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COB-2021-1658 CONTROL METHODS FOR SWITCHING BETWEEN COEXISTENT ATTRACTORS OF IMPACT OSCILLATOR

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Abstract. *The idea of taking advantage of nonlinear system dynamics to promote adaptability or improve performance is becoming a popular concept in science and engineering. One way to explore these dynamics is to have the ability to switch between coexisting solutions at will. For example, switching between non-impacting and impacting solutions can enhance drilling processes or avoid chatter in machining. Despite the potential applications, just a few control methods with such an objective are reported in the literature. Recently, two methods were proposed: the intermittent control and the time-delayed feedback control. These methods have completely different strategies to switch between coexistent attractors, and a comparison between them can lead to a deeper understanding of both controllers and guide selecting the adequate controller for an application. This work analyses the advantages and disadvantages of these methods to switch between stable coexisting solutions. A piecewise linear impact oscillator model is used as a typical multistable system to test controller capabilities. The comparisons are made in a scenario where the exchange is made between impacting and non-impacting solutions where period-1 and period-2 orbits coexist. The properties of the control methods are then analysed and discussed. Results show that, on the one hand, the time-delayed feedback method requires much less knowledge of the system to be applied but cannot perform the switch in some scenarios. On the other hand, intermittent control requires more system knowledge but can succeed in all analysed scenarios.*

Keywords: *Impact oscillator, control, nonlinear dynamics, multistability.*

1. INTRODUCTION

Recently, there is a growing interest in nonlinear behaviour in engineering systems. For example, the use of smart materials in structures (Hartl and Lagoudas, 2007; Janke, 2005), the introduction of nonlinearities to improve energy harvesting capabilities (Ai *et al*., 2019), the exploration of self-folding structures (Hernandez *et al.*, 2013), and the vibrations in drilling (De Moraes and Savi, 2019; Kapitaniak *et al*., 2015) are only a few examples of such studies.

In this scenario, the idea of a system that can respond to its environment to better perform its tasks was created, funding the area of adaptive systems. Usually, this adaptability is achieved by using smart materials (Hernandez *et al.*, 2013) that can change their parameters depending on the conditions, by the active application of control, or both. In all cases, the ability to exchange between configurations or modes is imperative for these systems applications.

Impact oscillator is a nonlinear system that was large and deeply studied to model adaptive behaviour. Its popularity is due to the great range of applications and the presence of impact in engineering systems as they can be found in the dynamics and operation in drill-string jarring (Skeem *et al*., 1979), seismic mitigation (Nucera *et al*., 2007), Resonance Enhanced Drilling (RED) (Wiercigroch, 2007), machining, among others. Recently, a new directly excited impact oscillator has been constructed, presenting a very high precision and multistability (Costa *et al*., 2020; Wiercigroch *et al*., 2020). The presented equipment can be used to study several control methods, being a great platform for studying impact systems.

One of the most interesting nonlinear phenomena observed in impact systems is multistability. This phenomenon can be observed when the system's response gravitates towards one of two (or more) attractors depending on the initial conditions. This phenomenon occurs in many systems, with and without impact, including rattling gears systems (Mason *et al*., 2009), impact oscillators (Liu *et al*., 2013), in rotor systems (Karpenko *et al.*, 2002; Chavez and Wiercigroch, 2013), two-bar truss (Savi and Nogueira, 2010), among others. Some of these systems can even benefit from switching between coexistent attractors such as smart structures (Kuribayashi *et al*., 2006; Salerno *et al.*, 2016) or energy harvesters (Barbosa *et al*., 2015).

Even though plenty of systems can benefit from a controller that switches between coexistent solutions, only a few of these methods are present in the literature. Liu *et al.* (2013) propose an intermittent control based on the system dynamics and proportional state feedback. They numerically study the application on an impact oscillator and only show an experimental transition between impacting and non-impacting solutions. The time-delayed feedback control (TDF) (Pyragas, 1992) is another method to switch between coexistent behaviours. It was initially proposed to be applied to chaotic systems and stabilise unstable periodic orbits. However, several authors managed to apply this control in different ways, such as bifurcation control (Costa *et al*., 2019; De Paula *et al*., 2012). TDF was recently used to switch between coexistent attractors in an impact oscillator, which shows the great potential of these applications.

