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STUDY OF ACCELERATIONS RESTRICTIONS ON A ROLLER COASTER

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Abstract. Roller coasters were created to amuse people. Since its creation in the 15th century in Russia, they have been evolved until the current format, which it is on most of them created of steel. But some continue to be made of wood. Although, even with all the evolution it has undergone since its inception, there are still cases of accidents and some of them even fatal. Therefore, nowadays with technology and new materials, the precision got to new levels, never achieved before. The design of a roller coaster must be very well calculated in order that, besides its structure and equipment remain intact and with excellent performance, passengers do not suffer health risks, with impacts and with accelerations and decelerations felt, which may occur along the route. The objective of this work was to propose a path profile for a roller coaster, in which passengers can ride comfortably, without suffering from the impacts of accelerations, materials and energies that are present along the path, so that they can enter into worry-free entertainment with health problems. Bibliographies related to the roller coaster theme were identified and studied. This made it possible to assess the needs for carrying out the project and the practices that must be respected, understanding how a roller coaster project is constituted, to reduce the impacts on the lives of passengers. Thus, the geometry and dimensions of a proposal for the tour's route profile were defined, based on what a passenger likes and expects on a roller coaster ride and data provided by the Sea World Entertainment group. However, some path / profile assumptions were defined in this work, given the confidentiality of some data. This profile was developed by excerpts constructed by mathematical functions and then discretized for evaluations of accelerations (and decelerations) and G forces. The constructions of the functions were supported with the use of the Maple program and discretization and evaluations with the Octave program. The equations for the calculations originated from the energy balances for each specific section of the roller coaster. These values were compared to the limits tolerable by the human body. The results obtained in the simulations were compared to international standards and, unfortunately, led to the conclusion that the proposed project cannot be approved. Problems in some sections would need to be corrected by changes in the profile and the reevaluated calculations

Keywords: Roller Coaster, G force in the bod, Force analysis, Energy balance, Work and energy

1. INTRODUCTION

According to Weisenberger (2013), the roller coasters were originated in Russia, in the 15th century, to entertain people in harsh winters. They were wooden constructions, that worked as large ice slides, increasing to heights of up to 21.34 meters with sand at the end of the falls, to stop passengers. They were evolving, until the 19th century, in France, when wheeled carts were inserted on the tracks becoming like current models.

Roller coasters can be classified into two types: wood and steel. As mentioned by Weisenberger (2013), wooden roller coasters are cheaper, but need more maintenance, unlike steel ones.

Even with all the evolution, there are still roller coasters accidents, sometimes even fatal. That is why, the project must be very well calculated and determined so that passengers do not suffer any risk to their health, with impacts and accelerations and decelerations felt, which may occur along the way. These accelerations that passengers may be subjected to during a tour, are restricted by international standards and regulations. The objective of this work is to understand these accelerations in which passengers can be submitted on a proposed layout of roller coaster, analyzing whether developed forces are within the limits of the human body.

2. METHODOLOGY

To develop this work, it was necessary to define the geometry and dimensions for the path. So that it was possible, some basis data were provided by the Sea World Entertainment group and used. However, some assumptions of paths/profiles were defined in this work, given the confidentiality of some data. It was identified what a passenger likes and expects on a roller coaster ride and, thus, a proposal for the runway profile was established. This profile was developed by stretches constructed by mathematical functions, which were discretized for evaluations of accelerations and G forces.

The constructions of the functions were supported using the Maple program and the discretization and evaluations with the Octave program. The equations were developed by the balance of forces at each specific stretch of the roller coaster and compared to the limits tolerable by the human body.

3. PROPOSED PROFILE TO THE ROLLER COASTERS

For the proposed roller coaster profile, the analysis considers the departure of the car at the top of the first drop and does not consider end boarding /landing passenger's area.

The complete route studied was divided into 14 sections. From section 1 to section 12, the track profile was defined in the vertical plane XZ, with X being the independent variable and Z being the dependent variable, defined by functions $z_i(x)$, with i being defined from 1 (for section 1) to 12 (for section 12). In sections 13 and 14, the track profile was defined in the horizontal plane YX, with Y being the independent variable and X being the dependent variable, defined by functions $x_i(y)$, with i being defined from 13 (for section 13) to 14 (for section 14). Thus, the fourteen sections were defined as follows: Section 1 (initial drop of 73 meters to 5 meters in height); Section 2 (ascent of the *loop* until the beginning of the inversion); Section 3 (inverted section of the *loop*, with the top in 59 meters high); Section 4 (section of the descent of the *loop*); Section 5 (horizontal straight at 5 meters high); Section 6 (climb up to 30 meters high); Section 7 (descent to a height of 5 meters); Section 8 (up to 25 meters high); Section 9 (descent to a height of 15 meters); Section 10 (descent to a height of 9 meters); Section 11 (descent to a height of 5 meters); Section 12 (horizontal straight at 5 meters high); Section 13 (left horizontal curve with a radius of 40 meters); and Section 14 (horizontal curve on the right with a radius of 40 meters).

