26th International Congress of Mechanical Engineering
COBEM 2021
ABCM Associação Brasileira de Engenharia e Ciências Mecânicas
November 22-26, 2021, Virtual Congress, Brazil

COB-2021-2322
ANALYSIS OF LOW FREQUENCY VIBRATIONS IN HERMETIC COMPRESSORS

Eng. Me. Pedro Motta Nunes
Eng. Dr. Arcanjo Lenzi
Federal University of Santa Catarina
pedromottanunes@hotmail.com
arcanjo.lenzi@ufsc.br

Eng. Dr. Ricardo Mikio Doi
Nidec Global Appliance
ricardo.m.doi@nidec-ga.com

Abstract. *Companies increasingly need to develop innovative and quality products in a short period of time and reducing the number of prototypes for testing. At this point in the modern era, it is essential to have model control to predict responses of system. One of the major engineering problems is attenuation of the levels of noise and vibrations emitted by the compressor housing, especially noise at low frequencies, which is one of the focuses of the analysis range of this work. The objective of this work was to develop an analytical model of the equations of motion of rigid body of the internal block-motor assembly supported from 4 springs which couple the system to the housing, for an analysis of low frequency vibrations, of a hermetic compressor. Thus, the model was used to implement an optimization method to obtain new assembly configurations for the internal assembly in order to minimize the low frequency responses of the housing.*

Keywords: *Prediction of responses, Compressor vibration, Multibodies, Analytical model, Optimization*

1. INTRODUCTION

Throughout the years, domestic refrigerator hermetic compressors manufacturers have been improving compressor designs in the aspects of initial assembly configuration, mass reduction of components, geometric alterations and new spring position configurations. The majority of hermetic compressors are made with their internal components supported by four springs, for ease of assembly in production line, as shown in Figure 1. In the past, some models were supported by 3 springs. A common engineering problem in hermetic compressors is the need to attenuate noise and vibration levels transmitted by the compressor. This is seen in clients' requirements of acceptable noise levels and acoustic comfort when the compressor is in operational conditions in a household refrigerator — specially in low frequencies. The main focus of analysis of this work are the vibration levels of the compressor. Figure 1 shows the internal components of a compressor.

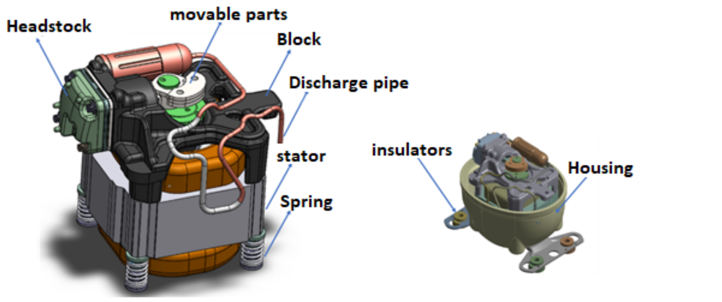

Figure 1. Alternative hermetic compressor.

Reciprocal piston compressors are naturally unbalance machines, which vibrate and transmit dynamic loads to the refrigeration system through the base-plate which supports it, as well as by the ducts that transport the refrigeration gas (PORTO, 2010). With the current presented situation, it can be seen that there's a need to develop an analytical model in which represents the behaviour of a hermetic compressor in a household application, focused on the low frequency vibration bands, in function of design parameters.

This work has the objective of developing an analytical model that represents the dynamic behaviour in low-frequency bands of the compressor, by way of using the multi-body method, considering the inner components of the compressor as rigid bodies. The responses in the mass center of the inner components, as well of the outer housing are calculated in both steady-state and transient regimes. We will focus on the rotation frequency and it's main harmonics.

The first step of the method is kinematic analysis of the loads coming from the moving parts of the compressor (namely the connecting rod, piston and crankshaft). The inertial dynamic loads of each element generate forces on the superior and inferior bearings of the axle, which provokes vibration in the compressor's block, which is then transmitted to the housing, as shown in Figure 2.

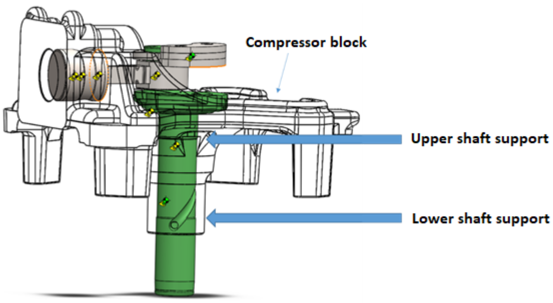

Figure 2. Configuration of moving parts and block.

