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COMPARISON BETWEEN QUAD AND DOUBLE-DOUBLE LAMINATES IN THE BUCKLING DESIGN OF A CARBON/EPOXY WING STRUT

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Abstract. The strut-braced wing configuration has received prominence in the aeronautical industry today, mainly due to the development of the SUGAR aircraft project of the Boeing-NASA partnership. The configuration is a wing with high elongation and with a truss, called strut, which connects the wing to the fuselage. In this way, it is possible to obtain wings with large wingspans and small thicknesses. However, the design of the strut becomes decisive for the viability of the configuration, since when in compression, it needs to resist a high buckling load. Thus, the use of composite materials is necessary to guarantee resistance to buckling without the substantial increase in weight. On the other hand, the conventional laminate, called quad for having the orientations 0, ±45 e $90°$, currently used in the aeronautical industry, must follow a series of design practices that make the choice of the optimum laminate unfeasible. The 10% rule stands out, which requires that the final laminate present at least 10% of its layers in each of these orientations. Despite not being documented in a standard, thus restriction is adopted to guarantee a minimum portion of the laminate's transversal stiffness. In addition, quad laminates are difficult to manufacture and do not allow the suppression of layers for weight relief, since the removal of some layer from the laminate alters its properties. Thus, a new family of laminates called double-double appears, in which the layers have orientations $\pm\varphi$, $\pm\psi$, in which the angles can assume any desired values, which allows optimization laminate with less complexity, less weight and manufacturing advantages. Therefore, it was proposed in this work to compare the structural design of the strut in conventional laminates quad and double-double from the composite carbon-epoxy material T300-F934, in which the truss is subjected to compressive uniaxial load. As a result, it is obtained the laminates quad and double-double $[0_{56}/45_2/-45_2/90_4]$ and $[(\pm7/\pm16)_8]_s$ respectively, where the quad laminate resulted in 28% increase in weight in relation to the optimal unidirectional laminate and the double-double laminate by 14%. Thus, the double-double has an intermediate number of layers between the optimal unidirectional and quad laminates and, although small, the recompositing of the stiffness portion in the transverse direction in the double-double laminate can be considered significant, proving the viability of these laminates for the aeronautical industry.

Keywords: Strut-braced wing, SUGAR VOLT, Buckling, Quad Laminate, Double-Double Laminate.

1. INTRODUCTION

Recent studies (Meinicke, 2019), regarding aerodynamics, indicate that wings for future aircraft will have greater elongation, will be lighter and more flexible and an example with these characteristics is the strut braced wing concept. This configuration has received prominence in the aeronautical industry in the last few years due to the development of the project Subsonic Ultra Green Aircraft Research (SUGAR) by Boeing's partnership with NASA. The strut braced wing concept is characterized by a wing with large span and high elongation, which reduces induced drag and allows performance improvements. To avoid an increase in structural weight, which could make the configuration unfeasible, a truss connecting the wing to the fuselage, called strut, is used as a way to reduce wing's bending moment and undesirable aeroelastic effects. Thus, the relevance of this new configuration and the fact that the strut design is essential for the feasibility and good performance of the aircraft motivated the study presented in this work.

For most of the flight and in maneuvers with a positive load factor, the strut works in tension. However, for the cases of landing and maneuvering with a negative load factor, the structure is subject to a high compression load. As it is a slender structure, there is a concern about a possible buckling failure. Thus, the strut design using conventional materials to resist buckling would result in a heavy structure, which could penalize the configuration in relation to the cantilever wing and make the aircraft unfeasible. Therefore, the use of composite materials becomes a viable and necessary alternative for strut design due to the characteristics of these materials of high rigidity and strength and low density. In this way, it becomes possible to guarantee the necessary rigidity without major increases or even a reduction in structural weight compared to a conventional wing.

Currently, the use of composites is already widespread in the aeronautical industry, but there are still difficulties in relation to the optimization and manufacture of parts made of these materials. Due to industry practices, the so-called

conventional or quad laminate has restrictions that make it impossible to use the optimum laminate. This conventional laminate has layer orientation at 0, ±45 and 90 degrees and is required to be symmetrical and balanced. In addition, the 10% rule applies, which requires at least 10% of the layers of the final laminate to be facing each of the main directions of reinforcement. These limitations affect the performance of the laminate, mainly because it cannot be tapered to save weight, since the removal of some layer in the stack changes its properties.

