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BAYESIAN PROBABILISTIC APPROACH FOR RAILWAY BRIDGE MODEL UPDATING

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Abstract. In structural engineering, there are many cases which design data is insufficient or even nonexistent. Thus, there is a demand to assess the structural integrity of these constructions, usually depending on experimental field data. In order to provide information for helping on assessing the structural safety, researches have given special attention to model updating techniques, as strategies based on finite element models to simulate the behavior of the structure. The main idea of the model updating is to improve the match between theoretical model and experimental results. In this context, Bayesian approach can be an efficient tool in estimating parameters of interest. A Bayesian methodology summarize the degree of information regarding these parameters by means of probability distributions. This technique also takes into account errors and uncertainties inherent to the update process. In this work, it is proposed to estimate unknown design variables by means of the Bayesian numerical model updating based on experimental modal responses. A real railway bridge structure is studied. One numerical model of the railway bridge is updating in order to estimate two parameters of interest of structure: vertical stiffness of the supports and modulus of elasticity of concrete. In addition, the standard deviation of errors between experimental and numerical measurements is estimated, in order to understand the potential of Bayesian inference. Finally, it is presented that in the creation of numerical models to assess real structures, one must be very sparing in order not to make a problem overly complex and, as result of, high computational cost.

Keywords: Bayesian model updating, Finite Element model, Parameters identification, Structural safety, Railway bridges

1. INTRODUCTION

Structural safety has gained attention in the field of engineering, peculiarly for structures such as bridges and viaducts, so that development of computers and mathematical tools was fundamental to overcome several imbroglios in structural analysis.

In the approach of structures with complex behavior, the use of numerical models based on Finite Element (FE) methods allows to simulate a real structure and evaluate its effects. The process of development a FE model can involve many simplifications related to geometry, discretization, boundary conditions and mechanical properties of the material that may cause errors (Ribeiro, 2012). In these cases, the use of experimental data, such as dynamic measurements, is important to validate the numerical model, as reported by Ren et al. (2004). Thus, the process of numerical model updating aiming to match the numerical and experimental responses and to estimate the parameters of interest. The updated numerical model can be used with confidence in related analyzes to the structure, to predict of dynamic responses under new operating speed or under new load scenarios, or for damage identification, proceeding from Structural Health Monitoring (SHM). The application of the SHM in railway bridges can be found in Kolakowski et al. (2011), Vagnoli et al. (2018) and Meixedo et al. (2021).

In the context of methods for updating models, the solution can be done by probabilistic inferences such as Bayesian approach. Bayesian methods summarize the degree of information of the parameters of interest through probability distributions, so that the combination of a priori knowledge with the probability of experimental observations allows to obtain a posterior probability distribution that reflects on the estimation of the parameters of interest. In other words, the increase of information of measurements in the process of updating models implies in the reduction of uncertainties related to the estimation parameters (Behmanesh and Moaveni, 2016). Recent works based on Bayesian approach to FE model updating of bridges can be found in Behmanesh and Moaveni (2016), Mustafa and Matsumoto (2017) and Jang and Smith (2017).

Regarding of Bayesian perspective, the Markov Chain Monte Carlo (MCMC) methods may overcome the difficulties of working with computationally expensive models in estimating the parameters of interest and, thus, make the solution viable (Kass et al., 1998). One important algorithm based on MCMC is the Metropolis-Hastings, widely used in the literature. The Metropolis-Hastings algorithm is executed by the independent distribution of each parameter and uses an

acceptance condition, thus having the advantage of working with problems of obtaining the full conditional distribution is not feasible (Kass et al., 1998).

This paper reports the Finite Element model updating of an existent railway bridge based on experimental vibrational responses identified by Rodrigues (2005). One numerical model developed by Silva (2010) is studied in order to estimate two parameters of interest of the structure, vertical stiffness of the supports and modulus of elasticity of concrete, as well as the errors between the experimental and numerical measurements. The numerical model updating involves the Bayesian approach by means Markov Chain Monte Carlo techniques. Finally, this study evaluates some advantages and limitations of the Metropolis-Hastings algorithm.

2. BAYESIAN INFERENCE

2.1 Classical Bayesian framework

Bayesian inference summarizes the degree of information regarding of the parameters of interest through probability distributions, i.e, in the Bayesian perspective, the assumptions regarding the estimation parameters are updated for a probability distribution from a set of measurement data (Behmanesh and Moaveni, 2016).