After kinematic analysis, the body movement equations are built in matricial equations, in order to calculate the displacement of the inner parts and the housing. With these matrices calculated, frequency domain and steady-state responses can be obtained using the direct solution method. This way, it is possible to predict the displacement amplitudes of the low-frequency spectrum.

Furthermore, dynamic analysis follows, which consists in obtaining the transient response caused by shaft velocity gradients. This is done in order to evaluated the behaviour of the compressor in start-stop situations. In order to perform this analysis, it is paramount the development of a model that also takes in consideration the motor's torque and gas-compression force loads in the shaft's movement equations, so that the angular velocity can be calculated in each time step.

The matrices referring to the equations of motion of the system are now developed, in order to calculate the displacement amplitudes of the compressor's internal assembly (kit) and the housing. With the motion matrices defined, it is possible to obtain results in the frequency domain, in steady state, from the direct solution method, predicting the displacement amplitudes for the first frequencies of the system.

A later stage of the dynamic analysis consists of the calculation in transient regime, considering the variation in the shaft speed, in order to evaluate the behavior of the compressor at the motor stop. For this analysis, it is necessary to create a model that also considers the engine torque and the gas compression force in the shaft movement equation, to calculate the angular velocity at each instant.

Model Considerations:

- Compressor's inner components (rotor, shaft, connecting rod, crankshaft, stator, piston, block and counterweight) are considered rigid bodies.
- The discharge tube and suspension springs are considered to be massless elastic bodies with damping.
- The inertial forces of the rotor, shaft, connecting rod and piston are calculated from their respective accelerations in relation to the center of mass of the block.
- The effect of pressure in the discharge line is considered to be insignificant.
- Body forces due to gravity field are not considered.
- The generation of movement of the compressor's motor is represented by the torque curve – motor speed.
- The phenomena in question is considered to be linear in regards to vibration and displacement fields.

2. MULTI-BODY SYSTEM DYNAMIC ANALYSIS

Mechanical systems are those made of joined subsystems subjected to loads. A multi-body system is a set of mechanical subsystems called bodies or components. The movement of components are kinematic and only translation or rotation movement is allowed (SHABANA, 1998). The rigid-body elements of the compressor are the rotor, stator, shaft, crankshaft, counter-weight, connecting rod, piston, headstock, block and housing. The elastic elements are the suspension springs and the discharge duct.

2.1 Efforts generated by moving parts

The main loads responsible for the compressor vibration field are inertia from each element and the systems' unbalance. This kinematic analysis takes as basis the work done by Rodrigues (2003), Fulco (2008) and Hamilton (1982). The loads are evaluated from the translational acceleration from the mass centers of the elements and the angular acceleration from the moving parts, as can be seen in Figure 3.

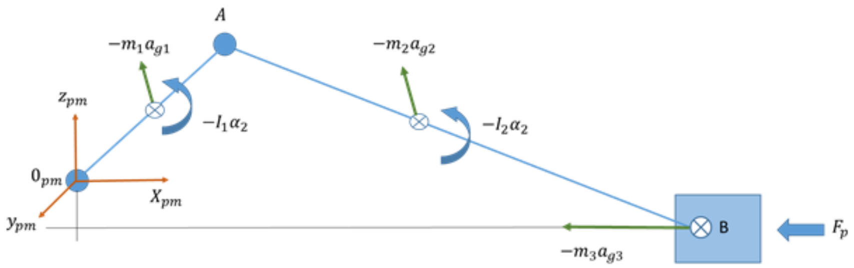

Figure 3. Displacements of the mass centers of the connecting rod, crankshaft and piston

The vector $\{f_{0pm}(t)\}$, calculated via Newton's second law is the sum of forces and moments around the moving parts coordinate axis, as shown in equation 1

$$\{f_{0pm}(t)\} = [M]\{\ddot{s}(t)\} = [M][a(t)]. \tag{1}$$

To finish the calculation of the loads acting on the compressor, a rotation of loads to the center of mass of the inner component is needed, and it's done using the rotation matrix $[\pi^{kit,pm}]$:

$$\{f_{kit}(t)\} = [\pi^{kit,pm}]\{f_{0pm}(t)\}, \tag{2}$$

The loads of the counter-weight are added to the resultant load vector in the coordinate axis with origin in 0_{pm}.