Thus, a new family of laminates is proposed by Tsai (2019). The laminate known as double-double has two stacking orientations ±φ, ±ψ, where the angles can take on any desired values. In addition, the double-double laminate can be substantially homogenized and tapered to reduce weight, as symmetry is naturally achieved by homogeneity. In addition, the angles of double-double laminates form a continuous field, in addition to having improvements in manufacturability.

Therefore, the objective of this study was to compare the strut design of a strut-braced wing aircraft with characteristics similar to the SUGAR project design in quad laminate with the practices of the aeronautical industry and the strut design in double-double laminate.

2. LITERATURE REVIEW

2.1 Double-double laminates

According to Tsai (2019) and Vermes et al. (2021), some characteristics and advantages of the double-double laminate stand out. Quad laminates are very difficult to design and manufacture because the sub laminate of the building block is very thick. It can be 12 to 20 layers thick with imposed midplane symmetry. On the other hand, double-double is a maximum of 4 layers thick in its building block, which results in a thinner laminate.

As a consequence of the smaller thickness of the sub-laminates, the double-double layers are thin or evenly dispersed. Homogenization is achieved when, mathematically, $[A^*] = [D^*]$ and $[B] = 0$. Once these dispersion and homogenization are obtained, there is no longer need for midplane symmetry. Quad laminate can also achieve homogenization, however with much more layers.

The difference between reaching a homogenized state by a quad $[0_5/\pm45_2/90]$ and its replacement by a DD $[\pm0/\pm50]$, for example, is shown in Figure 1. It is observed that for a quad laminate, they are more than 400 layers are needed, while for the DD only 32 layers.
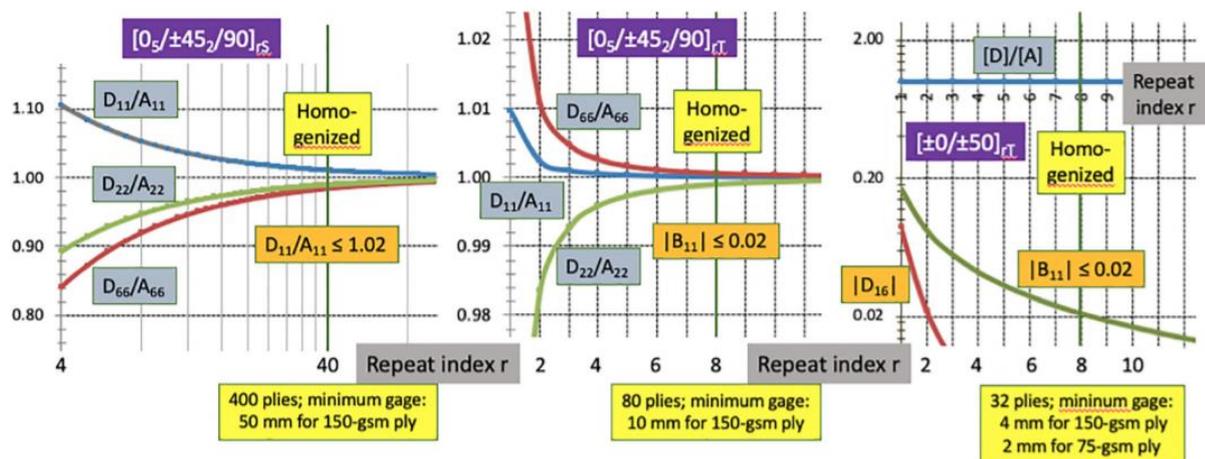

Figure 1. Homogenization: more than 400 layers needed for quad; 32 for DD. Source: Vermes et al. (2021)

