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# DEVELOPMENT OF HEAT AND MASS TRANSFER MODELLING FOR COUNTER-FLOW COOLING TOWER

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**Abstract.** *The present work aims to model and design a counter-flow cooling tower equipment. The cooling tower is designed to meet the thermal load of a single-effect absorption chiller for a thermal power plant. The methodology is based on a discretization method that divides linearly the cooling tower in several increments. The software Engineering Equation Solver is used to develop the heat and mass transfer modelling, therefore, allowing to predict the thermodynamic states of the air stream and cooling water across the cooling tower. The humidity ratio, dry-bulb and wet-bulb temperatures, make-up water, heat and mass transfer rates, tower characteristic and other thermodynamic parameters are analyzed for each increment. The results show how different input data can influence not only the sizing of the cooling tower but also the heat and mass transfer across this equipment. For the experimental thermal system under a range and approach of 5.6 and 4°C, respectively, the cooling tower will transfer heat rate around 1968 kW, presenting a make-up water of 2614 kg/h and tower characteristic of 0.959. The results from this work will enlighten how to physically model and simulate properly the cooling tower rather than treat it as black box modelling.*

**Keywords:** *Cooling Tower, Discretization Approach, Heat and Mass Transfer, Engineering Equation Solver.*

## 1. INTRODUCTION

Cooling tower is a well-known thermal dissipative equipment which rejects heat to the environment in several different applications, since heating, ventilation, and air conditioning (HVAC) systems to power plants and cogeneration facilities (Kröger, 2004; Hensley, 2009).

In the past, it was utilized the hydrosphere technique to discharge heat to the environment in industrial plants due lowest costs and simplest operating requirements. Notwithstanding, over the years, cooling towers had been assuming an essential role as good alternatives because of the complex restrictions regarded to the utilization of natural water in heat exchangers. Usually, amounts of hot water were returned to the river, ocean, or dam, impacting negatively the local ecosystems and generating environmental problems (Kröger, 2004).

Bearing in mind that cooling tower became more frequent in the industrial facilities around the world, it provoked an interest on analyzing the performance of this dissipative equipment. Hence, urging more researches about the performance of this heat sink. In the literature, there are a variety of reliable theoretical books providing mathematical modelling under simplifying assumptions that allows to predict the physical phenomena inside the cooling towers (Threlkeld, 1970; Stoecker and Jones, 1985; Perry *et al.*, 1997). Besides that, more detailed theoretical approaches can be encountered in Kloppers (2003) that discusses a few methods such as Merkel, Shebyshev, Pope and effectiveness with number of transfer units ( $e$ -NTU) analyses for cooling tower modelling.

Regarding more complex cases, there is a study that analyzed the performance of the cooling tower in the rain zone by using experimental drop size data with computational fluid dynamics (CFD) simulations (Pierce, 2007). The loss coefficient and Merkel number were evaluated during the simulations, permitting to validate some correlations from the literature. Another practical investigation is reported by Herman (1991) who analyzed the

performance of the cooling tower fan based on field data. In his work, experimental data from the site were used to determine the minimum set point temperature of the cooling water at the condenser’s inlet based on minimum energy consumption. Also, it was validated with an analytical modelling developed by Joyce (1990) for a chilled water system, comparing experimental results with theoretical approach.

Although theoretical analyses are fundamental for cooling tower modelling, it is crucial to expose here that some manufacturers had been providing practical guidelines in order to select and perform different cooling towers based on some characteristics, such as type of components, materials, design and operation considerations (Mulyandasari, 2011). Furthermore, handbooks from ASHRAE (2008) and Stanford’III (2016) provide practical information about the main technical aspects of cooling towers combined with HVAC systems as well as the required theory that is necessary to model mathematically.

In this paper, a mathematical modelling for a wet-cooling tower with mechanical draft and counter-flow configuration is carried out based on a linear discretization method. The goal is to investigate how the air stream is thermodynamically changing across the cooling tower. Usually, when the cooling tower is treated as a black-box modelling, it is assumed a saturated air stream at the exit of the cooling tower. This consideration is not completely true, but seems to be a reasonable practical assumption. The major contribution is to present the mass and heat transfer modelling combined with thermodynamic principles and linear discretization approach as a better alternative on analyzing the thermodynamic states and other parameters inside the cooling tower. The Engineering Equation Solver (EES) software is used due to the thermodynamic library for humid air properties. Additionally, EES platform is friendly-user, which permits to develop easily mathematical equations.

## 2. CASE STUDY

Regarding the case study of this work, it is derived from a Research and Development project (ANEEL PD-06483-0318/2018) in partnership with a thermal power plant called Luiz Oscar Rodrigues de Melo (UTE LORM), which is located at Linhares, Espírito Santo, Brazil. This facility generates electric power through 24 generator sets of Wärtsilä 20V34SG engine of 8.7 MW.

In short, an experimental thermal system is designed and proposed to operate with one engine, as it can be seen in Figure 1. The major components of the new thermal system are single-effect absorption chiller, cooling coil, auxiliary heat exchanger, waste heat recovery unit, and cooling tower.

