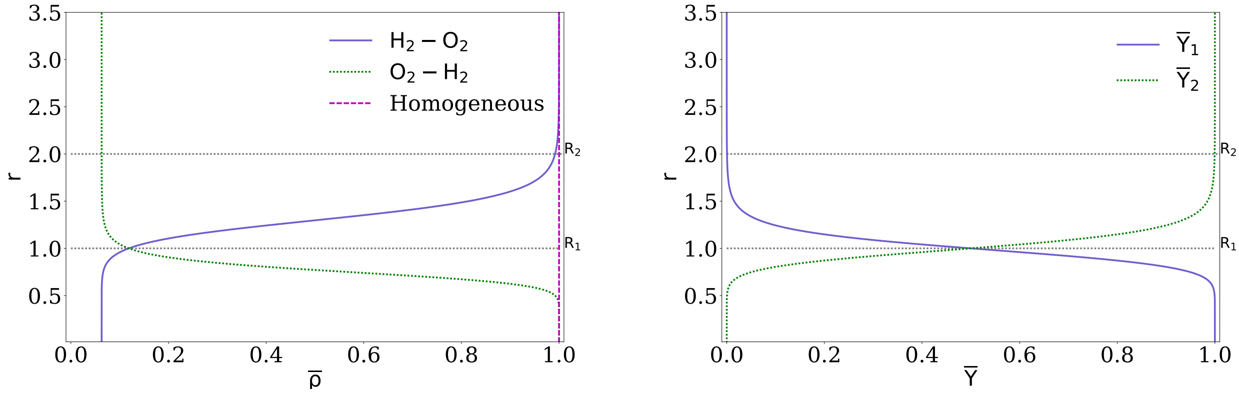
The mass fraction profiles is defined by

$$\bar{Y}_1 = \frac{1}{2}\left\{1 + tanh\left[b_1\left(\frac{R_1}{r} - \frac{r}{R_1}\right)\right]\right\} \tag{15}$$

and

$$\bar{Y}_2 = 1 - \bar{Y}_1. \tag{16}$$

The b_1, related to the momentum thicknesses, and h are the same used in the velocity profile.



(a) Density base flow

(b) Mass Fraction base flow

Figure 2. Density base flow and Mass fraction profiles for different binary coaxial jets, non-dimensionalized with respect to the oxygen jet at the $r = 0$ temperature, $\Gamma = 2$ and $h = 0.7$.

3. Results

In this section the results from the Linear Stability Analysis and High Order numerical simulations are presented for binary jets formed by H_2 and O_2 species and evaluated in different base flow configuration parameters for the radii ratio Γ and the velocity ratio h.

3.1 Linear Stability Analysis of Binary Coaxial Jet

Using linear stability analysis both for compressible and low mach number regime, the purpose of this study is to identify the role of chemical species, oxygen (O_2) and hydrogen (H_2), in the stability characteristics of coaxial jets.

A low Mach number formulation, $M_0 \approx 0$ allows to neglect the effects of compressibility given by the use of different species that changes the speed of sound and given at the Mach number of the inner jet $M = 0.6558$, and allows to only concentrate in the effects of the gradient of velocity and density.

Fig. 3a presents the growth rates for compressible and low Mach number coaxial jet using different species and using a homogeneous configuration without density or temperature gradients in the base flow. The low Mach number configuration can be distinguished by the use of a line thickness but the same color and line type than the compressible cases. Differently than what happens with the change in Γ and h, that modified a single mode of the homogeneous coaxial jet Perrault-Joncas and Maslowe (2008), the change in the density profile induced by the use of different species in the formation of the jets, changes both modes I and II. This can be observed both in the low Mach number as well the compressible cases.

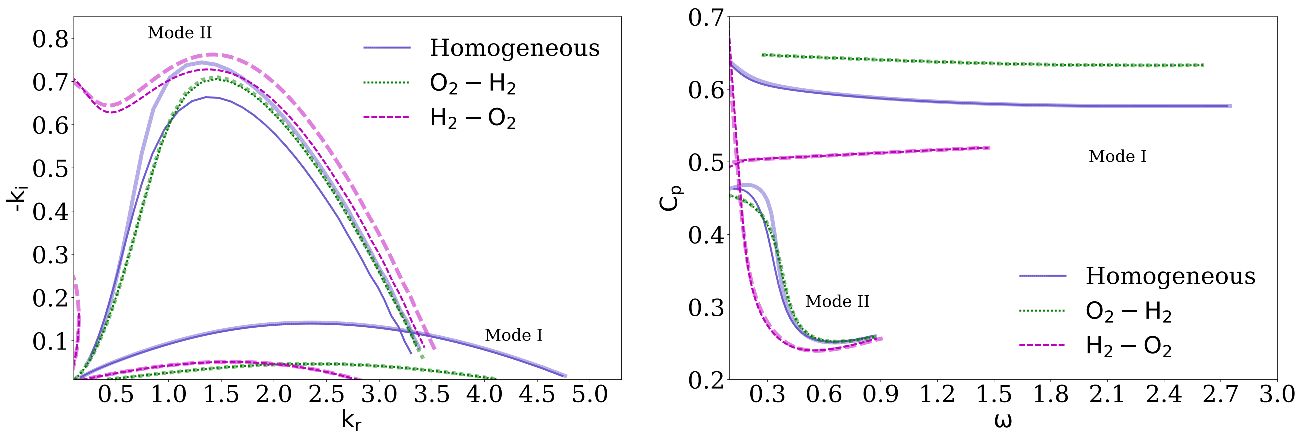
As can be seen in Fig. 3a, the growth rates of the second mode are higher when the oxygen is positioned in the outer jet, and smaller when it is positioned in the inner jet. However, in both cases, the growth rates are larger than the homogeneous case both in the compressible and the low Mach number cases. Furthermore, when the low Mach number approximation is used to compare the results, the largest amplification rate is given by the $H_2 - O_2$ configuration, which is consistent with Kozusko *et al.* (1996) results that concluded that when the heavier species is in the lower velocity stream the amplification rates are larger. This results show that the compressibility effects of hydrogen, which has a speed of sound almost 4 times the oxygen speed of sound, is not so pronounced in reducing the amplification of the jet.