Besides, ply drop can be done one layer at a time. For quad laminate, it needs to be two layers at a time to maintain midplane symmetry. Also, the stiffness of the DD may remain the same. For quad laminate, stiffness and strength change as each set of layers is removed as can be seen in Figure 2.
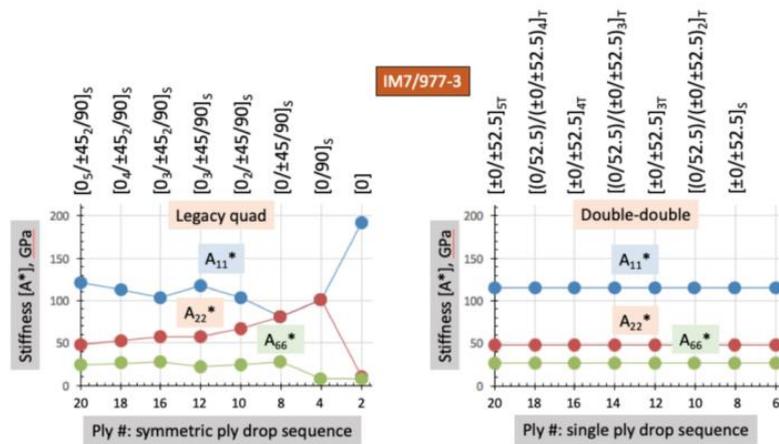
Figure 2. Quad vs Double-double: variable vs constant stiffness. Source: Tsai et al. (2019)

3. METHODOLOGY

3.1 Boundary conditions

As the structural design of the strut is included in the aircraft design, some initial parameters are previously defined and these analyzes are beyond the scope of this work and will only be considered valid. These definitions include the strut chord and profile, the attachment point of the strut to the wing, and the strut not being a lifting surface. From the profile and the chord, the length and height of the strut box are defined.

For the most conservative analysis possible, it is assumed that the ends of the structure are both simply supported. Also, it is considered that the strut is subjected to a compressive uniaxial load, being the main load for the design, since the buckling failure is critical in the analysis. It was checked whether the load acting due to drag would be considerable in the design, but the order of magnitude is much lower than the compressive load analyzed. Besides that, the buckling load is assumed uniform over the area of the strut and the strut is not pressurized. Finally, the use of bracing known as jury to reduce the buckling length of the structure was considered.

Details of the SUGAR's structure can be seen in Figure 3. It can be seen in the image that the bracing divides the structure into two: it is assumed one of 5.5 m and the other of 4.5 m. Therefore, the analysis will consider the length of 5.5 m.
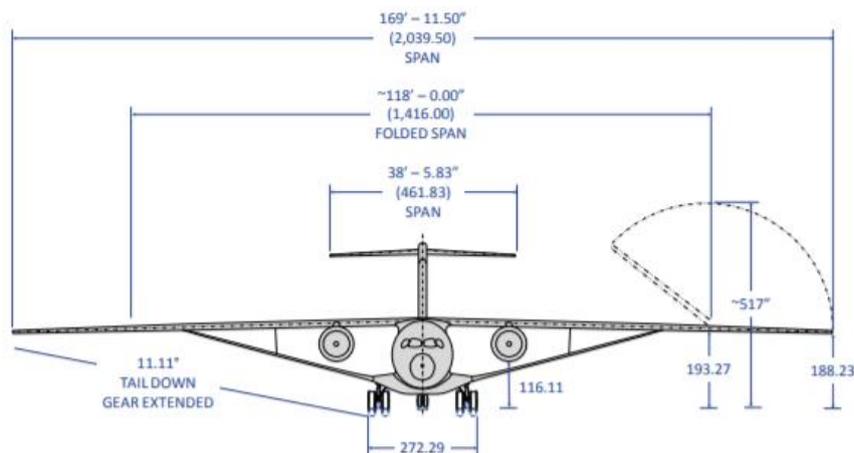

Figure 3. Front view of the aircraft with the strut and jury structures. Source: Bradley et al. (2012).

3.2 Material properties

The properties of the composite material used in the analyzes were values related to T300 - F934, carbon fiber/epoxy composite, obtained from Tsai (2019). The defined mass density is an average value obtained in MInus (2005). Values can be seen in Table 1.

Table 1. T300-F934 material properties.

