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BREAKUP OF THIN LIQUID SHEET WITH A VISCOUS INTERFACE

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Abstract. *Thin liquid films play a big role in many real-life applications and are of indisputable interest to scientific and industrial researchers. Understanding the physical mechanisms contributing to the stability of thin liquid films is a challenging problem, as thin films' flows present a fluid-fluid interface which is free to deform. The interface is bounded between two liquids or a liquid and a gas, typically having its own dynamic properties from which surface tension effects and complex interfacial rheological behavior arises. Instability effects are driven by long-range intermolecular forces, also known as van der Waals attractions, and might result in the rupture of the layer. In this work, a computational investigation of the breakup dynamics of a stationary thin liquid sheet bounded by a passive gas with a viscous interface is presented. The Arbitrary Lagrangian-Eulerian method (ALE) is used to track the interface position. The rheological behavior of the viscous interface is modeled by the Boussinesq-Scriven constitutive law, and the numerical solution is obtained through the Finite Element Method. The results show that thin liquid film stability is influenced both by surface rheology and disjoining effects and that the viscous character of the interface delays the sheet breakup, leading to more stable films.*

Keywords: *thin liquid sheets; viscous interfaces; Boussinesq-Scriven; interfacial rheology; surface flow*

1. INTRODUCTION

The dynamics of thin liquid films have fascinated scientists over many decades, presenting several interesting multiphysics effects (Craster and Matar, 2009; Kondic et al., 2020). They are central to numerous areas of engineering, geophysics, and biophysics, can occur over a wide range of length scales in nature, such as lava flows and snow avalanches (Balestra, 2018), tear-film thinning and tear breakup (Zhong et al., 2019) and the liquid lining protecting the pulmonary airways (Grotberg, 1994), and also in many industrial processes such as solid state thin film lithium-ion battery systems (Dudney and Neudecker, 1999), high-resistance thin film resistors, atomization, soft-lithography methods, and several coating techniques such as dip, roll, slot, spin and curtain coating (Becerra and Carvalho, 2011; Bazzi and Carvalho, 2019).

Thin liquid films can generate a host of fascinating behaviors, as they present a fluid-fluid interface which is free to deform. The interface is bounded between two liquids or a liquid and a gas, typically having its own dynamic properties from which surfaces tension effects and complex interfacial rheological behavior arises. Thin films are key elements in the manufacturing of several electronic devices, such as nanoscale lubricant coating of head-disc interfaces in hard disk drives (Izumisawa and Myung, 2002), next-generation ultrathin electronics (Takeda et al., 2016) and also in industrial applications such as aerosol droplet generation (Collins et al., 2012), inkjet printing (Chung et al, 2019) and monodisperse emulsions fabrication (Shah et al., 2008). In nature, the rupture of thin liquid films bounded by a solid substrate and a free surface is one of the core mechanisms in common health conditions, such as the dry eye syndrome (DES), in which instabilities lead the tear film to quickly breakup, leading to a poor coat of the eye surface. The tear film is typically considered a three-layered film of a very thin lipid layer on its free surface, a relatively thick aqueous layer and a bound mucin layer on the bottom, and DES is widely accepted to be caused by hyperosmolarity and tear film instability (Zhong et al., 2019). Another occurrence of pathologies due to thin film instability is the respiratory distress syndrome, where the lungs of prematurely born infants are not sufficiently mature to produce adequate quantities of pulmonary surfactant, giving rise to respiratory difficulties associated with airway closure, pulmonary edema and mechanical damage of the airway linings (Jensen and Grotberg, 1992).

The flow of free thin liquid sheets is an intrinsic example of multiphase flow. The temporal evolution of fluid phases in a multiphase flow is often a very difficult task to predict due to their dependence on several parameters such as geometry, fluid properties and flow regime, and are commonly characterized in dispersed flows and separated flows (Kassar, 2016). Numerical simulations of multiphase flows rely mainly on a Multi-Fluid model and a One-Fluid model approaches, in which the first model consider mass, momentum and energy conservation equations for each phase through a phase average procedure and the dynamics of each phase is coupled with each other through interfacial interaction terms; whereas the latter model solves only one set of conservation equations for the whole domain, usually done by tracking the interfaces between the fluid phases (Prosperetti and Tryggvason, 2007). The appropriate values of fluid properties are used according to each phase region and the coupling between phases is established by the proper description of surface tension forces.