As the field grows, a better understanding of the controllers designed for switching between coexistent attractors is still required. This would facilitate the design process of existing controllers and open pathways to the proposal of new control strategies. Thus, this article compares the intermittent and TDF control methods to switch between coexistent attractors in an experimental impact oscillator. Both types of control are applied to the oscillator numerically and experimentally. The scenarios of both controllers are discussed, highlighting the advantages and disadvantages of each one of them. Results can further guide the design decisions when dealing with the control of adaptive systems.

2. CONTROL METHODS

Two control methods are analysed to verify their ability to switch between coexisting attractors. The first method is the intermittent control proposed by Liu *et al.* (2013), which was already designed to perform such a task. The second method is TDF (Pyragas, 1992) which is not originally designed to perform such tasks and only recently this new application was proposed.

2.1 Intermittent control

The intermittent control is designed to exchange between coexisting attractors by eliminating the dynamics of one attractor and introducing the dynamics of the other with the adition of a proportional feedback when a defined criterion is met. This idea is highly successful and ideally can always bring the system to the desired solution. Also, the criterion adopted can be defined to minimise energy consumption, control signal or a combination of these factors.

Considering an evolution function $\boldsymbol{f}(\boldsymbol{x})$ of an observable system, the control signal $\boldsymbol{u}$ for this control strategy can be defined as:

$$\boldsymbol{u} = -\boldsymbol{f}_{\boldsymbol{dyn}}(\boldsymbol{x}, \boldsymbol{x}_{\boldsymbol{targ}}) + \boldsymbol{K}\boldsymbol{e}, \tag{1}$$

where $\boldsymbol{x}_{\boldsymbol{targ}}$ is the desired behaviour of the system, $\boldsymbol{K}$ is a positive definite matrix proportional gain matrix, $\boldsymbol{e} = \boldsymbol{x}_{\boldsymbol{targ}} - \boldsymbol{x}$ is the error and $\boldsymbol{f}_{\boldsymbol{dyn}}$ is the difference between the dynamics of the target behaviour and the dynamics of the system response defined as:

$$\boldsymbol{f}_{\boldsymbol{dyn}} = \boldsymbol{f}(\boldsymbol{x}) - \boldsymbol{f}(\boldsymbol{x}_{\boldsymbol{targ}}), \tag{2}$$

The first term of Eq. (1) inputs the desired dynamics into the system while eliminating the current behaviour, while the last term of the equation is a proportional feedback used to bring the solution closer to the desired behaviour basin of attraction.

The verification of the control success can be simply done by differentiating the error with respect to time obtaining the error evolution:

$$\dot{\boldsymbol{e}} = \frac{d}{dt}(\boldsymbol{x}_{\boldsymbol{targ}} - \boldsymbol{x}) = \boldsymbol{f}(\boldsymbol{x}_{\boldsymbol{targ}}) - (\boldsymbol{f}(\boldsymbol{x}) + \boldsymbol{u}(\boldsymbol{x})) = -\boldsymbol{K}\boldsymbol{e}, \tag{3}$$

which has an exponential decaying solution if $\boldsymbol{K}$ is a positive definite matrix. Hence, the system will always converge to the desired response as the error tends to zero.

Even though the control law of Eq. (1) can exchange between any given pre-existent orbit, its direct application may not be the best solution or even possible in a real application. Restrictions may be in place, such as limitations on energy consumption, control signal, or even actuation velocity. Hence, a better strategy is to wait for the system to present a more favourable state to initialise the control. This can be defined as the point where the system is closer to the desired behaviour or when the controller will consume the least energy. One example of this strategy would be waiting for the system solution to be at a certain trigger distance $\delta_{trigger}$ from the target orbit, resulting in the following control law:

$$\boldsymbol{u} = \begin{cases} \boldsymbol{0} & , \text{if } |\boldsymbol{x} - \boldsymbol{x}_{\boldsymbol{targ}}| > \delta_{trigger} \\ \boldsymbol{K}\boldsymbol{e} - \boldsymbol{f}_{\boldsymbol{dyn}}(\boldsymbol{x}, \boldsymbol{x}_{\boldsymbol{targ}}) & , \text{if } |\boldsymbol{x} - \boldsymbol{x}_{\boldsymbol{targ}}| < \delta_{trigger} \end{cases}. \tag{4}$$

If this criterion is used and the system can meet it at some point, the controller ideally can always bring the system to the desired response. However, if restrictions are applied to the intermittent control, for example, a limited control signal, one of two outcomes can be expected: the controller manages to transfer the system to the target solution by successive waiting and actuation periods dependent of the criteria or the controller cannot transfer the system to the target solution due to its limitations and the stability of the current solution. Finally, it is important to highlight that as the target behaviour is a response of the system, the control signal of the intermittent control will always tend to zero after the exchange of attractors.