Based on the Bayes' theorem, the main idea is to obtain a distribution of conditional probability ($X|Y$), from a priori information of the structural parameters X to be updating and of the probability of observation of measurements Y given a fixed set of parameters X :

$$p(X|Y) \propto p(X)p(Y|X), \quad (1)$$

where $p(X|Y)$ is the posterior probability density function (PDF), $p(X)$ is the prior probability and $p(Y|X)$ is called likelihood function. A common type of measured data Y includes vibrational data, such as natural frequency responses. Assuming the data of measurements of natural frequencies for a mode of vibration m by λ_m , applied in the likelihood function, the error e_{λ_m} between the experimental and numerical measurements is assumed to have zero-mean Gaussian distributions and standard deviation σ_{λ_m} (Behmanesh et al., 2015):

$$\tilde{\lambda}_m - \lambda_m(x) = e_{\lambda_m} \sim N(0, \sigma_{\lambda_m}^2), \quad (2)$$

where $\tilde{\lambda}_m$ denotes the experimental natural frequencies and $\lambda_m(x)$ are the numerical natural frequencies responses.

$$p(\tilde{\lambda}|x, \sigma) = \prod_{m=1}^{N_m} p(\tilde{\lambda}_m|x, \sigma_{\lambda_m}^2) = \prod_{m=1}^{N_m} p(\tilde{\lambda}_m|\lambda_m(x), \sigma_{\lambda_m}^2), \quad (3)$$

in this equation, N_m is the total number of identified modes and $p(\tilde{\lambda}_m|\lambda_m(x), \sigma_{\lambda_m}^2)$ is the value obtained by the probability distribution function with mean $\lambda_m(x)$ and standard deviation σ_{λ_m} at mode m . In the case of having N_t number of independent measures related by the mode m , the likelihood function can be extended as:

$$p(\tilde{\lambda}_1, \dots, \tilde{\lambda}_{N_t}|x, \sigma) = \prod_{t=1}^{N_t} \prod_{m=1}^{N_m} p(\tilde{\lambda}_{tm}|\lambda_m(x), \sigma_{\lambda_m}^2), \quad (4)$$

where the sub-index tm indicates the measurement t from the identified mode m . The posterior probability density function of updating parameters can be approximated numerically using Markov Chain Monte Carlo (MCMC) samples or analytically through asymptotic approximations (Behmanesh et al., 2015).

2.2 Markov Chain Monte Carlo approach

In the context of Bayesian inference, the basic idea of the MCMC is to build a Markov chain over the space of the parameter of interest in order to build the posterior probability distribution. A sequence of random parameters X^0, X^1, X^2, \dots is called a Markov chain in a space \mathbb{R} for every measurable set if:

$$q(X^{k+1}|X^k, X^{k-1}, \dots, X^0) = q(X^{k+1}|X^k) \quad (5)$$

In this way, for the next point X^{k+1} is obtained a distribution $q(X^{k+1}|X^k)$ which depends only on the current state of the chain, X^k , i.e, it does not depend on the history of the chain X^0, X^1, \dots, X^{k-1} . The distribution $q(\cdot | \cdot)$ is called the transitional kernel of the chain and, applied to the regularization conditions, the Markov chain will 'forget' its initial state $q(\cdot | X^0)$ and will eventually converge to a stationary distribution (Gilks et al., 1995). In this case, the stationary distribution is the posterior PDF.

2.2.1 Metropolis-Hastings

The Metropolis-Hastings algorithm is an important class of MCMC. It simulates samples of a probability distribution using the joint density function of the parameters of interest (Sköld and Roberts, 2003). In the first step of the algorithm, an initial value x_k for each parameter of interest is usually sampled from the prior probability distribution, thus, the implementation of the Metropolis-Hastings algorithm is given by the following steps:

1. Choose an initial value for $x_k \in \mathbb{R}$ and set $k = 1$
2. Do $\hat{x} = x_k + \omega$
3. Calculate:

$$\alpha = \min \left(1, \frac{p(\hat{x}|y)}{p(x_k|y)} \right) = \min \left(1, \frac{p(y|\hat{x})p(\hat{x})}{p(y|x_k)p(x_k)} \right) \quad (6)$$

4. Sample u from the uniform distribution on $[0,1]$
5. If $\alpha \geq u$ then
accept the proposal: $x_{k+1} = \hat{x}$
Else
reject the proposal: $x_{k+1} = x_k$
6. Do $k = k + 1$ and go to step 2. Repeat this process until the defined sample size.

where ω is the perturbation of the parameter of interest, often sampled from a zero-mean Gaussian distribution and standard deviation γ ; $p(\hat{x}|y)/p(x_k|y)$ is the full joint density and α is the acceptance probability.

