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NEURAL NETWORKS FOR AERODYNAMIC MODELING IN FLIGHT SIMULATIONS

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Abstract. *The control laws of a fly-by-wire system are the algorithms present on the flight controls computer. They interpret the pilot commands and sensor measures to efficiently command the control surfaces, taking into consideration the operational limits and flight control envelope. The control of any system presumes the existence of a plant, which refers to the aircraft's model, that holds utmost importance for the development of representative flight simulators and improvement of the control laws. The modeling of this plant is the responsibility of the flight mechanics team, which uses traditional approaches, based on the literature, wind tunnel, and industry experience. During the flight tests campaign, some maneuvers are selected for comparison of the modeling, generating data to be used in the identification of the system parameters. This process is developed with consolidated techniques, such as output error method, filter error method, equation error method, maximum likelihood, and, recently, neural networks. Neural networks can work as function approximators, creating a structure that is capable of predicting outputs based on input values, without needing an a priori knowledge of the system. Furthermore, they bypass the solution of equations of motion for the prediction of flight coefficients, becoming an attractive solution for the analysis of highly non-linear dynamics. In this work, a neural network is discussed for the estimation of the aircraft's longitudinal aerodynamic coefficients. The estimated parameters are used to evaluate the aerodynamic model performance in comparison to the real flight dynamics and aid in the proposition of a more suitable model. Due to the lack of available flight data, a computational model of an aircraft is described as an alternative for artificial flight data generation, and the data obtained from the model is conditioned to match the real signal conditioning obtained by flight data, based on the availability of aircraft sensors and the signal noise relations.*

Keywords: *Neural Networks, Aerodynamic Model, Aircraft Parameter Estimation, System Identification*

1. INTRODUCTION

The importance of having a trustworthy mathematical model that represents the dynamics of an aircraft is present in applications such as control law design, validation of wind tunnel testing, analytical prediction, development of a flight simulator, prediction of full design of the aircraft, and verification of system performance. A fine estimate of the aerodynamic coefficients related to the resultant forces and moments on the aircraft is the basis of such models and therefore has been the subject of matter of research in the last decades (R. K. Chauhan, 2017). There are, mainly, three ways of determining aerodynamic derivatives of an aircraft as seen in the literature: theoretical methods, wind tunnel testing, and flight testing (J. Raol, 1995).

The task of finding the best model that represents the dynamics of an aircraft is a problem of system identification. Applications of this kind can be seen as an approach to the inverse problem, trying to find the question to an answered solution. System Identification is a research area that studies techniques of mathematical modeling for systems in which little or no knowledge is acquired (Aguirre, 2007).

The identification of aerodynamic coefficients using flight test data has been solved by many different methods throughout the years. The first approaches for the problem were studied in 1947, with the use of frequency-time response to obtain a graphical representation for analysis, followed by parameter estimation methods applying the non-linear least squares. Improvements came in the late sixties and early seventies because of the availability of digital computers and progress in system identification, as described by Klein (1989). Since then other methods were developed, such as equation error methods, maximum likelihood for non-linear systems, filter error method with extended Kalman filter, output error method, and, recently, applications based on Neural Networks.

The history of Artificial Neural Networks dates back to the year 1940, with the first studies about how the human brain works and how it could be used in favor of mathematical models. However, Artificial Neural Networks only began to draw attention in the late eighties, when the algorithm for backpropagation was developed and the work of Hornik *et al.* (1989) proved that neural networks are universal approximators for mathematical functions, thus proving the adaptive capacity of neural networks to learn from examples. This contributed to the use of neural networks in applications of pattern recognition, data mining, sequence recognition, system identification among many others.

There are two different approaches when dealing with the system identification purposes with neural network. The first one is based on the dynamic estimation of the parameters, where the state variables in an instant k are used as inputs and the values in an instant k+1 are used as outputs for the neural network training. This results in a neural network that works as a black box model of the aircraft model, and can be compared to the technique employed in the output error method and maximum likelihood methods. The second approach is to implement a neural network for a regression kind of estimation, where the time relation between the input and output variables are not important, therefore not relying on the aircraft dynamics. It is a type of curve fitting, such as the equation error method, that has the advantage of not requiring initial estimates of parameters, unlike conventional methods (Das *et al.*, 2010). In this approach, Neural Networks can also bypass the need of solving the equations of motion, making them a promising field of research for calculation of aerodynamic coefficients, especially when dealing with highly nonlinear phenomena (Peyada and Ghosh, 2009)

