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IDENTIFICATION OF CONTACT FAILURES IN COMPOSITE MATERIALS VIA BAYESIAN INFERENCE AND REDUCED MODELS

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Abstract. *In several engineering applications, heat conduction in a composite medium has been analyzed, such as thermal insulation, corrosion protection and layered compounds, which offers new opportunities to adapt structures to meet different requirements in modern building materials . In addition, the formulation and solution of problems that allow assessing the adherence between two or more materials is of great importance in several fields, such as electronics, telecommunications, aviation, defense and oil, among others. Given the importance of non-destructive detection of adhesion failures in laminated composites and the use of infrared thermographic images for this purpose, efforts have been made to ensure that the knowledge of heat transfer is applied so that quantitative analyzes can be made possible. It is not always possible to identify adhesion or adhesion failures using only qualitative tests, since temperature gradients in contact failure regions are generally very small. In some situations, the thickness of the material and its glass transition temperatures prevent the occurrence of large gradients and cause the flaws to be identified only by thermal image. In this work, a two-dimensional heat conduction problem is modeled in a single domain in a multilayered medium with isolated side edges, heat flux on the lower surface and heat exchange by natural convection on the upper surface. A reduced model, that represents an approximation of the complete model, is developed using the Improved Lumped Formulation and it involves only the adhesive layer that joins the two layers of the material analyzed in the original problem. This methodology aims to find a low computational cost reduced model and use it to estimate contact failures in multilayered materials. The forward problem solution gives us the temperature profile in one of its surfaces and it will be used in the inverse problem as simulated experimental measures, by adding a noise to the solution. The solution of the inverse problem is an estimated thermal conductivity function with spatial variation. The objective of this work is to detect contact failures in composite materials using a reduced model in the inverse problem to reduce the computational cost. Since the thermal conductivity of the adhesive is considerably grater than the air's, in the case of the presence of a contact failure, the estimated thermal conductivity function must present a significant variation around the failure position. The methodology adopted was able to estimate the contact failure.*

Keywords: *heat conduction, laminated composite, identification of contact failure, single domain, Markov Chain Monte Carlo.*

1. INTRODUCTION

The application of composite materials in engineering has become very common, as their manufacture allows a combination of mechanical properties of two or more different materials according to the need for use, thus obtaining a material with improved specific properties (CHUNG, 2003). Therefore, its use can be observed in various fields of engineering, such as civil construction, electronics and thermal applications. The growing interest in the application of a multilayer media makes it important to know the phenomena that occur at the interface of these layers. However, in these materials, the presence of contact failures between layers may occur, characterized by the presence of an air bubble in the region of the adhesive. A quality of adhesion between layers of composite media can be evaluated by means of a heat conduction analysis, in which a study of the temperature behavior of these materials is carried out. A practical way to carry out this study is to heat a sample and monitor the temperature decay. In this way, it is possible to detect possible material failures (Grosso *et al.*, 2013, 2016; Abreu *et al.*, 2014a,b; Mascouto *et al.*, 2020).

In this work, a two-dimensional heat conduction problem, originally proposed by Mascouto *et al.* (2020), is modeled in a single domain in a multilayered medium with isolated side edges, heat flow on the lower surface and heat exchange by natural convection on the upper surface. In the work of Mascouto *et al.* (2020) was observed a high computational cost to obtain the solution of the complete model forward problem, solved using CITT, and the inverse problem. Therefore, a methodology involving a reduced model is formulated aiming the reduction of the computational cost and the analysis of a contact failure due to lack of adhesive between the layers. It involves a model with Lumped formulation that represents an approximation of the complete problem, but it is considered only the part of the adhesive that joins the two layers of a composite material. The solution of the inverse problem is a function with spatial variation of the thermal conductivity and the contact failure will be characterized by a significant variation of such property around the failure region. It is noteworthy that this work does not deal with estimation of actual thermal properties, but with identification of contact failures in composite materials.

2. PHYSICAL PROBLEM AND MATHEMATICAL FORMULATION

In this work, the physical problem involves a two-dimensional heat conduction in a laminated composite with three different layers and materials according to Fig. 1, with width in x-direction L_x and height in y-direction L_y. The adhesion failure is represented by the presence of an air bubble ($L_{x_b} \times L_{y_b}$) among the adhesive, that might be caused during the manufacturing process. The height of the thermal insulation, the adhesive (and the air bubble) and the metal is, respectively, L_{y_c}, L_{y_b} and L_{y_a}. The air bubble width is L_{x_b}; the remaining distances to the left and to the right of the air bubble are, respectively, L_{x_a} and L_{x_c}.

The physical phenomenon involves a heat flux (q'') at $y = 0$ while the opposite surface is subjected to a heat exchange by natural convection with the environment, with convection heat exchange coefficient h and with ambient temperature T_∞. The functions $f(x)$ and $g(x)$ model the thermal insulation-adhesive interface and metal-adhesive interface, respectively.