The descent and ascent sections, except for the *loop* (sections 2 to 4), were developed with cubic polynomial $z_i(x)$ functions: $z_i(x) = c_1x^3 + c_2x^2 + c_3x + c_4$, where i is the descent section or the climb analyzed (\neq of 2 to 4). The choice of using a cubic polynomial function allows a better transition between each stretch (in most cases, as will be seen in the analysis), as it has a second continuous derivative. The sections 2 to 4 were determined by a *loop*, with 36 meters in the entrance radius (section 2), a second radius of 18 meters in the curvature towards the peak (section 3) and, at the exit of the *loop*, back at the same radius as the entrance, 36 meters (section 4).

4. ANALYSIS

To carry out the analyzes, the vehicle contains a mass of 200 kg, with four people on board, each with 136 kg. For the analysis of the accelerations, an initial speed of 1 m/s was established at the beginning of the first fall (start of Section 1). This value was defined after some analyzes with higher initial velocities where the analyzed parameters obtained great divergences in relation to the G forces that the body can support, defined by norm F2291 (2006).

Through the balance of forces and energies, it was possible to carry out calculations, for each position of the cart and passengers, by stretch (i), according to the equations in Table 1.

The sum of the mass of the car (m_c) and the people (m_p) that are in it, are considered as the total mass (m_t) and the calculations of the forces are based on this value. The incidence of the weight force, which is the action that the cart undergoes on Earth, was analyzed; the normal force, responsible for keeping the car on track; the drag force, which will contribute to the loss of acceleration and speed, due to the fluid in which the roller coaster is (the air), the rolling resistance force (due to the action of the wheels on the rails) and the frictional force on the cart bearings.

In the stretch of the *loop*, the resistive forces are placed in the direction of the acceleration A_t (frontal acceleration), but in opposite directions. The normal force is always perpendicular to the track and the weight is always vertical and down. Thus, in the balance of forces in the tangential and normal directions, the values of the angles of the slopes between 0° and 90° , between 90° and 180° and 0° were considered, which leads to projections of the weight that contribute and do not contribute to the accelerations. The same behavior applies to weight work. These contribution situations are or are not adjusted through the signs of these variables in the equations of normal force and accelerations.

With the action of forces in a space, it was possible to calculate the work of each of them. Thus, with the balance of the acting forces, including the dissipative forces, the total work was calculated.

The human body has a limit of exposure to force G in a defined period for each of the axes. Tables 2 to 4 highlight these maximum allowed Gs values that can be supported by the passenger. In some cases, the addition of some items, such as the head support or some restriction on displacement is enough, so that there is re-accommodation and no interference in the person's health. In other cases, the project should have its calculations redone.

5. RESULTS

For the initial speed of 1 m/s and for the parameters established to analyze the ride, some results were obtained and analyzed. The main parameters established for the analyzes are shown in Table 5. The profiles of each stretch are shown in Figure 1.