The Feed-Forward Neural Network stands out as one of the most selected architectures and is the basis for several kinds of research, as highlighted by Raol and Jategaonkar (1995). The work of Ghosh *et al.* (1998) is a great example of a method for estimating the aircraft parameters from simulated and flight data, with validated results. The way of extracting the coefficients from the neural network is based on the Delta method where the trained network could determine the derivatives of an aircraft by submitting small positive and negative perturbations of a signal to the network. These small perturbations are the called delta values and the corresponding output is the response of the referred output to that perturbation, therefore the relation between output and input would give an estimate of the derivative. However, a near-normal distribution of the estimated values should be observed by most of the parameters, and the proposed scheme for calculating confidence levels in the estimates does not always work. This lead to a study of an improvement of the Delta method in the work of Singh and Ghosh (2013), called the Modified Delta method, that proposed it could be possible to obtain variation in the value of an aerodynamic coefficient due to variation in only one of the motion/control variables, while the other variables are kept constant. Therefore, the output of the neural network would be a variation of the coefficient due to the variation of the input, or, in other words, the derivative itself. This method obtained estimates with lesser standard deviation than the Delta method, being useful for estimating parameters from both stable and unstable aircraft.

Although it leads to satisfying results, the Delta and Modified Delta methods depend on a finite difference approximation, applied in a posterior phase of signal processing. This post-processing can be seen as the use of a linear technique for estimating the aerodynamic coefficients, therefore interfering in one of the main advantages of a Neural Network: the capability of working with nonlinear systems. The works of Das *et al.* (2010) and Mohamed and Dongare (2018) employ a technique that uses the partial differentiation of the Neural Network outputs in relation to the inputs as a way to estimate the aerodynamic derivatives during the Network training, giving an exact first-order derivative, instead of an approximation, as the solutions obtained via Delta and Modified Delta method.

This paper proposes to further explore the advantages of the partial differentiation method for the identification of the longitudinal aerodynamic derivatives from artificially generated flight data. The data is obtained from a nonlinear Simulink representative model of the aircraft dynamics and a proper treatment of the outputs is performed, to match the real conditions of the signals obtained from aircraft sensors. Then, a Neural Network architecture and algorithm are presented for this specific application, and the estimated derivatives are compared with their nominal values for analysis of the signal matching and the estimation errors.

This paper is organized in three main sections: Section 2 describes the methodology applied for the data generation and Neural Network development; Section 3 presents the discussion about the obtained results, and Section 4 addresses the conclusion of the paper and future works.

2. METHODOLOGY

2.1 Data Generation

The first step for the development of a Neural Network model is the proper selection of a data set for both training and validation of the Network. This paper aims to study applications of parameter estimation based on artificial flight data obtained from simulations. An aircraft dynamic nonlinear model, based on a long range high performance business aircraft developed in the work of Cruz and Kienitz (2007) and further explored in the work of de Bonfim Gripp (2015) was used for the data generation. The equations used in the model are heavily based on the work by Muir *et al.* (1997) and the rigid body equations of motion, coordinate transformations, airspeed calculation, atmospheric properties, and the gravity force are the same presented in the aforementioned paper.

This paper focuses on the estimation of the longitudinal derivatives of the explored aircraft model, therefore the presentation of the aerodynamic forces and moment coefficients will be focused on the longitudinal dynamics, written as:

$$C_L = C_{L_0} + C_{L_\alpha}\alpha + C_{L_{\delta_e}}\delta_e + C_{L_{\delta_{ih}}}\delta_{ih} + C_{L_q}\frac{q\bar{c}}{2V} \quad (1)$$

$$C_m = C_{m_0} + C_{m_\alpha}\alpha + C_{m_{\delta_e}}\delta_e + C_{m_{\delta_{ih}}}\delta_{ih} + C_{m_q}\frac{q\bar{c}}{2V} \quad (2)$$

These equations are based mainly on the angles of motion and input signals, being α the angle of attack, q the pitch rate, δ_e the elevator displacement angle and δ_{ih} the horizontal stabilizer displacement angle. In the left side of the equations are either the lift coefficient, C_L or the pitching moment coefficient C_m. The focus of this paper is mainly on the derivatives of these parameters in relation to the aerodynamic and control parameters, also presented in the equations.

This aircraft model make use of a change in the horizontal stabilizer incidence angle to help alleviate the necessary forces for trimming the aircraft in certain conditions. However, since this deflection is kept constant in the trimmed value during the simulations, the lift and pitching moment contributions and the derivatives due to this parameter were treated as a bias during the parameter estimation process.