Notice that in this sequence, \hat{x} is only dependent on the last iteration x_k . This is the Markov property, and so this sequence is called Markov chain (Hoff, 2009).

2.2.2 MCMC Diagnostics

The main idea of the Monte Carlo Markov Chain is to obtain a sequence of parameters of interest X^0, X^1, X^2, \dots that converge to a stationary distribution, in this case, the posterior distribution. Thus, the challenge is to assess whether the chain achieves stationarity and the convergence of this parameters. Convergence and correlation issues of MCMC sampling may be generated by overfitting or data redundancy resulting from the application of a small sample size to a complex problem, in addition to cases in which the prior distribution of parameters is not very informative (Congdon, 2007).

There are some graphic techniques to assess the convergence of parameters, such as time series graphs and histograms. The time series graphs show the estimations of the MCMC at each iteration of the algorithm and are important to visualize how the Markov chain moves in the parameter space over time. This graph is also important because it can allow observing regions of high rejection of proposals, so that the estimated value of the parameter is confined to such regions. For an efficient approximation, if the initial value of the parameter is not in the high-density region, the beginning of the trace shows consecutive steps in one direction until reaching a region where the parameters assume very close values. In this case, the initial data referring to the non-stationary region can be discarded from the sample. This procedure is known as the burn-in sample. On the other hand, if the graph plot shows a similar pattern throughout the process, it is not convenient to discard the initial sample data. The histograms indicate the frequency of estimation of the data. When the algorithm reaches stationarity, the histogram presents a higher frequency of data around the stationary region.

One way to assess the quality of data in a sample is through the autocorrelation function ACF. In this case, the ACF graph shows the values of the autocorrelation function along the time series of t steps, denoted by lag- t . For Markov chains with a variability of data, the sample correlation is quickly reduced to a value close to zero as t increases. On the other hand, high values of ACF for high numbers of t indicate the presence of a high degree of correlation and slow variability of the Markov chain (Roy, 2020). Vrugt (2016) presents that a sample has a high correlation when for lag- t greater than 5, the correlation is greater than 0.80, which characterizes it as a sample with low data variability.

3. CANELAS RAILWAY BRIDGE

Canelas Bridge is selected for this study. The structure is part of the Portuguese railway line, near Aveiro, in Portugal, and has a total length of 72 m between the axes of supports. This structure consists of 6 simply supported spans, each 12 m long, shown in Figure 1. It is a composite steel-concrete of the type filler beam, composed of two half independent concrete slab decks, each supporting a railway track. Both slab decks are 6.2 m wide, with 4.5 m of 0.7 m thick slab with nine HE500B metal beams spaced 0.475 m apart. The ends of the slabs are composed of consoles 1.7 m wide and linearly variable in thickness between 0.3 and 0.7 m. The bridge deck is also equipped with a ballast guard consisting of a beam

0.60 m high and 0.3 m wide, located between the concrete slab and the side console. Ballast, sleepers and rails are placed on the board. The ballast guard makes the ballast structure to be confined to the board. All spans of the board are fixed on one side and mobile on the other side, made of neoprene involving stainless steel sheets and supported under each HE500B profile, totaling 9 support devices. In this work, the span called P5-E2 is studied, i.e., the gap closest to the landfill towards Porto. The longitudinal view and the respective cross section of this bridge are illustrated in Figure 2. The main reason for choosing this type of bridge is due to the fact that the structural system of concrete is the most used in railway bridges in Brazil.