Focusing on the requirements to implement the intermittent control, the full knowledge of the system is necessary. To calculate the control, one needs a time evolution model of the system $\boldsymbol{f}$, the knowledge of the system current state $\boldsymbol{x}$ and the full knowledge of the target solution $\boldsymbol{x_{targ}}$, as well as the phase of the excitation for non-autonomous systems. Hence, this controller can be effective to promote the transfer between orbits but demands a high amount of information and calculations to do so. On a real implementation, these requirements can lead to high costs, the introduction of a delay in the control signal or even the inability of the controller to promote the exchange.

2.2 Time-delayed feedback control

The time-delayed feedback control was initially designed to stabilise unstable periodic orbits (UPOs) embedded on a chaotic attractor. The main idea of the TDF method is to use the delayed signal of a system on a tracking control instead of some reference response $\boldsymbol{x_{targ}}$. By assuming an observable of the system $\boldsymbol{y}$, these leads to control signal $\boldsymbol{u}$ given by:

$$\boldsymbol{u} = \boldsymbol{K}[\boldsymbol{y}(t-\tau) - \boldsymbol{y}(t)], \tag{5}$$

where $\boldsymbol{K}$ is a proportional gain and τ is the period of the delay. Note that if the system is on any periodic behaviour with period τ or $/n$, $n \in \mathbb{N}^*$, in other words $\boldsymbol{x}(t) = \boldsymbol{x}(t-\tau)$, then:

$$\boldsymbol{y}(t-\tau) - \boldsymbol{y}(t) = 0. \tag{6}$$

Hence, from Eq. (6), the control signal $\boldsymbol{u}$ tends to zero as the system solution approaches this periodic behaviour. The target UPO is stabilised by setting the parameters τ to the targeted UPO period τ_s and a proper gain $\boldsymbol{K}$.

Despite its simplicity, the TDF can be difficult to design as the delay parameter τ demands the knowledge of the targeted UPO period τ_s and the gain $\boldsymbol{K}$ requires a stability analysis of the UPO under control. Also, this control can present difficulty to stabilise high period orbits or UPOs with great instabilities.

Recently, it was proposed that the TDF control can make the system switch between different solutions by modifying its delay parameter τ to the desired response period τ_s, even though it was not originally designed to perform such a task. This application of TDF also has a low knowledge requirement as it only needs the knowledge of the period of the targeted orbit and an observation of the system $\boldsymbol{y}$. This is due to the fact that the matrix gain $\boldsymbol{K}$ for this controller can be set by a try and error method to switch between coexisting attractors as the targets are already stable.

However, TDF still presents some drawbacks that can disable its ability to perform exchange between orbits. The first and most critical setback is that the TDF method cannot perform exchanges from a period τ_s solution to a period $n\tau_s$ orbit, with $n \in \mathbb{N}$. From Eq. (6), one can conclude that if the system has a behaviour that satisfies $\boldsymbol{y}(t) = \boldsymbol{y}(t - n\tau_s)$, the control signal stays null despite not being on the targeted behaviour. Another problem is to set control gain $\boldsymbol{K}$ high enough to push the system away from its initial behaviour and low enough to not destabilise the target attractor. Finally, if the target of the control is a UPO, the calculation of Floquet exponents to set the parameter $\boldsymbol{K}$ becomes necessary, which requires additional knowledge of the system time evolution $\boldsymbol{f}$ and the target orbit $\boldsymbol{x_{targ}}$.

Some of these problems can be avoided by having good protocols for adjusting the control gain $\boldsymbol{K}$ and by acquiring more knowledge about the system. However, the inability to perform the transition from a period τ_s solution to a period $n\tau_s$ orbit cannot be easily avoided.