However, since values of the lift and pitching moment coefficient can not be obtained from a direct measure of aircraft sensors, manipulation of the variables was performed to represent the real signal conditions in which the parameters of interest could be obtained from real flight data. Based on the measures of accelerometers, inertials, and force sensors, the values of lift coefficient and pitching moment coefficient can be obtained from the following equations:

$$a_z = -\frac{\bar{q}s}{m}C_L\cos\alpha - \frac{F_e}{m} \quad (3)$$

$$\dot{q} = \frac{\bar{q}s\bar{c}}{I_y}C_m + \frac{pr}{I_y}(I_z - Ix) + (r^2 - p^2)\frac{I_{xy}}{I_y} + \frac{F_e}{I_y} \quad (4)$$

Where a_z is the vertical acceleration, m is the aircraft mass, s the wing area, $\bar{c}$ the aerodynamic chord, and F_e the engine thrust. Also, p represents the roll rate and r the yaw rate of the aircraft, whilst I represents the inertia.

The value of the pitching rate derivative was obtained by derivation of the pitch rate with an auxiliary filter developed to work as a derivative function. This was achieved by using a transfer function with a zero in the origin and a pole far from the origin, obtained by setting a low value for the parameter τ, so that the transfer function works as a derivative in the range of frequencies from zero to the pole frequency.

$$H(s) = \frac{s}{\tau s + 1} \quad (5)$$

For the generation of the data set, the aircraft was initially trimmed in a stable condition and then a series of control commands were applied to excite the longitudinal modes of the aircraft. While keeping the horizontal stabilizer in a fixed position, elevator doublets were applied to excite the short period mode of the aircraft. Finally, a noise of low amplitude was added to the signals extracted from the model, to simulate the real dynamics of the aircraft sensor signals and the obtained signals were gathered together to form a data set for the training and validation of the Neural Network. The performance of the Network is evaluated with the application of noise to the signals, to observe how it deals with this implication.

Since the inputs and outputs have different amplitude ranges throughout the generated data set, it is important to make a normalization to each of the signals from the data set so that the Neural Network can deal with signals that vary within the same range during the training procedure. Finally, the data set is split into a training set, used for the training process of the Network; a validation set, used for the validation of the trained Neural Network; and a test set, used for the analysis of the updated Neural Network's performance.

2.2 Neural Networks

Neural Networks are structures composed basically of two main components being the neurons and the connectors between them, that are organized in layers that contain multiple neurons each. The information that is inputted to the Neural Network is processed from each neuron from a layer to all neurons from the posterior layer, through the weight between these connections. Each neuron performs basically two operations: a weighted sum of all the information brought from the neurons of the previous layers plus an added bias and a nonlinear operation performed by an activation function, that is responsible for giving Neural Networks its nonlinear characteristics.

Figure 1 shows the structure of a Feed-Forward Neural Network, a structure where the information is passed only in a straight path from the input to the output, following the previously explained processing behavior. Therefore, the output of a Neural Network can be obtained by matrix multiplication of the previous layers with the connection weights summed

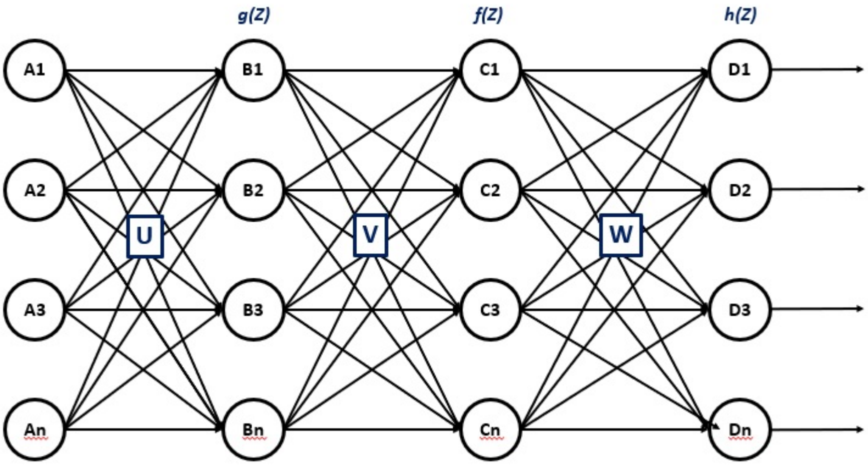

Figure 1. Feed Forward Neural Network Structure

with a bias, followed by the application of a nonlinear activation function. This series of operations can be described by the following equations:

$$B = g(U^T A) + b_1 \tag{6}$$

$$C = f(V^T B) + b_2 \tag{7}$$

$$D = h(W^T C) + b_3 \tag{8}$$

And, therefore, the relation between the output and the input of the Neural Network can be given as:

$$D = h(W^T(f(V^T(g(U^T A) + b_1)) + b_2)) + b_3 \tag{9}$$

The described equations presents the Neural Network processing of a set of inputs, generating an output that represents the Network interpretation of the given data set. Therefore a Neural Network needs to be trained so that it can be capable of correctly map the inputs to the processed outputs. This training is performed with a back-propagation algorithm, where an estimated output is compared with a real output value by means of a cost function and the result dictates how the weights and biases of the Network should be updated at each training iteration to reduce this cost value, by analyzing the gradient of each of these weights and biases values in relation to the calculated cost.

