26th ABCM International Congress of Mechanical Engineering
November 22-26, 2021. Florianópolis, SC, Brazil

COB-2021-1531
ERROR PROPAGATION IN PARAMETRIZATION METHODS FOR NACA AIRFOILS

João Victor Barros dos Santos
Guilherme Amaral do Prado Campos
Vinícius Gomes Pereira Siston
Centro Federal de Educação Tecnológica Celso Suckow da Fonseca campus Nova Iguaçu - Departamento de Engenharia Mecânica, Estrada de Adrianópolis, nº1.317, Santa Rita, Nova Iguaçu, RJ, CEP 26041-271, Brazil
joaovictor.96@hotmail.com, guilherme.campos@cefet-rj.br, viniciusgpsiston1@gmail.com

José Luiz Zanon Zotin
Centro Federal de Educação Tecnológica Celso Suckow da Fonseca campus Itaguaí - Departamento de Engenharia Mecânica, Rod. Gov. Mário Covas, s/n, Santana, Itaguaí, RJ, CEP 23812-101, Brazil
jose.zotin@cefet-rj.br

Abstract. *Over the past decade, wind turbines have been improving their Benefit-Cost Ratio mainly due to the evolution of higher performance aerodynamic profiles, obtained through increasing studies in the optimization area applied to aerodynamics. In the literature, some methods of airfoil parameterization have been used to help the process of wing profile optimization, such as the Bezier method, PARSEC method, the Bezier-PARSEC method, the Sobieczky method, the Modified Sobieczky method, among others. These methods aim to guarantee the original profile geometry with a reduced number of control points, resulting in an optimization process with a lower computational cost. In this article, the Bezier and PARSEC parameterization methods were compared to verify the error propagation behavior for nine symmetrical NACA profiles and different Reynolds number values. A previous study concluded that the Bezier method has an advantage over PARSEC, as it presents a parameterized profile with a smaller number of control points and good approximation. However, the study was limited to just one profile model and a single Reynolds number. Therefore, the present work sought to understand whether the Bezier method remains the most suitable for different symmetrical airfoils as well as the most suitable for the optimization analysis. Within this analysis, the number of control points was evaluated by comparing the geometric error of the original profile with the parameterized profile. The pressure and lift coefficient errors were also quantified by implementing the XFOIL program in MATLAB to guarantee the similarity in the aerodynamic properties between the original and parameterized airfoil. As a result, there was a linear increase in the geometric error as a function of both the NACA profile thickness and Reynolds numbers.*

Keywords: *NACA, Bezier Curve, PARSEC, Parametrization.*

1. INTRODUCTION

The aerodynamic evolution of cars and aircraft has contributed to improving its performance and efficiency, which leads directly to the reduction of fuel consumption and emissions. The same process occurs in the energy sector, with the geometric improvement of the blades of gas, wind, hydrokinetic and oscillating hydrofoil turbines, allowing for improvements in efficiency and, consequently, an increase in energy generated from renewable sources.

Within this context, in the literature, several studies assess the wing section with the best behavior in each operational situation, such as the studies proposed by Schubel and Crossley (2012), Mohamed *et al.* (2019) e Kumar and Saini (2016). According to Hashem and Mohamed (2018), S1046 airfoils have better results in vertical-axis wind turbine (VAWT) of the Darrieus H type, with 3 blades and a cycloidal surface diffuser. While Horizontal-Axis Wind Turbine (HAWT) with flexible blades and different cross sections have difference, when compared to HAWT with rigid blades (Hoogedoorn *et al.*, 2010).

Given the well-known importance of wing section profiles, those shapes have been studied to provide a better aerodynamics performance. Often, to achieve the expected results, an optimization method is applied to generate new coordinate points so that the airfoil geometry, defined with fewer points, has the same characteristics as the original one. The parametrization methods are an excellent alternative to describe a curve with a smaller amount of points. As an example, Santos *et al.* (2020) performed an analysis of the symmetric profile NACA 0015, for Re = 5×10^5, comparing two parametrization methods, PARSEC and Bèzier. Their work concluded that the Bèzier method with 8 control points has a less geometric error and better aerodynamic performance than the other cases studied.

With a large number of methods presented by literature, this paper sought to compare the two main methods applied to wing sections: the Bèzier curve and the Parametric Section method (PARSEC). Those methods were used to describe a list of nine symmetrical NACA airfoils and then compared with the original one, described by Abbott and Von Doenhoff (2012). Beyond a geometric evaluation, aerodynamic simulations were performed with the aid of the XFoil software. Therefore, the paper aims to evaluate the behavior of both parametrization methods for different thicknesses of symmetrical wing sections and verify the absolute error obtained for each method and wing section.