Moreover, the breakup of thin liquid films bounded by two free surfaces, or free liquid films, is of fundamental importance, as they permit the generalization of various dispersed systems (Sheludko, 1967; Thete et al., 2015, Vegad et al., 2019). Intermolecular van der Waals attraction between molecules in the liquid can cause surface fluctuations, which lead the film to destabilize and rupture despite the stabilizing action of surface tension (Thete et al., 2016). Furthermore, the ultimate lifetime of liquid films is determined by two processes: thinning and growth of surface fluctuations, and the rupture gives rise to a finite time singularity in the governing equations. These surface fluctuations are due to the geometric configuration of thin liquid sheets, which results in a difference in the chemical potential with respect to large volumes of the same fluid, culminating in changes in all intensive thermodynamic properties. In both industry and nature, these surface fluctuations are often stabilized by the presence of surface-active agents (surfactants) that can extend the life span of thin films in a considerable manner. The response of the interface in these cases is strongly dependent on the chemical composition of the surfactants, which may reduce the drainage rate of the thin films according to the mechanisms by which the surface-active agents operate (Bhamla et al., 2017).

2. LITERATURE REVIEW

As previously mentioned, the growth of surface fluctuations can be spontaneous or induced, driven by long range intermolecular attractions known as van der Waals forces that trigger the rupture process. These intermolecular attractions arise from fluctuations in the electromagnetic field present in the medium, thus resulting in changes on intensive thermodynamic properties. In view of the above, Lifshitz (1955) developed a rigorous macroscopic theory for the interaction of bodies whose surfaces are brought within a small distance. Then, Dzyaloshinskii et al. (1959) derived general formulas for the thermodynamic quantities such as chemical potential of thin liquid films under the action of van der Waals forces.

2.1 Thin liquid sheets

The instability of free liquid sheets was first analyzed by Taylor (1959), aiming to understand wave propagation on the surface of free films. The author identified two main dynamical modes for freely suspended sheets that are associated with the film rupture, namely a symmetric mode – the motion of opposite surfaces of the film toward each other – and an antisymmetric mode – buckling of the interface while maintaining a constant thickness. Later on, Brown (1960) carried out an investigation to understand the behavior of a thin liquid sheet falling on a moving surface, describing the curtain coating process. Then, Sheludko (1967) observed that the amplification of spontaneous fluctuations in free liquid films are operative for sheet thickness ranging between 10 ηm and 100 ηm, and that nonlinearities arise due to long range intermolecular effects. The understanding of thin film rupture was extended theoretically by the derivation of a nonlinear evolution equation for the thickness of a thin film on a solid substrate by Williams and Davis (1982).

Prévost and Gallez (1985) took a similar approach and investigated the nonlinear effects on the stability of free film rupture, and declared that the squeezing mode firstly reported by Taylor is the more appropriate mode to describe the mechanism of rupture of a free liquid film. Moreover, Lin et al. (1990) performed linear stability analysis of a viscous liquid sheet and showed that there are two independent modes of instability, namely sinuous and varicose modes, in the presence of ambient gas. Through approximations of long wavelengths, Erneux and Davis (1993) considered the Navier-Stokes equations with an extra term to encompass the van der Waals attraction to derive asymptotically a system of governing nonlinear evolution equations for longitudinal velocity and film thickness. Ida and Miksis (1996) then solved numerically the set of equations proposed by Erneux and Davis (1993) and examined the dominant balances in the evolution equations using similarity-times solutions in the temporal and spatial vicinity of rupture.

The influence of the chemical composition of a free surface has long been recognized to change its dynamics (Scriven, 1960), thus asserting the importance of interfacial rheology for the understanding of free surface phenomena. Jensen and Grotberg (1992) used lubrication theory to study the spreading rate of a localized monolayer of insoluble surfactant on the surface of a thin viscous film, in the limit of weak capillary and weal surface diffusion. De Wit et al. (1994) performed an analogous study, in which they analyzed the dynamics of free liquid films with insoluble surfactants. They derived a system of nonlinear evolution equations asymptotically from the full Navier-Stokes equations for free films and incorporated the effect of van der Waals attraction, capillary forces and Marangoni forces due to gradients of surface tension.