Table 1. Equations Developed for the Analyzes

Description	Equation	Nº
Position on the track for the (i) up and down stretches	$z_i(x_i) = c_{i1}x_i^3 + c_{i2}x_i^2 + c_{i3}x_i + c_{i4}$	(1)
Track position for loop stretches	$z_2 = z_{o2} - \sqrt{(R_1)^2 - (x_2 - x_{o2})^2}$	(2)
	$z_3 = z_{o3} + \sqrt{(R_2)^2 - (x_3 - x_{o3})^2}$	(3)
	$z_4 = z_{o4} - \sqrt{(R_1)^2 - (x_4 - x_{o4})^2}$	(4)
Track position for horizontal stretches	$z_i = c_i = 5$	(5)
Slope in radians from tangents to the profile curve	$\text{alpha1_rad}_i = \text{atan}(3c_{i1}x_i^2 + 2c_{i2}x_i + c_{i3})$	(6)
Slope in degrees from tangents to the profile curve	$\text{alpha1_degrees}_i = z_{o2}/180\pi$	(7)
Curvature radius (ascending and descending stretches)	$\rho_i = \left[1 + \left(\frac{dz}{dx}\right)^2\right]^{3/2} / 180\pi$	(8)
Curvature radius (horizontal stretches)	$\rho_i = 10^{10}$	(9)
Radius of curvature (circumference legs in loop)	$\rho_i = R_1, \text{ou } R_2, \text{ ou } R_3$	(10)
Normal force	$N_i = P \cos(\text{alpha1_degrees}_i) + m_t \left(\frac{v_i^2}{\rho_i}\right)$	(11)
Displacement along the track profile (from i - 1 to i)	$s_i = \sqrt{(z_i - z_{i-1})^2 - (x_i - x_{i-1})^2}$	(12)
Weight work	$W_{P_i} = P \text{sen}(\text{alpha1_rad}_i) s_i$	(13)
Rolling resistance	$F_{r_i} = fN_i$	(14)
Rolling resistance work	$W_{F_r} = F_{r_i} s_i$	(15)
Friction force on bearings	$F_{b_i} = \mu N_i$	(16)
Friction force work on bearings	$W_{F_b} = F_{b_i} s_i$	(17)
Speed	$v_i = \sqrt{\frac{2}{m_t} [E_{c_{i-1}} - W_{P_i} - W_{F_{r_i}} - W_{F_{b_{i-1}}}]}$	(18)
Kinetic energy	$E_c = (1/2)m_t v_i^2$	(19)
Drag force	$F_{d_i} = \frac{1}{2} \sqrt{\rho_{ar} A_f C_d} v_i^2$	(20)
Drag force work	$W_{F_{d_i}} = F_{d_i} s_i$	(21)
Front acceleration	$a_{t_i} = \frac{1}{m_t} [P \text{sen}(\text{alfa1_rad}_i) - F_{r_i} - F_{b_i} - F_{d_i}]$	(22)
Normal acceleration	$a_{n_i} = \frac{1}{m_t} [N_i + P \cos(\text{alfa1_rad}_i)]$	(23)
Front G force felt	$G_{t_i} = 1 + a_{t_i}$	(24)
Normal G-force felt	$G_{n_i} = 1 + a_{n_i}$	(25)
Instant of time in each position	$\text{time}_i = \text{time}_{i-1} + \left \frac{v_{i-1} - v_i}{a_{t_i}} \right $	(26)

Where:

- zo2 = center of circumference of *loop* rise;
- zo3 = center of the circumference of the inverted *loop* portion;
- zo4 = zo2 = center of circumference of *loop* descent;
- R1 = radius at the curvature at the entrance and exit of the *loop* (Strech 2 and 4);
- R2 = radius in the curvature of the inverted section of the *loop* (Strech 3).

Table 2. Maximum Allowable Time for Acceleration on the Gx Axis

Maximum Acceleration + Gx / -Gx	+6.0 G	+4.0 G	+3.0 G	+ 2.5 G	-3.5 G	-2.5 G	-2.0 G	-1.5 G
Maximum allowed time	0.8 s	2.0 s	6.6 s	Undetermined time	1.8 s	1.0 s	Undetermined time	

Table 3. Maximum Allowable Time for Gy Acceleration

Aceleração Máxima +-Gy	+3.0 G	+ 2.0 G
Maximum allowed time	0.8 s	Tempo indeterminado

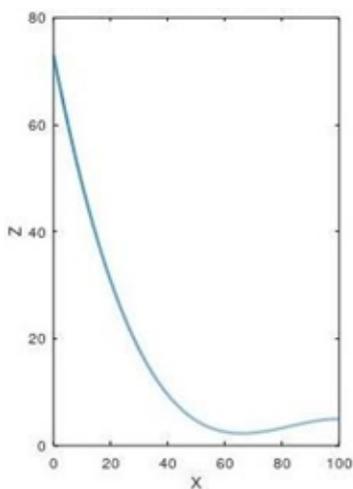
Table 1. Maximum Allowable Time for Acceleration on the Gz Axis

Maximum Acceleration + Gz / -Gz	+6.0 G	+4.0 G	+3.0 G	+ 2.0 G (if preceded by 3s or more than - Gz)	+2.0 G	-1.5 G	-1.1 G
Maximum allowed time	0.8 s	2.0 s	6.6 s	3.5 s	Undetermined time	1.0 s	Undetermined time