This is most evident in the $O_2 - H_2$ configuration, where the compressibility does not change visually the growth rates. In this case, the ambient where the coaxial jet is ejected is formed by hydrogen too, making the speed of sound larger that the inner jet with oxygen. This implies that it can be considered as the low Mach number case. Remember that the speed of sound of reference is with respect to the oxygen stream and the Mach number of the inner jets at $r = 0$ is $M = 0.6558$, if this Mach number was measured with respect to the hydrogen the values will be $M \approx 0.16$, where the compressible effects are really negligible.

The second instability mode (Mode II), originated by the instability of the outer shear layer, must be little affected by the use of the different species. Once, it is in the inner shear layer where the gradient of density actually places and where the binary effects over the growth rate must be more evident (Mode I). However, in Mode II, the fact of using Hydrogen as an inner jet shows traces of absolute instability in the lower real wave numbers, 3a.

It is important to note in Fig3a that amplification rates of the first mode when two species are considered are much smaller than the amplification rates of the homogeneous cases, approximately one third lower, this contrast with a behavior observed in the mixing layer, where the amplification rates when the heavier gas is located in the slow stream are the largest.

The phase velocity of both modes is presented in Fig. 3b. When the oxygen is located in the inner jet, the phase velocities are higher and the perturbation travels at the speed of the fast stream. The opposite can be obeserved when the hydrogen is located in the inner jet the phase velocities are smaller. Another important comment is about the dispersive behavior of Mode II, which is much more dispersive than the Mode I. One more time, the compressible effects, when hydrogen is used as coaxial jet specie, do not play a important role at Mach number of $M = 0.6558$ for the phase velocity, Fig 3b. The first mode for $H_2 - O_2$ and $O_2 - H_2$ configurations, with is non-dispersive and have smaller growth rates, is almost a neutral mode. For the Mode II the configuration $H_2 - O_2$ has a lower phase velocity than the homogeneous and $O_2 - H_2$ configurations, which are very similar.



(a) Growth rates (b) Phase velocity

Figure 3. Effect of species configuration and compressibility (thinner lines) on the coaxial jet Modes growth rates ($-k_i$) and on phase velocity C_p. With $\Gamma = 2$ and $h = 0.7$.

3.2 Effects of radii ratio Γ.

By changing the radii ratio the amount of fluid in the outer jet can be increased in relation to the mass flow in the inner jet changing the momentum $\bar{\rho}\bar{w}$ of the second jet stream, Fig. 4.

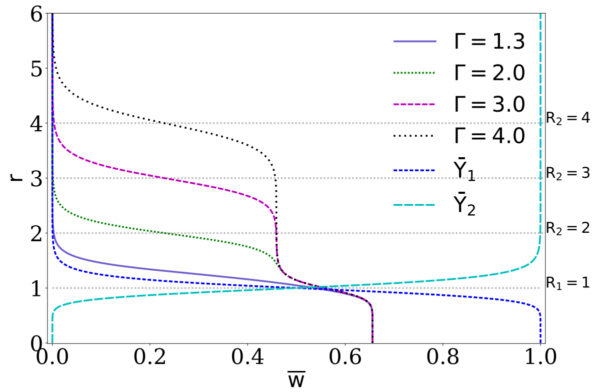
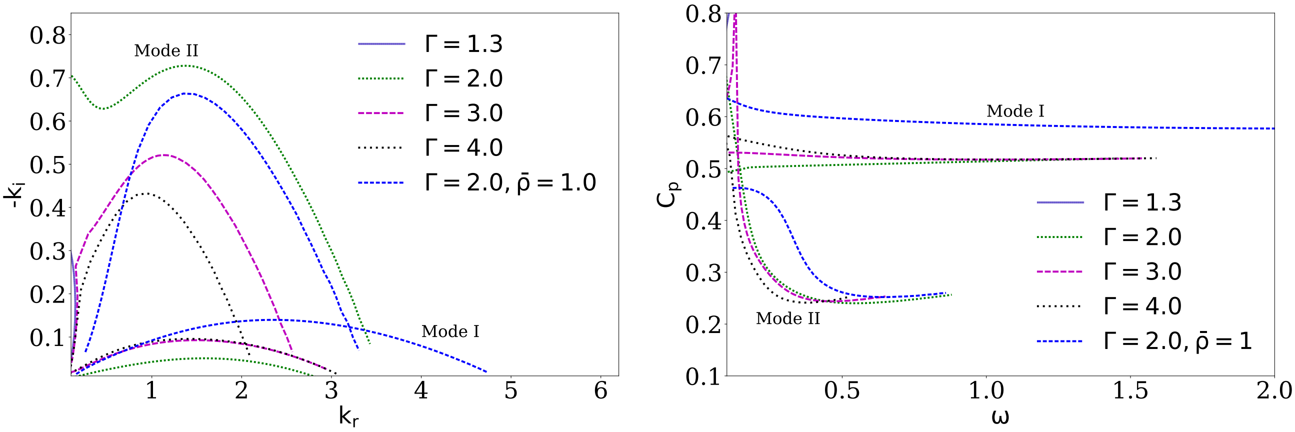