For applications of Neural Network that explore the regression of data, such as the one explored in this paper, the common choice of cost function is the mean squared error of the outputs in comparison to the target value, described by Eq. 10, where D represents the output predictions of the Neural Network and $\hat{y}$ are the actual values extracted from the data set:

$$MSE = \frac{1}{m}\sum_{i=1}^{m}(D - \hat{y})^2 \tag{10}$$

The choice of activation function for each layer, the Neural Network architecture and the learning rate of the neurons is of utmost importance for the performance of the Network in mapping the input and outputs, and must be studied carefully to find a suitable combination.

The inputs for the model are the angle of attack α, the pitch rate q and the elevator deflection δ_e, while the outputs are either the lift coefficient C_L or the pitching moment coefficient C_m. The stabilizer deflection δ_{ih} is considered as a bias during the estimates, since it is kept constant during the excitation maneuvers.

$$Input = [\alpha \; \frac{q\bar{c}}{2V} \; \delta_e] \tag{11}$$

$$Output = [C_L] \text{ or } [C_m] \tag{12}$$

In this paper a Feed-Forward Neural Network is proposed for the task of estimating the aerodynamic and control derivatives of the longitudinal mode of an aircraft, using an Adam optimizer, alongside with the selection of hyperbolic tangent as the activation function of the middle layer and linear function as the activation function of the output layer. The choice of architecture and learning rate was based on the analysis of the validation cost obtained from a Monte Carlo analysis, where a combination of different possible Neural Network architectures were explored.

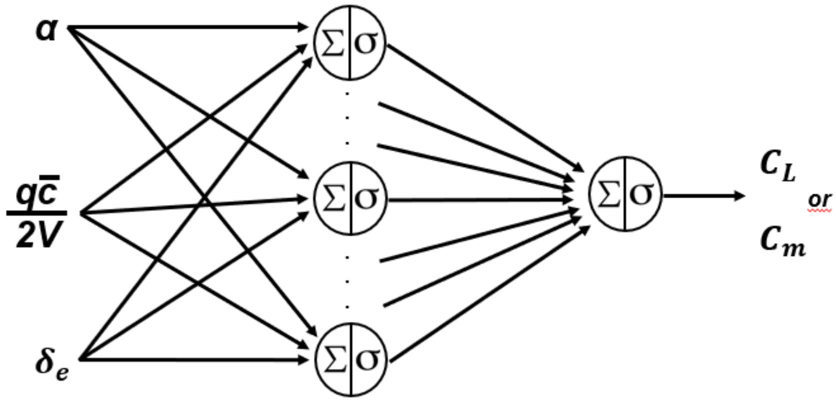

Figure 2. Neural Network inputs and outputs

2.3 Partial Differential Method

The training process of a Neural Network is what makes it possible for the Network to represent the mapping function between their inputs and outputs (Das *et al.*, 2010). In this work, the inputs represent the control and motion variables whereas the outputs inherit the aircraft longitudinal forces and moment values. Therefore, the partial differential of the Neural Network outputs in relation to the input vector directly represents the stability and control derivatives terms in the equations of forces and moments. These values can be obtained by direct manipulation of the Network weights with respect to the activation function of each layer. Moreover, the equations for the solution can be obtained by exploring the partial derivative of equation 9. Thus, we have:

$$\frac{\partial \mathrm{D}}{\partial \mathrm{A}} = \frac{\partial \mathrm{D}}{\partial \mathrm{C}} . \frac{\partial \mathrm{C}}{\partial \mathrm{B}} . \frac{\partial \mathrm{B}}{\partial \mathrm{A}} \tag{13}$$

The partial differentials between the layers can be represented by the derivative of the activation function applied to the weights:

$$\frac{\partial \mathrm{D}}{\partial \mathrm{C}} = \mathrm{W}^{\mathrm{T}} \tag{14}$$

$$\frac{\partial \mathrm{C}}{\partial \mathrm{B}} = \mathrm{f}'(\mathrm{V}^{\mathrm{T}}) \tag{15}$$

$$\frac{\partial \mathrm{B}}{\partial \mathrm{A}} = \mathrm{g}'(\mathrm{U}^{\mathrm{T}}) \tag{16}$$