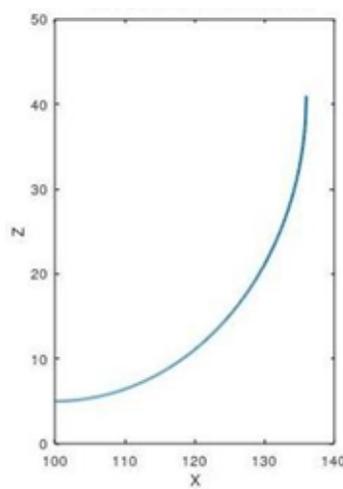
Table 5. Main Analysis Parameters

Parameter	Variable	Value
Mass per person in kg	m_p	136
Car mass in kg	m_c	200
Total mass of car with people in kg	m_t	$4m_p + m_c$
Gravity acceleration in m/s^2	g	9.81
Total weight of car with people in Newton	P	$m_t g$
Bearing friction coefficient	f	0.001
Bearing friction coefficient	m_i	0.001
Air density in kg/m^3	ρ_{ar}	1.225
Front area of the car	A_f	0.49
Drag coefficient	C_d	1
Minimum height of the runway in meters	H_{chao}	5
Circle radius (loop entry)	R_1	36
Circle radius (inversion of loop)	R_2	$R_1/2$
Circle radius (loop output)	R_3	40
Angle of initial slope of tangent to runway	$\alpha_{initial_degrees}$	110

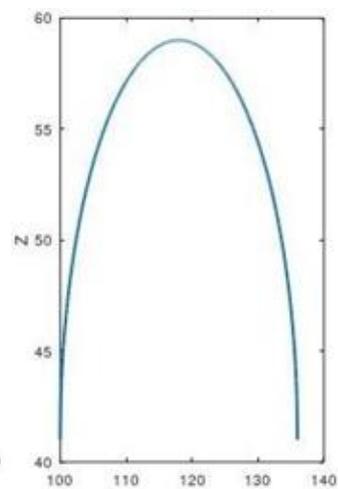
Profile 1



Profile 2



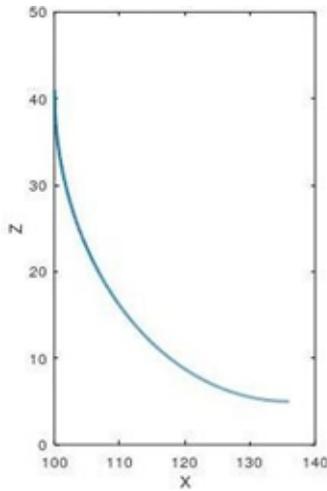
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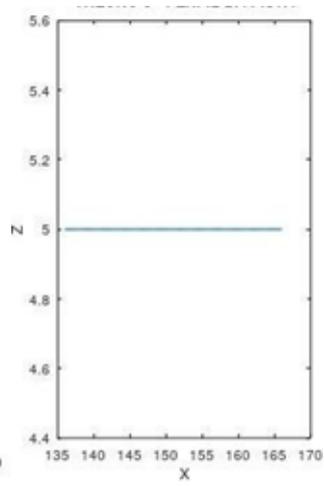
Profile 4

Profile 5

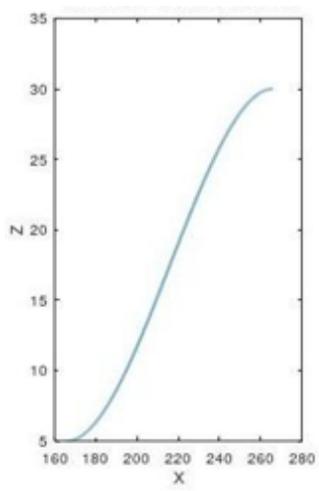
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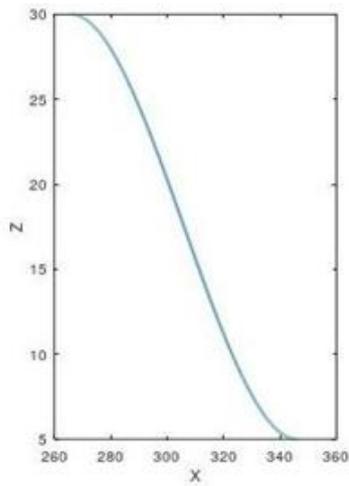
Profile 7