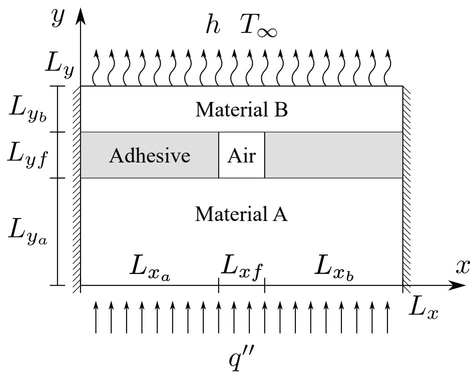


Figure 1: Schematic representation of the physical problem.

The heat conduction equation formulated in a single domain can be written as (Ozisik, 1987; Incropera *et al.*, 2014)

$$\rho(x,y)\; c_p(x,y)\; \frac{\partial T(x,y,t)}{\partial t} = \frac{\partial}{\partial x}\left(k(x,y)\; \frac{\partial T(x,y,t)}{\partial x}\right) + \frac{\partial}{\partial y}\left(k(x,y)\; \frac{\partial T(x,y,t)}{\partial y}\right) \tag{1}$$

where ρ is the density, c_p is the specific heat and k is the thermal conductivity.

The boundary and initial conditions are expressed by

$$\left.\frac{\partial T(x,y,t)}{\partial x}\right|_{x=0} = \left.\frac{\partial T(x,y,t)}{\partial x}\right|_{x=L_x} = 0\,, \qquad \text{for } x=0\;,\; x=L_x \text{ and } t>0 \tag{2}$$

$$-\,k(x,y)\;\left.\frac{\partial T(x,y,t)}{\partial y}\right|_{y=0} = q''\,, \qquad \text{for } y=0 \text{ and } t>0 \tag{3}$$

$$k(x,y)\;\left.\frac{\partial T(x,y,t)}{\partial y}\right|_{y=L_y} + h\,T(x,y,t)\big|_{y=L_y} = h\,T_\infty\,, \qquad \text{for } y=L_y \text{ and } t>0 \tag{4}$$

$$T(x,y,t)\big|_{t=0} = T_0\,, \qquad \forall\; x\,,\; y \text{ and } t=0 \tag{5}$$

where q'' is the heat flux, h is the heat transfer coefficient and T_0 represents the initial temperature.

2.1 METHODOLODY

Several multidimensional diffusion problems, involving partial differential equations system, and/or with complex geometries, imply non-analytical and difficult solutions, which may require a high computational cost. Therefore, it is of interest for the engineering application, to propose simplified formulations for the equations system by reducing the number of independent variables in cases of problems with more than one variable. This is done by performing an integration of the partial differential equations in one or more spatial variables, but preserving some information in direction integrated, provided by the the boundary conditions. Different levels of approximation can be used, starting from Classical Lumped to improved formulations, such as the Hermite approximation, which consists of approximating an integral based on the values of the integrand and its derivatives in the limits of integration, in the form (Hermite, 1878; Mennig *et al.*, 1983; Cotta and Mikhailov, 1997).

With this approach, only the part of the adhesive is considered, with a heat flow on the lower surface and convection on the upper surface with the thermally insulated edges. In the inverse problem, the thermal conductivity function is estimated along the adhesive region and synthetic experimental measurements obtained from the solution of the forward problem of the complete model on the lower surface will be used, but the likelihood function will be calculated using the reduced model. The failure detection will be according to the behavior of the estimated thermal conductivity function. It is expected that, around the region of failure, this property will vary, since the thermal conductivity of air is significantly lower than that of the adhesive. A schematic of this model is shown in Fig. 2.

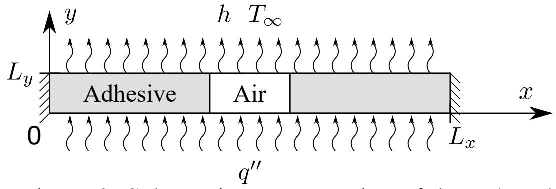


Figure 2: Schematic representation of the reduced Model.

The equation that governs the physical problem in a single domain formulation is written as

$$\rho(x)\ c_p(x)\ \frac{\partial T(x,y,t)}{\partial t} = \frac{\partial}{\partial x}\left(k(x)\ \frac{\partial T(x,y,t)}{\partial x}\right) + \frac{\partial}{\partial y}\left(k(x)\ \frac{\partial T(x,y,t)}{\partial y}\right) \tag{6}$$

with boundary an initial conditions

$$\left.\frac{\partial T(x,y,t)}{\partial x}\right|_{x=0} = 0\ , \qquad \text{for } x = 0 \text{ and } t > 0 \tag{7}$$

$$\left.\frac{\partial T(x,y,t)}{\partial x}\right|_{x=L_x} = 0\ , \qquad \text{for } x = L_x \text{ and } t > 0 \tag{8}$$

$$-\ k(x)\ \left.\frac{\partial T(x,y,t)}{\partial y}\right|_{y=0} = q''(x)\ , \qquad \text{for } y = 0 \text{ and } t > 0 \tag{9}$$

$$k(x)\ \left.\frac{\partial T(x,y,t)}{\partial y}\right|_{y=L_y} + h\ T(x,y,t)\big|_{y=L_y} = h\ T_\infty\ , \qquad \text{for } y = L_y \text{ and } t > 0 \tag{10}$$

$$T(x,y,0) = T_0\ , \qquad \forall\ x\ ,\ y \text{ and } t = 0 \tag{11}$$

2.1.1 *Classical Lumped Formulation*

The Classical Lumped Formulation is a method that allows us to adopt a simplification of the temperature in space. In sufficiently thin materials the temperature gradient can be neglected according to the Biot number, which must be less than 0.1. Then a uniform temperature in space is considered. However, in many practical engineering applications, the number of Biot is higher, which does not allow the application of this method without losing some information regarding the space. In these cases, an Improved Lumped Formulation is used, which takes into account the temperature gradient present in the physical phenomenon (Su *et al.*, 2009; Knupp *et al.*, 2012; Orlande *et al.*, 2014).