3. IMPACT OSCILLATOR

The detailed description of the experimental apparatus and its design process is presented in Wiercigroch *et al.* (2020), while the configuration utilised on this work is given in Figure 1. The oscillator is fixed upon a base that provides stability and alignment of the components. The rig includes a stabilising rigid structure mounted on the base to suppress any spurious external vibrations that may affect the main mass (highlighted in grey). The leaf springs (coloured in red) are clamped in one end to the main mass (coloured in grey) by grooved plates and in the other end between two beams and a grooved base. This ensures the proper alignment of the leaf springs. A strong permanent cylindrical neodymium magnet is attached to one side of the main mass by a stainless steel rod and fixed by two stainless steel nuts. The magnet itself is placed inside an in-house built coil (coloured in orange), capable of generating a variable magnetic field that provides direct excitation to the main mass. The inner diameter of the coil is close to the diameter of the cylindrical magnet to improve the coupling between the varying field of the coil and the fixed field of the magnet, thereby limiting the nonlinear effects in the excitation system. The current I running through the coil is provided by a signal generator composed of two power supplies, a current amplifier and an NI® PCI-6251 board which produces the desired signal. There are impact beams (coloured in pink) on both sides of the main mass that can be inserted independently. The distance g between an impact beam and the main mass can be easily modified by a treaded bolt fixed to the tip of the beam.

The experimental data is collected by an acquisition system composed of an NI® PCI-6251 board and an in-house LabVIEW® program. The coil input current is measured by a multi-meter, with a precision of ± 0.001 A, while the displacement of the main mass is measured by an eddy current probe (coloured light blue) attached to the structure close to the base of the leaf springs. Accelerometers (coloured light blue) are placed on the structure, the main mass and the impact beam, as shown in Figure 1. Finally, a piezoelectric load-cell (coloured light blue) is also placed between the coil and the rigid structure to measure the reaction force due to the mass excitation.

The displacement X of the oscillating mass can be calculated by treating the system as a piecewise linear oscillator if $|X| < 6$ mm, as shown in Figure 2. Hence, it is considered that the system can be described by:

$$\ddot{X} = -\frac{k_1}{m}X - \frac{k_2}{m}(X-g)H_s(X-g) - \frac{c}{m}\dot{X} + \frac{F_{coil}(t)}{m}, \quad (7)$$

where H_s is a Heaviside step function, k_1 is the stiffness of the leaf springs, k_2 is the stiffness of the impact beam, c is an equivalent damping of the system, dot represents derivatives in relation to time and F_{coil} is the force applied by the coil to the system. Considering that the applied by the coil is a composition of the control signal and a sinusoidal excitation Eq. (7) becomes:

$$\ddot{X} = -\frac{k_1}{m}X - \frac{k_2}{m}(X-g)H_s(X-g) - \frac{c}{m}\dot{X} + \frac{A}{m}sin(2\pi f t) + \frac{u}{m}, \quad (8)$$

where A is the excitation amplitude, f is the frequency of the excitation, and u is the control signal.

Finally, if non-dimensional units are taken the equation of motion becomes:

$$x'' = -x - \kappa(x-1)H_s(x-1) - c^* x' + A^* \sin(f^* t^*) + u^*, \quad (9)$$

where $x = X/g$, $t^* = \omega_0 t$, $\omega_0 = \sqrt{k_1/m}$, $c^* = c/\sqrt{k_1 m}$, $\kappa = k_2/k_1$, $A^* = A/(k_1 g)$, $f^* = 2\pi f/\omega_0$, $u^* = u/(k_1 g)$, and prime represents derivatives in relation to time t^*.

The parameters used in the model are published in literature by Costa *et al.* (2020) and are summarised on Table 1.