Material Properties	Values
Modulus of elasticity in direction 1, GPa	148.0
Modulus of elasticity in direction 2, GPa	9.65
Stiffness modulus, GPa	4.55
Poisson's coefficient	0.30
Mass density, kg/m^3	2.0×10^3
Layer thickness, mm	0.125

3.3 Running loads

For the calculation of the running loads in the strut, the following loads are considered:
- Lift force distributed along the span of the wing;
- Wing inertia load;
- Engine load considered as concentrated;
- Weight of fuel allocated to the wing;

As a result, the problem becomes a hyperstatic structure with 3 components, wing, strut and jury. The critical condition of load factor n =-1 and safety factor FS = 1.5 was considered.

3.4 Analytical model

The analytical model used was developed in MATLAB software. Inputs are defined as the number of layers and the sequence of angles of the laminate to be analyzed and the minimum number of layers for laminates in which the critical buckling load is greater (and as close as possible) than the acting load is sought. Calculations are made for the flanges and column webs. The model flowchart is represented by Figure 4.
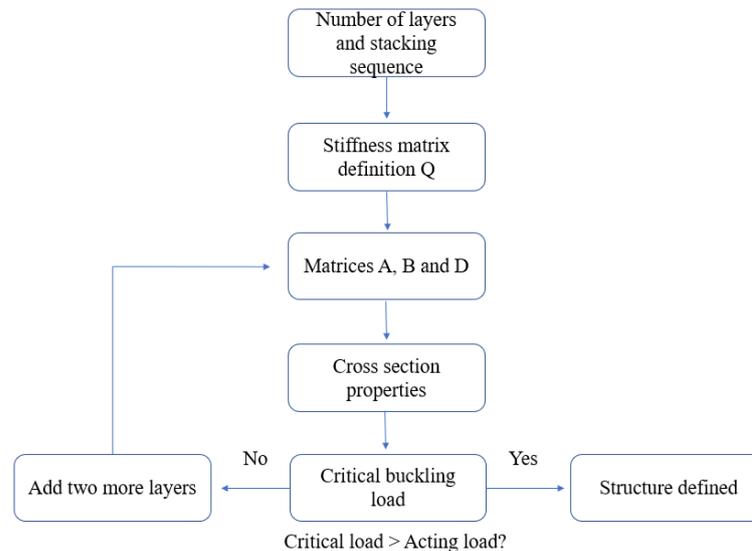

Figure 4. Laminate definition flowchart. Source: Authors (2021).

The model follows the following calculation sequence:
- The stiffness matrix of the material [Q] is defined;
- From the transformation ratios, the matrices A, B and D of the laminated flanges and webs are calculated;
- With the thicknesses and matrices, A and D, the resulting modulus of elasticity is defined due to the laminates of the webs and flanges and the moment of inertia of the cross section, and, consequently, the stiffness of the EI section;
- Finally, from the Euler equation, the critical buckling load for the column (both ends simply supported) is calculated;
- By comparing the critical buckling load and the acting compressive load, if the beam fails, two new layers are added to the laminate and the process is repeated.

3.5 Cross section stiffness EI

To calculate the moment of inertia of the cross section, a necessary parameter for the calculation of the critical buckling load by the Euler equation, the box is approximated to the strut by a rectangular shape. Furthermore, the 4 members of the section are assumed to have the same layup and thickness.
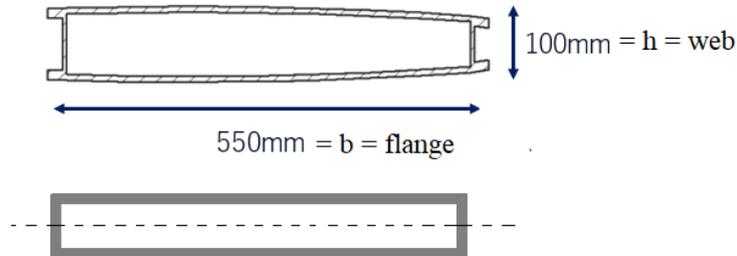

Figure 5. Cross section simplification. Source: Authors (2021).