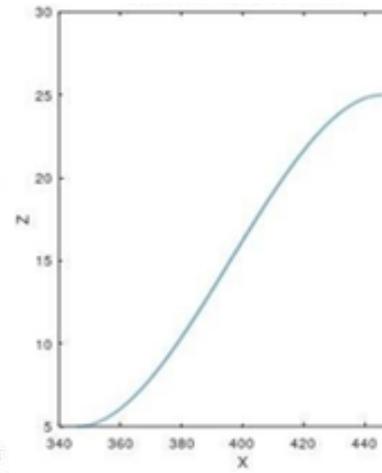
Profile 8



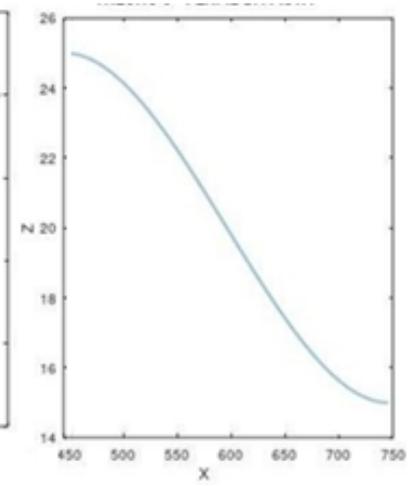
Profile 9



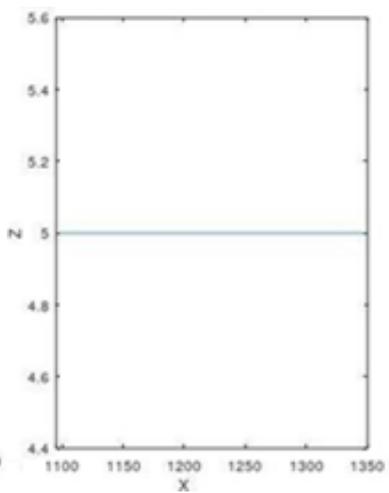
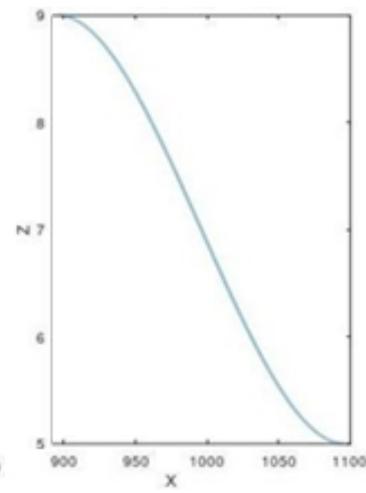
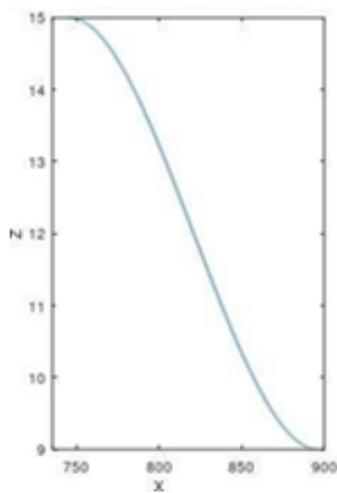
Profile 10



Profile 11



Profile 12



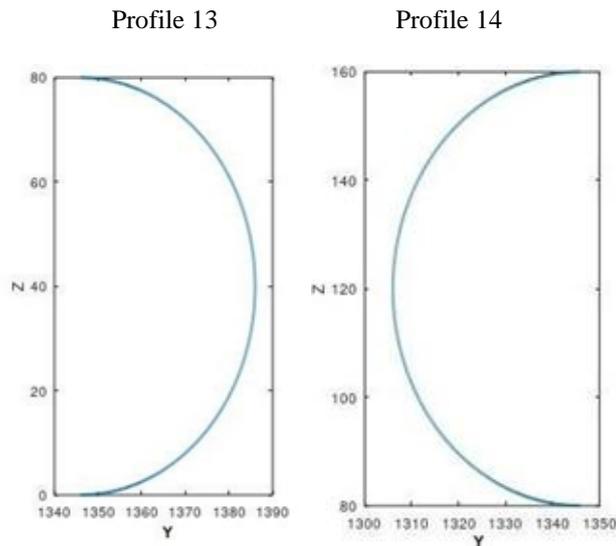


Figure 1. Track Profiles on the Sections

5.1. G-FORCE OVER THE STRECHES

In this section, will be seen how the G-Force impacts on the stretches, by showing some of them that generate more impact on construction of the roller coaster trail.

Figure 2 highlights the vertical and horizontal G-forces felt by the passenger along the stretch 2. The maximum vertical G-force felt was 6.5 G, around 100 m from the X axis. This value is above the limit for tolerance, which may cause risks to the passenger. The maximum frontal G force felt was 0.97 G, around 100 m on the X axis. This value is below the limit for tolerance, causing no risk to the passenger.