Considering the average temperature as

$$\bar{T}(x,t) = \frac{1}{L_y}\int_0^{L_y} T(x,y,t)\ \mathrm{d}y \tag{12}$$

to perform a lumped in the y direction the operator $\dfrac{1}{L_y}\displaystyle\int_0^{L_y}(\cdot)\,\mathrm{d}y$ is applied in the PDE (Eq. (6)), resulting in

$$\frac{\partial}{\partial x}\left(k_x\frac{\partial \bar{T}(x,t)}{\partial x}\right)+\frac{-h\,\bar{T}(x,t)}{L_y}+\frac{h\,T_\infty+q''(x)}{L_y}=\rho_x\,c_{px}\frac{\partial \bar{T}(x,t)}{\partial t} \tag{13}$$

with the boundary and initial conditions written as

$$\left.\frac{\partial \bar{T}(x,t)}{\partial x}\right|_{x=0}=0\,, \qquad \text{for } x=0 \text{ and } t>0 \tag{14}$$

$$\left.\frac{\partial \bar{T}(x,t)}{\partial x}\right|_{x=L_x}=0\,, \qquad \text{for } x=L_x \text{ and } t>0 \tag{15}$$

$$\bar{T}(x,0)=T_0\,, \qquad \forall\ x\,,\ y \text{ and } t=0 \tag{16}$$

Note that the problem is now dependent only on one variable in space.

2.1.2 *Improved Lumped Formulation*

In the classical lumped formulation the temperature gradient across the thickness is neglected. Therefore, only an average temperature for the whole body is considered and, in consequence, the temperatures on the upper and lower surfaces are equal to the average temperature (Su *et al.*, 2009; Knupp *et al.*, 2012; Orlande *et al.*, 2014).

However, in the improved lumped formulation, based on the integral pair equation approach, the temperature gradient across the thickness is not neglected, but taken into account as an approximation. The purpose of this formulation is to find an improved approximation for the contours from the average temperature obtained in the Classical Lumped Formulation. Thus, besides the approximation for the average temperature, we have also an approximation for each of the surfaces.

The approach considered in this work was the $H_{1,1}/H_{0,0}$ approach, in which the Hermite formulas $H_{1,1}$ and $H_{0,0}$ are used to approximate the average temperature $\bar{T}(x,y,t)$ and the integral of the temperature gradient in the direction y.

The upper surface temperature is written as

$$T(x,L_y,t)=T_s \tag{17}$$

for the lower surface

$$T(x,0,t)=T_i \tag{18}$$

The $H_{0,0}$ and $H_{1,1}$ formulas are applied to the Eq. (12), which results in

$$\bar{T}(x,t)\approx\frac{1}{2}\left[T_i+T_s\right]+\frac{L_y}{12}\left[\left.\frac{\partial T(x,y,t)}{\partial y}\right|_{y=0}-\left.\frac{\partial T(x,y,t)}{\partial y}\right|_{y=L_y}\right] \tag{19}$$

and

$$\int_0^{L_y}\frac{\partial T(x,y,t)}{\partial y}\,\mathrm{d}y=\left[T_s-T_b\right]\approx\frac{L_y}{2}\left[\left.\frac{\partial T(x,y,t)}{\partial y}\right|_{y=0}+\left.\frac{\partial T(x,y,t)}{\partial y}\right|_{y=L_y}\right] \tag{20}$$

The Eqs. (19) and (20) are solved together with boundary conditions Eqs. (7) and (8) to obtain the following relations between the surface temperatures and the approximate average temperature

$$T_s\approx\frac{2hL_yT_\infty+6k_x\bar{T}(x,t)-L_yq''(x)}{2hL_y+6k_x} \tag{21}$$

$$T_i\approx\frac{6hkL_y\bar{T}(x,t)-2hk_xL_yT_\infty+hL_y{}^2q''(x)+12k_x{}^2\bar{T}(x,t)+4k_xL_yq''(x)}{4hk_xL_y+12k_x{}^2} \tag{22}$$

3. INVERSE PROBLEM

In this work, we seek to estimate the vector $\mathbf{P}$, which represents the discrete values of the thermal conductivity (k) in the region where, theoretically, there should be only adhesive. Once the thermal conductivity of the air is different from the adhesive's, it must be clear whether there is a adhesion failure.