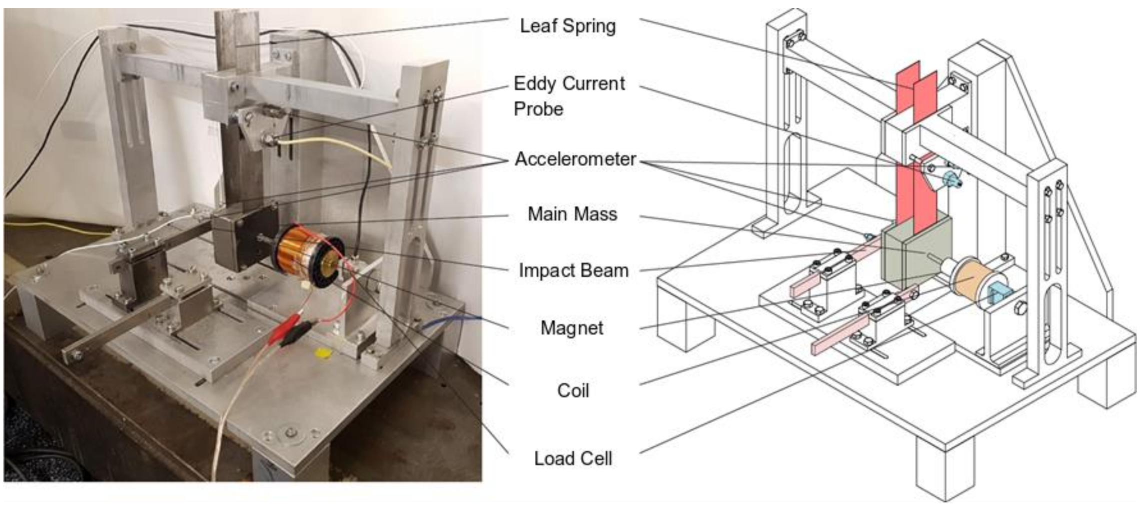

Figure 1. Schematic diagram (right) and the corresponding photograph (left) of the experimental rig. The main components of the system are highlighted as: sensors (eddy current probe, piezoelectric load cell and accelerometers mounted on the mass, frame and impact beam) in blue, coil in orange, main mass in grey, impact beams in pink, leaf springs in red and permanent magnet in white (Costa *et al.*, 2020).

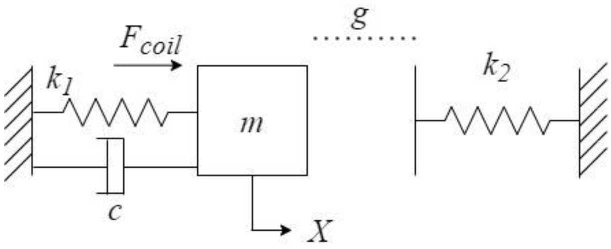

Figure 2: Schematics of the numerical piecewise linear model and its parameters.

Initially, the system dynamics is explored to look for scenarios where the intermittent control and the TDF method can be applied and compared. Hence, an experimental bifurcation diagram is conducted and presented in Figure 3. The diagram is constructed by two forward sweeps that start at different initial conditions and a backward sweep in frequency. Several behaviours are identified on the diagram. Initially, there is a coexistence of a period-1 non-impacting and a period-2 impacting solutions up to 7.1 Hz (Figure 3b). Afterwards, only the period-1 solution is verified until the first grazing incidence occurs. After grazing, the system presents a chaotic behaviour. After this small window of chaos, a period-5 with three impacts per orbit coexists with a period-2 solution up to 7.36 Hz (Figure 3c). After this frequency, the system has a different period-5 solution with two impacts per orbit coexisting with the same period-2 branch presenting only one impact per orbit (Figure 3d) lasting up to 7.50 Hz. Further increases in frequency make the period-5 solution vanish, leaving only the period-2 attractor. At 7.78Hz, the system presents a period-doubling bifurcation to a chaotic response which extends up to 8.28 Hz (Figure 3e). Finally, the chaotic motion gives way to a period-1 solution which lasts up to the end of the diagram.