2.2 Coupled equations of movement

The equations of motion of the compressor's inner component assembly in matrix form is given by:

$$[H_k]\{\ddot{s}(t)\} + [C_k]\{\dot{s}(t)\} + [K_k]\{s_k(t)\} = \{f(t)\}, \tag{3}$$

where

- Inertia Matrix – $[H_k]$

$$[H_{kit}]_{6x6} = \begin{bmatrix} [\ m_{kit}\ ] & \vdots \\ \vdots & [I_{kit} \end{bmatrix}$$

- Damping matrix – $[C_k]$

$$[C_k^m] = \begin{bmatrix} C_x & \vdots \cdots & \vdots \\ \vdots \cdots & C_y & \vdots \\ \cdots & \vdots \cdots & C_Z \end{bmatrix} \tag{4}$$

- Stiffness matrix – $[K_k]$

$$[k_i^{kit}] = \begin{bmatrix} [\pi^{b,m}]\,[k_m^m]\,[\pi^{b,m}]^T\,[IT_m^b] \\ [T_m^b]\,[\pi^{b,m}]\,[k_m^m]\,[\pi^{b,m}]^T\,[IT_m^b] \end{bmatrix} \tag{5}$$

and $[T_m^b]$ represents a translation/rotation transformation matrix.

- Load vector – (f_k)

Equation 6 presents the six possible types of rigid body movement, therefore, the uncoupled system has six degrees of freedom.

$$\{s_k\}^T = \{y, x, z, \emptyset, \theta, \varphi\} - deslocamentos \tag{6}$$

The inertial matrix of the internal assembly considers the inertial properties of the system as:

$$H_{kit} = H_{block} + H_{headstock} + H_{axle} + H_{counterweight} + H_{crankshaft} + H_{connectingrod} + H_{piston} + H_{rotor} + H_{stator}. \tag{7}$$

With the expression for H_{kit} calculated, next step is coupling the movements of the inner components to the housing through the elastic components. This coupling of bodies is shown in Figure 4.

