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COB-2021-1603 SENSITIVITY ANALYSIS OF THE TORSIONAL VIBRATION OF A DRILL STRING USING SOBOL INDICES

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Abstract. *We are interested in the nonlinear torsional dynamics of a drill string. The aim of this work is to investigate the torsion vibration severity sensitivity with respect to the system parameters. A one degree-of-freedom system is considered, where an imposed rotational speed is imposed at the top and a nonlinear bit-rock interaction occurs at the bottom. Some parameters of the system are considered uncertain (rotation at the top, weight-on-bit, bit-rock interaction parameters) and a probabilistic model is constructed from them. The Sobol strategy is employed to obtain the global sensitivity indices. To better visualise the results, a normalisation is proposed where the maximum variance obtained for all the analyses is considered as a reference to construct the sensitivity maps. The results show that the response of the system is more sensitive to a specific parameter of the bit-rock interaction, and is less sensitive to the rotational speed at the top and the weight-on-bit.*
Keywords: *global sensitivity, Sobol index, stick-slip, torsional vibration, drill-string dynamics*

1. INTRODUCTION

The process of drilling oil wells is expensive. It involves several technical challenges such as controlling a nonlinear dynamics and the difficulty of obtaining accurate real-time measurements. In addition, there are several sources of uncertainties that have been amply studied previously Lobo *et al.* (2019); Ritto (2015); Ritto *et al.* (2017, 2010); Real *et al.* (2018); Ritto *et al.* (2009). Several investigations have assessed the severity of torsional vibration in the drill-string considering two operational parameters: the top drive speed Ω and weight on bit W_{bit}. It is possible to observe boundaries that separate regions with different levels of torsional oscillations Ritto *et al.* (2017); Aguiar *et al.* (2019); Wu *et al.* (2010); Zhu *et al.* (2014).

Global sensitivity analysis has been applied in several areas to provide accurate estimations about how uncertain parameters contribute to the model response. Applications include areas such as hydrological modelling meng Song *et al.* (2013); Devak and Dhanya (2017) and evaluation of geometrical parameters of well-bores, bottom hole assembly, and bits Sarker *et al.* (2017); Arvani *et al.* (2015); Jadoun (2009); Hough (1986).

This research aims to quantify how the uncertainty contained in the bit-rock interaction model and the column operating regime affect the torsional vibration severity. The model chosen to simulates the torsional dynamics of the drill string was the torsional pendulum model. The model is subjected to uncertainties both in the bit-rock interaction and in the operating regime, and the variability of the torsional vibration severity is calculated as a function of these sources of uncertainty. Finally, we identify operating regions in which the uncertainty in the bit-rock interaction and the operating regime has low incidence on the severity of the torsional vibration.

This paper is organised as follows. Section 2presents a detailed description of the torsional pendulum model and the bit-rock interaction model used in this research. Section 3.describes the Sobol method and postulates a normalisation of the method by sub-domains to allow the quantitative comparison of several operational regime sub-domains. Subsequently, in section 4.we present the results based on three comparative scenarios. First, a scenario where the uncertainty is only considered in the parameters of the bit-rock interaction model, then a scenario where only operational parameters are uncertain and, finally, both bit-rock interaction parameters and operational parameters are considered uncertain simultaneously. The conclusions are made in the last section.

2. TORSIONAL MODEL

The structure of the drill-string is composed of two predominant geometries: Drill-Pipes (DP), which are slender pipe sections that are added to the drill-string as the drill progresses; and the Bottom Hole Assembly (BHA), which is stiffer structure than the DP and has the function of supporting various types of equipment such as lateral stabilisers, measuring equipment, and the drill bit. Figure 1 shows a schematic representation of the drill-string, where Ω is the rotational speed imposed on the surface, $\dot{\theta}_{bit}$ is the bit rotational speed and T_{bit} is the torque of the bit-rock interaction.

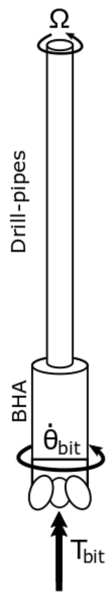


Figure 1. Representation of the drill-string.