Figure 4. Mass fraction $\bar{Y}_1$ and $\bar{Y}_2$ base flow profiles together with base flow axial velocity profiles obtained using different Γ ratios.

The stability characteristics of these different Γ configurations can be seen in Figs. 5 for the $H_2 - O_2$ arrangement. The second mode is strongly affected by the choice of Γ, having the largest amplification rate with the $\Gamma = 2$. It seems that absolute instability appears for the lowest Γ ratios and practically are not found in $\Gamma = 4$. Mode I also change with Γ, with a reduction in the growth rate with respect to the homogeneous case. The phase velocities for both modes are little affected by Γ. The phase velocities for the binary cases are smaller than the phase velocities of the homogeneous case, in Fig. 5b the Mode I is almost non-dispersive.

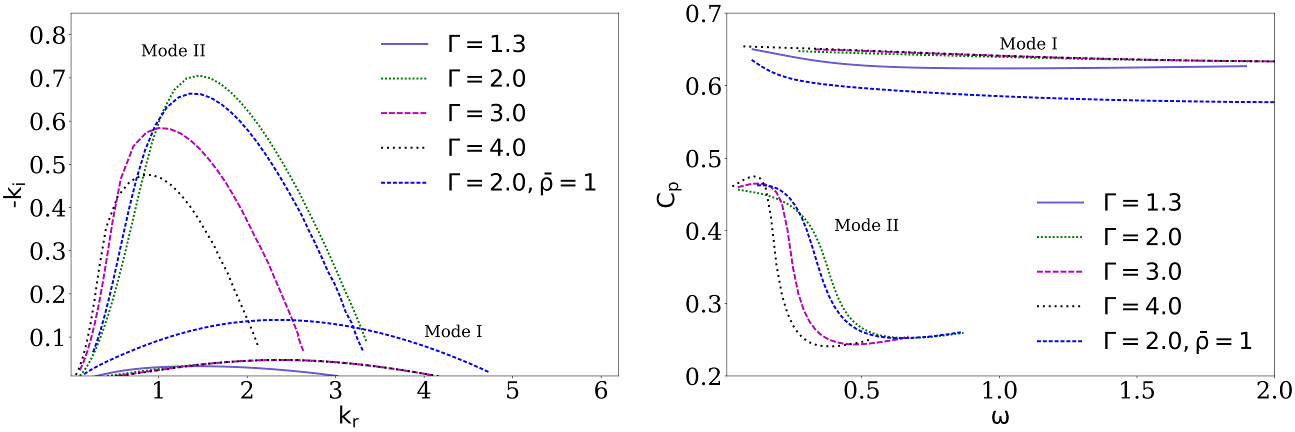


(a) Growth rates k_i (b) Phase velocity $C_p = \omega/k_r$

Figure 5. Effect of Γ ratio on instability characteristics of the coaxial binary jet configuration $H_2 - O_2$ with $h = 0.7$.

Similar results are observed for the $O_2 - H_2$ arrangement, with a more pronounce effect of Γ on the phase velocity of Mode II.

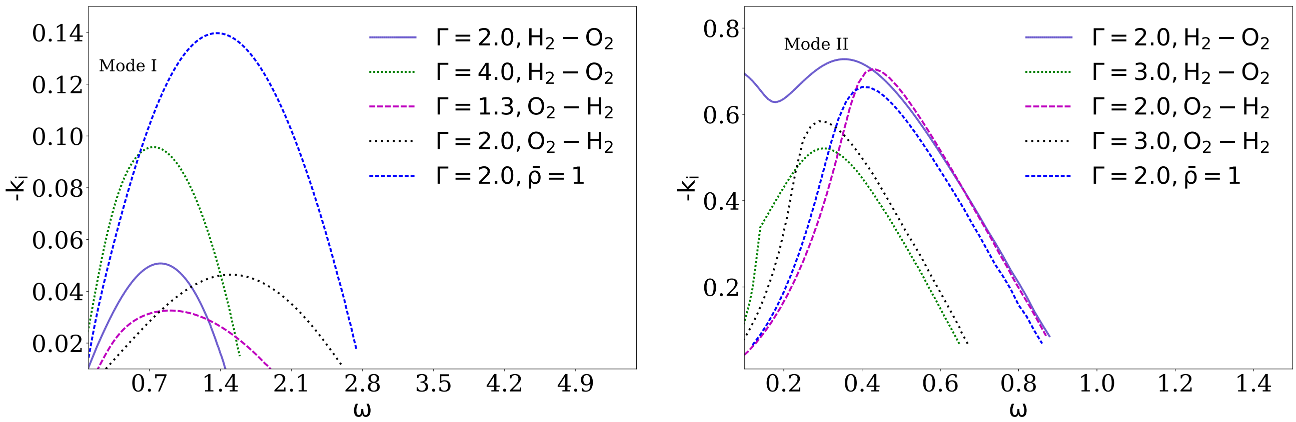
Is important to note that there is a important behavior when $\Gamma = 1.3$, there is only one unstable mode. When the set up of the coaxial jet is $H_2 - O_2$ this mode behaves like the second instability mode, however when the configuration is $O_2 - H_2$ this single mode behaves like the first instability mode and its growth rates are almost zero. This is consistent with Kozusko *et al.* (1996) results when the heavier species is in the faster velocity stream the amplification rates are smaller. In the homogeneous case this mode behaves like the Mode II, Perrault-Joncas and Maslowe (2008)