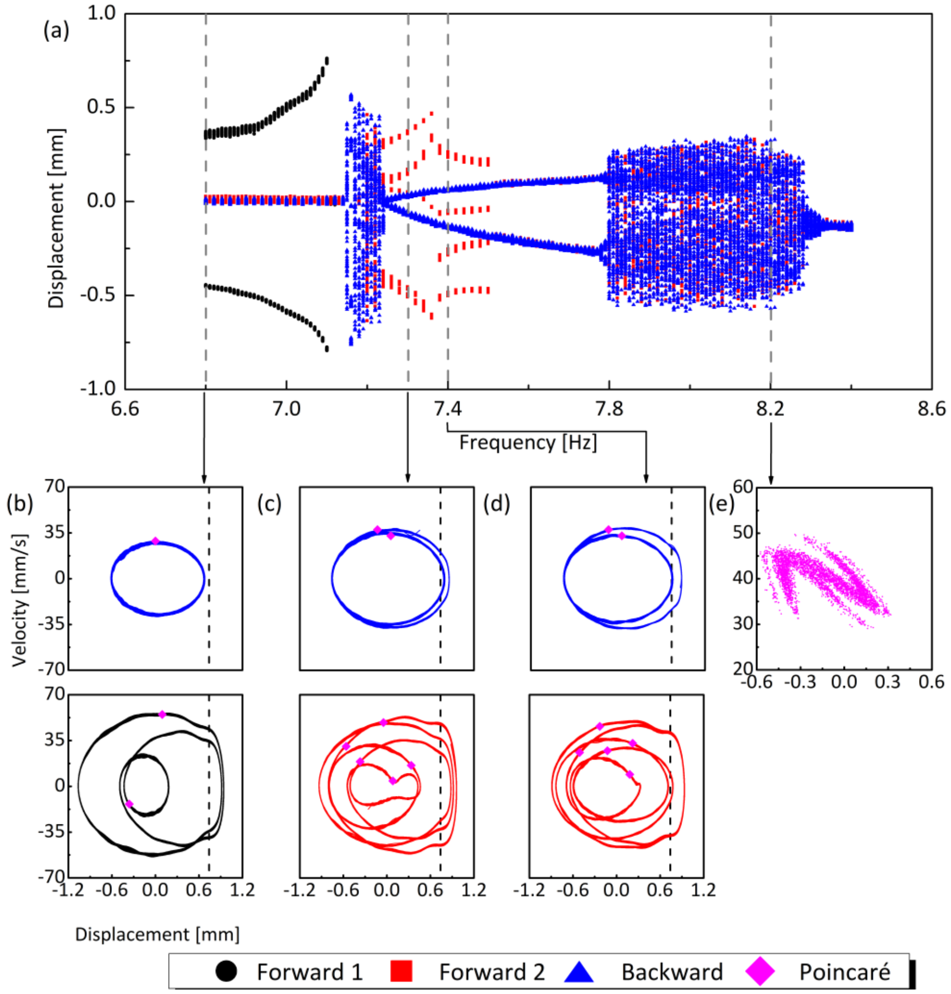

Figure 3. Experimental bifurcation diagram and state spaces to select the control cases. Dashed grey lines represent the frequencies where the state spaces are taken and dashed black lines represent the impact boundary. Phase diagrams (b-e) are taken at $f = 6.8, 7.3, 7.4$ and 8.18 Hz respectively.

Analysing the diagram the 6.8 Hz coexisting attractors are selected to study the ability of the intermittent and TDF control to switch between coexistent attractors. These two attractors are selected as they are impacting and non-impacting solutions of the system and can represent several applications in engineering.

Table 1. Parameters for the piecewise linear model of the impact oscillator rig.

Parameter	Value	Parameter	Value
g	0.74 mm	m	1.325 kg
k_1	4331 N/m	c	0.272 kg/s^2
k_2	87125 N/m	A	1.16 N

4. COMPARISON BETWEEN THE CONTROLLERS

In this section, the TDF and intermittent controllers are compared on their ability to exchange between stable periodic orbits of the impact oscillator. As it is only considered that the actuation is provided by the coil the gain matrixes for both controllers become:

$$K = \begin{bmatrix} 0 & 0 \\ K_p & K_v \end{bmatrix}, \tag{10}$$

where $K_p > 0$ and $K_v > 0$ are the scalar gains related to displacement and velocity, respectively. It is also considered for both controllers that the system is observable and that $\boldsymbol{y} = \boldsymbol{x}$. Control gains are set by try and error method where initially gains are set to zero and afterwards incremental increases are made until the exchange is achieved. If the experiment safety limits are reached and the exchange is not achieved it is considered that the control cannot perform the exchange.