On the other hand, it is important to consider the normalization of the inputs and outputs when making the calculations, therefore:

$$\frac{\partial \mathrm{D}}{\partial \mathrm{A}} = \frac{\partial \mathrm{D}}{\partial \mathrm{D}_{\mathrm{norm}}} \times \frac{\partial \mathrm{D}_{\mathrm{norm}}}{\partial \mathrm{A}_{\mathrm{norm}}} \times \frac{\partial \mathrm{A}_{\mathrm{norm}}}{\partial \mathrm{A}} \tag{17}$$

Where the output normalization matrix $\frac{\partial \mathrm{D}}{\partial \mathrm{D}_{\mathrm{norm}}}$ is a diagonal matrix, in which the terms that fill the diagonals are the output normalization parameters and the input normalization matrix $\frac{\partial \mathrm{A}_{\mathrm{norm}}}{\partial \mathrm{A}}$ is also a diagonal matrix, however, filled with the inverse of the input normalization parameters.

Therefore, by combining equations 13, 14, 15, 16 and 17 it is possible to get the equation that gives the aerodynamic derivatives obtained by the partial differential method.

$$\frac{\partial \mathrm{D}}{\partial \mathrm{A}} = \frac{\partial \mathrm{D}}{\partial \mathrm{D}_{\mathrm{norm}}} . \mathrm{h}'(\mathrm{W}^{\mathrm{T}}) . \mathrm{f}'(\mathrm{V}^{\mathrm{T}}) . \mathrm{g}'(\mathrm{U}^{\mathrm{T}}) . \frac{\partial \mathrm{A}_{\mathrm{norm}}}{\partial \mathrm{A}} \tag{18}$$

The strategy for obtaining the estimated aerodynamic stability and control derivatives will rely on applying the equation 18 on the trained Neural Network. These derivatives are then compared to the nominal values extracted from the aircraft model and the force and moment signals are reconstructed based on equations 1 and 2, to measure the performance of the estimates obtained with this approach.

3. RESULTS AND DISCUSSION

The simulation data was generated by exciting the short period dynamics of the aircraft model developed in Simulink around the trimmed condition described in table 1. A different set of maneuvers consisted of elevator doublets were applied to the aircraft, to obtain the excitation of the longitudinal modes. Then, with the proper normalization of the obtained data set, it was split into training, validation, and test sets for the Neural Network. The training and validation data sets can be observed in Fig. 3.

	Weight (kg)	**CG (%MAC)**	**q_Pa**	**Mach**	**Gamma (deg)**
Trim Conditions	33800	25	3696	0.3	0