The dynamic of the drill-string is influenced by the boundary conditions applied on the surface, the mechanical properties of the drill-string and the bit-rock interaction. The torsional model considers only the rotational degrees of freedom. This is a good approximation when the lateral and axial vibration are small Ritto *et al.* (2017).

2.1 Torsional pendulum model

In this section, the one degree-of-freedom torsional pendulum model is presented. The equation of motion is given by

$$I\,\ddot{\theta}_{bit}(t) + c\,\dot{\theta}_{bit}(t) + k\,\theta_{bit}(t) = T(\Omega) + T_{bit}(\dot{\theta}_{bit}), \tag{1}$$

where t is the time, I is the drill-string inertia, c is a damping constant, k is the torsional stiffness, θ_{bit} is the angle of rotation of the bit, Ω is the nominal rotation speed at the top of the drill-string, T is the torque produced by Ω, and T_{bit} is the torque of the bit-rock interaction. The inertia is written as

$$I = \frac{1}{3}\rho J_p L_p + \rho J_b L_b, \qquad J_p = \frac{\pi}{2}(r_{po}^4 - r_{pi}^4), \qquad J_b = \frac{\pi}{2}(r_{bo}^4 - r_{bi}^4), \tag{2}$$

where J_p and J_b are the polar moments of inertia of the DP and BHA, r_{po} and r_{pi} are the outer and inner radius of the DP, r_{bo} and r_{bi} are the outer and inner radius of the BHA, L_p and L_b are the length of the DP and BHA, and ρ is the density of the drill-string material. Considering the BHA a rigid structure, k can be calculated as a function of the shear modulus G, J_p and L_p:

$$k = \frac{G J_p}{L_p}\,. \tag{3}$$

Additionally, the torque induced by Ω is defined by:

$$T(\Omega) = k\,\Omega\,t + c\,\Omega\,. \tag{4}$$

2.2 Bit-rock interaction model

The bit-rock interaction was modelled as presented in Nogueira and Ritto (2018):

$$T_{bit}(\dot{\theta}_{bit}) = \mu\,R_{bit} W_{bit}\left(tanh(\alpha_o\dot{\theta}_{bit}) + \frac{\alpha_1\,\dot{\theta}_{bit}}{1+\alpha_2\,\dot{\theta}_{bit}^2}\right), \tag{5}$$

where μ, α_o, α_1 and α_2 are positive constants, R_{bit} is the radius of the bit, $\dot{\theta}_{bit}$ is the rotational speed of the bit, and W_{bit} is the axial force applied to the bit which is assumed constant.

3. SOBOL INDEX

Considering a model $Y = f(X_1, X_2, ..., X_k)$ where Y is a scalar representing the model response as a function of X_k independent random parameters. The first-order variance, as a function of X_i, can be defined as:

$$V_{X_i}(E_{\mathbf{X}_{\sim i}}(Y|X_i)), \tag{6}$$

where X_i denotes the random variable i, and $\mathbf{X}_{\sim i}$ denotes a matrix with samples of all other random variables but fixing X_i. Thus, the calculus of $E_{\mathbf{X}_{\sim i}}$ implies assess the mean of Y for samples $\mathbf{X}_{\sim i}$ but keeping X_i fixed. On the other hand, in Eq. 6, the calculation of the variance V_{X_i} is done over all possible sampled values of X_i. This relation defines the first-order variance of the model response Sobol (1993); Janon *et al.* (2014); Sobol (2001); Saltelli *et al.* (2010), and considering the total variance of the model $V(Y)$, the first-order Sobol index S_i can be defined as:

$$S_i = \frac{V_{X_i}(E_{\mathbf{X}_{\sim i}}(Y|X_i))}{V(Y)}. \tag{7}$$

Given the law of total variance Saltelli *et al.* (2007), in Eq. 8 the variance is decomposed into the first-order variability $V_{X_i}(E_{\mathbf{X}_{\sim i}}(Y|X_i))$ induced by X_i, plus residual variability $E_{X_i}(V_{\mathbf{X}_{\sim i}}(Y|X_i))$ originating from the interaction of X_i with all other random variables $\mathbf{X}_{\sim i}$:

$$V_{X_i}(E_{\mathbf{X}_{\sim i}}(Y|X_i)) + E_{X_i}(V_{\mathbf{X}_{\sim i}}(Y|X_i)) = V(Y). \tag{8}$$

