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# DIRECT NUMERICAL SIMULATION OF THE FLOW PAST SMALL CAVITY INSIDE LAMINAR BOUNDARY LAYER

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**Abstract.** Typically, the presence of a small cavity placed inside the boundary layer alters the Tollmien-Schlichting wave amplitude growth, anticipating the transition location. However, some recent studies carried out a parametric search and identified a range of flow configurations where the transition location changes abruptly to near downstream of the cavity, the so-called bypass transition. Therefore, the current study aims to perform a two-dimensional direct numeric simulation to investigate better the aspects involved in this threshold of transition forms. The geometric and flow parameters chosen for the simulation were: cavity aspect ratio,  $L/D = 5$ , depth,  $D/\delta_c^* = 4$ , Reynolds and Mach number,  $Re_{\delta_c^*} = 1405$  and  $M_\infty = 0.065$ , respectively, were employed to compare with experimental data provided in the literature. An in-house code solves the compressible Navier-Stokes equations since acoustic resonance can arise. The results exhibit a strong stream-wise oscillation just downstream the cavity. The temporal frequency of this unsteady motion agrees with reported by literature within 4% of relative deviation and differs from the expected Tollmien-Schlichting phase frequency for current flow conditions. The results suggest that the Rossiter second mode instability causes such oscillatory motion and may play a role in bypass transition.

**Keywords:** transition, small cavity, DNS, flow instability.

## 1. INTRODUCTION

The boundary layer over an aircraft surface causes friction forces that account for a great portion of the vehicle's total drag. This is intensified when the laminar boundary layer becomes turbulent. Thus, pushing the turbulence transition as downstream as possible may result in significantly increased aircraft performance, for instance, fuel efficiency, according to Schneider (2001). In the two-dimensional boundary layer, which is found in several parts of an aircraft, e.g. wings and nacelles, the transition is dominated by the Tollmien-Schlichting (T-S) mode, after Tollmien (1931) and Schlichting (1933). This mode oscillates and, depending on the flow conditions, a certain range of frequencies become unstable, leading to the laminar breakdown. However, several irregularities are found in a real aircraft surface, such as roughness, excrescences, steps, and cavities. Those irregularities affect the flow, therefore the main scope of this study regards to investigate the impact of a small cavity on the boundary layer stability.

Early works regarding cavities, e.g. Roshko (1955), Krishnamurty (1955), Plumlee *et al.* (1962), Rossiter (1964) and East (1966), studied a resonance mechanism inside the cavity. As the boundary layer reaches the cavity leading edge, it becomes a shear layer that oscillates and impinges on the trailing edge of the gap. The impingement causes acoustic waves that propagate inside the cavity and destabilizes the shear layer, closing the feedback cycle. Some parameters such as Mach number, free-stream velocity, and cavity aspect ratio, dictates the frequency of this mode, denominated Rossiter mode. Sarohia (1977) studied the oscillation of the cavity with the incoming laminar boundary layer. The author proposed a non-dimensional geometric parameter that determines whether the flow is steady or oscillatory. Gharib and Roshko (1987) employed experimental investigation of the flow over an axisymmetric cavity. It was identified a mode characterized by a large increase in drag, and it was named "wake mode".

Recently, some works research the impact of Rossiter modes on laminar boundary layer stability. Colonius *et al.* (1999), through direct numerical simulation (DNS), identified the three different flow regimes over open cavities: the steady motion, the shear layer (Rossiter) mode, and the wake mode. It was suggested that the latter is associated with separation at the cavity trailing edge. Theofilis (2000) carried out linear stability analysis of the flow given by a DNS. The author concluded that the base-flow calculated by the simulation is decisive for the amplification of unstable modes, although the circular frequency converges readily. Rowley (2000) employed a proper orthogonal decomposition (POD) analysis on the DNS results. Comparison between the modes captured by POD and linear stability theory (LST) applied for Kelvin-Helmholtz instability showed agreement for high frequencies modes. de Vicente *et al.* (2014) and Meseguer-Garrido *et al.* (2014) employed an extensive work on the stability of tri-dimensional disturbances over two-dimensional cavity flow. Mathias and

Medeiros (2018), using DNS, investigated the influence of certain parameters on the flow stability employing the Bi-Global method. Accordingly to the authors, decreasing the displacement thickness, in respect to cavity depth, destabilizes the shear layer and selects which Rossiter mode (first, second, etc.) will dominate the oscillation. The acoustic emissivity influences the temporal growth of the mode.