The cooling tower is strategically used to meet the dissipative heat transfer rate from the absorption chiller, and provide simultaneously cooling effects at the auxiliary heat exchanger. This configuration allows further reduction of the temperature on engine cooling water at the radiator’s outlet.

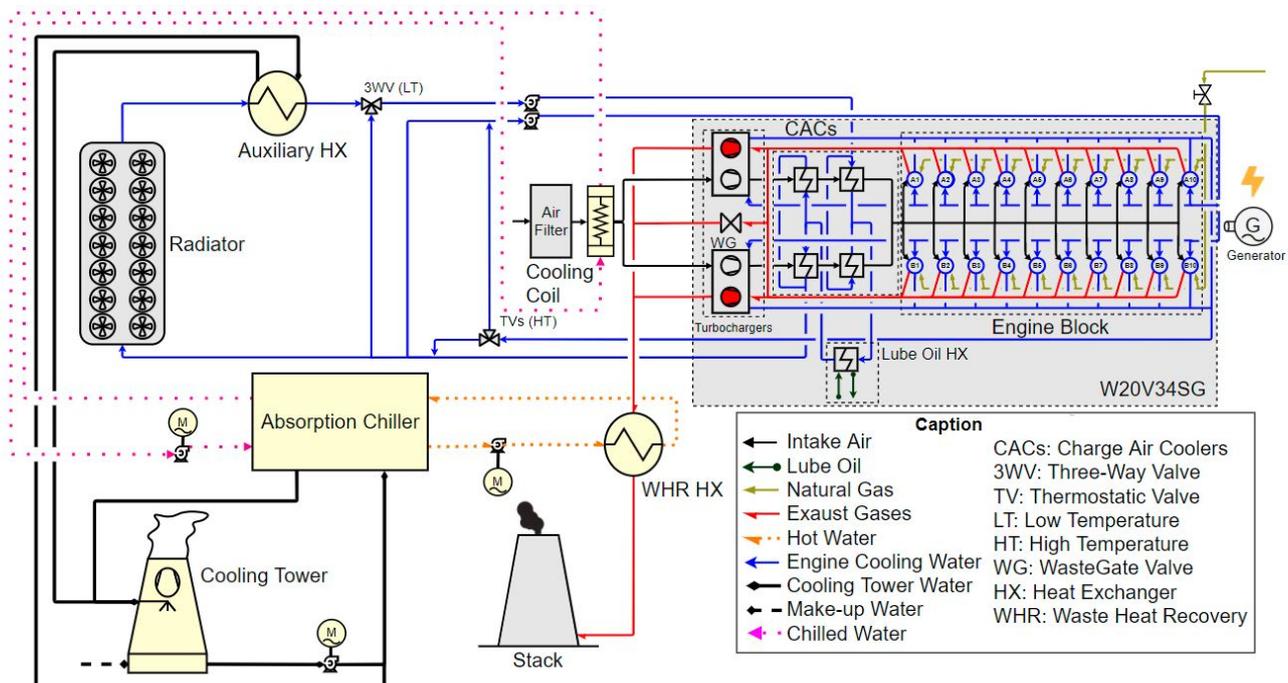


Figure 1. Flowsheet of the experimental thermal system coupled to the Wärtsilä 20V34SG engine.

As the cooling tower depends on the ambient site conditions, it is required to track the wet-bulb temperature over a period of time. For this case study, the climate data are consulted from a nearby meteorology station to the thermal power plant (INMET, 2020). Figure 2 represents the wet-bulb temperature profile throughout the year. The critical ambient condition is assumed when there is the highest peak of wet-bulb temperature ( $T_{wb,amb} = 27.81^\circ\text{C}$ ). During the simulations, an investigation, regarding the ambient conditions, is developed in order to analyze the performance and sizing of the cooling tower unit.