Besides S_i, the total Sobol index S_{Ti} is introduced to evaluate the effect of X_i on the total variability of the model. For each independent random parameter, S_{Ti} accounts the first-order effect of X_i plus the highest order effects of X_i on total variability $V(Y)$. Rearranging Eq. 8 as a function of $\mathbf{X}_{\sim i}$, the first-order effect of $\mathbf{X}_{\sim i}$ is $V_{X_i}(E_{\mathbf{X}_{\sim i}}(Y|X_i))$, and the residual effect of $\mathbf{X}_{\sim i}$ is $E_{X_i}(V_{\mathbf{X}_{\sim i}}(Y|X_i))$, this gives the total Sobol index:

$$S_{Ti} = \frac{E_{\mathbf{X}_{\sim i}}(V_{X_i}(Y|\mathbf{X}_{\sim i}))}{V(Y)} = 1 - \frac{V_{\mathbf{X}_{\sim i}}(E_{X_i}(Y|\mathbf{X}_{\sim i}))}{V(Y)}. \tag{9}$$

Since $V_{\mathbf{X}_{\sim i}}(E_{X_i}(Y|\mathbf{X}_{\sim i}))$ is the first-order effect of $\mathbf{X}_{\sim i}$, thus $V(Y)$ minus $V_{\mathbf{X}_{\sim i}}(E_{X_i}(Y|\mathbf{X}_{\sim i}))$ accounts for the contribution of all terms that include X_i in the variance Saltelli *et al.* (2010).

The Sobol indices S_i and S_{Ti} can be interpreted in terms of the expected variance reduction; hence, $V_{X_i}(E_{\mathbf{X}_{\sim i}}(Y|X_i))$ is the expected variance reduction if X_i is kept fixed and $E_{\mathbf{X}_{\sim i}}(V_{X_i}(Y|\mathbf{X}_{\sim i}))$ is the expected variance reduction if all random variables except X_i can be kept fixed, accordingly, $V_{X_i}(E_{\mathbf{X}_{\sim i}}(Y|X_i))$ and $E_{\mathbf{X}_{\sim i}}(V_{X_i}(Y|\mathbf{X}_{\sim i}))$ represent the upper and lower marginal variances respectively.

3.1 Normalisation of the Sobol Index

The Sobol sensitivity analysis was applied to this problem to evaluate how uncertainties induced by the bit-rock interaction, as well as the choice of operating variables Ω and W_{bit}, may affect the severity of torsional vibration.

Considering the torsional vibration severity maps, in which the severity visualisation strategy is to represent it as a function of Ω and W_{bit}, the calculus of Sobol indices is distorted by the choice of each Ω and W_{bit} pair. That happens because we have different severity scales depending on the map region analysed.

To overcome it, the severity map domain was divided into sub-domains and the Sobol indices were normalised as a function of the total variability of each sub-domain $V(Y_{sd})$. Besides, in each sub-domain the uncertainty in the bit-rock interaction parameters (μ, α_1, α_2) and the variation of the operating parameters (Ω, W_{bit}) are taken into account. The expressions of the normalised indices are:

$$S_{i,sd} = \frac{V(Y_{sd})}{\max(V(Y_{sd}))}\left[\frac{V_{X_i}(E_{\mathbf{X}_{\sim i}}(Y_{sd}|X_i))}{V(Y_{sd})}\right], \tag{10}$$

$$S_{Ti,sd} = \frac{V(Y_{sd})}{\max(V(Y_{sd}))}\left[1 - \frac{V_{\mathbf{X}_{\sim i}}(E_{X_i}(Y_{sd}|\mathbf{X}_{\sim i}))}{V(Y_{sd})}\right], \tag{11}$$

where N_{sd} is the number of sub-domains, $sd = 1, 2, ..., N_{sd}$, and $\max(V(Y_{sd}))$ is the largest variability among all sub-domains.