Vaynblat et al (2001) extended the studies on the rupture dynamics by introducing two main rupture geometries, line rupture and point rupture. They considered a free viscous film exhibiting both simple Newtonian and more complex power-law rheology, and analyzed the self-similar dynamics that arise during sheet rupture. In order to further understand how the rheological properties of the liquid influence a free liquid sheet stability, Becerra and Carvalho (2011) proceeded with an examination regarding the flow of free viscoelastic liquid sheets occurring in curtain coating processes, and showed that high extensional viscosity due to rheological behavior of polymer solutions on the flow create more stable curtains. More recently, Thete et al. (2015) examined the self-similar evolution of free liquid films of power law fluids towards the rupture spatial and temporal location through numerical methods. They identified two regimes of thinning depending on the power-law index $0 < n \leq 1$ of the fluid, in which for $6/7 \leq n \leq 1$, the sheet thins and ruptures under the competition between inertial, viscous and van der Waals forces, and for $n < 6/7$, the dominant balance of forces changes and viscous force becomes asymptotically negligible and the sheet thins and ruptures under inertial, van der Waals and surface tension forces. Later on, Thethe et al. (2016) have also revealed that the scaling exponents for film thickness in any power-law index have the same value regardless of whether the film undergoes line or point rupture, complementing the findings of Vaynblat et al. (2001). Furthermore, Bazzi and Carvalho (2019) addressed the effect of viscoelastic properties of polymer solutions of the breakup process of a free liquid sheet. They studied the response of stationary Newtonian and viscoelastic thin liquid sheets to planar and axisymmetric disturbances by extending the stability criterion proposed by Erneux and Davis (1993) for axisymmetric perturbations and Oldroyd-B liquids, and showed through numerical solutions that the effect of rheology slows down the perturbation growth drastically and thus slowing the sheet rupture.

Experimental analyses also show that thinner stable liquid curtains can be obtained with viscoelastic liquids, even though the underlining physical mechanisms are not completely understood. Karim et al. (2018) studied the effect of viscoelastic forces on the hole growth speed in a liquid curtain by high-speed visualization and showed that elastic stresses stabilize the liquid curtain by both the reduction on the retraction rim speed observed with viscoelastic solutions and by the delay on the growth rate of any disturbance that may lead a hole in the curtain. Moreover, Karim et al. (2019) performed a thorough analysis on the rheological behavior of dilute xanthan gum solutions and reveal that the dynamics of curtain breakup is governed by the shear viscosity at a characteristic deformation rate. They also showed that for Newtonian liquids, the stability increases as the characteristic viscosity rises. Additionally, Karim et al. (2020) proposed the use of a two-layer configuration with a shear-thinning liquid as the bottom layer and a viscoelastic liquid as the top layer to delay the breakup of the liquid sheet and to reduce the minimum film thickness. They stated that the thickness reduction is a function of the viscoelastic stress of the top layer liquid, and reported a film thickness reduction of 60% for an aqueous solution of polyethylene oxide with 0.1 wt% as the viscoelastic fluid. The additional mechanisms from rheology therefore contribute on the delay on the growth rate of the perturbation in a free thin liquid film, leading to substantial changes in the dynamics with respect to Newtonian fluids.

Also, a growing number of emerging technologies involve the manipulation of liquid metals from millimeter to sub-micron scales, such as additive manufacturing, electronics, solar cells, plasmonic-related applications, among others, in which inertial effects play a big role on the thin film dynamics and lubrication theory cannot be used to describe the local flow close to rupture (Kondic et al., 2019; Moreno-Boza et al., 2020). In this context, González et al. (2015) put in evidence the effects of inertia and the bidimensional aspects of a flat liquid film extended over a solid plane and performed numerical simulations of the nonlinear Navier-Stokes equations and analyze to which extent the linear predictions can be applied for cases involving inertial and different aspect rations of the film. They found that inertia does not lead to new regions of instability, but their results show that the growth rates of the instability decrease as inertial effects are stronger.