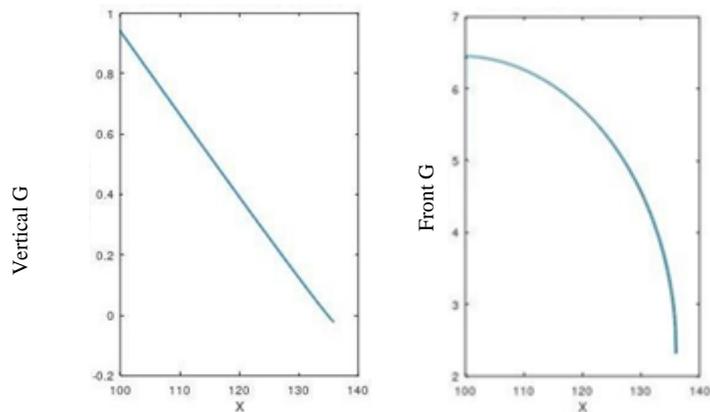


Figure 2. Incidence of G's Forces in Strech 2

Figure 3 highlights the vertical and horizontal G-forces felt by the passenger along the stretch 3. The maximum vertical G-force felt was 3.5 G, around 140 m from the X axis. This value is below the limit for tolerance, causing no risk to the passenger. The maximum frontal G force felt was 2 G, around 100 m traveled on the X axis. This value is below the limit for tolerance, causing no risk to the passenger.

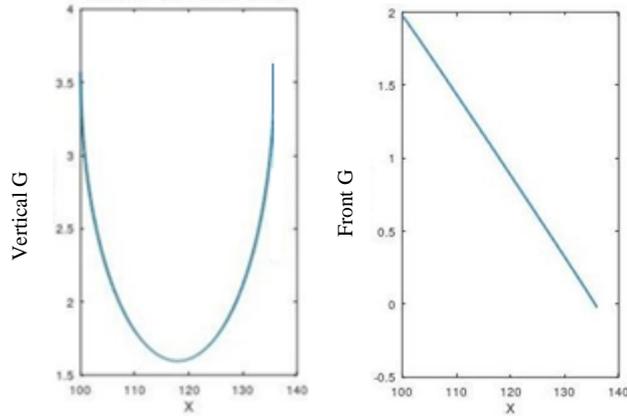


Figure 3. Incidence of G's Forces in Stretch 3

Figure 4 highlights the vertical and horizontal G-forces felt by the passenger along the stretch 4. The maximum vertical G-force felt was 6.15 G, around 140 m from the X axis. This value is above the limit for tolerance, causing risks to the passenger. The maximum frontal G force felt was 2 G, around 100 m traveled on the X axis. This value is below the limit for tolerance, causing no risk to the passenger.

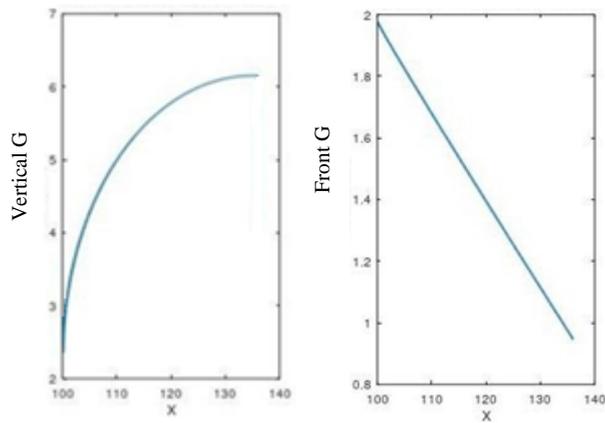


Figure 4. Incidence of G's Forces in Stretch 4

Figure 5 highlights the G forces, vertical and horizontal, felt by the passenger along the stretch 5. The maximum vertical G force felt was 4.2 G at the exit of the *loop* to 1 G during the entire displacement of the stretch 4. These values are below the limit for tolerance, causing no risk to the passenger. The maximum frontal G force felt was 0.953 G. This value is below the tolerance limit, causing no risk to the passenger.