In this way, the moment of inertia of the section is calculated:

$$I = \frac{bh^3}{12} - \frac{(b-2t)(h-2t)^3}{12}, \tag{1}$$

where b is the length of the section, h is the height, and t is the thickness of the walls.
Then, the laminate modulus of elasticity of each member of the cross section is calculated. It is defined:

$$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} \\ A_{12} & A_{22} & A_{26} \\ A_{16} & A_{26} & A_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_{xy} \end{Bmatrix}, \tag{2}$$

For balanced laminates, know that $A_{16} = A_{26} = 0$. Then:

$$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} & 0 \\ A_{12} & A_{22} & 0 \\ 0 & 0 & A_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_{xy} \end{Bmatrix}, \tag{3}$$

For uniaxial loading in the global x-axis direction:

$$N_x = A_{11}\varepsilon_x + A_{12}\varepsilon_y, \tag{4}$$

$$0 = A_{12}\varepsilon_x + A_{22}\varepsilon_y, \tag{5}$$

Solving the previous equation:

$$\varepsilon_y = -\frac{A_{12}}{A_{22}}\varepsilon_x, \tag{6}$$

Replacing equation (6) into equation (4):

$$N_x = A_{11}\varepsilon_x - \frac{A_{12}^2}{A_{22}}\varepsilon_x, \tag{7}$$

$$N_x = \varepsilon_x \left(A_{11} - \frac{A_{12}^2}{A_{22}} \right) = \sigma_x t, \tag{8}$$

$$\sigma_x = \bar{E}_x \varepsilon_x = \frac{1}{t}\left(A_{11} - \frac{A_{12}{}^2}{A_{22}}\right)\varepsilon_x,$$
(9)

Therefore, the resulting modulus of elasticity of the laminate can be calculated by:

$$\bar{E}_x = \frac{1}{t}\left(A_{11} - \frac{A_{12}{}^2}{A_{22}}\right),$$
(10)

4. RESULTS

The calculation was based on the classical laminate theory and it was used the geometric parameters of the column and the properties of the composite material chosen T300-F394 carbon-epoxy for the analyses. As both flanges and webs will have the same layup and the same thickness, the following results are for the laminate of both members.

4.1 Analytical model of the quad laminate without application of the 10% rule

For a uniaxial compressive load, it is known that the best orientation is for all layers at 0 degrees, which was defined in the analytical model previously. The result laminate is represented by $[0_{28}]_s$ and matrix A referring to this laminate are described in Eq. (11).

$$[A] = \begin{bmatrix} 1042 & 20.4 & 0 \\ 20.4 & 67.9 & 0 \\ 0 & 0 & 31.9 \end{bmatrix} GPa \cdot mm,$$
(11)

Thus, the properties of the column cross section and the equivalent laminate were calculated. Finally, from the Euler equation for columns with simply supported ends, the critical buckling load was calculated and, with the assumed mass density for the laminate, the structure mass for 10 m was calculated, considering that all the strut is manufactured with the same layup and thickness.

4.2 Analytical model of the quad laminate with application of the 10% rule

With the application of the 10% rule, it becomes necessary to add layers in the orientations ±45 and 90. Thus, keeping the 56 layers at [0], it will be necessary to add at least 8 layers in the orientation [±45] and 8 layers in the orientation [90]. The resulting laminate will be $[0_{28}/+45_2/-45_2/90_4]_s$ with the proportions: 78% layers at [0]; 11% layers at [±45]; 11% layers at [90]. Despite the increase in layers in the directions [±45/90] compared to the previous laminate, it is not possible to remove layers from the orientation [0] and maintain the necessary rigidity of the laminate. The matrix [A] referring to the laminate are, respectively:

$$[A] = \begin{bmatrix} 1097.47 & 59.8 & 0 \\ 59.8 & 262.5 & 0 \\ 0 & 0 & 74.6 \end{bmatrix} GPa \cdot mm,$$
(12)