Table 1. Trimming condition established for the simulations

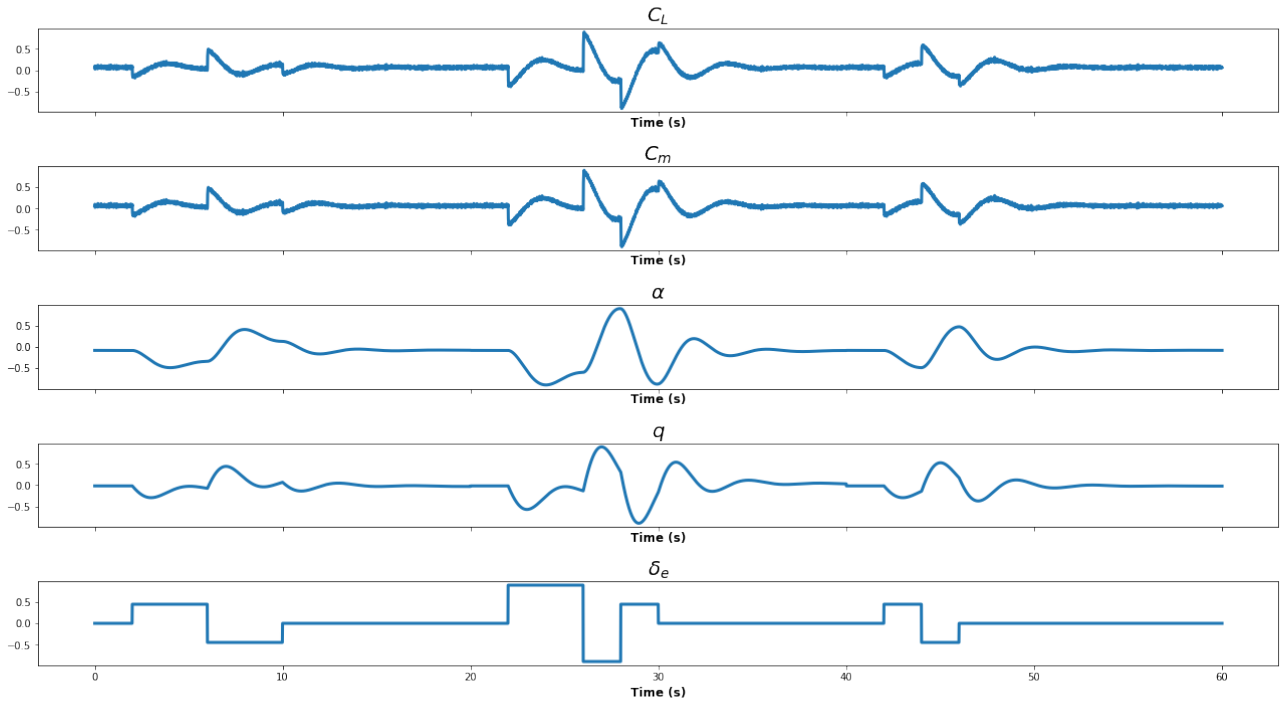

Figure 3. Normalized training and validation data set

The input vector for the Neural Network was constructed based on the longitudinal motion and control variables, whilst the output vector is either the lift or pitching moment coefficient, therefore leading to the development of two different Neural Network architectures. Each Neural Network was trained with the normalized training data sets gathered from the simulations and had its performance evaluated based on the calculation of the mean squared error of the predicted data for the validation set.

During the training process, the aerodynamic derivative estimates are updated at each iteration, based on the partial differentiation of the outputs in relation to the inputs. At the end of the iterations, the estimated derivatives are employed in Equations 1 and 2 to reconstruct the output signal and evaluate the fitness of the proposed model.

3.1 Lift Coefficient Estimation

The first step towards the development of the Neural Network for the lift coefficients prediction was the choice of a proper Network architecture. The solution proposed was a Monte Carlo simulation to evaluate a series of combinations of possible architectures, combining different numbers of layers and neurons in each of these hidden layers. The results showed that a single hidden layer with approximate 12 neurons was the best choice for this application. The initial learning rate for the Adam optimizer was chosen as 0.01.

After the training process, the test data was used for comparing the measured outputs with the Neural Network estimates. It is possible to see that, even in the case where a 5% noise was applied in the signals, as shown in Fig. 4, the Neural Network was able to give a good estimate of the outputs based on the provided input information.

The training process was done with 1200 iterations and the estimation of the derivatives was updated in each iteration. Figure 5 shows the evolution of the estimation of the aerodynamic derivatives in relation to the lift coefficient during the

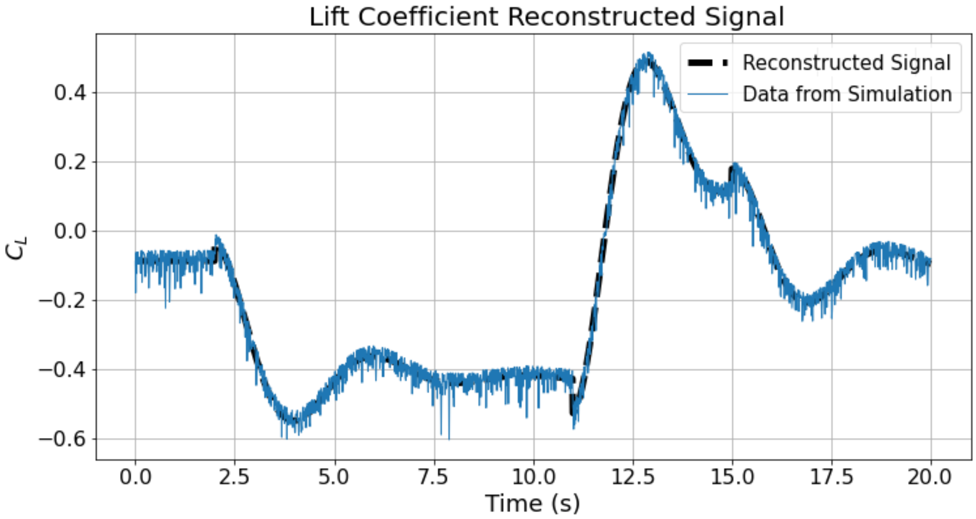

Figure 4. Estimate of the Neural Network compared with the real values

training procedure, for the case where noise of 5% amplitude was applied.