2. PARAMETRIZATION METHODS

Before presenting the optimization method developed, it is necessary to understand how parametrization methods can be used to describe airfoils geometries with less information when compared to the original function used to describe them. Thus, in this section, two different parametrization methods (Bèzier and PARSEC) will be described and then compared with the original NACA series airfoil profile.

2.1 NACA 4-digit series

According to Abbott and Von Doenhoff (2012), the NACA 4-digit airfoils' family can be described using some analytical equation to calculate the camber of the mean-line airfoil section and the section's thickness distribution along the chord. Fig. 1 illustrates a standard geometry profile defined as NACA MPXX. The camber (M), the position of camber (P), and the thickness distribution (XX) values are the fundamental geometric references used.

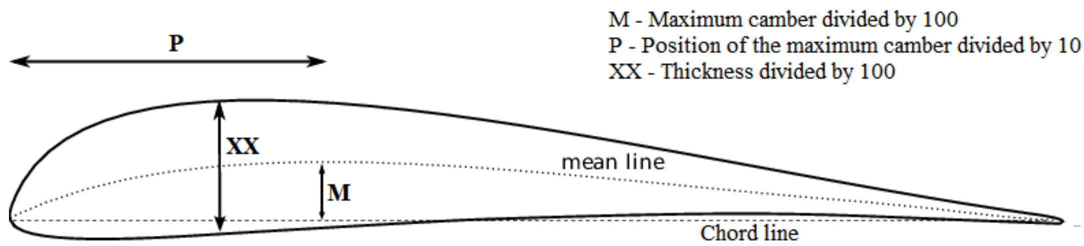


Figure 1: Representation of a NACA 4-Digit Series - NACA MPXX, adapted from Ayton (2016).

Camber Line:

$$y_c = \frac{M}{P^2}(2Px - x^2) \qquad (0 \leq x < P) \tag{1}$$

$$y_c = \frac{M}{1-P^2}(1 - 2P + 2Px - x^2) \qquad (P \leq x \leq 1) \tag{2}$$

Thickness:

$$y_t = \frac{T}{0.2}(a_0 x^{0.5} + a_1 x + a_2 x^2 + a_3 x^3 + a_4 x^4) \tag{3}$$

$$a_0 = 0.2969 \quad a_1 = -0.126 \quad a_2 = -0.3516 \quad a_3 = 0.2843 \quad a_4 = -0.1015 \tag{4}$$

Upper Surface:

$$x_u = x_c - y_t \sin\theta \qquad y_u = y_c + y_t \cos\theta \tag{5}$$

Lower Surface:

$$x_l = x_c + y_t \sin\theta \qquad y_l = y_c - y_t \cos\theta \tag{6}$$

where, $\theta = \arctan\left(\frac{\mathrm{d}y_c}{\mathrm{d}x}\right)$.

In order to provide a better leading and trailing edges resolution, and consequently better parametrization results, we use a cosine spacing function, as suggested by Santos *et al.* (2020), to define the abscissa point distributions. Therefore, a greater amount of point in the airfoil edges can be achieved.

2.2 Bèzier curve

The Bèzier curve is a mathematical description of any curve, that is commonly used to parametrized airfoils (Salunke *et al.*, 2014). The method is a variant of B-Spline description, and enables the generation of complex curves from a few control points with good precision, a desired behavior for optimization methods. The equations used in this paper for these methods were developed by Mohebbi (2014) as shown above:

$$P(t) = \sum_{i=0}^{n} B_i J_{n,i}(t), \tag{7}$$

where,

$$J_{n,i}(t) = \frac{n!}{i!(n-i)!} t^i (1-t)^{n-1}. \tag{8}$$

Based on the original airfoil geometry (section 2.1), it is possible to calculate the control points (B_i) by re-written Eq. (7) in matrix form.

$$[P(t)] = [J(t)]\,[B]\,. \tag{9}$$

Defining m as the number of chosen points on the airfoil surface, and n as the Bèzier curve degree, then Eq. 9 can be rewritten as:

$$[P(t)]_{nx2} = [J(t)]_{mx(n+1)}\,[B]_{(n+1)x2} \tag{10}$$

From that point, some algebraic changes can be made as,

$$[B]_{(n+1)x2} = [J(t)]^{-1}_{mx(n+1)}\,[P(t)]_{mx2} \tag{11}$$

$$[J(t)]^{T}_{(n+1)xm}\,[P(t)]_{mx2} = [J(t)]^{T}_{(n+1)xm}\,[J(t)]_{mx(n+1)}\,[B]_{(n+1)x2} \tag{12}$$