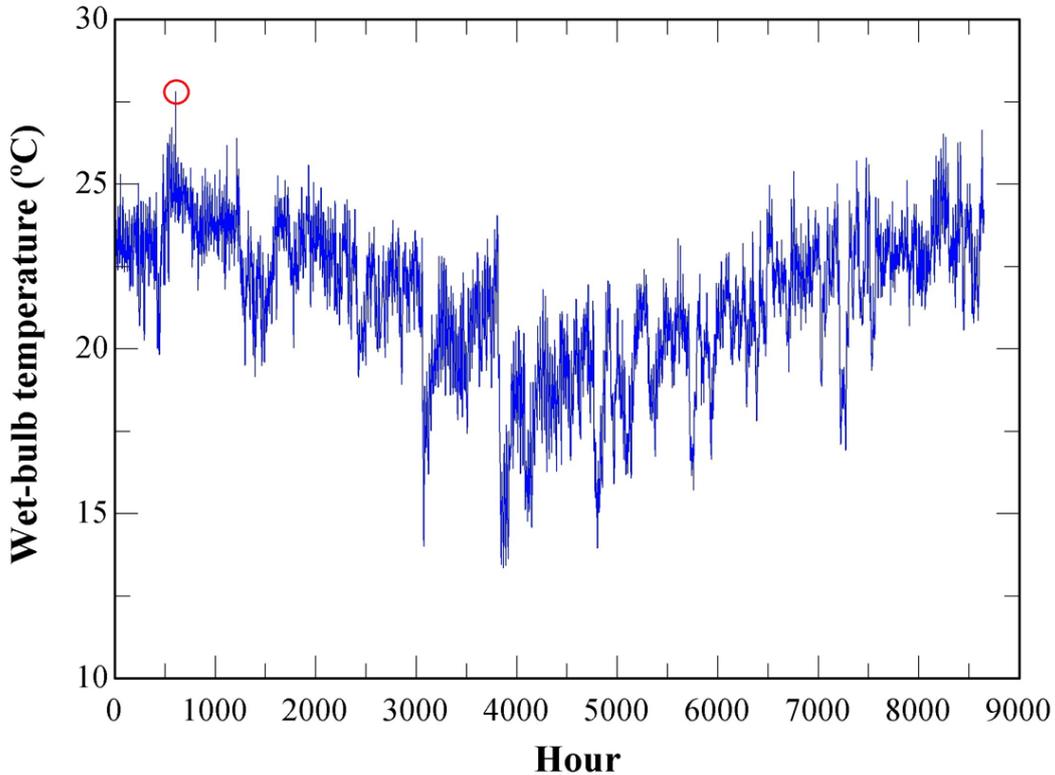


Figure 2. Wet-bulb temperature from 01/26/2019 to 01/25/2020.

### 3. METHODOLOGY

The methodology of this work is based on mass and heat transfer fundamentals, and thermodynamic principles. Moreover, a linear discretization technique is applied in order to analyze the thermodynamic states of the air and water streams across the cooling tower (Stoecker and Jones, 1985).

The following assumptions are set to simplify and support the modelling: (i) steady state, (ii) kinetic and potential energy are negligible, (iii) no pressure losses in each discretization, (iv) humid air in contact with water film is in saturated condition, (v) water droplets leaving the tower due induced draft are not accounted in this modelling, (vi) humid air stream is not saturated at the tower's outlet, and (vii) evaporation process is considered, thus, mass balance must be corrected for water mass flow in each discretization.

There are mainly five parameters in cooling tower modelling that must be presented; tower characteristic ( $NTU$ ),  $L/G$ , approach, range and wet-bulb temperature (Perry *et al.*, 1997). The first parameter is regarded to the tower sizing, also known as number of transfer units or tower characteristic. The second parameter  $L/G$  is water mass flow rate ( $L$ ) divided by air mass flow rate in dry basis ( $G$ ). The third parameter (approach) is the difference between water temperature leaving the tower ( $T_{out,w}$ ) and wet-bulb temperature ( $T_{in,wb}$ ) entering the tower, as represented in Eq. (1). Range is the difference between inlet and outlet water temperatures at the tower, as explicated in Eq. (2). Finally, wet-bulb temperature is dependent on ambient conditions.

$$approach = T_{out,w} - T_{in,wb} \quad (1)$$

$$range = T_{in,w} - T_{out,w} \quad (2)$$

The cooling tower is presented here as a counter-flow configuration with induced draft, as shown in Fig. 3. The humid air and water streams flow vertically in opposite directions (Kröger, 2004; Stanford'III, 2016).