The literature also presents experimental results that quantify the impact of the N-factor ( $\Delta N$ ), proposed by van Ingen (1956), caused by the presence of a small cavity immersed on the boundary layer. Perraud *et al.* (2014) proposed a semi-empirical correlation to estimate the N-factor variation based on geometric parameters and flow conditions. Experiments executed by Forte *et al.* (2015) confirmed the proposed correlation. Beguet *et al.* (2017) gathered results from several experimental investigations employed at ONERA, including a diagram exhibiting the threshold region of the parametric space which determines rather the cavity triggers bypass transition or anticipates the transition by altering the N-factor. Crouch and Kosorygin (2020) carried out an experimental investigation of the impact on boundary layer stability caused by the presence of gap, step, and isolated roughness. Accordingly to the author, the impact on N-factor for a shallow cavity is well modeled by the correlation for the backward-facing step. The semi-empirical correlation for cavity was presented in Crouch *et al.* (2020). It was identified that for shallow cavities the influence on transition is dominated by cavity depth non-dimensionalized by displacement thickness ( $D/\delta^*$ ), whereas for deep cavities,  $\Delta N$  depends exclusively on the gap length ( $L/\delta^*$ ). Bypass transition is also reported by the author and the frequency when bypass occurs is about five times the frequency associated with T-S mode.

Therefore, the actual work simulates via DNS the flow over a small cavity placed inside the laminar Blasius boundary layer. The scope of the parameters selected aims at cavity with aspect ratio  $L/D = 5$ , with  $D/\delta_c^* = 4$ , specifically at the threshold region where bypass transition occurs. The parameters associated to flow conditions are similar to the survey by Crouch *et al.* (2020). The goal is to investigate the possibility of the Rossiter instability being present in such flow conditions.

## 2. METHODOLOGY

The DNS is employed through an in-house code developed by the own research group. Details of the implementation of the code is provided by Bergamo (2013), Martínez (2016) and Mathias (2017). To reduce the computational cost, the solver runs in Fortran 90 and counts with processing parallelization, performed via 2DECOMP&FFT library, described in Li *et al.* (2010). Pre- and post-processing is carried out through Matlab.

The code solves the non-conservative two-dimensional (2D) compressible Navier-Stokes equations. As acoustic resonance within the cavity can cause the Rossiter instability, the solver must assess compressibility effects. Besides, because Rossiter instability is essentially a two-dimensional phenomenon, it was employed a 2D DNS for practical reasons. Thus, it is assumed null spanwise velocity and null derivatives for all flow variables in such direction as well.

The sixth-order spectral-like compact differentiation method evaluates the spatial derivatives, as proposed by Lele (1992), and the fourth-order Runge-Kutta method performs the time marching. A low-pass filter attenuates numeric noise due to aliasing effects caused by the motion transport of different length scales.

The computational domain is represented by a structured mesh with refinement in decisive regions. Buffer zones are employed next to open boundaries to avoid reflection and to provide a suitable far-field condition, through spacing substantially increased and reduced accuracy order of spatial derivatives. Selective frequency damping implemented accordingly with Åkervik *et al.* (2006) acts on the buffer zones during the simulation to improve the damping of reflective waves.

Figure 1 exhibits every tenth consecutive grid point of the mesh employed for the current work and the limits of the computational domain. The solid magenta line represents the free-slip wall, whereas the solid green line indicates the solid wall. Blue dashed lines display the boundaries of the buffer zones.

Figure 2 shows the spacing between nodes for  $x$  direction (stream-wise) in Fig. 2a, and for  $y$  direction (wall-normal) in Fig. 2b. The yellow and purple dashed lines, in Fig. 2a, indicate where the cavity's leading and trailing edge are located, respectively. The axes are non-dimensionalized by the boundary layer displacement thickness over the cavity's leading-edge,  $\delta_c^*$ . The mesh has  $1146 \times 141$  grid points in stream-wise and wall-normal directions, respectively. In  $x$  axis, the minimum spacing is 0.05 located over the cavity's trailing edge, while in  $y$  axis, equivalent spacing occurs close to zero. Considering the low Mach number employed for simulation ( $M_\infty = 0.065$ ), caution was taken regarding the mesh spacing, especially next to the cavity's corners and flat plate's leading edge.

Dirichlet boundary condition occurs at inflow, for uniform temperature and velocity over the cross-flow direction. The Neumann condition is applied in this boundary for pressure, with a null derivative across the stream-wise direction. At solid walls, isothermal boundary layer condition is employed for energy. For velocity, both no-slip and non-penetration conditions are imposed, while at the free-slip wall, only the latter occurs. At the outflow, pressure is constant and the second derivative of the remaining variables is null. The open boundary, which is opposed to the flat plate, sets the second derivative of all variables to zero. The numeric solution of the Blasius equation provides the initial condition.