2.2 Interfacial rheology

For an open system of variable interface area, the minimum work required to evolve the free surface in time is the product of the interfacial tension and the change in interface area. The adsorption of surfactants at low concentrations at some or all the interfaces in the system therefore significantly changes the amount of work required to change those interfaces (Rosen and Kunjappu, 2012). To that sense, interfacial rheology deals with the flows and additional surface stresses and deformations at liquid interfaces induced by these complex mechanical quantities. Complex fluid interfaces are often characterized by the presence of amphiphilic molecules such as surface-active agent (surfactants), proteins and particles that induces microstructures with significant mechanical strength, along with possible thermodynamic complexities. Therefore, the response of interfaces against deformations depends on their composition and main mechanical quantities such as elasticity and viscosity of dilatation and shear, respectively (Krägel et al. 2010; Fuller and Vermant, 2012).

A rheologically complex interface is often modeled through interfacial constitutive models that account the extra and deviatoric contributions to the interfacial stress from the interfacial microstructure. Thus, a general expression for the interfacial stress for a complex interface is given by (Slattery et al., 2007):

$$\sigma_S = \sigma(\Gamma, T) I_S + \sigma_E \ , \tag{1}$$

in which $\sigma(\Gamma, T)$ is the interfacial tension which depends on the excess concentration Γ and temperature T, I_S is the surface unit tensor that transforms every vector into its component tangential to the interface and σ_E is the surface extra stress. A general form of equations for a linear dependence of stress on rate of strain in the interface was first introduced by Boussinesq (1913) and later generalized by Scriven (1960) to account for the evolution of Newtonian fluids in the interfacial state. Thus, for a liquid-like viscous interface, the extra stress can be described by the Boussinesq-Scriven model:

$$\sigma_E = (\kappa_S - \eta_S)(\nabla_S \cdot u)I_S + 2\eta_S D_S \ , \tag{2}$$

Where κ_S is the interfacial dilatational viscosity, η_S interfacial shear viscosity, $\nabla_S = I_S \cdot \nabla$ is the surface gradient operator, u is the velocity vector on the interface and $D_S = 1/2\ (\nabla_S\, u \cdot I_S + I_S \cdot (\nabla_S\, u)^T\ )$ is the surface rate-of-deformation tensor. The Boussinesq-Scriven model is a Newtonian model for viscoelastic interfaces derived from a differential geometry approach in tensor form.

2.3 Interface tracking

The One-Fluid model approach is often implemented for numerical analyses of separated flows, as the prediction of dispersed flows through the aforementioned approach will require extremely fine grids, thus increasing the computational cost of the solution drastically. There are several methods to track interfaces in literature, which can represent the interface explicitly or implicitly and proceed with the tracking though a Lagrangian or Eulerian approaches. The Front-Tracking method (Unverdi and Tryggvason, 1992) and the Marker and Cell (MAC) method (Harlow and Welch, 1965; Daly, 1967) are examples of tracking strategies employed through a Lagrangian reference system and track and follow the interface according to the local velocity explicitly and implicitly, respectively. Both methods require a dynamic deletion or addition of particles when stretching or shrinking of the interface occur, and the MAC method requires a large number of particles to track the interface, which is computationally expensive.

The high computational cost of the tracking of large number of particles led to the development of methods that rely on different strategies to track the free boundary. The use of a marker (color) function in an Eulerian reference system instead of Lagrangian particles led to the development of approaches such as the Level-Set (LS) method and the Volume of Fluid (VOF) method. The LS method (Osher and Sethian, 1988) identifies the interface through a smooth signed distance function γ, in which the interface lies in the zeroth level and its evolution is performed by advecting γ at each time step. However, the magnitude of $\nabla\gamma$ can become too large or too small close to the interface, leading to inaccurate calculations of variables dependent of the interface position such as the normal vector and mean curvatures. Moreover, the method may also not conserve mas in flows with significant vorticity or with high deformation of the interface. To mitigate these problems, the distance function γ must be reinitialized in every time step after the interface evolution process.