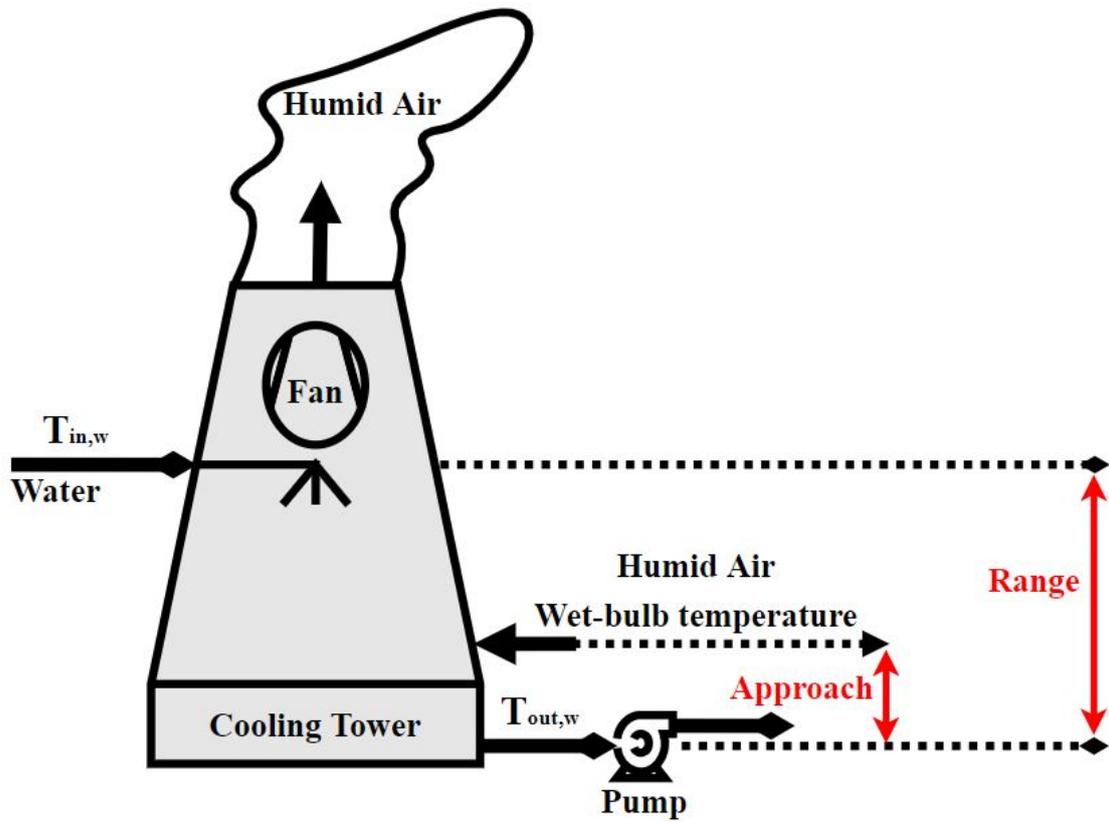


Figure 3. Counter-flow cooling tower with induced draft.

In consonance with Stoecker and Jones (1985), it is set a linear discretization technique. Then, the temperature difference for each increment ( $\Delta T_{inc}$ ) is calculated by Eq. (3), which is in function of desired number of increments ( $n_{CT}$ ), and inlet and outlet water temperatures ( $T_{in,w}$  and  $T_{out,w}$ ). Figure 4 illustrates the discretization method in the cooling tower equipment.

$$\Delta T_{inc} = \frac{T_{in,w} - T_{out,w}}{n_{CT}} \quad (3)$$

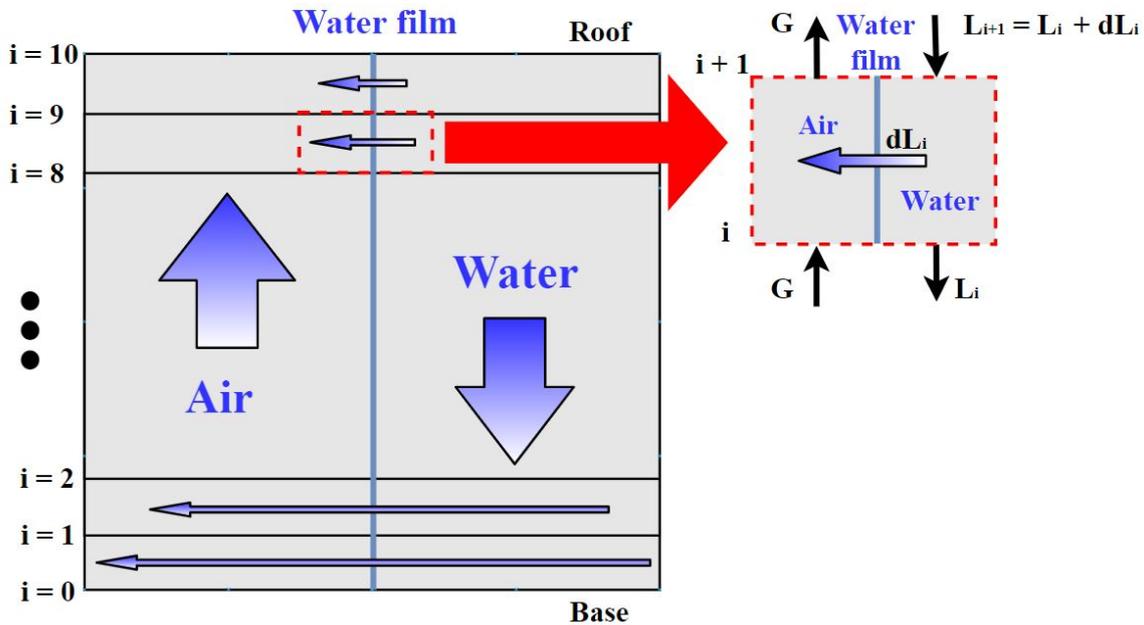


Figure 4. Discretization in the cooling tower modelling.