(a) Growth rates k_i (b) Phase velocity $C_p = \omega/k_r$

Figure 6. Effect of Γ ratio on instability characteristics of the coaxial binary jet configuration $O_2 - H_2$ with $h = 0.7$.

A direct comparison between the different arrangements with respect to the effect of Γ is presented in Fig. 7 and allows a conclusion about the arrangement that promotes the best mixing between oxygen and hydrogen. In Fig. 7, it is evident that the largest amplification rates are reached when the oxygen is in the slower jet stream, in the outer jet, which is consistent with results founded in a mixing layer by Kozusko *et al.* (1996). However in the first instability mode, the homogeneous case shows larger growth rates. This behavior can be explained by the finite quantity of species that can be place in a coaxial configuration, in mixing layer both stream are infinite. For the second mode this limitation is less evident because it is formed by almost a homogeneous density profile, which is also infinity on one side. Nonetheless the influence of the species in the growth rates in this mode are evident, achieving the largest amplification rates with $\Gamma = 2$ and apparently with the onset of absolute instability. When the oxygen is located in the fast stream the amplification rates are the smallest, leading to the almost annihilation of Mode I and smaller growth rates of Mode II, when compared with the other results.



(a) First Instability mode, Mode I

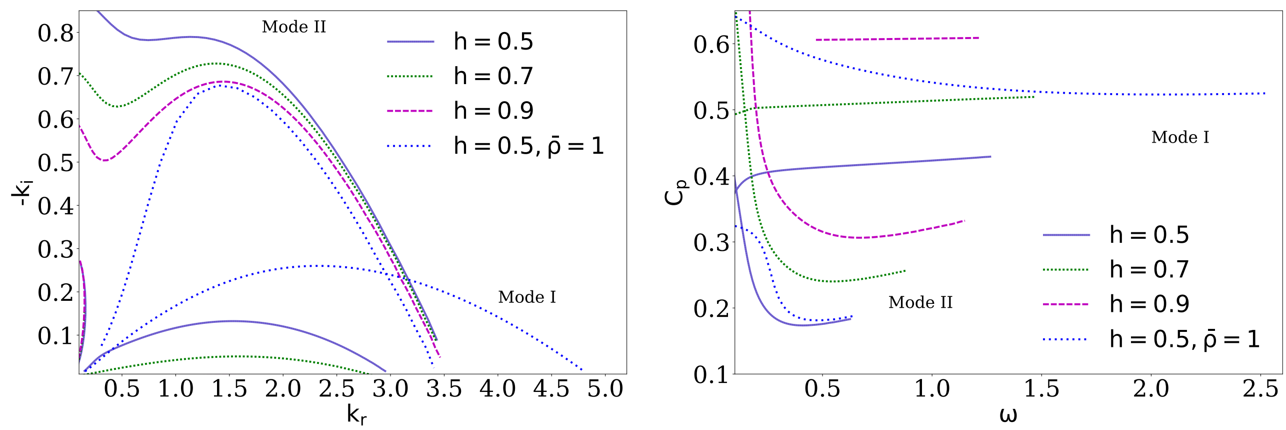
(b) Second Instability mode, Mode II

Figure 7. The most relevant cases evaluated to illustrate the effect of Γ ratio on of the growth rate and phase velocity of coaxial binary jets with $h = 0.7$.

3.3 Binary Coaxial jet, effects of velocity ratio h.

After the effects of Γ ratio were verified in binary coaxial jet, the effects of the velocity ratio will be presented. As was done for homogeneous case, with constant density profile, different h relation were evaluated using two binary configurations with oxygen and hydrogen. Fig. 8a shows different h ratios in a $H_2 - O_2$ configuration and are compared with the most unstable case of a homogeneous coaxial jet, $\bar{\rho} = 1$. Differently than happened in the homogeneous cases the change in the h changes notably the growth rates and the behavior of the second instability mode. Increasing h the amplification rates decrease, being similar to the unchanged Mode II of the homogeneous cases, however the traces of absolute instability are present in low frequencies.

Regarding Mode I, as was observed with the radii relation, the biggest amplification rates are reached with the homogeneous case, but its interesting to note that as the h relation modified significantly the first mode, increasing h decrease Mode I amplification until it becomes stable for $h = 0.9$. The phase velocity, Fig. 8b, as was expected with the smaller h ratios allows the perturbation to travel slower, this apply to both modes. Its interesting to highlight the $h = 0.5$ case, which was the lowest velocity reached for the Mode I in this work both for binary as homogeneous coaxial jet. For $h = 0.9$ the neutral waves travel at a higher velocity. Again Mode II is more dispersive in relation to the behavior almost non-dispersive of Mode I when a binary coaxial jet are considered.