Finally,

$$[B]_{(n+1)x2} = \left[[J(t)]^{T}_{(n+1)xm}\,[J(t)]_{mx(n+1)}\right]^{-1}[J(t)]^{T}_{(n+1)xm}\,[P(t)]_{mx2} \tag{13}$$

The equation described by Mohebbi (2014) as in Eq.(13), can be used to provide the control points required to generate the intended wing section geometry. Thereby, those B_i can be re-used in Eq. (7) to recreate the airfoil section and then compared it with the original shape.

2.3 Parametric Section method (PARSEC)

Another method that uses geometric parameters is the Parametric Section method (PARSEC) developed by Sobieczky (1999), which attempts to represent an airfoil by eleven geometric parameters, as shown in Fig.2. Compering with Bèzier Curve, this method can provide a safer way to reproduce the shape and aerodynamics characteristics from the wing section. However, it has a fixed number of control points to describe the curve, and Bèzier can generate the airfoil with fewer points, which could be a better approach when the goal is an optimization process.

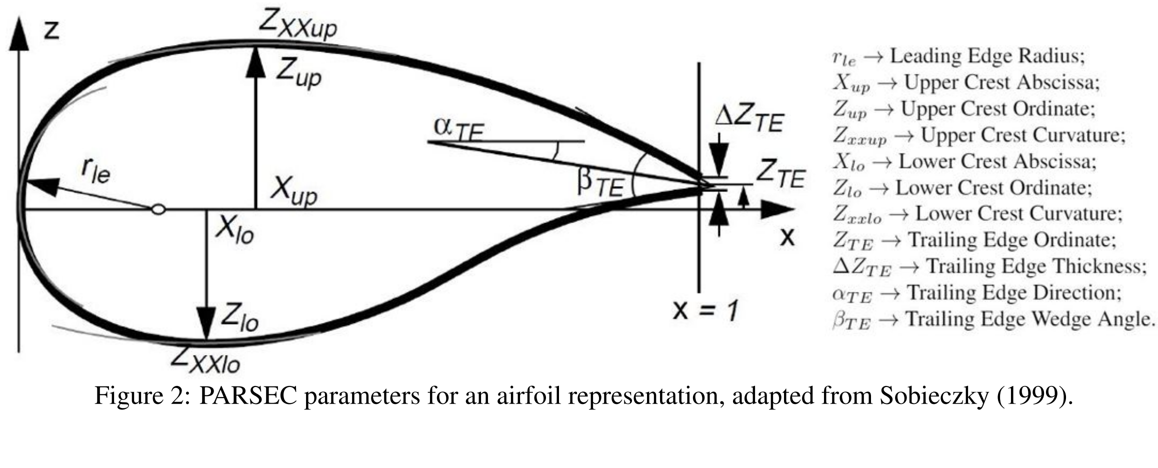


Figure 2: PARSEC parameters for an airfoil representation, adapted from Sobieczky (1999).

Using PARSEC the airfoil can be parametrized mathematically by two polynomial equations as,

$$y_e\left(x_e\right)=\sum_{k=1}^{6}\alpha_{ek}\mathbf{x}_e^{k-\frac{1}{2}} \qquad y_i\left(x_i\right)=\sum_{k=1}^{6}\alpha_{ik}\mathbf{x}_i^{k-\frac{1}{2}} \tag{14}$$

Where aek and aik are the coefficients from the Eq.(14) and can be found analytically using the original geometric parameters from the airfoil and the limits known for the trailing and leading edges ($\alpha_{i1}=-\alpha_{e1}$ for symmetrical airfoils). Thereby, those values are useful to recreate the wing section to compared it later with the original airfoil shape, as the NACA 4-Digit section suggests.

3. METHODOLOGY

The NACA 4-Digit equations and both parametrization methods (PARSE and Bèzier curve) were implemented in a MATLAB script together with XFoil software, in the same way proposed by Santos *et al.* (2020). However, in this case, we include the different airfoil thickness distributions and the Reynolds numbers from the fluid flow. The XFoil was developed by Drela (1989) to provide an easy way to evaluate some aerodynamics characteristics with low computational processing, using panel methods, linear vorticity, and boundary layer functions. In addition, an iterative process was performed to enable the results' evaluation.