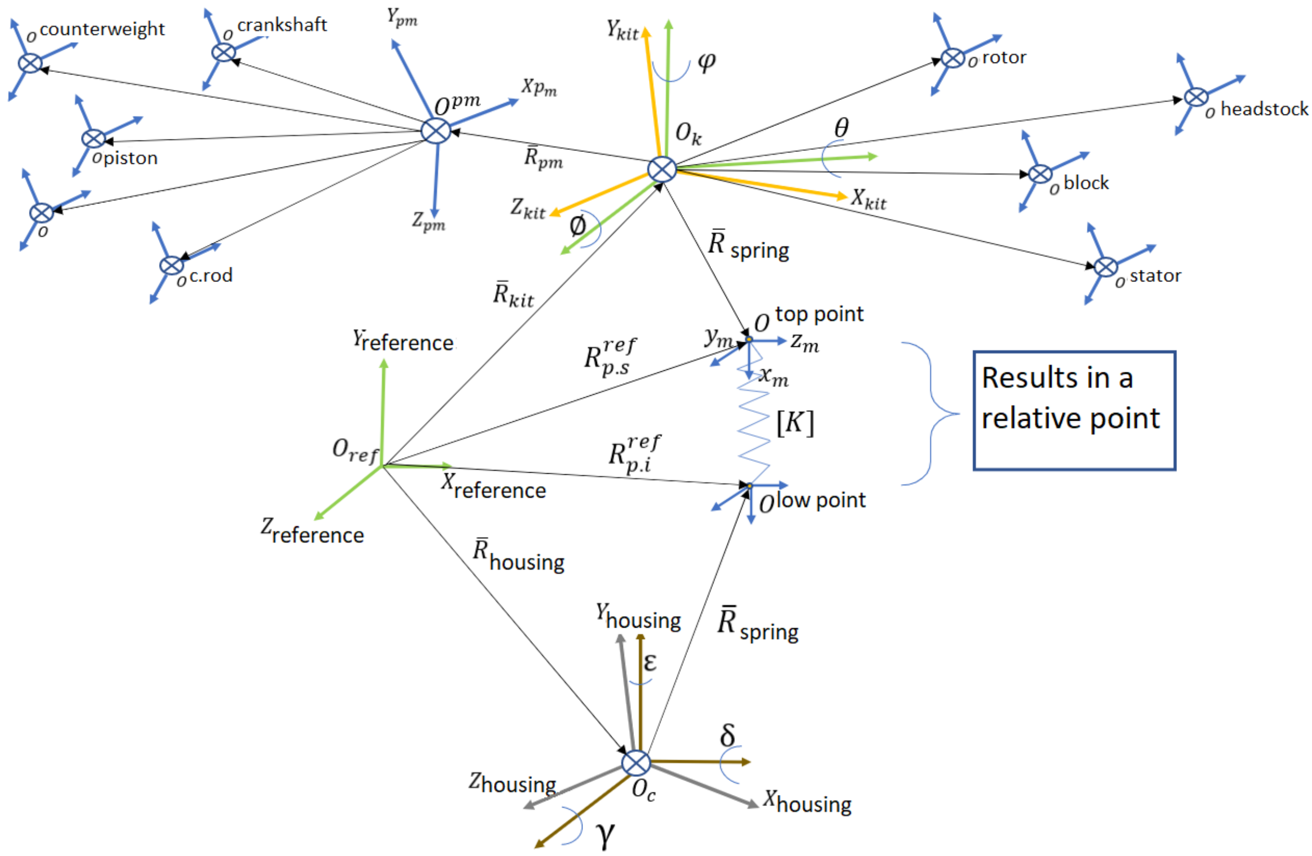

Figure 4. Position vectors in relation to reference bodies and axe.

The reference of the coordinate systems, O_{ref}, is common to the two bodies. Their position vectors are named, respectively, $\bar{R}_{housing}$ and $\bar{R}_{kit}$. With this approach it is possible to extract the displacements of the bodies in relation to the reference coordinate system, with the influences of these elastic elements. The new coupled equation of motion is defined using the equation:

$$[H_{sa}]\{\ddot{u}_{sa}(t)\} + [C_{sa}]\{\dot{u}_{sa}(t)\} + [K_{sa}]\{u_{sa}(t)\} = \{f(t)\} \tag{8}$$

where the subscript "sa" means coupled system.

3. ANGULAR MOVEMENT OF THE AXLE

To further complement the model of the compressor, the variation of the angular velocity of the shaft is considered.

3.1 Constant axle angular velocity

For the frequency domain dynamic analysis, a constant axle angular velocity is considered. That is:

$$\dot{\beta}(t) = \omega_0, \tag{9}$$

implying an angular acceleration $\ddot{\beta} = 0$.

3.2 Time-variant axle angular velocity

By analyzing the free body diagram of the axle-rotor system, it is possible to define a new differential equação for the angular position $\beta(t)$:

$$[I_R + M_p k_1^2]\ddot{\beta}(t) + M N_p k_1 k_2 \dot{\beta}(t)^2 + \left(C_{connectingrod} + C_p k_1 \dot{\beta}(t)\right) = T_m + k_1 F_p, \tag{10}$$

where F_p represents the gas compression induced torque, C_p is the piston damping coefficient, M_p is the piston mass, T_m the motor's torque, I_R the moment of inertia of the center of mass of the rotor and C_R the rotor's damping coefficient.

3.2.1 Pressure load at the piston

Figure 5 presents the idealized thermodynamic process in a P-V diagram that happens in the compressor. This diagram serves as a basis for the determination of the pressure load at the piston's cylinder, which has a mobile part angle determined by the expression:

$$V = V_c + \frac{\pi D^2}{4}[R(1 - \cos\beta(t)) + L(1 - \cos\alpha)] \tag{11}$$

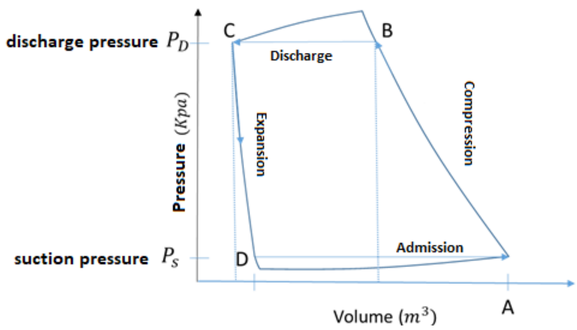

Figure 5. P-V diagram of the compression cycle.