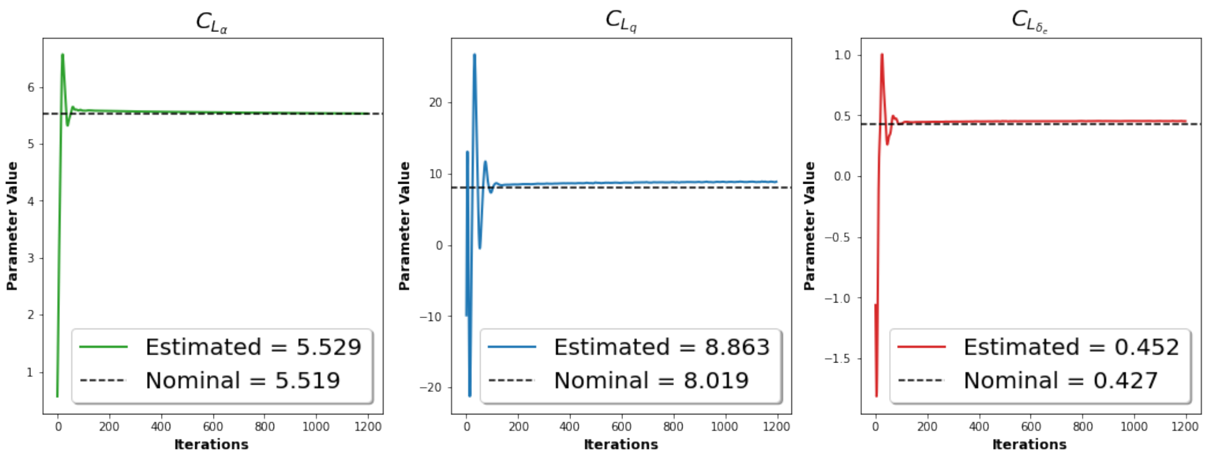

Figure 5. Evolution of the lift coefficient derivatives during the training process

Finally, Table 2 presents the obtained results for the estimate of the lift coefficient related derivatives, for both the cases when the data set contained zero or a 5% white Gaussian noise added. It is possible to see that the estimates for both cases are satisfactory, concluding that the neural partial differentiation is suitable for this application. The estimates tend to get farther from the nominal values when the data set contains noise, according to what is expected, however, the obtained values are still approximate to the nominal values, therefore leading to the same conclusions previously stated.

Parameters	**Nominal Value**	**Noise = 0%**	**Noise = 5%**
C_{L_α}	5.519	5.567(0.114)*	5.529(0.120)*
C_{L_q}	8.019	8.029(0.459)*	8.863(0.193)*
$C_{L_{\delta e}}$	0.427	0.431(0.007)*	0.452(0.013)*
C_{L_0}	0.359	0.352(0.001)*	0.353(0.015)*

(Standard Deviation)

Table 2. Lift Coefficient Estimates

3.2 Pitching Moment Coefficient Estimation

The estimation of the pitching moment coefficient followed the same steps applied for the estimation of the lift coefficient derivatives, as explained in the previous section. The Monte Carlo analysis of the possible combinations of architectures lead to the conclusion that an architecture composed of one hidden layer, containing 10 neurons combined with a learning rate of 5.10^{-4} presented as a appropriate combination.

A good generalization characteristic was also observed for this Neural Network, as it properly predicted the test data after the training procedure was concluded with the data set extracted from the simulations, as shown in Figure 6. The training process was based on 1200 iterations over the training data. Figure 7 shows the evolution of the estimates of the aerodynamic coefficients.