3.1 Fluid and simulation parameters

Based on previous studies of oscillating hydrofoils (Kinsey *et al.*, 2011; Campos *et al.*, 2019; Santos *et al.*, 2020), Tab. 1 presents the fluid parameters used in this paper. Also, we use different NACA airfoil profiles in XFoil simulations to generate the aerodynamics data that will be used for further comparisons. Furthermore, it was considered a viscous, rotational, and incompressible fluid passing around the wing section.

Table 1: Fluid and foil characteristics

Parameter	**Mach**	**Fluid**	**Kinematic viscosity**
Value	0.006	Water	$4.2 \times 10^{-6}\ m^2/s$

All the NACA profiles used are symmetrical, with its thickness ranging from NACA 0012 and NACA 0020. With this list of nine airfoils, it will be possible to assess the absolute error of each method as a function of their thickness and the flow Reynolds Number.

3.2 Code flowchart

The main functionality of the code is based on an already validated code used in a previous analysis (Santos *et al.*, 2020), where different functions were developed for each step of the simulation. First, for each airfoil, the parameters M, P, and XX were used to generate the original curve, which will be compared with the others two parametrization methods.

After this first step, there are two ways to follow, depending on the method that will be approached. In the first case, considering a Bèzier curve, it is necessary to define the curve degree to calculate the control points and then generate the parametrized airfoil, as shown in Fig. 3.

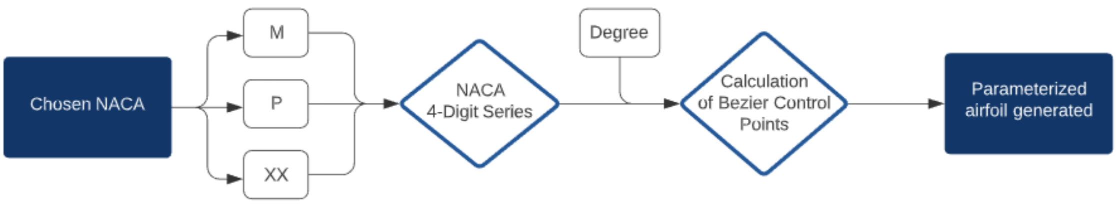


Figure 3: Bezier flowchart (Santos *et al.*, 2020)

In the second case, using the PARSEC method, the coordinates generated with the NACA 4-Digits equations are used to calculate the geometry parameters from the wing section, as illustrated in Fig. 4.

In both cases, there was an aerodynamic evaluation of each parametrized airfoil using XFoil software, resulting in aerodynamic data, such as pressure and lift coefficients, as presented in Fig. 5. Therefore, it is possible to assess the absolute errors from geometric or aerodynamic parameters between the original and parametrized airfoils. This is an

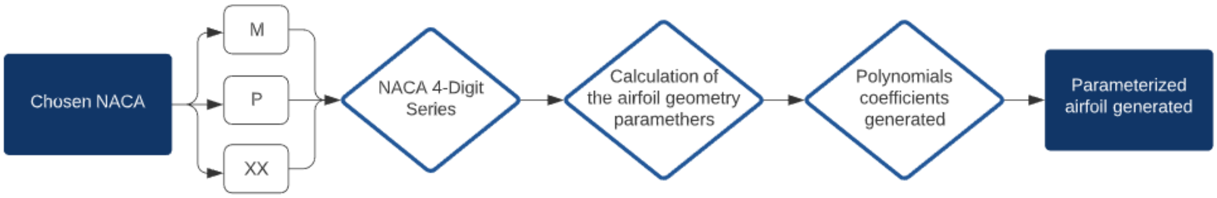


Figure 4: PARSEC flowchart (Santos *et al.*, 2020)

iterative process that allows us to know if the same parameters used in an optimization process are the best for different airfoil shapes.

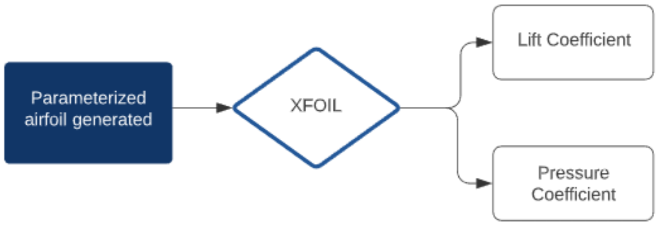


Figure 5: XFOIL flowchart (Santos *et al.*, 2020)

4. RESULTS AND DISCUSSION

With the processes and methods presented in the previous sections, the aerodynamic characteristics of each one of the aforementioned symmetric profiles could be performed based on the tabulated simulation parameters. Then, a comparison between average absolute errors of the parametrization processes, Bèzier and Parsec, from the different airfoil thicknesses was made, to enable a better understanding of the behavior of parametrization between different wings sections.