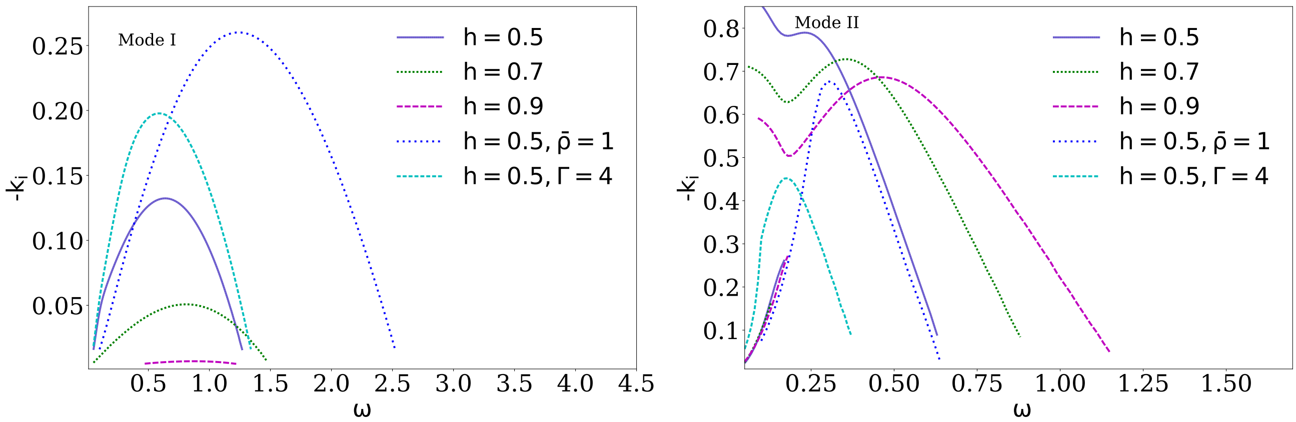


(a) Growth rates k_i.

(b) Phase velocity $C_p = \omega/k_r$.

Figure 8. Effect of h ratio on instability characteristics of the coaxial binary jet configuration $H_2 - O_2$ with $\Gamma = 2$. $\bar{\rho} = 1$ is the most unstable homogeneous case for Mode I.

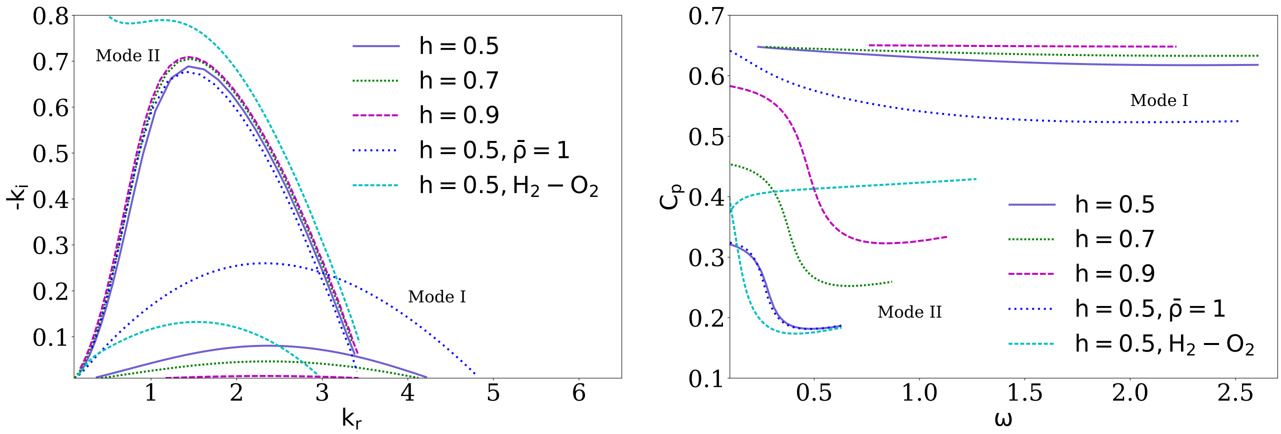
Changing to the $O_2 - H_2$ configuration, Fig. 10a, the stability characteristics of Mode II are not very sensitive to h



(a) Growth rates k_i. (b) Phase velocity $C_p = \omega/k_r$

Figure 9. Effect of h ratio on instability characteristics of the coaxial binary jet configuration $H_2 - O_2$ with $\Gamma = 2$. $\bar{\rho} = 1$ is the most unstable homogeneous case for Mode I and $\Gamma = 4$ with $h = 0.5$ has the most unstable mode I.

but the phase velocity increases with h (Fig. 10b). The amplification rates of Mode I are reduced with increasing h and are smaller with respect to the homogeneous and $H_2 - O_2$ cases, but the phase velocity is relatively insensitive and the disturbances are non-dispersive.



(a) Growth rates k_i. (b) Phase velocity $C_p = \omega/k_r$.