The VOF method (Prosperetti and Tryggvason, 2007) tracks the interface indirectly using a scalar marker α that denotes the volume fraction of the reference phase in each discrete element, taking $\alpha = 1$ in regions fully occupied by the reference phase and $\alpha = 0$ otherwise. The scalar marker function α is also known as the volume fraction function, and cells with $0 \leq \alpha \leq 1$ yields the interface position which is advected in time in an Eulerian manner. The VOF is one of the most widely used interface tracking methods and one of its advantages is that it presents good mass conservation properties due to its formulation. However, the accuracy on computation of curvature is still a key issue in the method, albeit several works aimed at overcoming the issue (Ghobadian, 1991; Ubbink and Issa, 1999; Gopala and van Wachen, 2008). The VOF and LS methods track the interface implicitly and have similar ways to advect their respective marker function, and the main difference between the methods is that the LS marker function γ is smooth, in constrast with the VOF step-like marker function α. Nonetheless, both methods can lead to inaccurate curvature computations, which are known to generate spurious currents that result in non-physical pressure fields (Klostermann et al., 2013). In short, Lagrangian descriptions allow an easy tracking of free surfaces and facilitates the treatment of materials with time-dependent constitutive relations but are unable to follow large distortions of the computational domain without remeshing operations, whereas Eulerian descriptions can handle large distortions in the continuum motion with relative ease, but generally at the expense of precise interface definition and resolution of flow details (Donea et al., 1982).

The shortcomings of purely Lagrangian and Eulerian descriptions lead to the development of an alternative technique that combines the best features of both descriptions in a mixed manner. This technique is known as the Arbitrary Lagrangian-Eulerian description. The arbitrary Lagrangian-Eulerian description represents a generalization of the two previous methods and treats the computational mesh as a reference frame which may be moving with an arbitrary velocity ω (Hughes et al., 1981). If $\omega = 0$, the reference frame is fixed in space and corresponds to the Eulerian coordinate system, whereas $\omega = v$ indicates that the reference frame moves in space at the same velocity as the grid points, corresponding to the Lagrangian reference system. For the case $\omega \neq v \neq 0$, the reference system is called arbitrary Lagrangian-Eulerian (ALE) frame and moves in space at a velocity ω. The ALE reference frame is identified by its instantaneous position vector ξ, which is a priori arbitrary and consequently independent of the motion of the mesh grid points. The position vector ξ is linked to the material variables indirectly through the material coordinates in the initial configuration of the computational mesh. Thus, the ALE description is taken as a mapping of the initial configuration of the computational mesh into the current configuration of the reference frame.

3. METHODOLOGY

In this work we consider the One-Fluid model to numerically analyze the breakup dynamics of a stationary free thin liquid sheet in a 2D configuration, considering the effects of interfacial tension and van der Waals forces and under a symmetric perturbation regarding Taylor's dynamical modes. Thus, we can ease the computational cost of the simulation by considering a symmetry line on the x axis. Also, the fluid is incompressible and the system is isothermal. We consider that the surrounding gaseous phase is inert and therefore does not interact with the thin liquid sheet. Marangoni advection and gravity effects are neglected in our study, and the interfacial rheology is taken in two cases, namely a passive interface with constant interfacial tension and no extra stress and a viscous interface and long-range van der Waals intermolecular forces are considered as de-stabilizing mechanism of the system. The interface is thought of as a sharp interface and the extra stress term of the viscous interface is modeled by the Boussinesq-Scriven constitutive law. The system is implemented through the Finite Element Method and it is tracked through the Arbitrary Lagrangian-Eulerian method. The numerical methods are implemented in Python through the FEniCs open-source library.

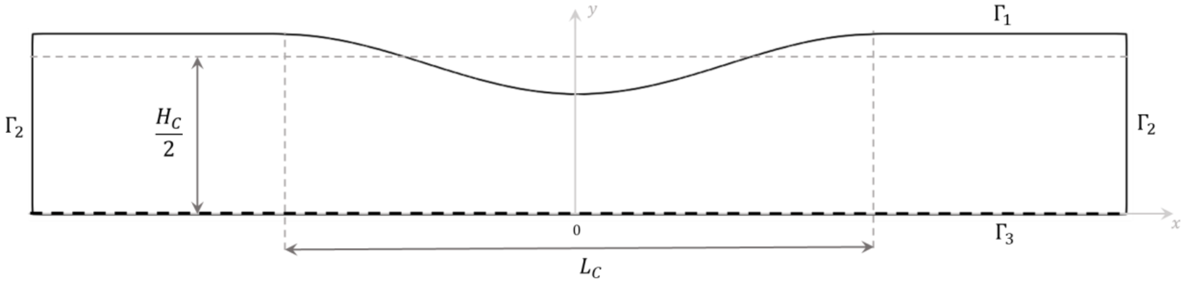

Figure 1. Initial configuration of the stationary free thin liquid sheet.