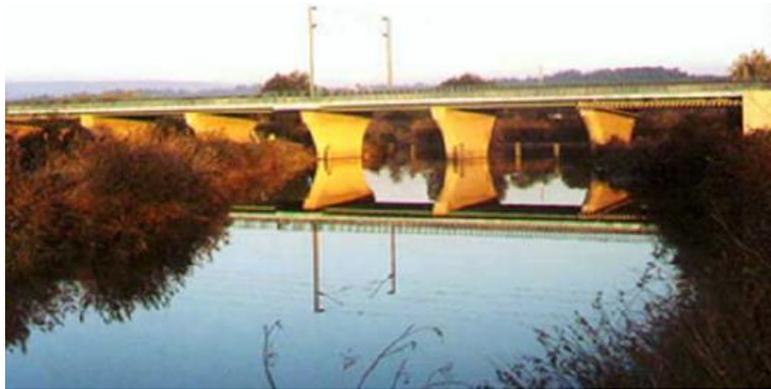
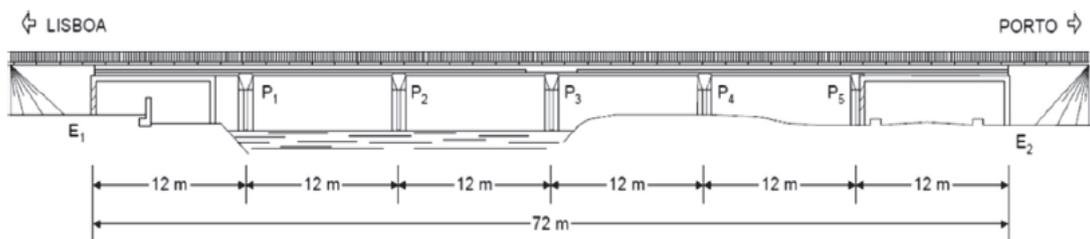
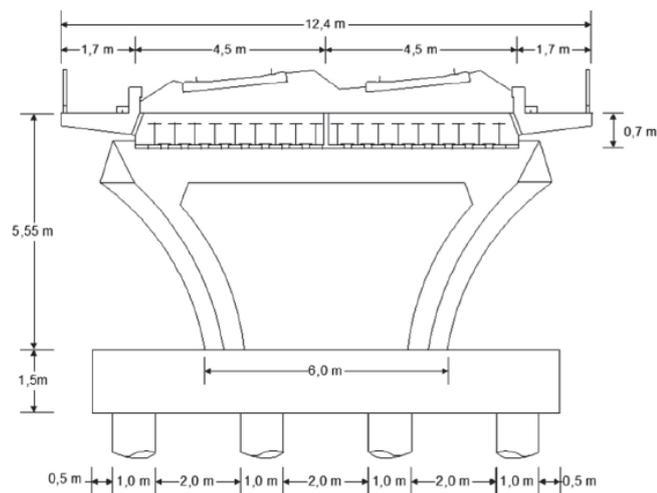


Figure 1. General view of the Canelas railway bridge (Rocha et al., 2016).



(a) Longitudinal view



(b) Cross section

Figure 2. Canelas railway bridge (Rocha et al., 2015).

3.1 Past Research

The Canelas bridge has been the target of experimental and numerical studies by some researchers. Presented in 2004, Rodrigues' doctoral thesis (Rodrigues, 2005) was supported by the Portuguese laboratory LNEC and addressed the

suitability of the Canelas railway bridge to increase the maximum operating speed of alpha-commuter trains from 140 km/h to 220 km/h. The study involved conducting in situ tests to assess the dynamic characteristics of the structure and the dynamic effects of rail traffic. The first vertical vibration mode and the first torsional mode of the bridge sections were identified, which corresponding values of the natural frequencies are 8.60 and 15.60, respectively. These natural frequencies will be used in this study for model updating.

Silva (2010) studied the dynamic behavior of the Canelas bridge using numerical models developed in ANSYS and coupled in Matlab for solving dynamic equilibrium equations. The dynamic analysis under moving loads corresponding to several trains that operate on the Canelas bridge, was applied and compared with results from major static analysis. Three finite element models of the bridge were developed, from the simplest to the most complete model. Based on the experimental responses of the first vibrational modes of structure, studied by Rodrigues (2005), numerical models were updated in relation to the parameters: elasticity modulus of the concrete, elasticity modulus of the ballast, ballast density and vertical stiffness of supports. For this update, a sensitivity analysis of the parameters was applied.

Rocha et al. (2016) published some studies on the dynamic behavior of the Canelas bridge were carried out by means of probabilistic approaches. The analysis of the sensitivity of the bridge dynamic responses as a function of its structural parameters was considered, as well as the effect of rail irregularities. For such simulations, the Monte Carlo method was applied. Thereby, Rocha demonstrated that the inertia of the cross section of the deck, the ballast density, the elasticity modulus of the concrete and the vertical stiffness of the supports were the parameters that most altered the bridge responses.