4.3 Double-double laminate

It is known that the best orientation of the layers to resist a uniaxial load is all [0]. Thus, a laminate that approaches the optimum is sought, that is, has the longitudinal stiffness component A_{11} close to that of the optimum laminate, but which restores, at least a little, the properties that were guaranteed by the 10% rule, that is, values of the shear and transverse stiffness components, A_{22} and A_{66}. To choose the angles, the sets were evaluated moving away from the angle of [0] and approaching [45], in order to ensure sufficient longitudinal stiffness for the laminate, but replacing the transversal and shear components in the plane. As a result, the double-double laminate $[(\pm7/\pm16)_8]_s$ was found. This laminate is made up of 16 sub-laminates, totaling 64 layers. The matrix [A] referring to the laminate are:

$$[A] = \begin{bmatrix} 1094.8 & 68.9 & 0 \\ 68.9 & 82.5 & 0 \\ 0 & 0 & 82.06 \end{bmatrix} GPa \cdot mm,$$
(13)

The properties of the resulting laminates are summarized in Table 2.

Table 2. Laminate properties.

Laminate Properties	Quad without 10%	Quad with 10%	Double-Double
Number of layers	56.0	72.0	64.0
Thickness, mm	7.0	9.0	8.0
Moment of inertia of the cross section, mm^4	1.74×10^7	2.14×10^7	1.94×10^7
Modulus of elasticity, MPa	1.48×10^5	1.20×10^5	1.29×10^5
Equivalent stiffness, MPa mm^4	2.58×10^{12}	2.57×10^{12}	2.52×10^{12}
Critical buckling load, kN	841.31	840.40	823.06
Mass, kg	178.08	227.52	202.88

5. DISCUSSION

The stiffness matrix for a layer of the chosen composite material is defined by:

$$[Q] = \begin{bmatrix} 148.87 & 2.9 & 0 \\ 2.9 & 9.71 & 0 \\ 0 & 0 & 4.55 \end{bmatrix} GPa \cdot mm, \tag{14}$$

The Tsai modulus for a sheet of the carbon-epoxy system used is:

$$\text{Tr}(Q) = 148.87 + 9.71 + 2 * (4.55) = 167.68 \text{ GPa}, \tag{15}$$

Normalizing the normalized stiffness matrix of the unidirectional laminate, designated by [A*], by the Tsai modulus, we have:

$$[A^*] = \frac{[A]}{t} = \frac{1}{7.0} \begin{bmatrix} 1042 & 20.4 & 0 \\ 20.4 & 67.9 & 0 \\ 0 & 0 & 31.9 \end{bmatrix} = \begin{bmatrix} 148.87 & 2.91 & 0 \\ 2.91 & 9.71 & 0 \\ 0 & 0 & 4.55 \end{bmatrix}, \tag{16}$$

$$[\overline{A^*}] = \frac{[A^*]}{\text{Tr}(Q)} = \frac{1}{167.68} \begin{bmatrix} 148.87 & 2.91 & 0 \\ 2.91 & 9.71 & 0 \\ 0 & 0 & 4.55 \end{bmatrix} = \begin{bmatrix} 0.888 & 0.017 & 0 \\ 0.017 & 0.058 & 0 \\ 0 & 0 & 0.027 \end{bmatrix}, \tag{17}$$

Thus, the trace of $\overline{A^*}$ is represented by:

$$\text{Tr}(\overline{A^*}) = 0.888 + 0.058 + 2 * (0.027) = 1, \tag{18}$$

This means that, of the total available stiffness, the longitudinal stiffness portion is 88.8%, the transverse stiffness portion is 5.8% and the in-plane shear stiffness portion is 5.4%.

For the quad laminate $[0_{28}/45_2/45_2/90_4]_s$, obtained with the 10% rule, commonly used in the aeronautics industry, the normalized stiffness matrices [A*] and its consequent normalization by the Tsai modulus $[\overline{A^*}]$ present the following results:

$$[A^*] = \frac{[A]}{t} = \frac{1}{9.0} \begin{bmatrix} 1097.47 & 59.8 & 0 \\ 59.8 & 262.5 & 0 \\ 0 & 0 & 74.6 \end{bmatrix} = \begin{bmatrix} 121.94 & 6.65 & 0 \\ 6.65 & 29.16 & 0 \\ 0 & 0 & 8.29 \end{bmatrix}, \tag{19}$$

$$[\overline{A^*}] = \frac{[A^*]}{\text{Tr}(Q)} = \frac{1}{167.68} \begin{bmatrix} 121.94 & 6.65 & 0 \\ 6.65 & 29.16 & 0 \\ 0 & 0 & 8.29 \end{bmatrix} = \begin{bmatrix} 0.727 & 0.04 & 0 \\ 0.04 & 0.174 & 0 \\ 0 & 0 & 0.049 \end{bmatrix}, \tag{20}$$