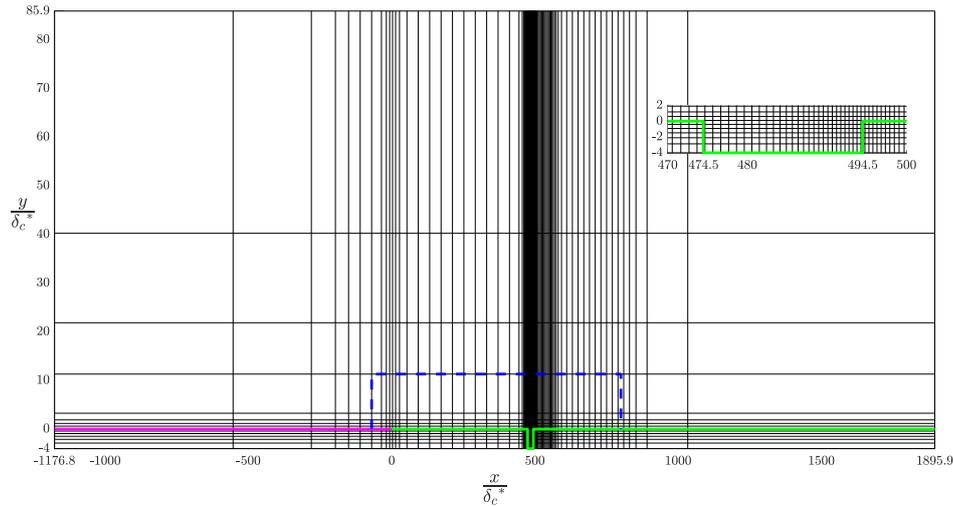


Figure 1. Computational mesh employed for DNS. An auxiliary plot on the superior right corner zooms in the cavity region.

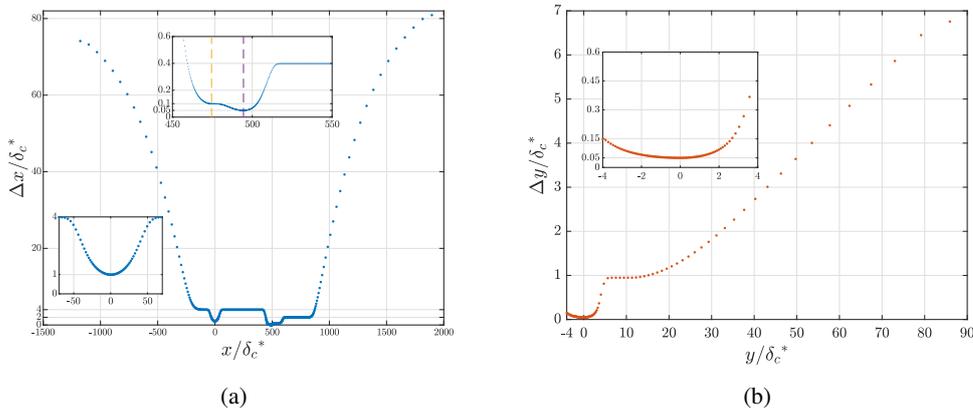


Figure 2. Mesh spacing for  $x$  axis in (a) and  $y$  in (b).

### 3. RESULTS

The flow parameters were chosen to reproduce the experiment of Crouch *et al.* (2020), specifically the *case C*. The Tab. 1 exhibits the parameters employed. Such case was chosen, because accordingly to the author, this parametric region is the threshold for bypass transition to occurs. The frequency measured experimentally also had higher value than the expected by a T-S mode, with non-dimensional frequency of  $f_{bypass} = 0.0367$ , where  $f_{bypass} = \frac{\hat{f}\delta_c^*}{U_\infty}$ , and  $\hat{f}$  represents the dimensional frequency measured.

Table 1. Flow parameters employed for the DNS.

$U_\infty$ [m/s]	$\delta_c^*$ [m]	$L/\delta_c^*$	$D/\delta_c^*$	$L/D$	$Re_{\delta^*}$	$M_\infty$	$T$ [K]
22.6	0.001	20	4	5	1405	0.065	299.05

From Fig. 3, a good agreement between the velocity profile and the Blasius profile over the cavity leading-edge. The simulation results presented an unstable state, i.e after an initial transient reached, oscillation occurs for all flow variables. Later, the unsteady motion saturates at a constant amplitude. Figure 4 shows the oscillatory components of velocity signal,  $\frac{u'}{U_\infty}$  and  $\frac{v'}{U_\infty}$ , measured by a probe located at the cavity trailing edge after reached the limit cycle.