3.2 Numerical model

The numerical model of the bridge is developed by the Finite Element model. Considering that this study basically focuses on the evaluation of the P5-E2 section, the representation of the entire length of the bridge and track becomes unviable due to the high computational cost of the numerical model. Thus, a simplification of the model with the inclusion of only the section under study can guarantee an adequate model and with a significant gain in processing time. In this case, one numerical model developed by Silva (2010) is studied. The simpler numerical model, with only one half-span, where the track is represented by an equivalent distributed load on the slab. In the numerical model performed, the pillars and the bridge embankment were not considered. In view of the analyzes to be carried out, the ANSYS 16.0 software is used in the APDL language for modeling.

Subsequently, the numerical model is calibrated by updating two properties of the bridge that are considered most relevant for possible changes in their dynamic behavior in modal terms: the stiffness of the supports and the modulus of elasticity of the concrete.

Regarding computational modeling, shell elements (SHELL63) were used to simulate the concrete of the slab, console and ballast guard. The beam elements (BEAM44) were considered to represent the steel profiles of the slab, and spring elements (COMBIN14) for elastic supports. The arrangement of the finite elements is given in relation to the center of gravity of the elements that make up the bridge structure, in the same way that the supports are located in the center of rotation.

The numerical model is a simplified model of the span under study, represents the half-board P5-E2 without the inclusion of track elements. In this case, the mass of the track; composed of ballast, sleepers and rails; it is simulated as uniformly distributed load on the shell elements of the slab, that is, it is considered as the dead load in which the stiffness of the track does not influence the model responses. The structure was discretized into 3335 elements and 2759 nodes. Figure 3 shows the numerical model C1 with its adaptive elements.

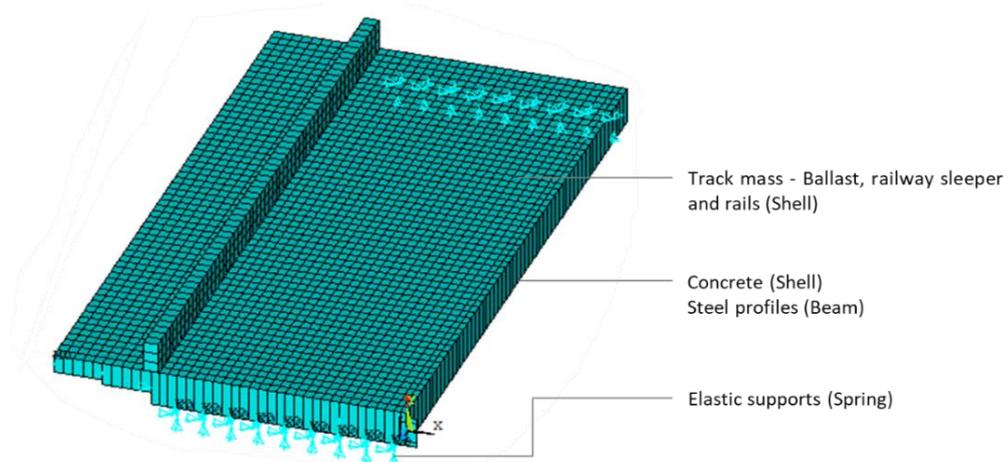


Figure 3. Numerical model of the bridge.

4. RESULTS

This chapter summarizes the main results obtained to Bayesian probabilistic approach in order to estimate the parameters of interest of the numerical model of railway bridge.

In the process of executing the algorithm, it is necessary to introduce data from the experimental measurements. For this purpose, the values of the experimental natural frequencies are simulated from the responses of the parameters, shown in Table 2. A total of 100 experimental responses is obtained based on perturbation of 4% of each parameter of interest. This perturbation implies in an error related to measurement process. Then, this work aiming to estimate the standard deviation of the experimental natural frequencies such as the physical parameters of the bridge structure. For this case, the exact standard deviation of the errors is $\sigma_{\lambda_1} = 0.1176$ Hz and $\sigma_{\lambda_2} = 0.2135$ Hz.