The Bayesian Inference is a method of statistical inference that essentially consists of using all the information available *a priori* in order to reduce uncertainty in decision-making problems. The Bayes' theorem, given by Eq. (23) is used to combine new information with previous information in order to form the basis of statistical processes (Kaipio and Somersalo, 2004; Orlande *et al.*, 2008).

$$\pi_{post}(\mathbf{P}) = \pi(\mathbf{P}|\mathbf{Y}) = \frac{\pi_{pri}(\mathbf{P})\pi(\mathbf{Y}|\mathbf{P})}{\pi(\mathbf{Y})} \tag{23}$$

where $\pi_{post}(\mathbf{P})$ is the posterior probability density, that is, the conditional probability of the parameters $\mathbf{P}$ given the measurements $\mathbf{Y}$; $\pi_{pri}(\mathbf{P})$ is the prior density, in other words, the coded information about the parameters prior to the measurements; $\pi(\mathbf{Y}|\mathbf{P})$ is the likelihood function, which expresses the likelihood of different measurement outcomes $\mathbf{Y}$ with $\mathbf{P}$ given; and $\pi(\mathbf{Y})$ is the marginal probability density of the measurements, which plays the role of a normalizing constant.

The vector of parameters is written as

$$\mathbf{P} = \{P_1, P_2, \ldots, P_N\} \tag{24}$$

and the available measurements' as

$$\mathbf{Y} = \{Y_1, Y_2, \ldots, Y_I\} \tag{25}$$

where N is the number of parameters and I is the number of measurements (Kaipio and Somersalo, 2004; Ozisik and Orlande, 2000; Ozisik, 1987). The vector $\mathbf{P} = \{P_1, P_2, \ldots, P_N\}$ represents the value of k in discrete nodes along the x direction at $y = 0.051$ (adhesion region medium point), or else, P_1 is equivalent to the value of k at $x = 0$ and P_N represents the value of k at $x = L_x$.

Considering the measurement errors as Gaussian random variables, with zero mean and known covariance matrix $\mathbf{W}$, and also additives and independent of the parameters $\mathbf{P}$, the likelihood function can be represented by

$$\pi(\mathbf{Y}|\mathbf{P}) = (2\pi)^{-I/2}|\mathbf{W}|^{-1/2} exp\left[-\frac{1}{2}(\mathbf{Y} - \mathbf{T}(\mathbf{P}))^T \mathbf{W}^{-1}(\mathbf{Y} - \mathbf{T}(\mathbf{P}))\right] \tag{26}$$

at which, $\mathbf{T}$ is the vector of estimated variables, obtained from the solution of the forward model with an estimate for the parameters $\mathbf{P}$.

The Metropolis-Hastings algorithm used was used in this work and its implementation starts with the selection of a movement distribution $q(\mathbf{P}^*, \mathbf{P}^{(t-1)})$ which generates a new candidate $\mathbf{P}^*$ given the current state $\mathbf{P}^{(t-1)}$ of the Markov chain. Once the distribution is selected, the Metropolis-Hastings algorithm is implemented following the steps (Kaipio and Somersalo, 2004; Ozisik and Orlande, 2000; Ozisik, 1987):

1. Sample a Candidate Point $\mathbf{P}^*$ from a jumping distribuition $q(\mathbf{P}^*, \mathbf{P}^{(t-1)})$;

2. Calculate

$$\beta = min\left[1, \frac{\pi(\mathbf{P}^*|\mathbf{Y})\; q(\mathbf{P}^{(t-1)}, \mathbf{P}^*)}{\pi(\mathbf{P}^{(t-1)}|\mathbf{Y})\; q(\mathbf{P}^*, \mathbf{P}^{(t-1)})}\right]; \tag{27}$$

3. Generate a random value U with uniform distribution on $(0, 1)$;

4. If $U \leq \beta$, define $\mathbf{P}^t = \mathbf{P}^*$; Otherwise, define $\mathbf{P}^t = \mathbf{P}^{(t-1)}$;

5. Return to step 1 in order to generate the sequence $\left[\mathbf{P}^1, \mathbf{P}^2, \mathbf{P}^3, ..., \mathbf{P}^n\right]$.

3.1 Delayed Acceptance Metropolis-Hastings

In the Delayed Acceptance Metropolis-Hastings, the information provided by the complete model is used. The regular Metropolis-Hastings algorithm is used to estimate the likelihood value using the reduced model. However, if a proposal state is accepted, another test of Hastings is performed with the complete model and, then, finally decide whether such state should be accepted. The DAMH algorithm is expressed as follows (Orlande *et al.*, 2008)

1. Sample a candidate point $\mathbf{P}^*$ from a proposal distribution $q(\mathbf{P}^*, \mathbf{P}^{(t-1)})$;
2. Calculate the acceptance factor with the reduced model

$$\beta = min\left[1, \frac{\pi(\mathbf{P}^*|\mathbf{Y})\; q(\mathbf{P}^{(t-1)}, \mathbf{P}^*)}{\pi(\mathbf{P}^{(t-1)}|\mathbf{Y})\; q(\mathbf{P}^*, \mathbf{P}^{(t-1)})}\right]; \tag{28}$$

