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PARAMETRIC STUDY OF HELMHOLTZ RESONATORS IN ACOUSTIC CAVITIES

Derick Fernando O. Fernandes
Manuel Nascimento D. Barcelos Júnior
University of Brasília, Brasília, Federal District
derickffernandes@gmail.com
manuelbarcelos@aerospace.unb.br

Abstract. *Currently, there are several forms of noise attenuation and, whether passive or active, all have specific applications and limitations, and it is up to the designer to select which optimizes the desired result. One of these possibilities is to base on the system's resonance effects through Helmholtz resonator assemblies. However, such devices we must design with specific parameters for a given critical situation. A given resonator's geometry and dimensions restrict acoustic effects to a frequency range. Therefore, it is fundamental to develop an analytical methodology, prior to the experimental and numerical analyses, that allows the efficient design of resonators and their matrix configurations for a specific acoustic system. Thus, based on known mathematical models, this paper develops a theoretical study of the impact on resonance frequency and sound pressure levels by changing layouts and parameters such as neck radius and length, cross-section geometries and series or parallel dispositions of Helmholtz resonators. The study establishes a mass-spring analogy, Newton's laws are applied to obtain the system's oscillatory dynamics, thus calculating its stiffness and resonance frequency (classical form). Subsequently, we develop an analytical methodology for the evaluation of resonator matrix configurations to quantify the sound transmission loss and the equivalent acoustic impedance in continuous cross-section cavities without internal flow and in cavities with constant pressure and volume flow at the duct intersection. An observation of the results shows that the resonance frequency decreases with the length of the resonator neck. For the matrix configurations, the growing behavior of both the equivalent system impedance and the sound transmission loss is observed directly to the number of resonators, tending to stabilize after a finite number of devices in parallel.*

Keywords: *Helmholtz resonators, Parametric Study, Acoustic resonance, Equivalent impedance*

1. INTRODUCTION

Helmholtz resonators are devices capable of efficiently producing sound attenuation around a resonant frequency by dissipative processes of sound pressure (Santos (2005)). This can be used in a matrix arrangement to elevate the acoustic properties of interest (Seo and Kim (2005)), an example of this configuration are the Liners, structures with acoustic and mechanical functions used in modern turbofan engines (Serrano *et al.* (2014)). However, these devices must be designed with specific geometric properties (Santos (2005)), since their acoustic effects are strongly associated with their dimensions. Therefore, it is essential to have analytical tools capable of estimating the geometric characteristics of devices for a preliminary design. Thus, the paper develops an analytical methodology based on the theory of planar propagation of sound waves capable of measuring the resonant frequency, transmission loss, and equivalent impedance of single-neck Helmholtz resonators in acoustic cavities (duct), evaluating the interference of geometric parameters and array arrangement on them. The magnitudes obtained by this methodology are compared with those obtained by the classical formulation, and further on the interference in these magnitudes is analyzed in relation to the length/diameter (l/d) ratio of the cavity and resonator neck dimensions.

2. METHODOLOGY

The study will be based on known formulations and physical laws. First, an analysis for different characteristic dimensions of a single-neck resonators, Fig 1, is performed to compare the values of the primary resonant frequency obtained by means of linear acoustic theory (classical form) and by analytical expressions of the theory of planar propagation of sound waves. Subsequently, using the analytical expressions of the theory of planar propagation of the sound wave, a study of the behavior of the transmission loss in the face of changes in the length-to-diameter (l/d) ratio of both the cavity and the neck of the resonators is performed. This is complemented by evaluating the resonant frequencies for three different cross sections (circular, square, and hexagonal) of the cavity of a single circular neck resonator, keeping the volume of both the cavity and the neck fixed. Lastly, the equivalent impedance of parallel and series resonator combinations are calculated

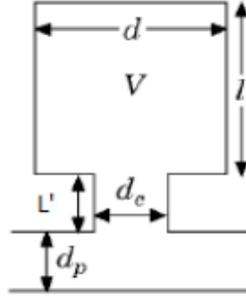with respect to the number of devices and different l/d ratios.