Table 2. Parameters of interest considered in the model updating

Parameters	Description	Unit	Exact value
Kv	Vertical stiffness of the supports	MN/m	309.6
Ec	Modulus of the Young of the concrete	GPa	38.8

For this numerical model, the Metropolis-Hastings perturbation ω of the parameters of stiffness is given with zero mean and applied a standard deviation around 1% of the exact value of the parameter of interest. Refer to parameters σ_{λ_1} and σ_{λ_2} , aiming to obtain satisfactory results regarding the correlation of error data, the applied standard deviation is 10% of the standard deviation of the simulated experimental measurement of natural frequencies λ_1 and λ_2 . For the mean and standard deviation of the prior of the stiffness parameters, the values are fixed with mean around the central value of the design limits of the parameters and with a standard deviation of 10% of the prior mean. The mean and deviation of the prior of parameters related to measurement errors are set in a very informative way, since the prior mean is given by the standard deviation of the simulated experimental measurements and the prior standard deviation of 10%. All probability distributions of input variables follow a Gaussian.

In Figure 4 is shown the graph of the time series of the parameters of interest. To assess the quality of the samples and verify the stationarity of the sampler, in Figure 5 is presented the ACF correlation graphs of the parameters. Finally, in Figure 6 is presented the histogram of the parameters of interest.

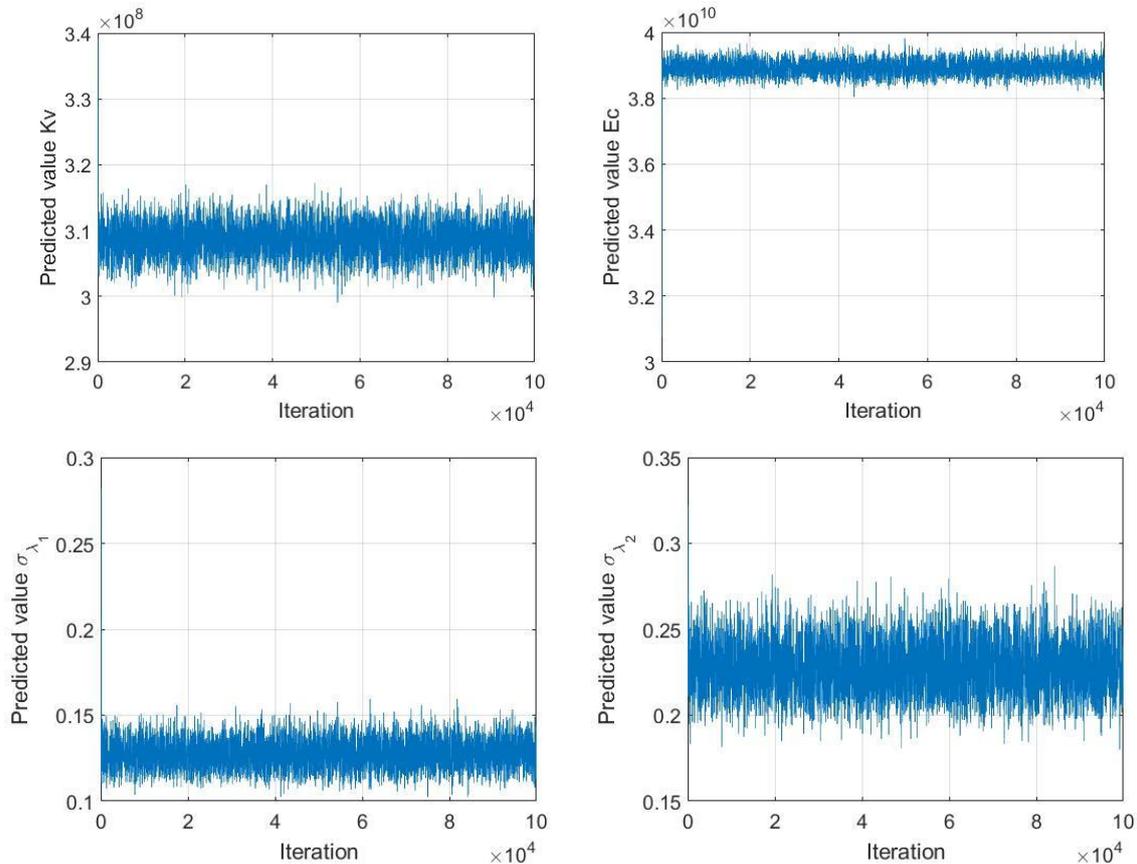


Figure 4. Time series graph of the parameters.