3. Generate a random value U with uniform distribution in $(0, 1)$;
4. If $U \leq \beta$, proceed to step 5; otherwise, set $\mathbf{P}^t = \mathbf{P}^{(t-1)}$ and return to step 1;
5. Calculate a new acceptance factor with the complete model

$$\beta_c = min\left[1, \frac{\pi_c(\mathbf{P}^*|\mathbf{Y})\; q(\mathbf{P}^{(t-1)}, \mathbf{P}^*)}{\pi_c(\mathbf{P}^{(t-1)}|\mathbf{Y})\; q(\mathbf{P}^*, \mathbf{P}^{(t-1)})}\right]; \tag{29}$$

6. Generate a random value U_c with uniform distribution in $(0, 1)$;
7. If $U_c \leq \beta$, define $\mathbf{P}^t = \mathbf{P}^*$; otherwise, set $\mathbf{P}^t = \mathbf{P}^{(t-1)}$
8. Return to Step 1 to generate the sequence $\left[\mathbf{P}^1, \mathbf{P}^2, \mathbf{P}^3, ..., \mathbf{P}^n\right]$.

Thus, the complete model, that requires a higher computational cost than the reduced model, doesn't need to be calculated for every new state proposed, only for those accepted using the reduced model.

The Total Variation prior is used in this work, and its form is given by (Kaipio and Somersalo, 2005; Orlande *et al.*, 2008)

$$\pi_{pri}(\mathbf{P}) \propto exp[-\gamma\, TV(\mathbf{P})] \tag{30}$$

where, for the present case

$$TV(\mathbf{P}) = \sum_{i=2}^{N-1} \Delta x\left[|P_i - P_{i+1}| + |P_i - P_{i-1}|\right] \tag{31}$$

in which $TV(\mathbf{P})$ is the Total Variation function and γ a regularization parameter.

Again, it is noteworthy that this work does not deal with estimation of thermal properties, but with identification of contact failure in composite materials accordingly to the thermal conductivity function obtained in the inverse problem solution. In the occurrence of an adhesion failure, such function must present a significant variation around its region. However, the function obtained should not be interpreted as an actual estimation of the thermal conductivity, but serves to analyze their behavior and infer about the existence of contact failure (presence of an air bubble in the adhesive region).

4. RESULTS AND DISCUSSION

In this section the solution of the inverse problem will be showed. The results were obtained through the *NDSolve* function from the software Wolfram Mathematica. The parameters used in the problem is presented in the Table 1 (Incropera *et al.*, 2014).

Table 1: Parameters used to solve the problem.

Parameter	Value	Unit
k_{ep}	0,87	
k_{ad}	0,7	$\left[\frac{W}{mK}\right]$
k_{air}	0,0263	
k_{ss}	13,4	
$(\rho \cdot c_p)_{ep}$	1,31	
$(\rho \cdot c_p)_{ad}$	1,75	$\left[\frac{MJ}{m^3K}\right]$
$(\rho \cdot c_p)_{air}$	1,17	
$(\rho \cdot c_p)_{ss}$	2,65	
$q''(x)$	5000	$W/(m^2K)$
h	15	$W/(m^2K)$
$T_0 = T_\infty$	20	$^\circ C$

The reduced model proposed to replace the complete model in the inverse problem proved faster. The forward problem solution of the complete model, as presented in Mascouto *et al.* (2020) obtained with CITT, was calculated 100 times inside a loop function to measure the total time of computing. On average, it takes 0,97 s for the forward problem to be calculated. On the other hand, the solution of the forward problem of the reduced model takes, on average, 0,032 s to be calculated, which means a reduction of 96,7%. As the direct problem needs to be computed for each new candidate generated to calculate the likelihood function (Eq. (26)) in the inverse problem, when adopting the reduced models there is a significant reduction in the computational cost.

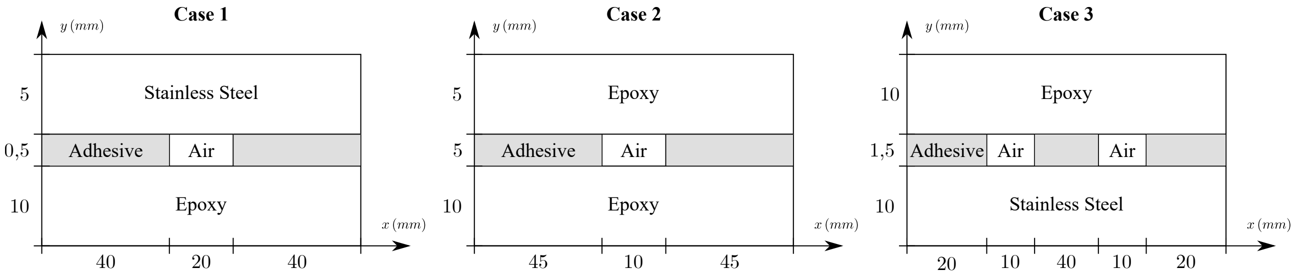
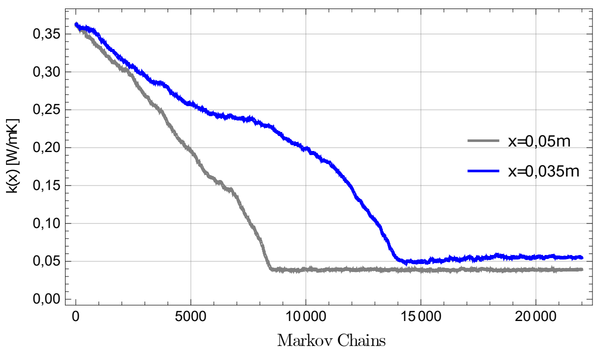
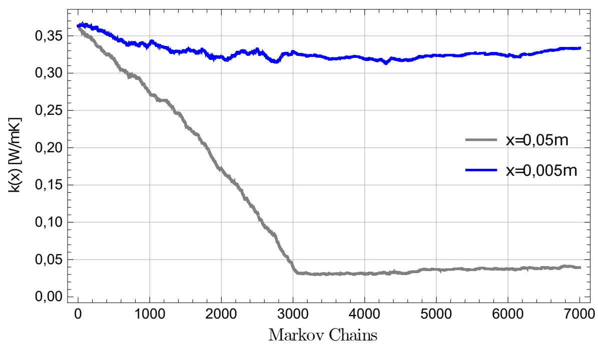