Figure 5 shows instantaneous contours of cross-flow unsteady velocity. The flow field exhibits a vortex emission caused by the cavity, which is responsible for the oscillatory signal displayed in Fig. 4. Given the size of the cavity, the vortex emission comprises two vortices over the cavity. Such results resemble the Rossiter second mode eigenfunction. After the reattachment of the shear layer downstream the cavity, the oscillation caused by the vortices presents amplitude that spatially decays as it convects downstream. One interesting fact is the profile of the unsteady stream-wise velocity,  $\frac{u'(y)}{U_\infty}$ ,

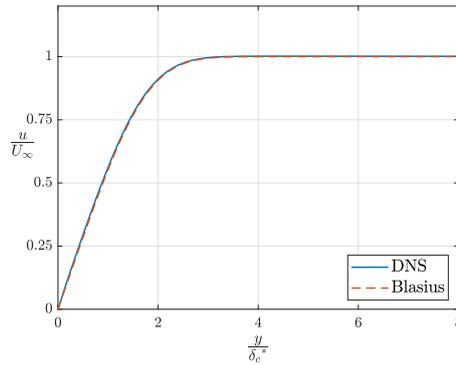


Figure 3. Comparison between the velocity profile and Blasius at the gap leading-edge.

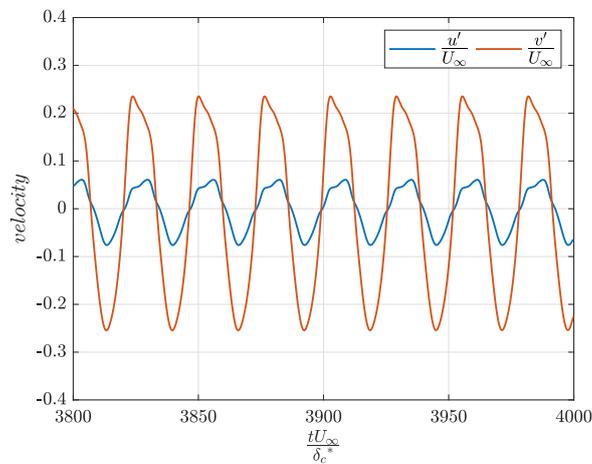


Figure 4. Non-dimensional time series of the stream-wise (blue) and cross-flow (orange) unsteady velocity signal normalized by freestream velocity, captured by a probe located at the gap trailing edge.

that is similar to the T-S eigenfunction, as shown by Fig. 6. However, its temporal frequency is higher than a T-S wave for these flow conditions and consequently, this mode is spatially stable.

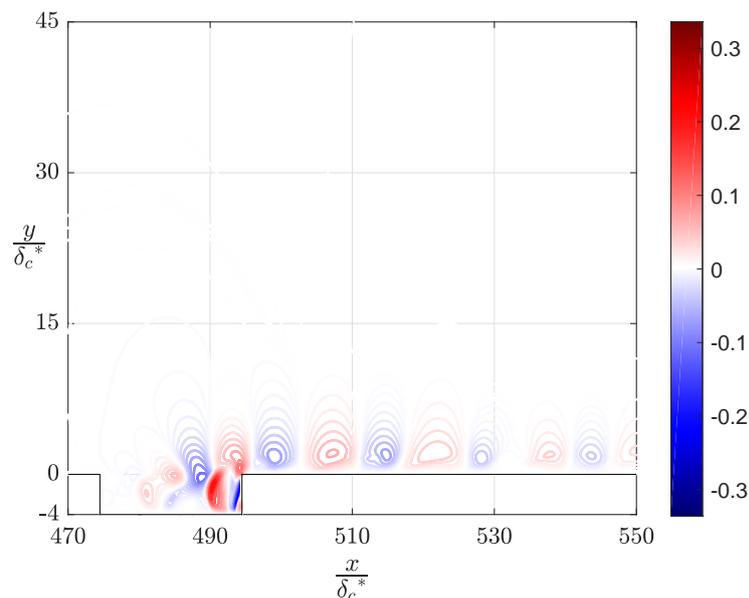


Figure 5. Instantaneous contours of cross-flow unsteady velocity  $\frac{v'}{U_\infty}$ .