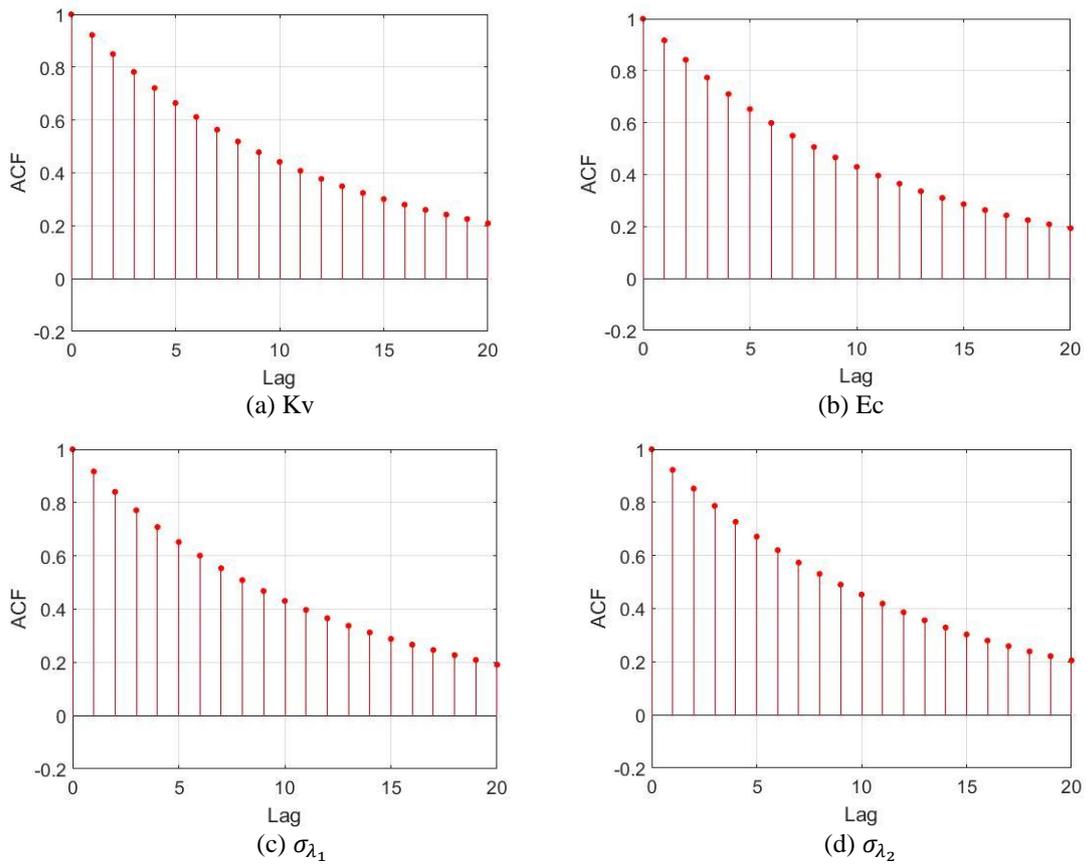


Figure 5. ACF graph of the parameters.