4. NUMERICAL RESULTS

This section presents the results of the sensitivity analysis using Sobol indices in the torsional pendulum model. We evaluate the sensitivity of the torsional vibration severity as a function of uncertainties in μ, α_1 and α_2, joint with the choice of the operational regime Ω and W_{bit}. The sensitivity of the model was evaluated in three scenarios:

1. Considering only the uncertainties in μ, α_1, and α_2 parameters in each sub-domain;

2. considering only the variation of Ω, and W_{bit} in each sub-domain;

3. and finally considering μ, α_1, α_2, Ω, and W_{bit} uncertainties in each sub-domain.

4.1 Global Sensitivity of the Torsional Pendulum Vibration Model

In this test the total analysis range of Ω is between 40 RPM and 240 RPM, this range is divided into 14 regions. Similarly, the total parsing range of W_{bit} is between 50 kN and 220 kN, this range is divided into 12 regions. Thus, 168 sub-domains are analysed in the generation of Sobol indices. The rock drill iteration parameters are sampled from the uniform distributions shown below:

$$\mu = U[0.032 \;\; 0.048] \quad \alpha_1 = U[1.6 \;\; 2.4] \quad \alpha_2 = U[0.8 \;\; 1.2]. \tag{12}$$

The shear modulus employed was $G = 70 \times 10^9$ Pa and the density of steel $\rho = 7850$ kg/m^3. The table 1 records the geometry of the analysed drill-string:

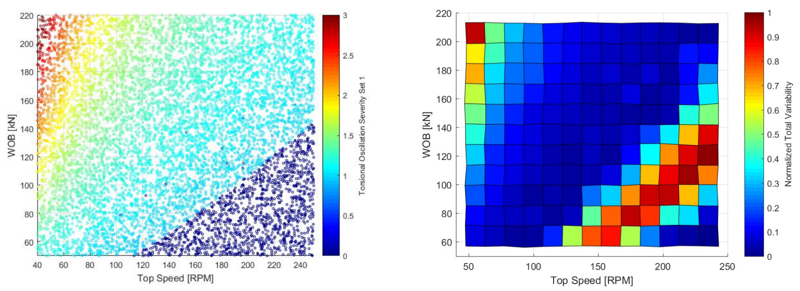
	D_o [m]	D_i [m]	L [m]
DP	0.12	0.095	1800
BHA	0.15	0.095	200

Table 1. Column geometry.

Due to size restrictions (2MB), we will move straight to the the sensitivity analysis of all the parameters.

Sensitivity evaluating all parameters

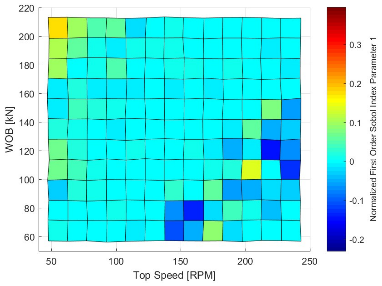
In this section sensitivity analysis has been done allowing μ, α_1, α_2, Ω and W_{bit} to be random variables. The objective of this joint analysis is to evaluate if the interaction between the uncertainties arising from the drill-rock interaction with the variability of the operation regime can bring larger order effects on the global sensitivity.