The next step is to determine the water temperature ( $T_{i+1,w}$ ) and the temperature of air in contact with the water film ( $T_{i+1,wf}$ ) for each increment, as shown in Eq. (4) and Eq. (5), respectively.

$$T_{i+1,w} = T_{i,w} + \Delta T_{inc} \quad (4)$$

$$T_{i+1,wf} = \frac{2 \cdot T_{i,w} + \Delta T_{inc}}{2} \quad (5)$$

Applying mass and energy balance for each increment, as well as, potential enthalpy difference, it is possible to determine the thermodynamic state of humid air stream in each increment's outlet. The corrected mass flow rate for water-vapour ( $dL_i$ ) is explicated in Eq. (6) while the liquid water ( $L_{i+1}$ ) is shown in Eq. (7). In order to calculate these previous equations, the humidity ratios ( $\omega_{i+1,a}$  and  $\omega_{i,a}$ ) for each increment are consulted according the psychrometry chart in EES internal library.

$$dL_i = G \cdot (\omega_{i+1,a} - \omega_{i,a}) \quad (6)$$

$$L_{i+1} = L_i + dL_i \quad (7)$$

The mass flow rate of make-up water ( $L_{mw}$ ) can be estimated by summing all mass flow increments. Thus, Eq. (8) is used for this simple calculation step.

$$L_{mw} = \sum_{i=0}^{n_{CT}-1} dL_i \quad (8)$$

First law of thermodynamics is applied in sequence, as represented in Eq. (9). On the left hand, there is an amount of heat transfer rate from the air stream in dry basis for each increment. This portion of thermal energy is equal to the heat transfer rate in the water stream.

$$G \cdot (h_{i+1,a} - h_{i,a}) = L_{i+1} \cdot h_{i+1,w} - L_i \cdot h_{i,w} \quad (9)$$

Where  $h_{i,a}$  and  $h_{i+1,a}$  are the enthalpy values in dry basis on the entry and on the exit of the air stream in each increment, respectively. Additionally,  $h_{i,w}$  and  $h_{i+1,w}$ , in that order, are the enthalpy values on the exit and on the entry of the water stream in each increment.

Subsequently, potential enthalpy difference is used in determining the tower characteristic for each increment ( $NTU_i$ ), which is defined in Eq. (10) to Eq.(12).

$$\frac{h_c \cdot A}{c_{pu}} \cdot \left[ \frac{h_{i+1,a} + h_{i,a}}{2} - \left( \frac{h_{i+1,wf} + h_{i,wf}}{2} \right) \right] = L_{i+1} \cdot h_{i+1,w} - L_i \cdot h_{i,w} \quad (10)$$

Where  $h_c$  is the convection heat transfer coefficient on the air stream,  $A$  is the heat transfer area,  $c_{pu}$  is the specific heat coefficient for constant pressure, and  $h_{i,wf}$  and  $h_{i+1,wf}$  are the enthalpy values on the entry and on the exit of the saturated air stream in contact with the water film, respectively.

$$NTU_i = \frac{h_c \cdot A}{c_{pu}} \cdot \frac{1}{L_{i+1}} \quad (11)$$

$$NTU_i = \left[ h_{i+1,w} - \left( 1 - \frac{dL_i}{L_{i+1}} \right) \cdot h_{i,w} \right] \cdot \frac{1}{\left[ \frac{h_{i+1,a} + h_{i,a}}{2} - \left( \frac{h_{i+1,wf} + h_{i,wf}}{2} \right) \right]} \quad (12)$$

By using sensible heat transfer rate and potential enthalpy difference definitions, the temperature of air stream on the exit of each increment ( $T_{i+1,a}$ ) is calculated by Eq. (13), where  $T_{i,a}$  is the temperature of air stream on the entry of each increment.

$$T_{i+1,a} - T_{i,a} = \frac{h_c \cdot A}{G \cdot c_{pu}} \cdot \left[ \frac{h_{i+1,a} + h_{i,a}}{2} - \frac{h_{i+1,wf} + h_{i,wf}}{2} \right] \quad (13)$$

The total heat transfer rate ( $\dot{Q}_{total}^{CT}$ ) across the cooling tower can be calculated by Eq. (14). For each increment, the vaporization heat ( $\dot{Q}_{i,Vap}^{CT}$ ) and sensible heat ( $\dot{Q}_{i,Sens}^{CT}$ ) rates are defined by Eq. (15) to Eq. (18).

$$\dot{Q}_{total}^{CT} = L \cdot (h_{n_{CT-1},w} - h_{0,w}) \quad (14)$$

The  $h_{i,support}$ , in Eq. (15), is an enthalpy calculation that assists to determine the vaporization heat rate for each increment.

$$h_{i,support} = f(AirH_2O; T_{i+1,a}; \omega_{i,a}; p_{i,a}) \quad (15)$$

$$\dot{Q}_{i,Vap}^{CT} = G \cdot (h_{i+1,a} - h_{i,support}) \quad (16)$$

The water mass flow rate for each increment is corrected in Eq. (17) due the evaporation process.

$$\dot{Q}_i^{CT} = L_{i+1} \cdot h_{i+1,w} - L_i \cdot h_{i,w} \quad (17)$$

Finally, for each increment, sensible heat rate can be calculated by subtracting the vaporization heat rate from the total amount of heat rate ( $\dot{Q}_i^{CT}$ ), as exposed in Eq. (18).

$$\dot{Q}_{i,Sens}^{CT} = \dot{Q}_i^{CT} - \dot{Q}_{i,Vap}^{CT} \quad (18)$$

The dry-bulb and the wet-bulb temperatures ( $T_{db,amb}$  and  $T_{wb,amb}$ ), the pressure ( $p_{amb}$ ) and the relative humidity ( $\phi_{amb}$ ) of the ambient site, with other parameters, are presented in Table 1. In sequence, some simulations are developed in order to analyse the thermodynamic states of the air and water streams across the cooling tower. The number of increments is set as 10 in this paper, however, if more precision results are desired, the number of increments must be increased.