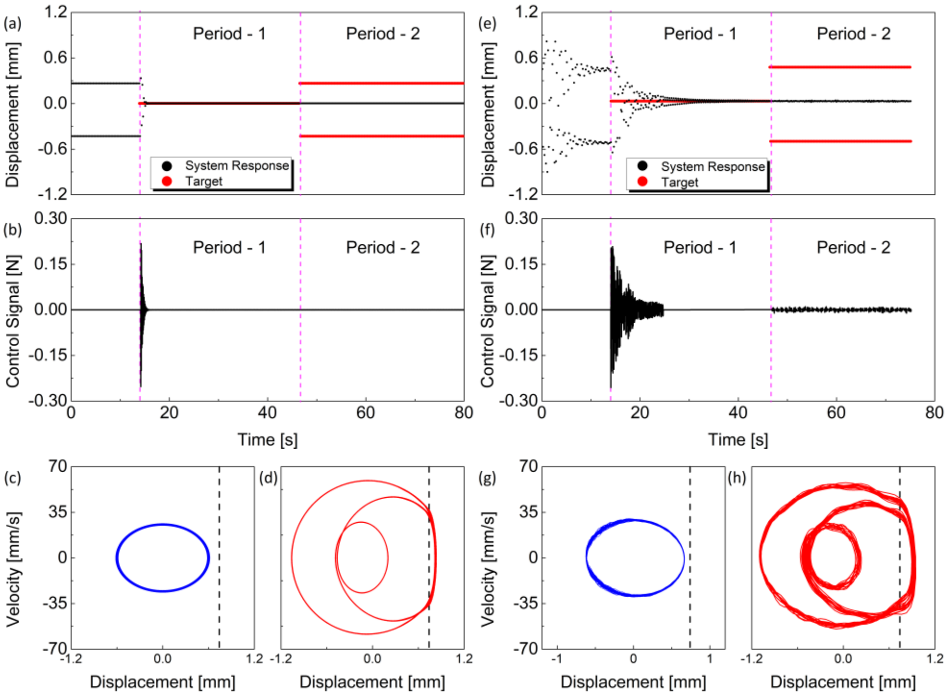

Figure 4. (a-d) Numerical results and (e-h) experimental results for the TDF control at the 6.8 Hz case. Dashed vertical magenta lines represent the exact moments when the controllers change their target orbit, while dashed black lines represent the impact boundary. (a)(e) Poincaré stroboscopic time history of position. (b)(f) Control signal time history. (c)(g) Numerical and experimental targeted period-1 orbit respectively. (d)(h) Numerical and experimental targeted period-2 orbit respectively.

The 6.8 Hz case is focused. The test initialises the system on the period-2 impacting attractor. Afterwards, the control is turned on to target the period-1 non-impacting solution with gains $K_p = 159.86$ N/m and $K_v = 2.42$ Ns/m. Finally, if the period-1 solution is achieved the control changes its target to the period-2 orbit with the same control gains. The controllers have up to 250 periods of excitation to switch between attractors. If the switch does not occur in this time span or the gains required to perform the exchange are larger than the safety limits of the experimental apparatus, the controller is considered to be unable to perform the exchange.

Figure 4 shows the TDF control. In numerical results (Figure 4a and 4b), the controller initially perform the exchange to the period-1 orbit without problems with a short transition time and low control signal. However, when targeting the period-2 orbit, the controller cannot perform the exchange as the control signal stays null. This is explained as the period-1 UPO also satisfies $\boldsymbol{x}(t) = \boldsymbol{x}(t - 2\tau)$ which disable the control signal. In experimental results (Figure 4e and 4f), the controller also brings the system to the period-1 orbit with a low control signal, but takes, approximately 125 excitation periods to perform the exchange. On the second exchange, the controller also cannot perform the exchange even with the presence of noise, as it does not destabilise the orbit or make the system diverge from the periodic behaviour. In fact, there was no value of control gains in which the controller would perform the exchange without destabilising the whole system and producing displacements greater than the safety limits for the experiment.

This case depicts the main problem of the TDF method to perform exchange between attractors. One may argue to increase the gain K of the controller to make the current behaviour unstable and, after destabilisation, change it again so the system may stabilise on the desired response. However, destabilising the attractor may turn the whole system unstable which is dangerous and can risk its integrity. Hence, a new time-delayed feedback method should be developed to deal with such cases.

The intermittent control is also applied. The control strategy is the same as the one used on the TDF controller. The same gains used on the TDF method were initially tried, however, due to difficulties on the experimental controller, the control gains were changed to $K_p = 60.37$ N/m and $K_v = 62.73$ Ns/m to perform the exchange and the controller is turned off on experimental results after the system is close to the solution. Figure 5 shows the numerical and experimental results obtained. On numerical results, the control signal is high, however, the transition time is only 1 excitation period and the system quickly goes to the desired behaviour. On experimental results, the control signal is high and, the system takes more time to stabilise the orbit due to the shutdown of the controller. The shutdown became necessary as the noise in the velocity signal was high and the system could not stabilise if the controller was turned on. Thus, the intermittent control can switch between the attractors in only one period of excitation in theory, however, in practice noise and other imperfections between model and system have a very high influence on the intermittent control ability to promote the orbit switch.