By assuming that the process is polytropic, the cylinder's pressure can be calculated as a function of the volume through the thermodynamic relation PV^n = constant.

With the cycle starting at the A point in the $P - V$ diagram, the compression pressure C_S can be calculated as:

$$C_S = P_s \left[V_c + \frac{\pi D^2}{4}(2R)\right]^n \tag{12}$$

The pressure in the cylinder during the compression process is calculated from the cylinder volume as

$$P = \frac{C_s}{V^n}, \tag{13}$$

before the discharge pressure point P_D is reached. The pressure in the cylinder is then assumed constant at P_D before $\beta(t) = \pi$. Therefore, the expansion constant C_D is defined as

$$C_D = P^D (V_C)^n. \tag{14}$$

For the expansion process, the pressure is calculated through the relation

$$P = \frac{C_D}{V^n} \tag{15}$$

before the suction pressure P_s is reached. The pressure in the cylinder is assumed constant at the point P_s before arriving $\beta(t) = 2\pi$.

3.2.2 Motor torque evaluation

The motor torque T_m used in the model is calculated analytically in regards to the variation of angular velocities of the axle, based on the parameters shown in Figure 6, through the tangent line in red, defined as:

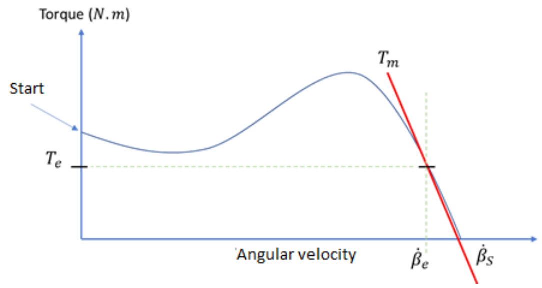

Figure 6. Torque vs Angular Velocity.

$$\overline{T_m} = \frac{T_e \dot{\beta}_e}{\dot{\beta}_s - \dot{\beta}_e} - \frac{T_e \dot{\beta}}{\dot{\beta}_s - \dot{\beta}_e} = V_1 - V_2 \dot{\beta(t)}, \tag{16}$$

where $\dot{\beta}_e$ and T_e represent, respectively, the shaft rotation speed and torque in steady state. $\dot{\beta}_s$ is called the synchronous rotation and f represents the network frequency.

4. SOLUTION OF THE EQUATION SYSTEM

4.1 Frequency domain analysis

To analyze the system in frequency domain, the following considerations are made:

- i) Constant axis angular velocity
- ii) Linearity of motion relations of the kit

Through these assumptions, the vector $\{s_g\}$ can be evaluated as

$$\{s_g\} = \left[-\omega^2 H_g + K_g\right]^{-1} \{F\}, \tag{17}$$

where $_g$ is composed by the six degrees of freedom (y, x, z, θ, ϕ, φ).

4.2 Time domain analysis

The transient regime of vibration of the compressor must be evaluated through time domain analysis, by considering the axle angular velocity as a function of time. For the present work, an explicit fourth order Runge-Kutta integration technique for the explicit solution was employed.

5. EXPERIMENTAL VALIDATION

5.1 Natural frequencies of the inner components supported by a rigid base

An initial experimental analysis was carried out in order to extract the natural frequencies of the system in order to validate the analytical results. Figure 7 shows the distribution of accelerometers positioned on the compressor block. Four tri-axial accelerometers, positioned at sufficiently spaced points were used for impact responses on the structure. The results of the first six natural frequencies compared to the analytical ones are presented in table 1. The model shows relevant convergence between models.