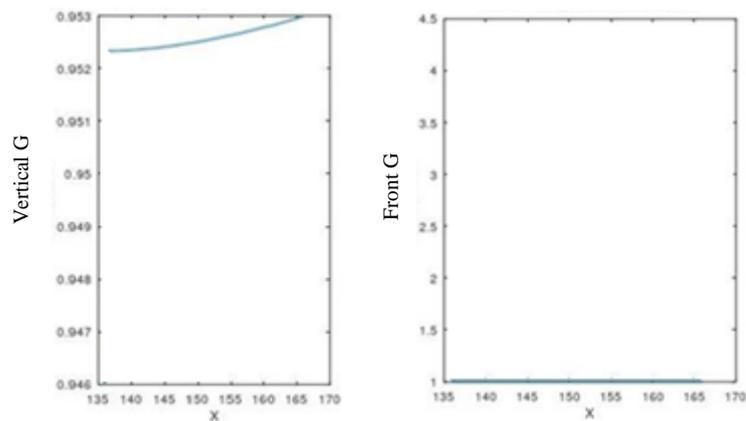


Figure 5. Incidence of G's Forces in Stretch 5

Figure 6 highlights the vertical and horizontal G-forces felt by the passenger along section 9. The vertical G-force, after goes along the fall, undergoes a slight decrease. Starting from the maximum value of 1.6 G to 1 G, when the angle is minimal and returned to 1, at the end of section 9. This value is below the limit for tolerance, causing no risk to the passenger for booth cases. Because the frontal G-force has its maximum point of approximately 0.98 G, at the beginning of the fall, returning to the same value at the end of it.

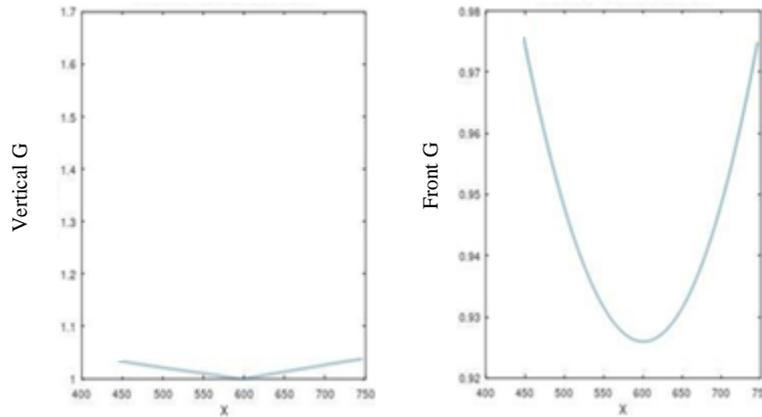


Figure 6. Incidence of G's Forces in Strech 9

Figure 7 highlights the vertical and horizontal G-forces felt by the passenger along the stretch 12. The maximum vertical G-force is felt at about 1.03 G at the end point of the fall at 5 m, remaining stable along the stretch, at 1 G. This value is below the tolerance limit, causing no risk to the passenger. The frontal G-force has its maximum point of 0.979 G, after a small jump of 0.974 G, due to the lateral acceleration being slightly less than gravity and losing speed along the path. This value is below the limit for tolerance, causing no risk to the passenger.

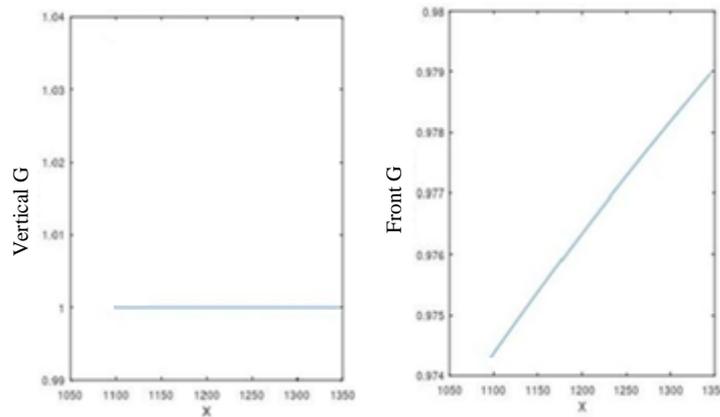


Figure 7. Incidence of G's Forces in Strech 12.

Figure 8 highlights the G forces, vertical and horizontal, felt by the passenger along the stretch 13. The maximum lateral G force is felt at approximately 2.4 G when changing the displacement of the X axis, to the Y axis, decreasing up to 2.05 G until the end of the roller coaster ride. This value is below the limit for tolerance, causing no risk to the passenger. The frontal G-force decays at point Y equal to 0 and rises slightly when approaching the end of the first curve. This value is below the limit for tolerance, causing no risk to the passenger.