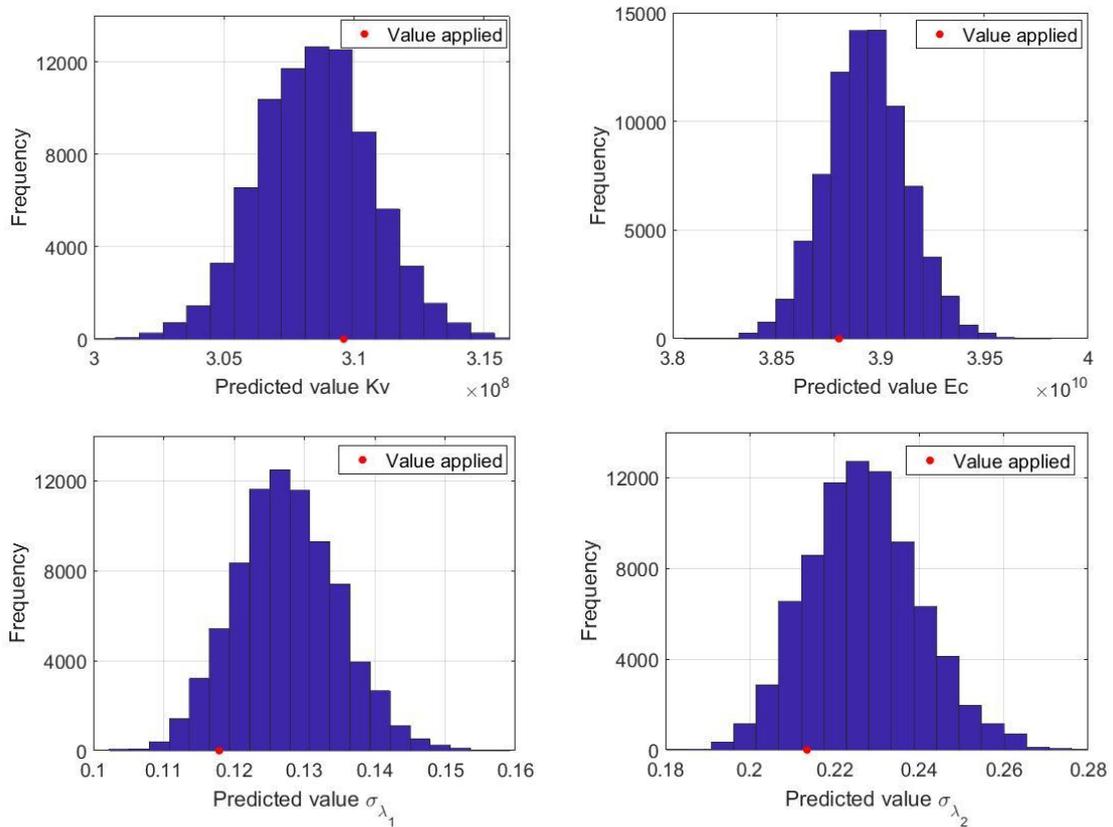


Figure 6. Histogram of the parameters.

According to the time series graph in Figure 4, the data for all parameters appears to be stationary. It is important to notice that this graph shown a high oscillation around the exact value of the standard deviation of the experimental measurements of natural frequencies. This results are due to the high perturbation ω of the parameter of interest of the Metropolis-Hastings algorithm.

Evaluating the sample autocorrelation function ACF of each parameter, presented in Figure 5, it is noted that the parameters do not present a high correlation, since for lag- t greater than 5, the ACF is less than 0.8, in order to characterize them samplers with good data variability and mixing, according to Vrugt (2016).

According to the histogram in Figure 6, it is possible to notice a higher density of points close to the exact value of the stiffness parameters K_v and E_c and a distribution closer to the shape of a Gaussian. For the error parameters, the point density is also close to the exact value of each parameter, despite the skewed that may have originated from the low number of measurements. In this figure, such as the time series graph, it is evidenced the high dispersion of data, especially for the standard deviation of the experimental measurements of natural frequencies.

It is important to mention the high computational cost of this simulation. The total computational time required to sampler this problem is approximately 120 hours for a 9-core 2.90 GHz computer. In the process of estimating the parameters of interest via Bayesian inference, the requirement of a large sample size is noted for the parameters to reach stationarity according to the initial data of the problem, a factor that implies in this high computational cost for the solution. In this context, the size of the perturbation of parameters in the Metropolis-Hastings technique associated with the mandatory number requires some initial tests so that the sample data showed low correlation and, consequently, were stationary. Thus, the incorrect choice of input variables required to Metropolis-Hastings algorithm can make the solution of poor quality and imply the non-stationarity of the parameters of interest.

Therefore, the Bayesian probabilistic approach to model updating of the numerical model shown satisfactory for this problem, despite of the high computational cost required to Monte Carlo Markov chain samples.

5. CONCLUSION

This paper presented a preliminary study on the use of Bayesian probabilistic approach to predict parameters of interest of a railway bridge using Finite Element model. The Metropolis-Hastings algorithm made it possible to sampler the posterior distribution. The results indicate that the parameters vertical stiffness of the supports and the modulus of elasticity of concrete can be accurately predicted, as well as the errors inherent to the experimental measurements. Moreover, the use of MCMC diagnostics was crucial to assess the stationarity and quality of samples.

The simplified numerical model updating presented in this research will serve as base model to be incrementally improved. Only Metropolis-Hastings was used to sample the posterior distribution in the Bayesian approach. Other algorithms can be applied and may be promising in order to reduce the computational cost of model updating. Furthermore, in this study, the model updating is given through the classical Bayesian approach. Aiming to extend the methodology to improve the quantification of uncertainties, the hierarchical update of parameters, proposed by Behmanesh et al. (2015), can be a powerful tool to estimate the uncertainties of the parameters in their entirety, such as the variation of physical parameters of the structure.

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7. RESPONSIBILITY NOTICE

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