5.2 Time Domain Results - Inner components and housing

A simulation of the compressor's working cycle was done in order to understand the displacement of the piston through the first 18 seconds of work. It's important to remind the reader of the 3 stages of analysis: motor start, steady state and the motor stop at second 10. The analytical response of the displacement, for the inner componentes through time is shown in Figure 8 for the x degree-of-freedom, which coincides with the movement of the piston. The z degree-of-freedom is

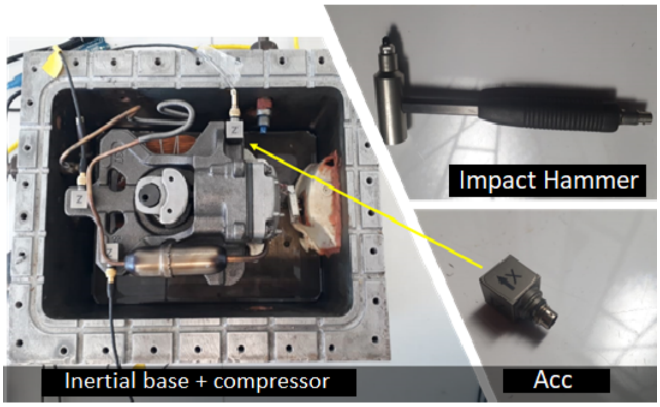

Figure 7. Apparatus used for the experimental test

MODE	Natural Frequencies (Hertz)	
	ANALYTICAL	EXPERIMENTAL
Longitudinal translation	5,1	4,6
transverse translation	5,7	4,9
Superior Translation	8,2	8,5
Longitudinal Rotation	13,2	13,4
Transverse Rotation	14	16,6
Top Rotation	14,4	17,3

Table 1. Comparative results

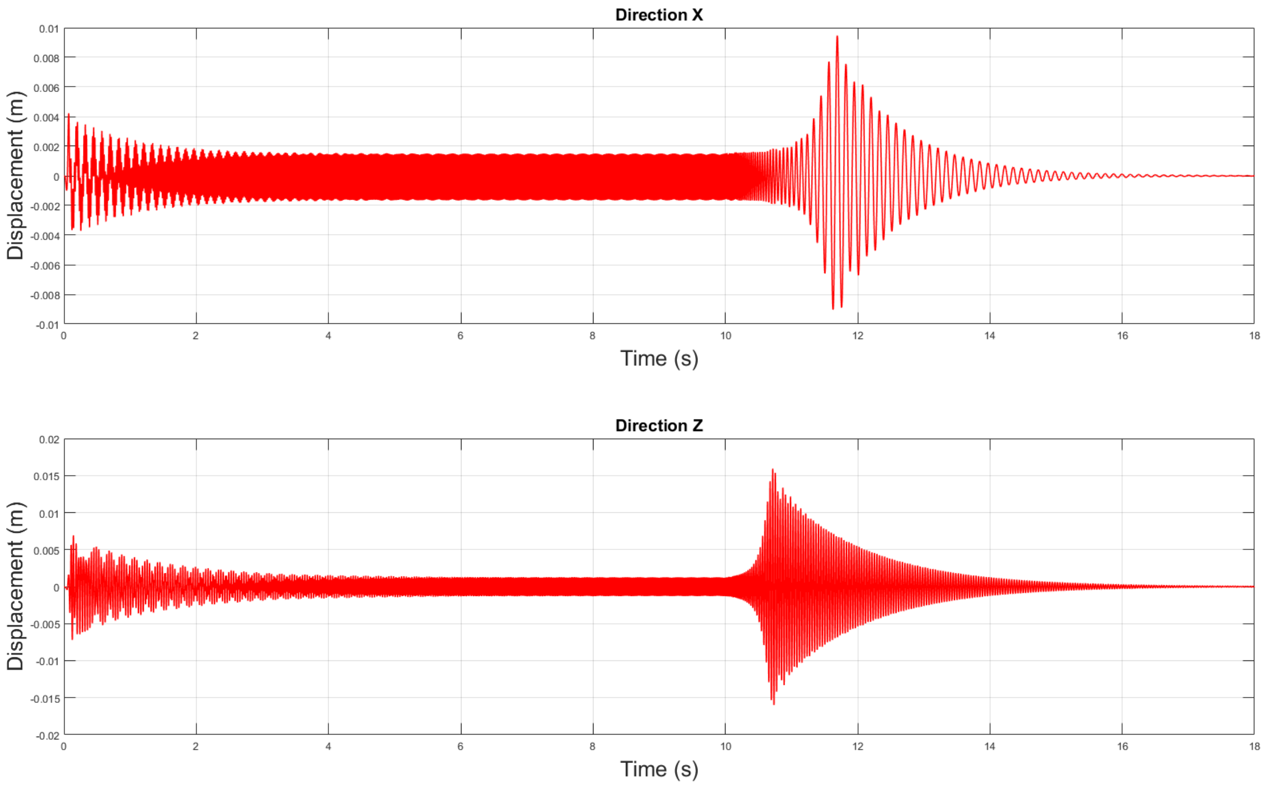

Figure 8. Displacements of the center of mass of the compressor inner assembly.