Figure 1 represents the initial configuration of the domain, in which H_C is the undisturbed film thickness, L_C is the perturbation wavelength and $\Gamma = \Gamma_1 \cup \Gamma_2 \cup \Gamma_3 \cup \Gamma_2$ is the domain boundary. The interface position is denoted as $h(x,t) \subset \Gamma_1$, the initial conditions used are $u(x,y,t=0)=0$ and $h(x,t=0)=\frac{H_C}{2}-\epsilon\cos(\pi x)$, where ϵ is the perturbation amplitude. According to Erneux and Davis (1993), in the limit of $H_C/L_C \ll 1$, the free thin liquid sheet is stable when

$$\frac{\mathrm{S}}{\mathrm{A}} \geq \frac{2}{\pi^2} \ , \tag{3}$$

where $S = \sigma_{\alpha\beta}\rho H_C/3\mu^2$ is a nondimensional constant that accounts the dimensional surface tension $\sigma_{\alpha\beta}$ and $A = \rho L_C \tilde{\mathrm{A}}/6\pi H_C^3\mu^2$ is a dimensionless constant which encompasses the van der Waals effects through the Hamaker constant Ã. Hence, the system configuration will take into account parameters that break the stability criterion presented in Eq. 3.

3.1 Governing equations

Considering the One-Fluid model, the conservation of mass, in the absence of mass transfer, is given by

$$\nabla \cdot \vec{u} = 0 \ , \tag{4}$$

The conservation of linear momentum is given by

$$\rho_b\left(\frac{\partial\vec{u}}{\partial t} + \vec{u}\cdot\nabla\vec{u}\right) = \nabla\cdot\boldsymbol{T} \ , \tag{5}$$

in which $\boldsymbol{T}(\vec{u},p) = -p\boldsymbol{I} + 2\mu_b\epsilon(\vec{u})$ is the Cauchy stress tensor and $\boldsymbol{\epsilon}(\vec{u}) = [\nabla\vec{u} + (\nabla\vec{u})^T]$ is the rate-of-deformation tensor. ρ_b and μ_b are the bulk density and viscosity, respectively. The boundary conditions are defined as follows: the motion of the interface is determined by a kinematic condition (Eq. 6) and a stress balance condition (Eq. 7) to account for the jump of the interface traction across Γ_1.

$$\Gamma_1: \quad \hat{n}\cdot u = \hat{n}\cdot u_S, \tag{6}$$

$$\hat{n}\cdot\boldsymbol{T} = \nabla_s\cdot\boldsymbol{\sigma_s} - (p_S + \Phi)\boldsymbol{I_s} \ , \tag{7}$$

where u_S is the surface velocity, p_S is the surface pressure and $\Phi = \tilde{A}/2\pi y^3$ is the long range intermolecular van der Waals potential. Left and right boundaries (Γ_2) are considered free to flow (Eq. 8) and bottom boundary (Γ_3) is considered a symmetry (Eq. 9).

$$\Gamma_2: \quad \frac{\partial u}{\partial x} = 0, \tag{8}$$

$$\Gamma_3: \quad u_y = \frac{\partial u}{\partial y} = 0, \tag{9}$$

3.2 Numerical method

The finite element method is a numerical technique to obtain an adequate representation model of continuous problems using a finite number of well-defined components. It relies on the process of subdividing all systems into their individual components or "elements," whose behavior is readily understood, and then rebuilding the original system from such components. Each of these elements are assumed to be precisely known, in which typical elements e_i associated with a specified number of nodes and vertices are organized in a common coordinate system (x, y) as an assembly of finite elements that represent a domain of interest. Furthermore, the assembly of elements describing the domain of interest is referred as a mesh.