Table 1. Input data for cooling tower modelling.

Parameter	Value
Approach (°C)	4.0
Range (°C)	5.6
$T_{wb,amb}$ (°C)	27.81
$\phi_{amb}$ (°C)	0.78
$T_{db,amb}$ (°C)	31.1
$p_{amb}$ (atm)	1.0
$L/G$	1.0
$n_{CT}$	10

#### 4. RESULTS AND DISCUSSION

In this section, the thermodynamic states of air and water streams, vaporization and sensible heat transfer rates, and other major parameters of the cooling tower unit are presented and discussed in detail.

Firstly, it is presented the air temperature and humidity ratio for each increment at the psychrometric chart, as represented in Figure 5. This result is in accordance with several literature reviews, thus validating this proposed cooling tower modelling (Threlkeld, 1970; Stoecker and Jones, 1985; Perry *et al.*, 1997; Kloppers, 2003; Kröger, 2004; Stanford'III, 2016).

The cooling tower modelling is simulated under four distinct conditions that are in function of the ambient relative humidity ( $\phi_{amb} = 0.2, 0.4, 0.6$  and  $0.78$ ). By analyzing Figure 5, three simulations with  $\phi_{amb}$  equal to 0.2, 0.4 and 0.6 present a smooth curvature with a concavity point. In addition, for the lowest wet-bulb temperature, there are more vaporization and sensible heat transfer rates across the cooling tower.

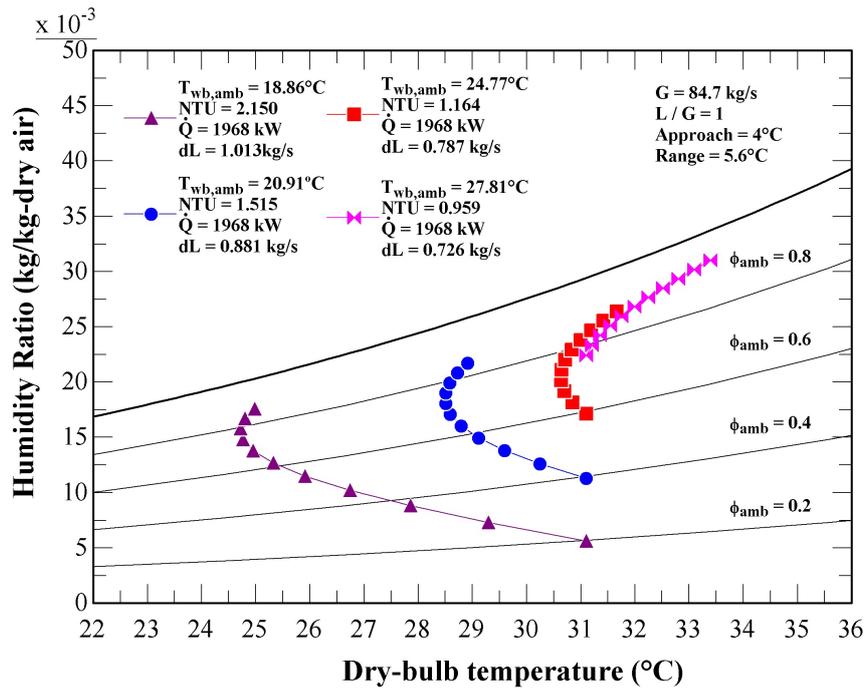


Figure 5. Determining thermodynamic states of air stream varying wet-bulb temperature at the cooling tower.

In Figure 6,  $L/G$  is varied from 0.5 to 2, which permits to verify that more heat is transferred at a higher ratio across the cooling tower. Therefore, a bigger tower is needed to allow this heat transfer. Moreover, as the  $L/G$  is increased, the thermodynamic states of the air stream are approaching the saturation line.