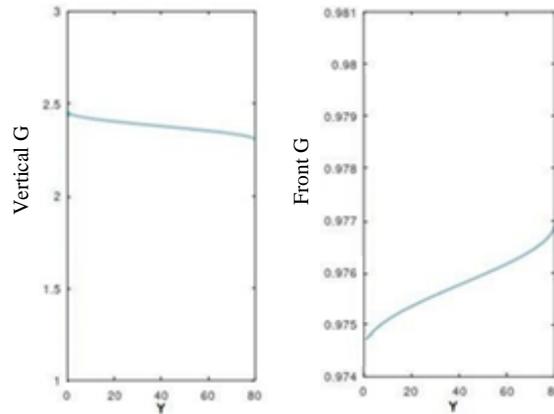


Figure 8. Incidence of G's Forces in Stretch 13.

Figure 9 highlights the G forces, vertical and horizontal, felt by the passenger along the stretch 14. The maximum lateral G force is felt at approximately 2.3 G in the transition between the axes, decreasing to 2.05 G by the end of the roller coaster path. This value is below the limit for tolerance, causing no risk to the passenger. The frontal G-force continues the growth of section 13, returning to an approximate value of the beginning of section 13, ending the roller coaster with 0.981 G. This value is below the limit for tolerance, causing no risk to the passenger.

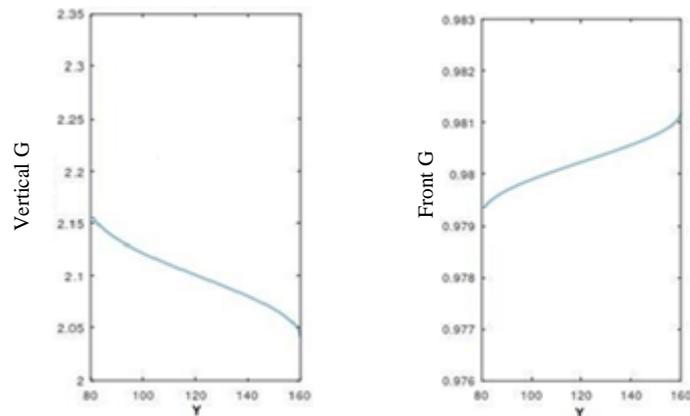


Figure 9. Incidence of G's Forces in Stretch 14.

6. CONCLUSIONS

The values obtained from the accelerations were compared to the limits tolerable by the human body, based on the established in tables 2 to 4, adapted from ASTM F 2291 (2006), international reference for roller coaster projects.

Even with care in choosing the polynomial functions of the third degree, to guarantee continuities in the transitions between each stretch, some variables described jumps. This was probably due to the inversions of the concavity of the curves and changes in the sign of the angle of the slope of the tangent to each analyzed point.

Another important fact was what happened in Section 3, where there was a loss of normal strength, indicating the possibility of the car detaching from the track. This behavior can be improved by increasing the initial speed at the beginning of Section 1, or by changing the profile. For all other stretches, the behaviors occurred within the expectation.

Regarding the levels of Gs obtained (normal, lateral and frontal), it can be said that the proposed profile, for the initial speed of 1 m/s at the beginning of section 1, can be considered safe for passengers for almost all stretches, as the values obtained were below acceptable limits. However, stretches 2 to 4 of the *loop*, showed acceleration values at the beginning and at the end above 6 G and at the top of Stretch 3 although it is an acceleration carried by the body, it is a too long exposure time.

Therefore, with these considerations, unfortunately the project could not be approved. The problems in Sections 2 to 4 would need to be corrected by a change in the profile and reassessed.

In view of the limitations of this work, it is suggested for future work to evaluate the accelerations felt by passengers along routes that include inclines on the rail in the horizontal stretches. It is also interesting to carry out works adding these same assessments in stretches with spiral falls.

Despite the importance for analysis of performing a comparison between literature results. It's not able to apply on this work, due to the difference into the stretches, once this one use real data based on what Sea World Parks provided.

7. REFERENCES

- ASTM International, 2001. *F 2137-2001 - Standard Practice for Measuring the Dynamic Characteristics of Amusement Rides and Devices*.
- ASTM International, 2006. *F 2291-2006 - Standard Practice for Design of Amusement Rides and Devices*.
- Halliday, D., Resnick, R., Walker J. "*Fundamentals of Physics Extended*", 8th ed. Rio de Janeiro: LTC, 2008.
- Hibbeler, R. C. "*Engineering Mechanics Dynamics*", 12th ed. São Paulo: Pearson Prentice Hall, 2011.
- Hunt, K., 2018. *Design Analysis of Roller Coasters*.
- Weisenberger, N, 2013. *Coasters 101 - An Engineer's Guide to Roller Coaster Design, Third Edition*.

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