At first, the geometric error found between the original airfoil and those generated by Bèzier and PARSEC method was analyzed for each of the symmetrical airfoils, with different thicknesses, listed. Thereby, as presented in Fig.6, a linear tendency was found in this error, where the linear coefficient decrease with the increasing of the number of control points (CP). Furthermore, the results also shows that the use of Bèzier with 8 control points, remains a better option compared with PARSEC, even with different airfoil thickness.

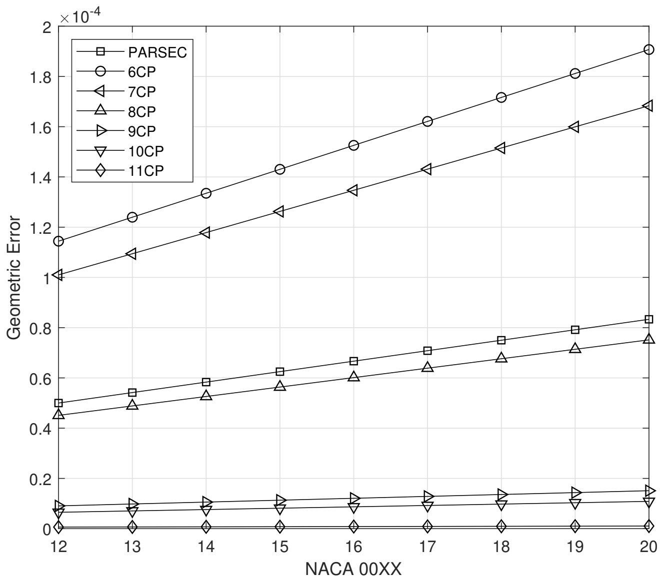
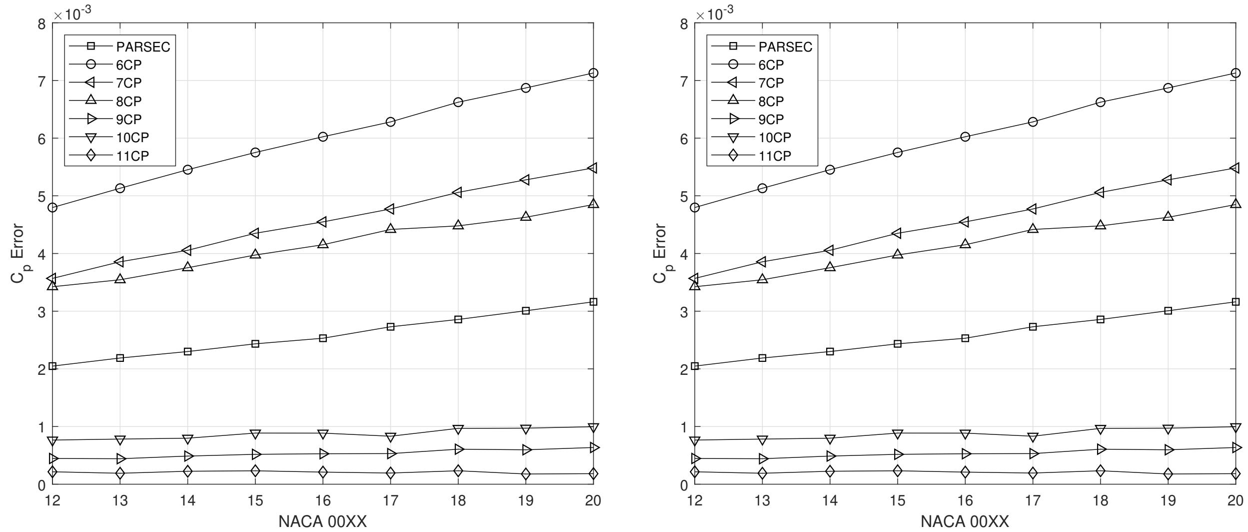


Figure 6: Average geometric error

The comparison between the averaged pressure coefficient error is an efficient way to verify if the parametrization method used provides a good approximation of the aerodynamics characteristics from the wing section. Therefore, this is an important factor to evaluate the best method. As PARSEC is based on the geometry of the airfoils, a closer approximation is foreseen, presenting lower average errors. Fig. 7 shows linear progressions, as well as in Fig.6, and enables the same previous analysis, in which the differences between Bèzier with 8 control points and PARSEC remain constant.