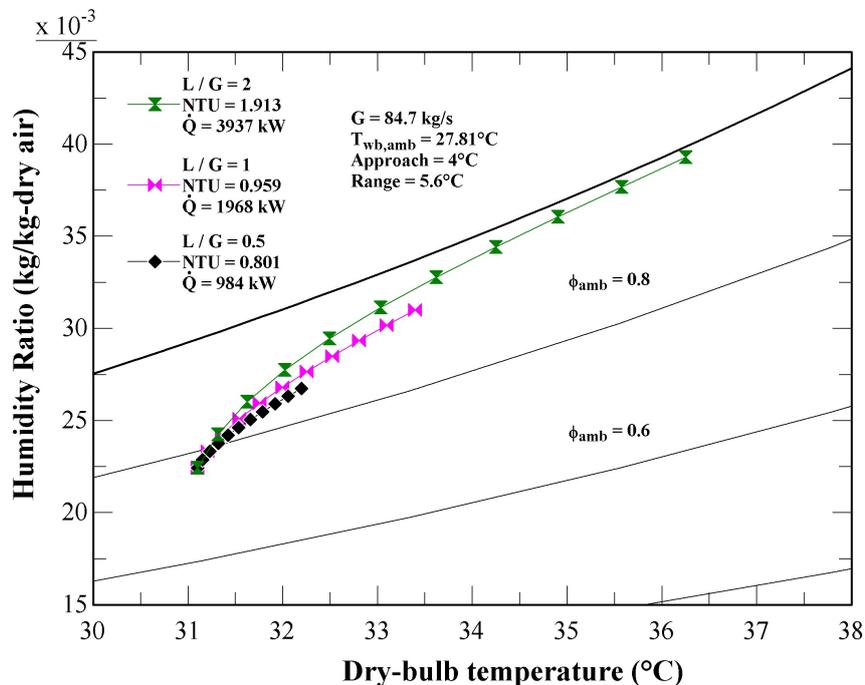


Figure 6. Determining thermodynamic states of air stream varying  $L/G$  at the cooling tower.

A comparison, taking into account two simulations ( $\phi = 0.78$  and  $0.6$ ), is developed in order to evaluate the behaviors of the thermodynamic states of the air and the water streams. The temperature chart of the air and the water streams is presented in Figure 7. This chart allows to note that the simulation with relative humidity of  $0.6$  is presenting a concavity point just after the third increment ( $n_{CT} = 3$ ). Thus, at the base ( $i = 0$ ), the air stream has a higher temperature than water stream until increment 3, thereby, transferring heat from air to water. Notwithstanding, after increment 3, the water temperature starts to become greater than air stream

temperature, hence, transferring heat from water to air. On the other hand, for relative humidity of 0.78, the sensible heat is always occurring from water to air.

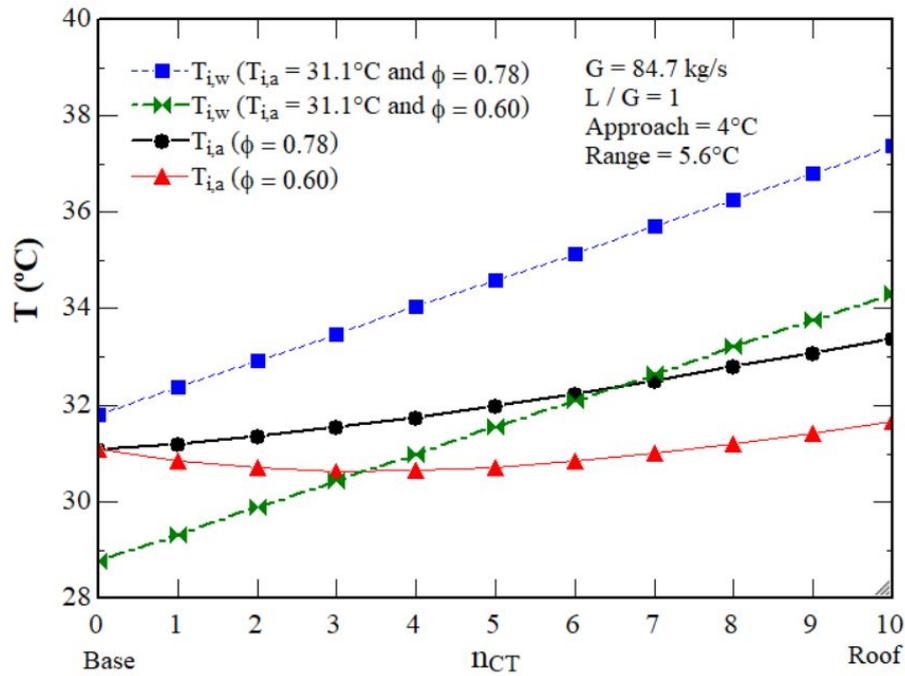


Figure 7. Temperatures of air and water streams across cooling tower.

The sensible heat changes direction at the same increment 3, as shown in Figure 8. Thereupon, the cooling tower modelling is responding accordingly to the mass and heat transfer principles by using potential enthalpy difference.

Another observation about this result is the amount of vaporization heat on the bottom of cooling tower that is higher than the top. By analysing this behavior, it can be presumed that there are more water-vapour evaporating nearby at the base.

Therefore, in Figure 9, it is observed that, indeed, the amount of evaporated water is highest at the first increment ( $i = 0$ ) due highest vaporization heat.