also shown, which represents the transversal movement of the piston. It is also possible to notice that, through the start of the motor, due to inertia of the components and initial torque being higher than the steady-state torque, higher amplitude vibrations are present in the start regime. After a few seconds, the kit vibration response converges to the steady-state level. After the motor stoppage, an even higher amplitude regime occurs.

For the housing, the relative displacement x and z are shown in Figure 10.

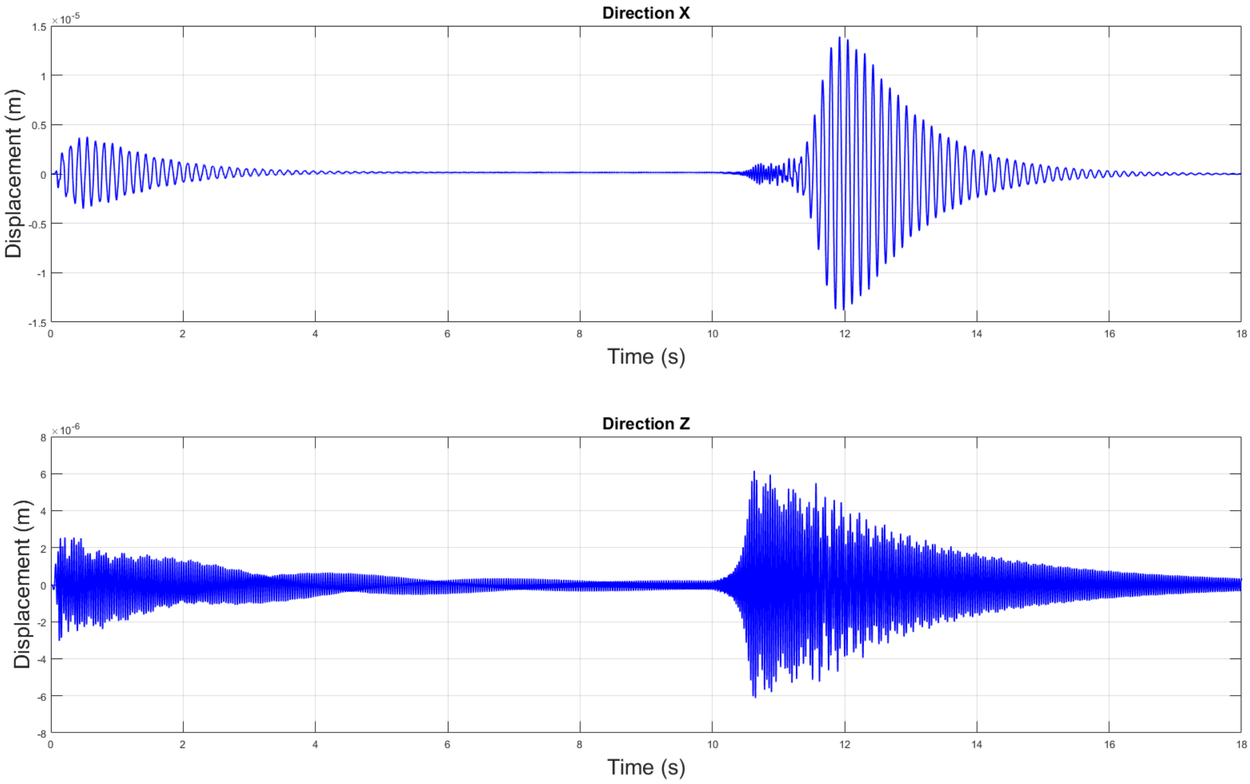

Figure 9. Displacements from a point on the top center of the housing.

The results shown in table 2 include those obtained over time for both the analytical and experimental models. It is worth remembering that the results obtained over time were transferred to the frequency domain to carry out the necessary comparisons in the motor rotation frequency. For this comparison, only the period in which the system is in permanent regime was considered.