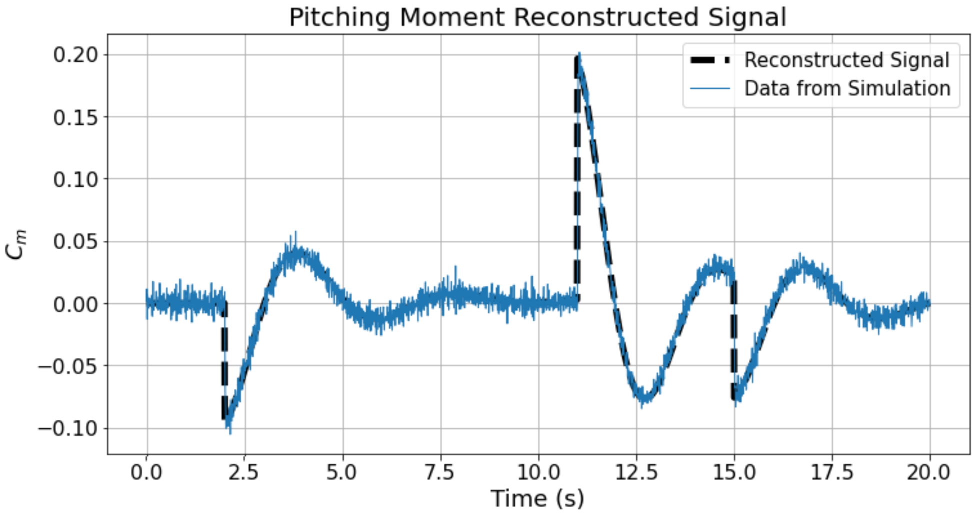

Figure 6. Estimate of the Neural Network compared with the real values

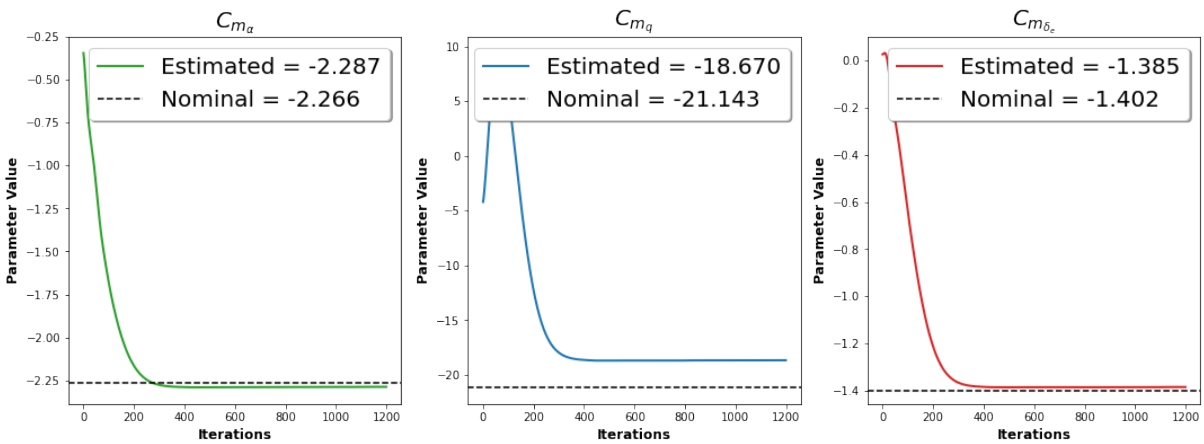

Figure 7. Evolution of the pitching moment coefficient derivatives during the training process

The results of the partial differentiation method for the prediction of the pitching moment related derivatives are gathered in Table 3, where it is possible to observe that a good match of the nominal values was obtained for both tested cases. The value of C_{m_q} presented a higher diversion if compared to the other derivatives, with this behavior present in both cases of data sets without and with 5% noise added. However, the conclusions taken are the same stated for the results obtained for the Neural Network developed for the estimation of the lift coefficient derivatives, in the sense that the partial differentiation method presented as a good alternative for this problem.

4. CONCLUSIONS

The neural partial differentiation method was employed for the estimation of the aerodynamic and control derivatives of the longitudinal modes of an executive aircraft. The data set was obtained from a Simulink model of an aircraft when it was submitted to a series of maneuvers that aimed to excite these longitudinal modes.

Two Neural Networks are proposed, one focused on the derivatives with respect to the lift coefficient, and the other focused on the pitching moment derivatives, using the same methodology of neural partial differentiation and a common algorithm for the Neural Network, based on an Adam optimizer. The architecture was composed of either one hidden layer which number of neurons was chosen based on a Monte Carlo analysis of the validation errors obtained during the

Parameters	Nominal Value	Noise = 0%	Noise = 5%
C_{m_α}	-2.266	-2.313(0.073)*	-2.287(0.074)*
C_{m_q}	-21.143	-19.773(0.941)*	-18.670(0.792)*
$C_{m_{\delta_e}}$	-1.402	-1.412(0.045)*	-1.385(0.049)*
C_{m_0}	-0.025	-0.024(0.006)*	-0.028(0.008)*
(Standard Deviation)			

Table 3. Pitching Moment Coefficient Estimates

analysis of multiple possible combinations.

The results of the Neural Networks predictions and the proposed estimate of the aerodynamic coefficients were found to be satisfactory, achieving values close to the nominal ones and keeping the efficiency of the method even for data sets with the presence of a low noise amplitude. Thus, this concludes the suitability of the neural partial differentiation method for parameter estimation of aircraft systems and highlights the versatility of applications that explore the use of Neural Networks, given their capacity of working with nonlinear systems without the need for a priori information about the system.

Future works based on this paper will aim to explore deep neural network architectures and observe the changes in the obtained results. A study focused in the online estimation of the aerodynamic coefficients will also be addressed. Furthermore, applications of high nonlinearity of aircraft dynamics shall be explored, taking further advantage of the Neural Network properties.

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