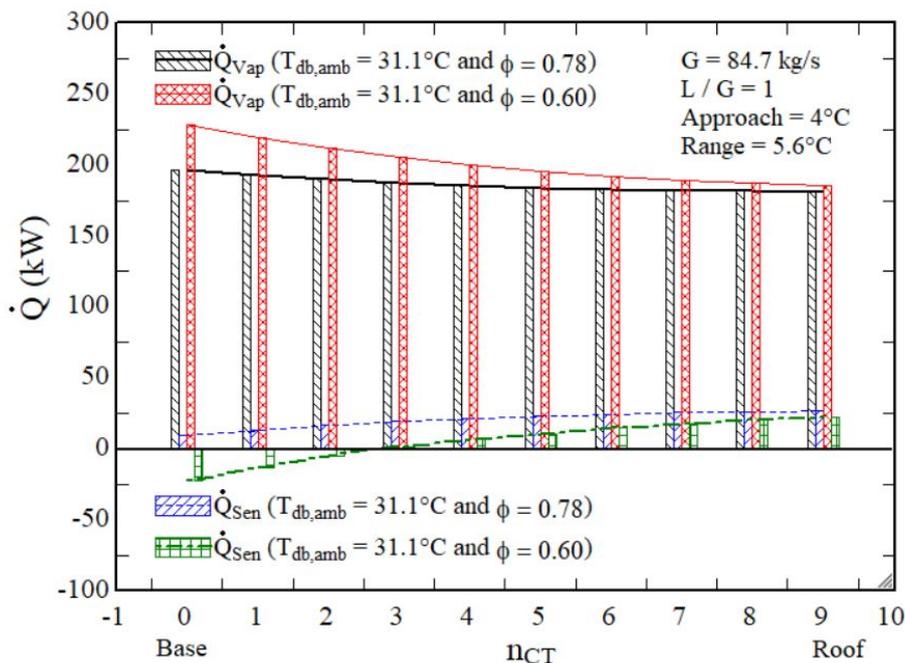


Figure 8. Vaporization and sensible heat transfer rates across cooling tower.

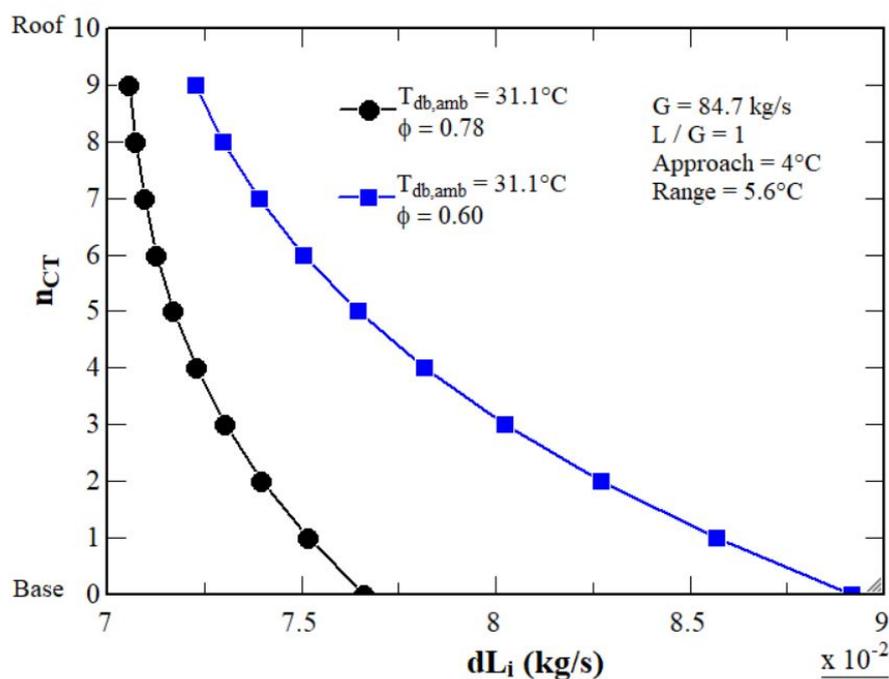


Figure 9. Water-vapour evaporating across cooling tower.

## 5. FINAL COMMENTS AND OUTLOOK

The purpose of this work is accomplished successfully, presenting reasonable results in means of heat and mass transfer principles, thermodynamic fundamentals and other parameters. This discretized cooling tower modelling is good enough to predict the thermodynamic states of the air and the water streams, the make-up water, the vaporization and the sensible heat transfer rates, and the tower characteristic. Therefore, allowing to evaluate in detail what is occurring across the cooling tower unit. The validity of the assumptions is responding accordingly several literature sources, as already described.

For the case study, the critical ambient condition requires a cooling tower unit of 1968 kW,  $NTU$  equal to 0.959 and make-up water of 2614 kg/h.

For future works, some enhancements in this modelling can be achieved, such as:

- Determine the heat transfer area of the filling inside the cooling tower (length, height and depth);
- Apply more complex and real assumptions to develop detailed equations in terms of mass and heat transfer principles;
- Explore the plume formation due condensation process in the air stream at the tower's outlet.

## 6. ACKNOWLEDGEMENTS

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