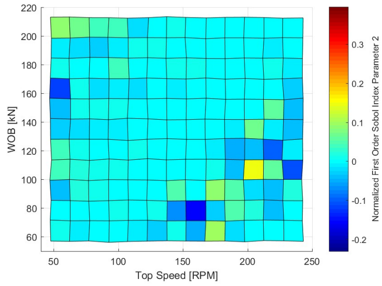
(a) Average torsional vibration severity (b) Normalised total variability

Figure 2. Average and normalised total variability maps of the torsional vibration severity

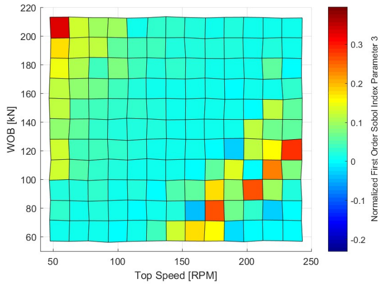
In the figure 2(a), to facilitate the analysis, the mean torsional vibration severity map is again presented. This map was generated with $\mu = 0.004$, $\alpha_1 = 2$ and $\alpha_2 = 1$. In this case, the normalised total variability, figure 2(b), presented a greater width in the high sensitivity region located at the interface of the region without torsional vibration, showing the joint effect of the uncertainties. The second region with the greatest variability was the region of high torsional vibration severity (red region in figure 2(a)).



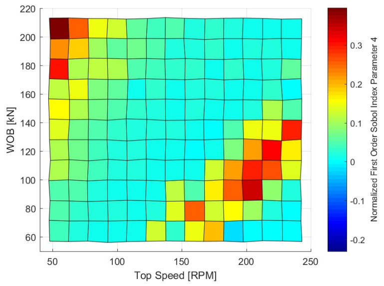
(a) Sobol index for Ω



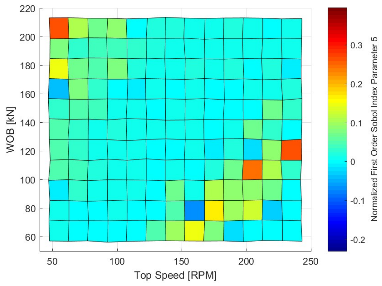
(b) Sobol index for W_{bit}



(c) Sobol index for μ



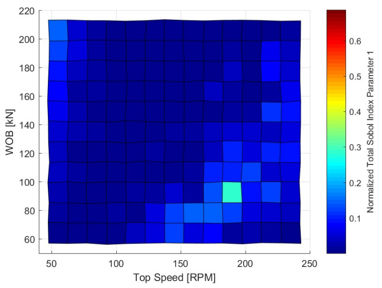
(d) Sobol index for α_1



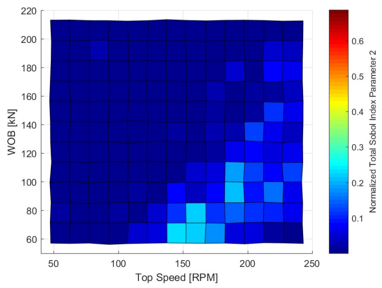
(e) Sobol index for α_2

Figure 3. Normalised First-order Sobol indices varying the parameters Ω, W_{bit}, μ, α_1 and α_2

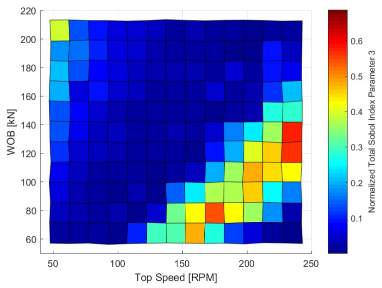
Figure 3 shows the first-order decomposition variability of the torsional vibration severity for each parameter. This confirms again that the torsional vibration severity is more sensitive to the uncertainty in the parameter α_1 both in the interface between the region without torsional vibration and with great torsional vibration severity. On the other hand, for some sub-domains, Ω and W_{bit} had negative first-order Sobol indices. In those sub-domains where the first-order Sobol indices have negative values, the variability introduced by the parameter in question tends to reduce the total variability in that sub-domain.