Although the PARSEC method shows lower pressure coefficient errors, the Bèzier method presents the possibility of a good approximation with a smaller number of points to describe a curve. Fig. 7 also shows the pressure coefficient errors similarity despite the Reynolds Number variation.



(a) Results with Re 2.5×10^5 (b) Results with Re 7.5×10^5

Figure 7: Average coefficient of pressure error

5. CONCLUSION

To be applied in an airfoil optimization, the parametrization methods presented (Bèzier and PARSEC) had their performance evaluated for different NACA profiles and Reynolds Number. Thereby, there was a comparison of geometric and pressure coefficient errors using the same method of previous works, which had already been validated.

After the accomplishment of all simulations, the results indicate that the recommendation of Santos *et al.* (2020) for the use of Bèzier with 8 control points, remains a good choice, even with different airfoil thickness. Furthermore, a linear behavior of the averaged pressure coefficient error was observed, increasing with the airfoil thickness. No significant error variation was verified for the assessed Reynolds Number range.

6. ACKNOWLEDGEMENTS

This optional section must be placed before the list of references.

7. REFERENCES

Abbott, I.H. and Von Doenhoff, A.E., 2012. *Theory of wing sections: including a summary of airfoil data*. Courier Corporation.

Ayton, L., 2016. “An analytic solution for gust-aerofoil interaction noise including effects of geometry”. *IMA Journal of Applied Mathematics*, Vol. 82, p. hxw038. doi:10.1093/imamat/hxw038.

Campos, G.A.P., Raptopoulos, L.S.C. and Dutra, M.S., 2019. “Mathematical model for flapping foil”. In *Proceedings of the 25nd International Congress of Mechanical Engineering - COBEM 2019*. Uberlândia, Brazil.

Drela, M., 1989. “Xfoil: An analysis and design system for low reynolds number airfoils”. In *Low Reynolds number aerodynamics*, Springer, pp. 1–12.

Hashem, I. and Mohamed, M., 2018. “Aerodynamic performance enhancements of h-rotor darrieus wind turbine”. *Energy*, Vol. 142, pp. 531–545.

Hoogedoorn, E., Jacobs, G.B. and Beyene, A., 2010. “Aero-elastic behavior of a flexible blade for wind turbine application: A 2d computational study”. *Energy*, Vol. 35, No. 2, pp. 778–785.

Kinsey, T., Dumas, G., Lalande, G., Ruel, J., Mehut, A., Viarouge, P., Lemay, J. and Jean, Y., 2011. “Prototype testing of a hydrokinetic turbine based on oscillating hydrofoils”. *Renewable Energy*, Vol. 36, No. 6, pp. 1710–1718.

Kumar, A. and Saini, R., 2016. “Performance parameters of savonius type hydrokinetic turbine–a review”. *Renewable* *and Sustainable Energy Reviews*, Vol. 64, pp. 289–310.

Mohamed, M., Dessoky, A. and Alqurashi, F., 2019. "Blade shape effect on the behavior of the h-rotor darrieus wind turbine: Performance investigation and force analysis". *Energy*, Vol. 179, pp. 1217–1234.

Mohebbi, F., 2014. *Optimal shape design based on body-fitted grid generation*. Ph.D. thesis, University of Canterbury, Christchurch, New Zealand.

Salunke, N.P., Juned Ahamad, R. and Channiwala, S., 2014. "Airfoil parameterization techniques: A review". *American Journal of Mechanical Engineering*, Vol. 2, No. 4, pp. 99–102.

Santos, J.V.B., Siston, V.G.P., Campos, G.A.d.P. and Zotin, J.L.Z., 2020. "Analysis of parametrization methods for aerodynamic profiles". In *Proceedings of the 18th Brazilian Congress of Thermal Sciences and Engineering - ENCIT 2020*. Online.

Schubel, P.J. and Crossley, R.J., 2012. "Wind turbine blade design". *Energies*, Vol. 5, No. 9, pp. 3425–3449.

Sobieczky, H., 1999. "Parametric airfoils and wings". In *Recent development of aerodynamic design methodologies*, Springer, pp. 71–87.

8. RESPONSIBILITY NOTICE

The authors are solely responsible for the printed material included in this paper.