Figure 3: Cases considered in the inverse problem.

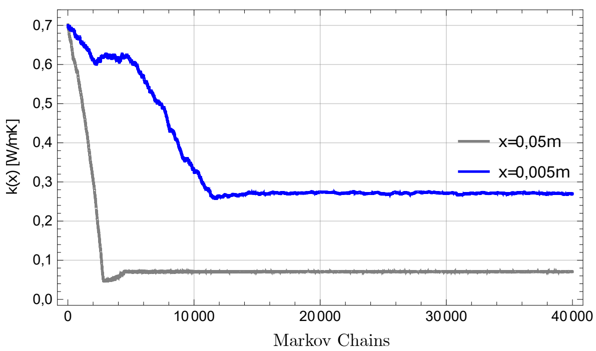
For the inverse problem, different cases were considered (Fig. 3) involving epoxy (noted by subscription ep) and stainless steel AISI 316 (noted by subscription ss); with different thicknesses; and failure dimensions and quantity. The Case 1 was solved using both regular Metropolis-Hasting (Case 1a) and Delayed Acceptance Metropolis-Hastings (Case 1b) algorithms; and Cases 2 and 3 were solved using only the regular Metropolis-Hastings algorithm. In all of them, the simulation ends at $t = 20000\,s$ with 40 steps in the time starting at $500\,s$ and 21 points in the space, discretized with $dx = 0,005\,m$ were considered. Furthermore, simulated experimental measurements were generated from the solution of the complete model by adding a Gaussian distribution noise with a mean of 0 and standard deviation of $0,05°C$. The experimental measurements are obtained on the lower surface of the complete model and compared with the temperature field on the lower surface of the reduced model in the likelihood function (Eq. (26)).



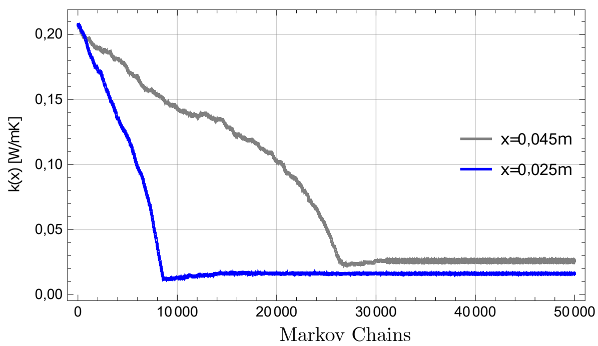
(a) Case 1a with regular Metropolis-Hastings.



(b) Case 1b with Delayed Acceptance Metropolis-Hastings.



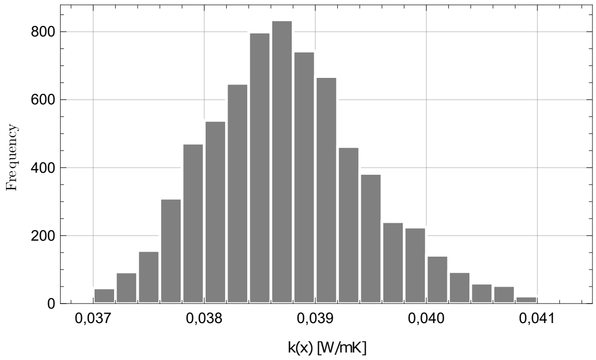
(c) Case 2.



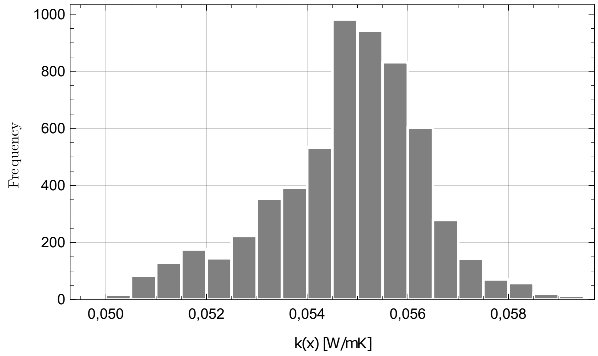
(d) Case 3.