The finite element space is thus a space of piecewise polynomial functions on a domain determined by the mesh, a finite dimensional space of polynomial functions on each element called shape functions and a set of functionals on the shape functions of each element called degrees of freedom (Arnold and Logg, 2014). Each degree of freedom is associated to a generalized face of the element and specifies a quantity which takes a single value for all elements sharing the face. In this thesis we decompose the domain into triangular unstructured elements and use P2-P1 element pairs, also referred as Lagrange or Taylor-Hood elements. These type of elements yields a stable finite element discretization and high-order accuracy of approximations.

4. PARTIAL RESULTS

Figures 2, 3, and 4 are results of the dynamics of free thin liquid sheet of aqueous solution with a passive interface, represented in a top-down order, considering $H_C = 10\mu\text{m}$, $\epsilon = 0.1\, H_C$ and $S/A = 1/\pi^2$ (see Telis et al., 2007 and Dadkhah et al., 2016 for reference values of bulk properties). Figure 2 represent the time evolution of the pressure field, whereas Figures 3 and 4 are the x and y components of the velocity field, respectively. The effects of long-range intermolecular van der Waals attraction depend on the thin sheet thickness, which yields in a pressure gradient from the region of minimum sheet thickness towards the sheet extremities that drives the drainage process.

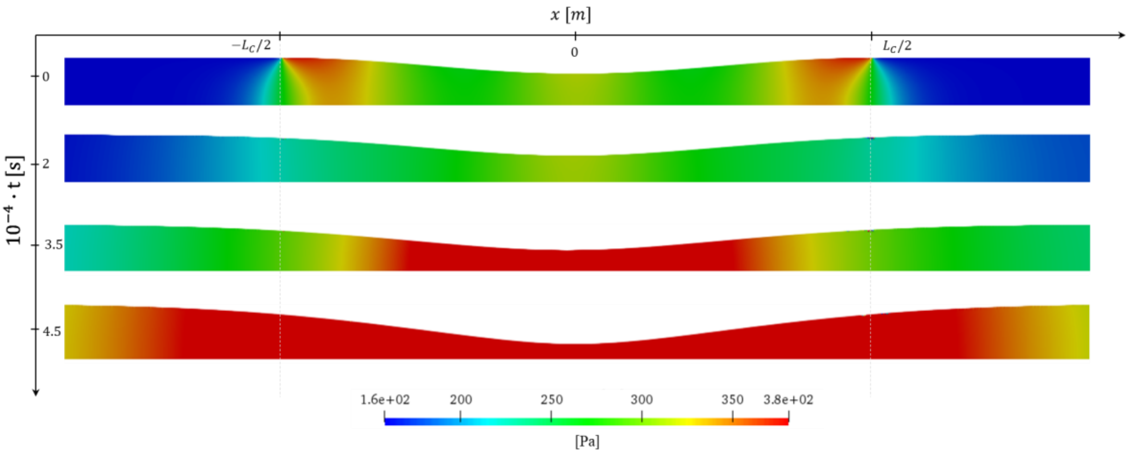

Figure 2. Evolution of the pressure field during the breakup process.

Moreover, the drainage of the thin liquid sheet is observable through the direction of the bulk flow. An outwards flow from $x = 0$ is portrayed in Figure 3, which indicates that the drainage effects are stronger as the thin film approaches the rupture. This behavior can also be observed in Figure 4, as the increase of the y component of the velocity field in the region of minimum thickness is resulted by van der Waals effects.