Figure 10. Effect of h on instability characteristics of the coaxial binary jet configuration $O_2 - H_2$ with $\Gamma = 2$. $\bar{\rho} = 1$ is the most unstable homogeneous case for Mode I, and $H_2 - O_2$ case with $h = 0.5$ is the most unstable mode.

3.4 High order simulation of Binary Coaxial Jet

In order to shown the binary effects on the vortical structures of unstable coaxial jets the most relevant cases founded with the linear stability theory will be evaluated with High Order Simulations. The cases simulated will be those with larger growth rates in the LST analysis, both for the $H_2 - O_2$ and $O_2 - H_2$ systems, with different radii ratios Γ and different velocity ratios h. In this case the most important parameter is the mass fraction Y_i of the species, which shows how the species are mixing due to the instabilities, which have as the principal role to combine the oxygen and the hydrogen.

The first relevant case is the $\Gamma = 2.0$ with higher amplification rates for Mode II in relation to the homogeneous cases. In Fig. 11 the mass fraction contours and the density contours of this configuration are shown using the axial symmetry of the coaxial jet. For $r > 0$ the mass fraction of the inner specie is plotted and for $r < 0$ the density is placed for the same time.

The LST shows that there are different cases where the growth rates of the unstable coaxial jet are larger allowing the mixing between the species. However as can be seen not all unstable modes promote the mixing between the species. In this case the first instability mode, Mode I, does not appear in $H_2 - O_2$ case. Although the first instability mode exits its amplification rate was no sufficient to form the K-H instabilities waves and mix the reactants.

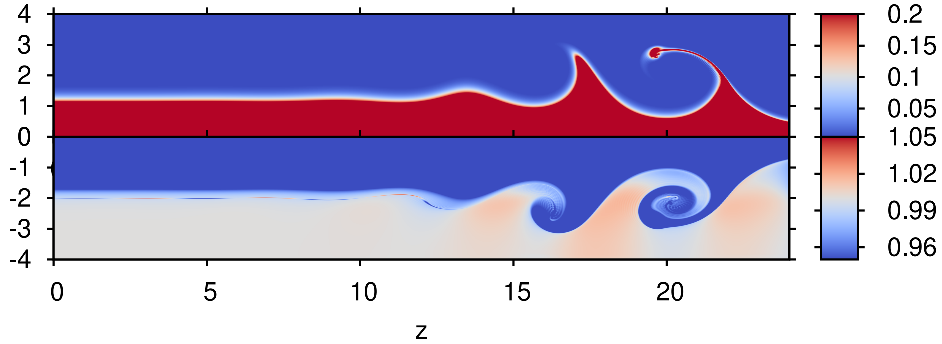


Figure 11. Mass fraction contours and density contours, showing the vortical structures for $H_2 - O_2$ coaxial jet at $t = 70$. Mass fraction contours are shown for $r > 0$ and density contour for $r < 0$. The radii and velocity ratios used were $\Gamma = 2$ $h = 0.7$, respectively.

In the same way, a coaxial jet formed by $O_2 - H_2$, with the same radii ratio $\Gamma = 2.0$ and the velocity ratio $h = 0.7$ was simulated, as shown in Fig. 12. As was expected the inner shear layer is almost stable, although the LST analysis has shown that this is an unstable shear layer but its small growth rates are not captured by the HOS. The outer shear layer is unstable and its growth is captured by the HOS. This is the only mode that allows the mixing between the species in the $O_2 - H_2$ configuration, differently that must be thought in the LST section.

The mixing between the species in this case is due to the growth of the vortical structures of the second mode when these structures achieve the inner radii, $R_1 = 1$, where in this case the oxygen is found. Although $O_2 - H_2$ configuration allows the mixing between the species, the amount of oxygen that left the inner jet is very small compared with the previous case and it depends completely on the growing of Mode II.

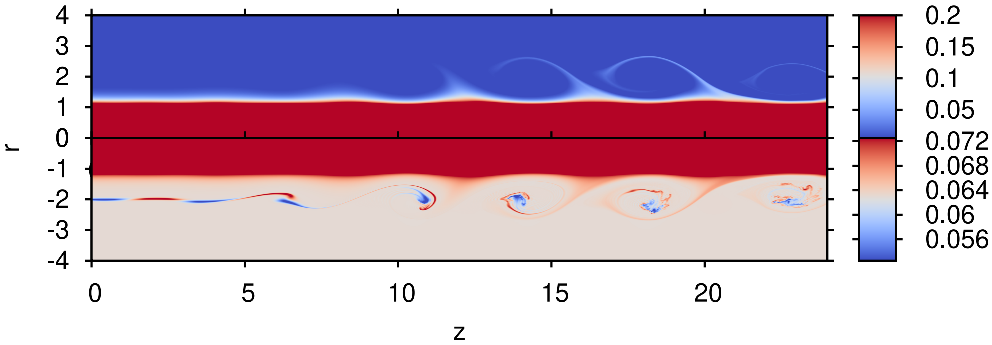


Figure 12. Mass fraction contours and density contours, showing the vortical structures for $O_2 - H_2$ coaxial jet at $t = 70$. Mass fraction contours are shown for $r > 0$ and density contour for $r < 0$. The radii and velocity ratios used were $\Gamma = 2$ $h = 0.7$, respectively.