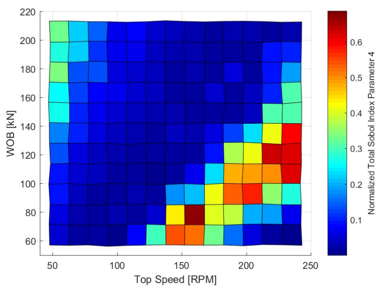
(a) Total Sobol index for Ω



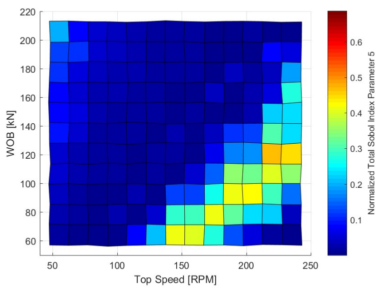
(b) Total Sobol index for W_{bit}



(c) Total Sobol index for μ



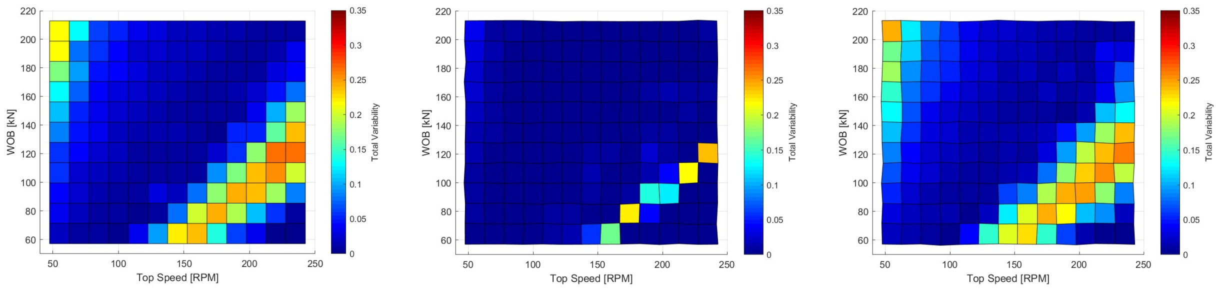
(d) Total Sobol index for α_1



(e) Total Sobol index for α_2

Figure 4. Normalised Total Sobol indices varying the parameters Ω, W_{bit}, μ, α_1 and α_2

The total Sobol indices for this case are shown in figure 4. Analysing the figures 4(a) and 4(b), it is evident that the total Sobol indices of Ω and W_{bit} do not have negative values in any of the sub-domains. This shows that even when the contribution of first-order of Ω and W_{bit} can be negative in some sub-domains, their interaction with the other uncertain parameters impact the global variability of the torsional vibration severity.



(a) Varying μ, α_1 and α_2 (b) Varying Ω and W_{bit} (c) Varying all the parameters

Figure 5. Maps of the total variability of the torsional vibration severity without normalisation

In order to be able to compare, in an absolute way, the total variability of the torsional vibration severity, in each of the three scenarios analysed with the torsional pendulum model, in figure 5 the total variability is presented for each case. The total variability shows that it is necessary to consider all the parameters to obtain an accurate estimation of the torsional vibration sensitivity. Either way, considering the uncertainty only in the parameters of the rock-drill interaction provided a good estimation of the global sensitivity.

5. CONCLUSIONS

In the analyses performed, considering uncertainty only in the parameters of the bit-rock interaction provided a good estimate of the global sensitivity. Although the calculation evaluating the sensitivity of all parameters together provided more accurate results, it is noteworthy that the regions identified as having low variability were similar in all cases.

The samples of μ, α_1, α_2, Ω and W_{bit} were generated considering 20% of uncertainty. Even though the variability introduced in the sampling of all parameters and variables was equal, the sensitivity of the torsional vibration severity to the parameters of the rock-drill interaction should be highlighted. In the drilling process, the uncertainty in the parameters μ, α_1, and α_2 represent the effect of how different types of bits behave with different lithologies.

In the sub-domains where some first-order Sobol indices showed negative magnitude, the total Sobol index exhibited positive values. Taking into account that the total Sobol indices consider the first-order effect of the random variable plus the effect of the interaction of that variable with the other random variables, this result highlights the importance of considering the impact of the interaction between random variables in the global variability of the problem.

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