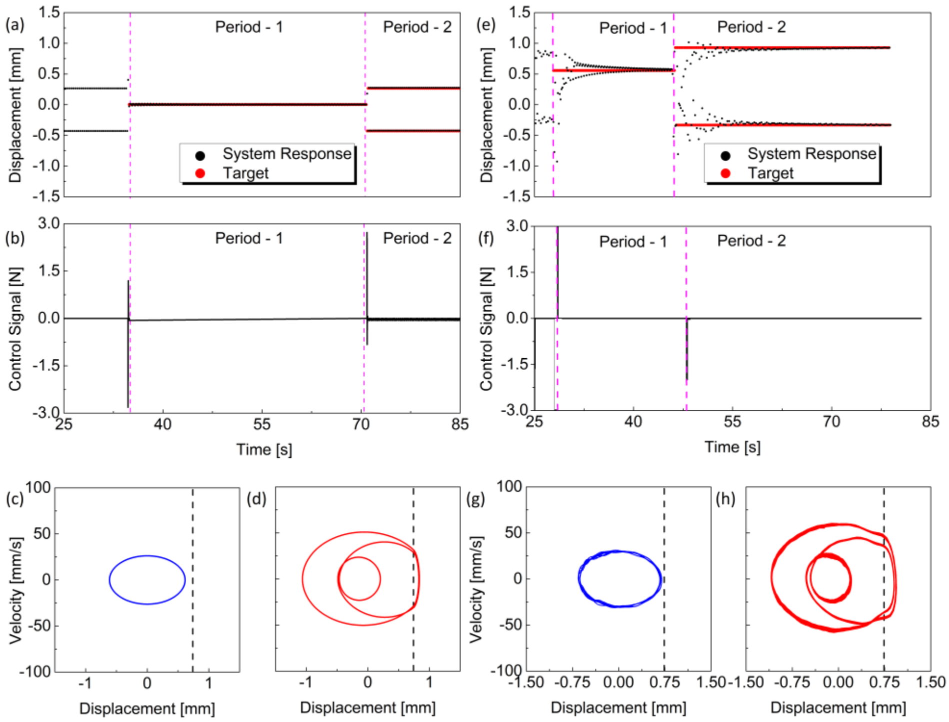

Figure 5. (a-d) Numerical results and (e-h) experimental results for the intermittent control at the 6.8 Hz case. Dashed vertical magenta lines represent the exact moments when the controllers change their target orbit, while dashed black lines represent the impact boundary. (a)(e) Poincaré stroboscopic time history of position. (b)(f) Control signal time history. (c)(g) Numerical and experimental targeted period-1 orbit, respectively. (d)(h) Numerical and experimental targeted period-2 orbit, respectively.

Comparing both controllers, one can verify that the intermittent control can provide a faster exchange than the TDF method for an ideal situation with full knowledge of the system and no noise. While, the TDF method has a low knowledge requirement, a low control signal, and some resistance to noise. It is also easier to implement on the experimental apparatus as it only requires a way to record data. However, it cannot perform the exchange from a period-1 solution to a period-2, for example. Thus, TDF can be used in cases where the controlled orbits do not have periods that are multiple from one another or at least when the objective is to reduce the period of response (Ex. from a period-2 to period-1). The intermittent controller is an alternative for cases where TDF does not work, as it can switch between any kind of attractors. It has some drawbacks, such as high knowledge requirements and a high control signal when compared to TDF. It was also more difficult to implement as it required the information about the system dynamics to be loaded to the control program and the tracking of the excitation phase.

5. CONCLUSIONS

In this work, the intermittent and TDF controllers are applied numerically and experimentally to an impact oscillator. Their properties are discussed and compared to help guide the design choices to construct a controller that can switch between coexisting attractors.

Results show that TDF has several advantages regarding knowledge requirements and more straightforward implementation. However, it cannot switch between any desired attractor. Thus, from a design point of view, TDF can be seen as a first design choice for some cases, but it is not recommended to switch between a period-n solution to a period-jn coexisting solution, with $j \in \mathbb{N}$ and $n \in \mathbb{N}$. In the cases where TDF fails, intermittent control is a reliable alternative to switch between the attractors. However, more detailed knowledge of the system and a more complex experimental setup is required for its implementation, which raises the costs of the application.

In summary, a new control method to switch between coexisting attractors is still required. This method should have the knowledge requirements and ease of implementation as TDF and manage to perform exchange between period-n and period-jn solutions.

6. ACKNOWLEDGEMENTS

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