When the radii ratio is larger, $\Gamma = 3.0$ or $\Gamma = 4.0$, the growth rates of the inner mode are larger by the reduction of the confinement cause by the outer jet. The second mode decrease its amplification, being smaller with a larger Γ ratio. Therefore, the case of $H_2 - O_2$ with $\Gamma = 3.0$ was simulated and the result is presented in Fig. 13. The possible effects of absolute instability are also present in this case, as in all binary cases with $H_2 - O_2$ configuration, however they are not so pronounced as with $\Gamma = 2.0$. Although the inner mode has larger amplification rates they continue to are not relevant to the mixing process.

For $O_2 - H_2$ the configurations with $\Gamma = 3.0$ or $\Gamma = 4.0$ are not interesting cases because increasing the radii ratio the growth rates of the second instability mode are reduced and it remains unaltered for the first mode in values that are not relevant for the mixing process, Fig. ref.

Turning now to the most relevant cases obtained changing the velocity ratio h for the different species configuration. The first case evaluated was $H_2 - O_2$ with $h = 0.5$ as can be seen in Fig. 14. This cases is important because it allows the growth of both modes, Mode I and Mode II, where Mode II has the largest amplification rates for for all cases tested,

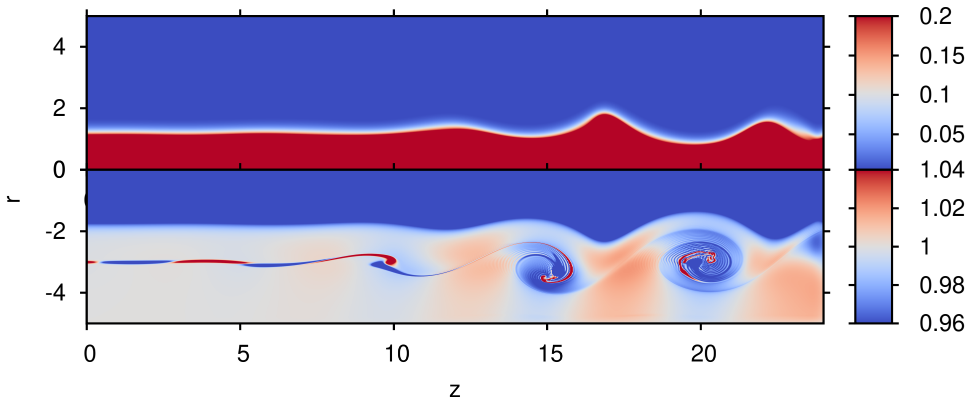


Figure 13. Mass fraction contours and density contours, showing the vortical structures for $H_2 - O_2$ coaxial jet at $t = 70$. Mass fraction contours are shown for $r > 0$ and density contour for $r < 0$. The radii and velocity ratios used were $\Gamma = 3$ $h = 0.7$, respectively.

including the homogeneous cases, Fig. 10. In the same way traces of absolute instability are observed as the other $H_2 - O_2$ cases and effectively the vortical structures are developed faster and these are larger than others cases. For this reason, this configuration will be the most promising for species mixing process.

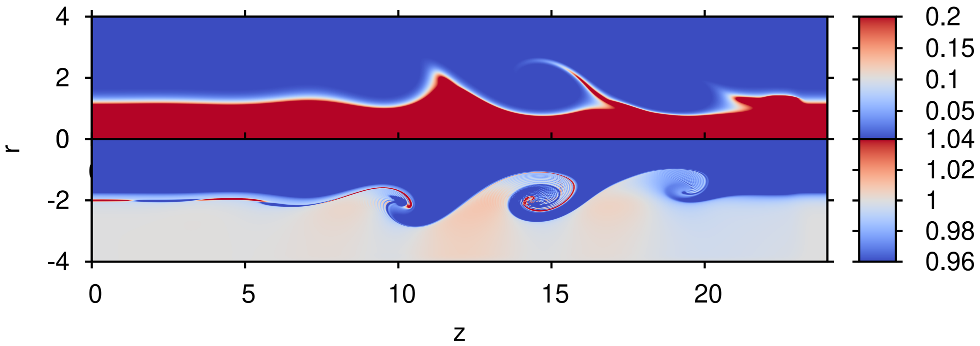


Figure 14. Mass fraction contours and density contours, showing the vortical structures for $H_2 - O_2$ coaxial jet at $t = 50$. Mass fraction contours are shown for $r > 0$ and density contour for $r < 0$. The radii and velocity ratios used were $\Gamma = 2.0$ $h = 0.5$, respectively.

An important point to discuss is that the first instability mode, Mode I, does not appear in none of the $H_2 - O_2$ cases that were simulated. Although the first instability mode exits its amplification rate was not sufficient to form the K-H instabilities waves. This is clear in Fig. 14, where Mode I does not appear due to the confinement effects making it smaller than the homogeneous counterpart.

To finished the HOS and looking for the improvement of mixing between species, the $O_2 - H_2$ with lower velocity ration $h = 0.5$ and with the radii ratio $\Gamma = 2$ was simulated. This radii ratio was chosen because in this configuration the confinement effects are not so pronounced and the outer mode has the largest growth rate. The inner mode has smaller growth rates, as can be seen in the LST section and specifically in Fig. 10. Therefore in the $O_2 - H_2$ arrangement with $h = 0.5$ and $\Gamma = 2.0$ is expected that the growth of Mode II leads to mix of the gases. Fig. 15 shows that the outer mode transport the oxygen situated in the inner jet and the vortical structures formed by the velocity gradients in the hydrogen outer shear layer reach the inner stream. However this configuration is similar to the case already simulated of $O_2 - H_2$ with $h = 0.7$, Fig. 12, since the second mode is not modified by the velocity ratio, but shows that the results of HOS and LST are consistent.