Figure 4: Evolution of the Markov Chains.

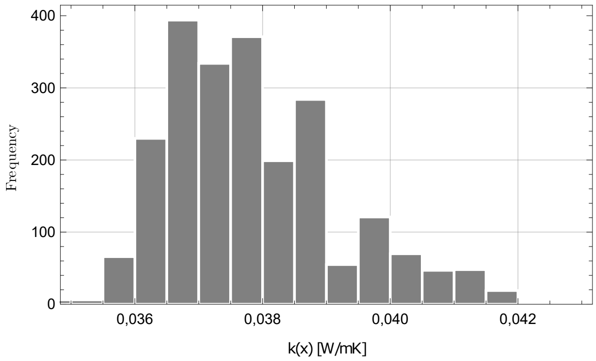
The Fig. 4 shows the evolution of the Markov Chains in two different positions in the material, one in the failure region and another outside of it. In the initial Markov Chain state was considered: the arithmetic mean of the air and adhesive thermal conductivities in the Case 1; only presence of adhesive in the Case 2; a value of $0,2\,W/(mK)$ in the Case 3. The histograms of the Markov chains presented are shown in Fig. 5.



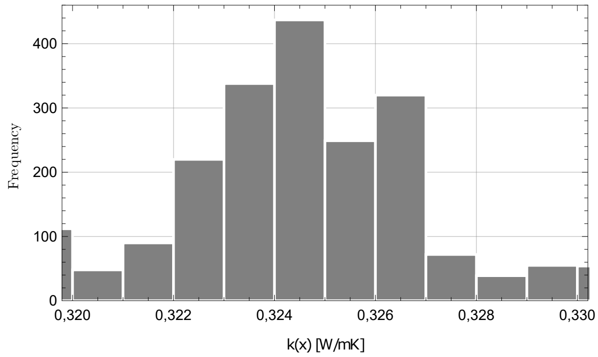
(a) Case 1a at $x = 0,05\,m$.



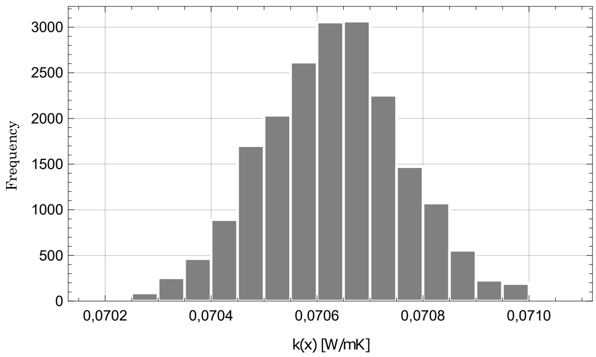
(b) Case 1a at $x = 0,0035\,m$.



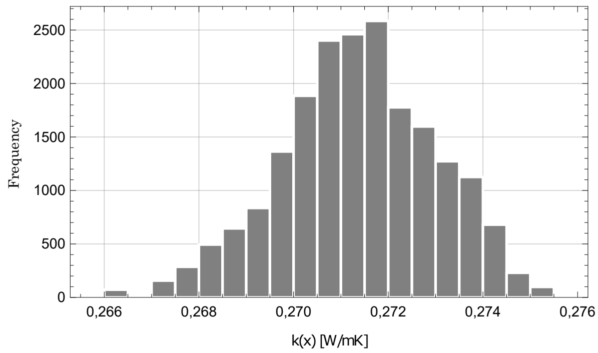
(c) Case 1b at $x = 0,05\,m$.



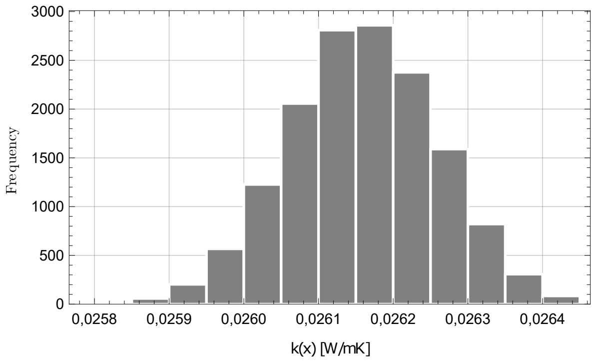
(d) Case 1b at $x = 0,0035\,m$.



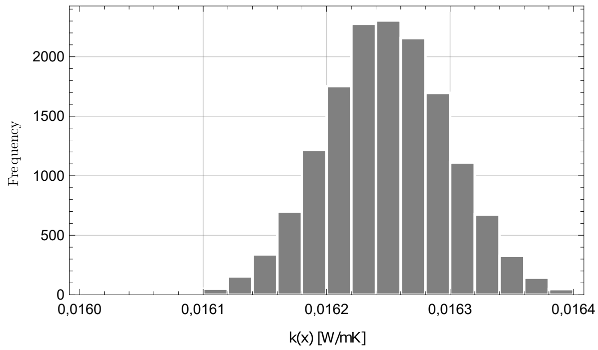
(e) Case 2 at $x = 0,05\,m$.