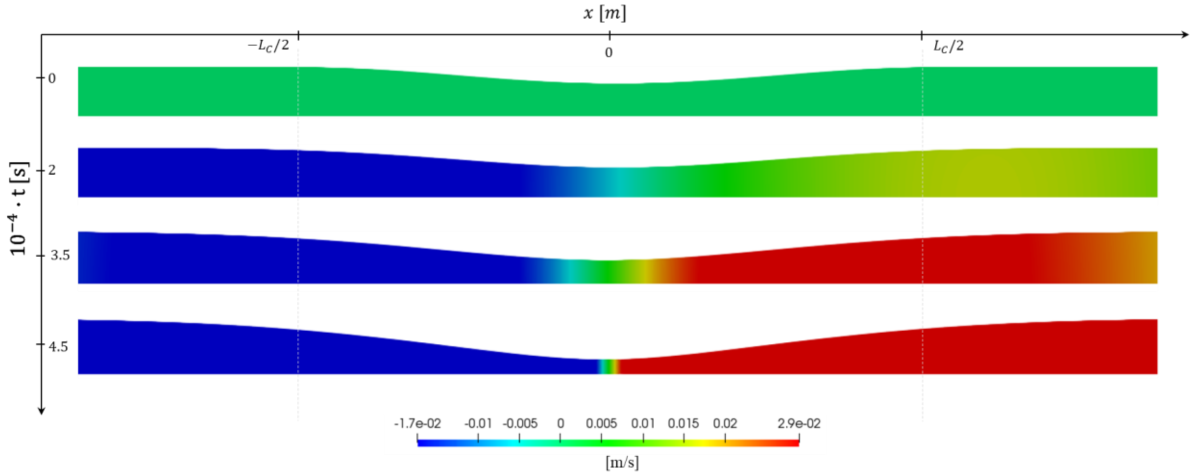

Figure 3. Evolution of the x component of the velocity field.

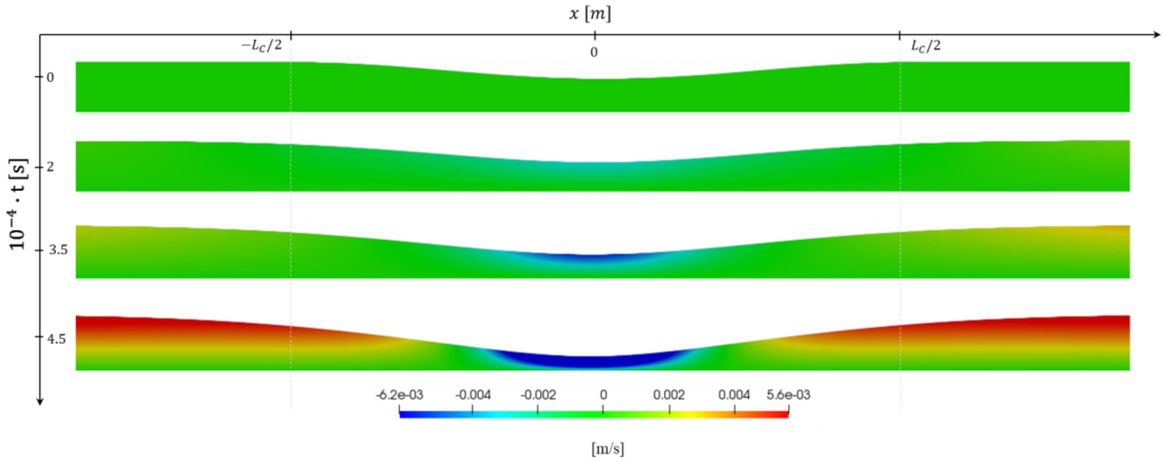

Figure 4. Evolution of the y component of the velocity field.

During the breakup process, van der Waals effects become more significant as the thin film thickens, which yields a nonlinear balance between interfacial tension effects and van der Waals forces. Figure 5 presents the time evolution of the minimal thickness region of the thin film, in which the nonlinear behavior of the system can be observed.

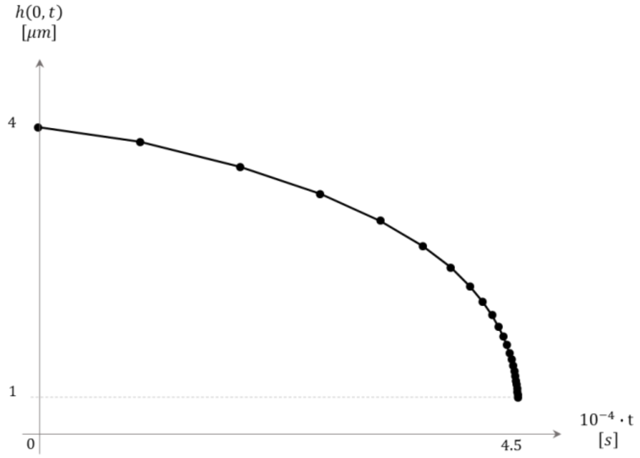

Figure 5. Evolution of the free thin sheet thickness at $x = 0$ at $S/A = 1/\pi^2$.

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