Direction	Displacement (µm)		Frequency (Hz)
	Analytical	Experimental	
y	2,7	3,4	58,5
x	3,8	4,2	
z	2,4	3	

Table 2. - Results for the coupled system.

6. SUSPENSION GEOMETRY OPTIMIZATION

Sensivity analysis show that some parameters are more important than others, namely the suspension spring geometry, the springs' stiffness, the number of springs and the inner component assembly total mass. As the objetive of this optimization is to analyze possible new configuration of assembly, the spring spatial configuration was chosen as the main variable of optimization to minimize the housing vibration field. The following possibilities for the springs were considered:

- Independently varying each spring's length.

- The angular position of each spring was constrained between $[-45^o, 45^o]$.

Figure 10 shows the new spring configuration. Configuration shown in Figure 10 presented a 36% reduction of vibration levels in relation to the initial configuration.

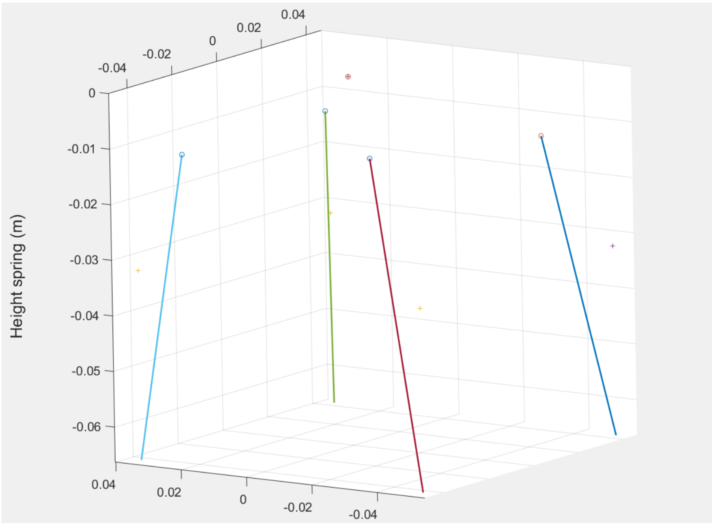

Figure 10. Possible new spring configuration

The configuration seen in Figure 12 provided a reduction of up to 36% in the vibration level of the housing in relation to the initial configuration in which the springs were positioned vertically.

7. CONCLUSIONS

The proposed analytical model considers the load strains that act on the internal assembly coming from the inertial component of the moving parts, gyroscopic effects of the rotor and variation of the angular velocity of the rotor due to the reaction loads caused by gas compression in the cylinder and piston. Due to the angular velocity gradients, the torque of the electric motor plays a part in the phenomena, being responsible for most of the angular velocity variation. Due to this transient regime, analysis was made in time domain.

During the rotation cycle of the rotor, in steady-state, this angular velocity variation is negligible and a constant rotation first order approximation is possible. In this type of analysis, loads due to gas compression and constant torque are not considered and the solution can be calculated through frequency analysis.

After the vibration analysis model was set, an optimization of the inner components configuration was done through the Genetic Algorithm, varying the spring spatial position and angle as parameters and the vibration field of the housing as objective function. A 36% reduction of vibration magnitude in RMS of the housing was obtained through this procedure.

8. REFERENCES

COUTO, P. R. C. Análise de mancais radiais hidrodinâmicos com aplicação em compressores herméticos de refrigeração. Tese de Doutorado. Universidade Federal de Santa Catarina. Florianópolis, Brazil. 2006.

DIAZ, F. S. Simulação do comportamento dinâmico do conjunto interno de compressores herméticos em baixas frequências. Dissertação de Mestrado. Universidade Federal de Santa Catarina. Florianópolis, Brasil. 2017.

FULCO, É. R. Esforços dinâmicos transmitidos de um conjunto eixo-rotor flexível ao bloco de compressores herméticos. Dissertação de Mestrado. Universidade Federal de Santa Catarina. Florianópolis, Brasil. 2008.

FULCO, É. R Modelos do comportamento dinâmico e vibroacústico de compressores herméticos / Érico Romera Fulco ; orientador Arcanjo Lenzi – Florianópolis, Sc, 2014.

SHABANA, A. A. Dynamics of Multibody Systems. 2ª Edição. ed. [S.l.]: Cambrige University Press, United Kingdom, 1998.