4. CONCLUSION

This work had as main objective the understanding of the stability characteristics of axisymmetric coaxial jets composed of different gases, specifically hydrogen and oxygen. To analyze the stability characteristics of coaxial binary jets

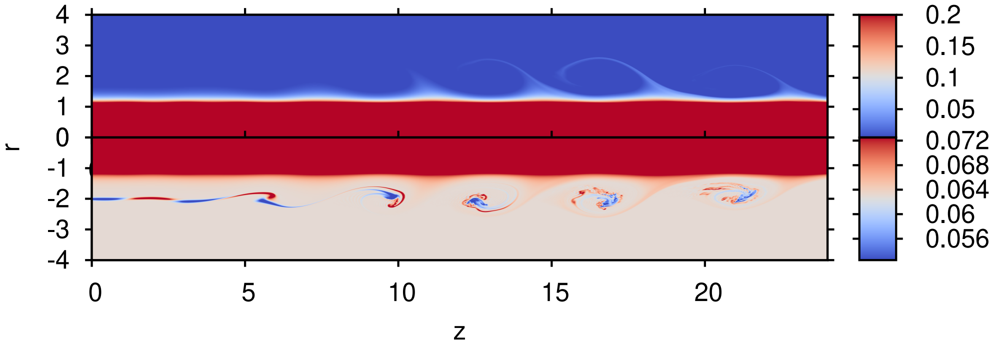


Figure 15. Mass fraction contours and density contours, showing the vortical structures for $O_2 - H_2$ coaxial jet at $t = 70$. Mass fraction contours are shown for $r > 0$ and density contour for $r < 0$. The radii and velocity ratios used were $\Gamma = 2.0$ $h = 0.5$, respectively.

the Linear Stability Theory and High Order Simulation approaches were used.

The cases where the hydrogen was used as species in the inner jet $H_2 - O_2$ the amplification rates of Mode II are larger than the homogeneous coaxial jet, contrarily to what happens in $O_2 - H_2$ configuration where the amplification rates are smaller than the homogeneous case. This agreed with the previous studies in a binary mixing layer, in which when the heavier species is in the lower velocity stream the amplification rates are larger, and vice versa.

However For Mode I, the binary mixing layer results can not be extrapolated for a coaxial binary jet, once the confinement effect, caused by the finite quantity of species that can be place in the inner jet, plays an important role. This effect reduces the amplification rates, which are smaller than the homogeneous cases, however the $H_2 - O_2$ configuration was shown to be more unstable than $O_2 - H_2$ configuration.

Using a low Mach number formulation, the compressible effects were neglected. This formulation together with the compressible formulation also allows to understand the compressible effects cause by the different speed of sound of the species. In the results, the use of the hydrogen, where the speed of sound is almost 4 times the oxygen speed of sound allows that the compressible effects, that reduce the amplification rates, to not be so pronounced. This was more evident in $O_2 - H_2$ configuration that may be considered as low Mach number case, due to fact that the hydrogen composes the outer jet and the ambient where the coaxial jet is ejected. Then, for the outer shear layer formed only by hydrogen the compressible effects are negligible. For the inner jet the mere use of hydrogen also reduced the compressible effects.

Using High Order Simulations (HOS) of the Euler equation as second way to analyses the stability characteristics of coaxial binary jets, the main results of the LST were simulated in order to view different effects neglected by this theory, as: nonlinearities due to mode interaction, the use of a realistic velocity and species profiles not based on canonical equations for the base flow and the visualization of the growth of the instabilities.

High Order Simulations (HOS) allow the visualization of the Kelvin-Helmholtz structures of both unstable modes of the homogeneous coaxial jet. In previous studies, only the LST was used to study the coaxial jets with these particular velocity profiles. The LST has shown that there are different cases where the growth rates of the binary coaxial jet mode may be important to allow the mixing between the species. However as was shown using HOS not all unstable modes promote the mixing between the species.

Other important result of the HOS was to show that not all unstable modes, in special Mode I, with apparent important growth rates calculated with LST, have larger vortical structures which promote the mixing between the species. If the inner mode amplification rates are smaller than $-k_i = 0.1$, these unstable modes grow very little and are not important to the mix of the oxygen and the hydrogen. In these cases the mix between the species depends exclusively on the growth of outer mode, Mode II. The outer mode transport the species situated in the inner jet when the grow of the vortical structures of the outer shear layer reach the inner stream.

High order simulations of the $H_2 - O_2$ cases with its different velocity ratios and radii ratios allows to evidence by the mass fraction contours that these cases are the most appropriated to mix the oxygen and the hydrogen by the larger development of the vortical structures of the outer mode. As have been seen in LST results the most unstable case of this configuration for Mode II are reached with $h = 0.5$ and $\Gamma = 2.0$ showing how the outer mode transport the hydrogen situated in the inner jet.

To confirm the principles of absolute stability present in $H_2 - O_2$ cases, observed both with LST and HOS it is necessary to conduct an absolute stability analysis.

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