(f) Case 2 at $x = 0,005\,m$.



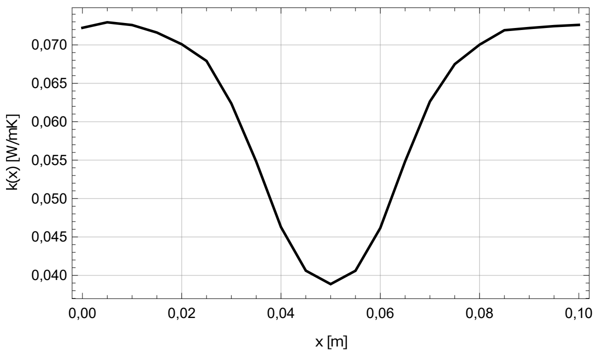
(g) Case 3 at $x = 0,045\,m$.



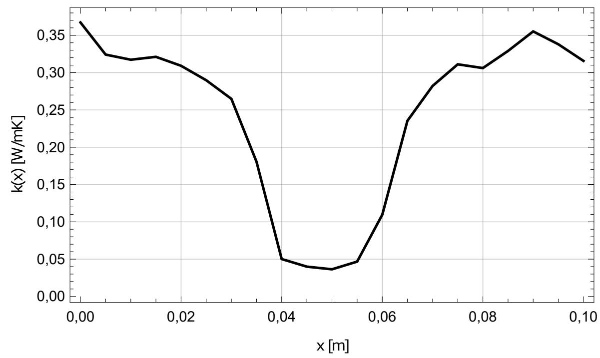
(h) Case 2 at $x = 0,025\,m$.

Figure 5: Histograms of the Markov chains.

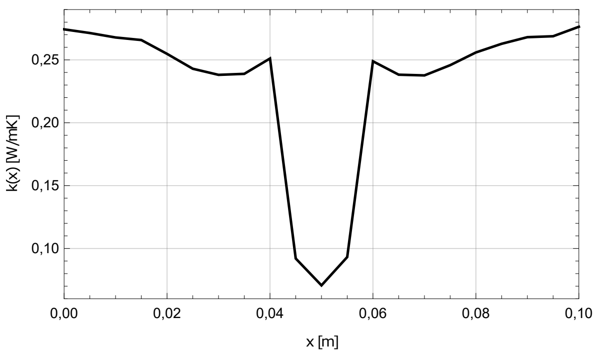
It is noticeable that, although, Case 1a and 1b deal with the same physical problem, the results are different. Which is explained by the fact that the regular Metropolis-Hastings deals only with the reduced model, differently of the DAMH, that deals with both the reduced model and the complete model, thus reducing the model error from the approximation of the reduced models. Despite this, the application of DAMH by itself was not enough to accurately estimate the actual values of the thermal conductivities of the adhesive and of the air, although it was able to detect the presence of contact failure. Fig. 6 shows the estimated functions obtained by the solution of the inverse problem considering the last: 5000 states in the Case 1a; 3000 states in the Case 1b; 20000 states in the Case 2; and 15000 in the Case 3. The 4 graphs presented shows the presence of failure with significant variation of the thermal conductivity around its region. Even with two failures, as shown in the Case 3, the method adopted was able to detect both failures.



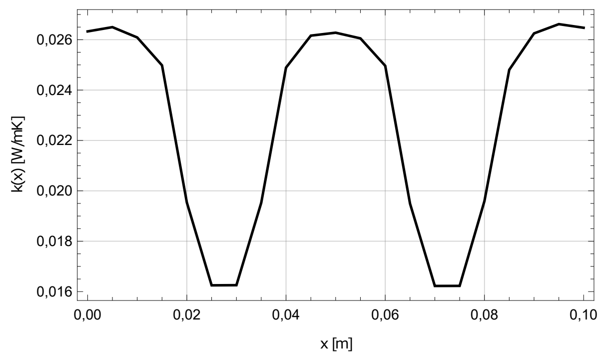
(a) Case 1a.



(b) Case 1b.



(c) Case 2.



(d) Case 3.

Figure 6: Arithmetic mean of the estimated parameters.

5. CONCLUSION

The main objective of this work was to identify contact failures in composite materials through the estimation of the thermal conductivity function from the solution of the inverse problem. For this, a reduced model was developed using Lumped formulation in order to reduce the computational cost required by the complete model in solving the inverse problem. As shown in the results, the solution of the forward problem of the reduced model was 96,7% faster to be computed when compared to the solution presented in Mascouto *et al.* (2020).

Three different cases were considered in the inverse problem and despite their physical differences, the same governing equations could be employed in all of them, thanks to the single-domain formulation, simply adjusting the functions with spatial variation of thermal properties. As the simplified model does not accurately reproduce the phenomena that occur in the complete problem, it was not possible to precisely estimate the thermal conductivities of air and adhesive by solving the inverse problem. However, it was possible to predict the behavior of the function of this property, denouncing the contact failures regions where there are considerable variations as shown in Fig. 6 in all three cases.

6. ACKNOWLEDGEMENTS

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