Thus, the trace of $\overline{A^*}$ is represented by:

$$\text{Tr}(\overline{A^*}) = 0.727 + 0.174 + 2 * (0.049) = 1, \tag{21}$$

Now you have a different proportion between the stiffness portions: 72.7% for longitudinal stiffness, 17.4% for transverse stiffness and 9.8% for in-plane shear stiffness. This is the result of the 10% rule that is used to guarantee a minimum of mechanical properties in all directions.

Using the double-double approach then provided a compromise between the optimal unidirectional laminate and the quad resulting from the 10% rule. The normalized stiffness matrices $[A^*]$ and their consequent normalization by the Tsai modulus $[A*]$ for the double-double laminate $[(\pm 7/\pm 16)_8]_s$ show the following results:

$$[A^*] = \frac{[A]}{t} = \frac{1}{8.0}\begin{bmatrix} 1094.77 & 68.96 & 0 \\ 68.96 & 82.56 & 0 \\ 0 & 0 & 82.06 \end{bmatrix} = \begin{bmatrix} 136.85 & 8.62 & 0 \\ 8.62 & 10.32 & 0 \\ 0 & 0 & 10.26 \end{bmatrix}, \tag{22}$$

$$[\overline{A^*}] = \frac{[A^*]}{\mathrm{Tr(Q)}} = \frac{1}{167.68}\begin{bmatrix} 136.85 & 8.62 & 0 \\ 8.62 & 10.32 & 0 \\ 0 & 0 & 10.26 \end{bmatrix} = \begin{bmatrix} 0.816 & 0.051 & 0 \\ 0.051 & 0.062 & 0 \\ 0 & 0 & 0.061 \end{bmatrix}, \tag{23}$$

Thus, the trace of $\overline{A^*}$ is represented by:

$$\mathrm{Tr}(\overline{A^*}) = 0.816 + 0.062 + 2*(0.061) = 1, \tag{24}$$

The result demonstrates a recovery of the portions of transversal and in-plane shear stiffness to, respectively 6.2% and 12.2%, if not compatible with those of the quad laminate, better than those of the optimal unidirectional laminate. The portion of longitudinal stiffness required to ensure the critical load of compatible buckling, in this case, is 81.6%. Despite being timid, the restoration of the portion of stiffness in the transverse direction in the double-double laminate can be considered significant, since that for the transverse direction small angle variations result in important resistance variations, which is the main motivation of the 10% rule.

Besides, it can be seen that the application of the 10% rule results in a 28% increase in the thickness of laminated flanges and cores and, consequently, in a 28% increase in the mass of the dimensioned structure in relation to the laminate oriented in [0]. The use of double-double laminate leads to a 14% increase in mass compared to the unidirectional laminate. It is observed that the DD laminate has a number of layers intermediate between the unidirectional and quad laminates. Added to this is the possibility of further optimizing the strut by performing a reduction of layers in the laminated section with a length of 4.5 m, which was not considered in this work. This reduction in thickness through ply-drop suppression is easily achievable in double-double laminates, but impossible to achieve in quads, keeping symmetry, balance and the 10% rule. Thus, it is proven that the double-double laminate is a viable alternative for the aeronautical industry. In addition to improvements such as better ease of optimization, better manufacturability, ease of homogenization, among others discussed above, the resulting laminate is lighter for the same strength and stronger for the same weight when compared to a quad laminate. Furthermore, it is interesting to use double-double laminates in more complex structures and with multiple acting loads, cases in which the advantages of DD will become more evident in the analysis process.

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