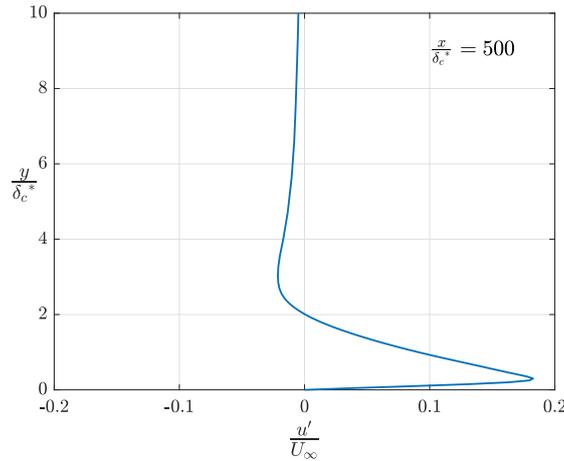


Figure 6. Instantaneous stream-wise unsteady velocity profile at  $\frac{x}{\delta_c^*} = 500$ .

Figure 7 displays the Fast Fourier Transform (FFT) of the stream-wise velocity signal shown in Fig. 4, after the Hanning window function applied on it and amplitude correction. The dominant component present in the non-dimensional frequency spectrum possesses a value of  $f = 0.0382$ , which corresponds to a normalized deviation of 4% relative to the experimentally measured by Crouch *et al.* (2020). Rossiter's semi-empirical formula, non-dimensional by the time scale, is given by Eq. 1 below.

$$f_{R_m} = \frac{\delta_c^*}{L} \frac{m - \gamma}{\frac{1}{K} + M_\infty} \quad (1)$$

Where,  $m = 1, 2, \dots$ , is the Rossiter mode number,  $\gamma = 0.25$ , a constant associated with phase between acoustic emission and vortex formation and lastly,  $K$ , the mean convective velocity of the vortex. For the parameters employed and the mean vortex convective velocity measured,  $K = 0.36$ , the second mode presented a non-dimensional frequency of  $f_{R_2} = 0.0308$ .

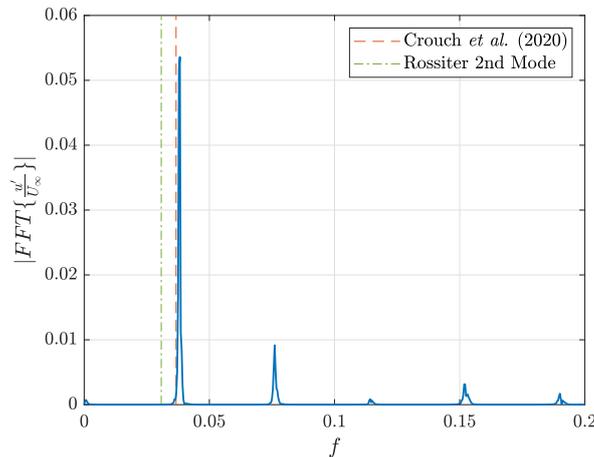


Figure 7. The non-dimensional frequency spectrum of unsteady stream-wise velocity signal measured at the cavity trailing edge.

#### 4. CONCLUDING REMARKS

A 2D DNS was performed aiming to investigate a possible Rossiter instability influence on bypass transition. For this, the flow parameters were chosen based on results obtained by Crouch *et al.* (2020). In these conditions, an oscillatory mode develops temporally due to the presence of the cavity. The stream-wise unsteady velocity profile just downstream the cavity resembles a T-S profile, although possesses a higher temporal frequency. The prediction given by Rossiter's semi-empirical formula presented a normalized deviation of about 20% with the actual results. This fact is expected, once the current parameters are distant from the parametric space covered by Rossiter (1964). However, a good agreement

with the frequency obtained by Crouch *et al.* (2020) was reached, with 4% of relative deviation. In a certain manner, it is reasonable to infer that these oscillations occur due to the Rossiter second mode instability, considering the resemblance between the resulting flow and the Rossiter eigenfunction reported in the literature and that at least the Rossiter second mode is present in this flow conditions. But, it is not clear whether the Rossiter mode causes the bypass transition. In addition, the presence of higher harmonics of the dominant frequency on the spectrum suggests non-linear effects, which is plausible considering the magnitude of the amplitudes. Lastly, a suggestion for future work is to employ a 3D DNS. Although the Rossiter mode is essentially a 2D mode, secondary instabilities may arise, besides the fact that the 3D baseflow may be slightly different from the actual 2D.

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