Figure 1. Basic geometry and characteristic dimensions of a Helmholtz resonator.

In figure 1, $\mathbf{d}$, $\mathbf{V}$ and $\mathbf{l}$ are, respectively, the characteristic length (diameter, side or double of the hexagon apotheme), the volume and the depth of the cavity. While, $\mathbf{d_c}$ and $\mathbf{L'}$ are the diameter and depth of the neck, respectively, and $\mathbf{d_p}$ the diameter of the main duct.

For all analyses the cavity and neck volume were set to 0.001 m³ and 1.963e-8 m³, respectively. For impedance analysis, a duct area with a circular cross section equal to 1.3854e-3 m² is considered.

3. MATHEMATICAL FORMULATION

3.1 Transmission loss and primary frequency of resonance by planar propagation of sound waves

For planar propagation of sound waves, the transmission loss of an acoustic element is calculated as (Selamet *et al.* (1994))

$$TL = 10 \log_{10} \left| \frac{C_{+,i}}{C_{+,tr}} \right|^2 \tag{1}$$

Where $\mathbf{C_{+,i}}$ and $\mathbf{C_{+,tr}}$ are complex constants representing magnitudes of incident harmonics and transmitted wave pressure, respectively. Assuming constant pressure, conservation of volume flow at the tube intersection, neglecting viscous effects, and incorporating wave motion in the volume and neck of the resonator, according to the classical one-dimensional acoustic field theory of Helmholtz resonators, the Transmission Loss is given by

$$TL = 10 \log_{10} \left| 1 + \frac{S}{2A_p} \left[\frac{1 + \varphi + (\varphi + 1)e^{-2ikL'}}{1 + \varphi - (\varphi + 1)e^{-2ikL'}} \right] \right|^2 \qquad \varphi = \frac{A_v}{S} \left(\frac{e^{2kil-1}}{e^{2kil+1}} \right) \tag{2}$$

where $\mathbf{A_p}$ is the cross-sectional area of the main duct, $\mathbf{k}$ is the wavenumber, $\mathbf{L'}$ is the neck length, S is the cross-sectional area of the neck, $\mathbf{l}$ is the cavity length, $\mathbf{A_v}$ is the cross sectional area of the cavity and $\mathbf{k} = \mathbf{2\pi/\lambda}$ the wavenumber. The equation can be rearranged to an equivalent trigonometric form as (Selamet *et al.* (1994))

$$TL = 10 \log_{10} \left[1 + \left(\frac{S}{2A_p} \frac{tan(kL') + \left(\frac{A_v}{S} \right) tan(kl)}{1 - \left(\frac{A_v}{S} \right) tan(kL') tan(kl)} \right)^2 \right] \tag{3}$$

The transmission loss of the resonator becomes infinite with the denominator in Eq. (3) close to zero, yielding an expression for resonant locations as follows.

$$tan(k_r L') tan(k_r l) = \frac{S}{A_v} \tag{4}$$

Furthermore, for a short neck, $\mathbf{k_r L'} \ll \mathbf{1}$ and $\mathbf{tan(k_r L')} \approx \mathbf{(k_r L')}$, then Eq. (4) can be simplified to

$$\left(\frac{A_v}{S} \right) \left(\frac{L'}{l} \right) k_r l = cot(k_r l) \tag{5}$$

From Eq. 4 and 5, the resonant frequency of the system can be calculated using the Newton-Raphson interactive method from an initial estimate until established convergence criteria are satisfied.

3.2 Primary frequency and Transmission loss of resonance by classical form

To the Helmholtz resonator attached to an anechoic terminated duct with no mean flow and wavelengths much larger than any characteristic dimension, the lumped parameter model in both the connector and volume gives the primary resonance frequency, f_r , reduces Eq. 5 to (Selamet *et al.* (1994))

$$f_r = \frac{c}{2\pi}\sqrt{\frac{S}{VL'}} \tag{6}$$

Furthermore, fixing the volume and letting both l and L' approach zero and assuming that $k_r L'$ is negligible in comparison with $1/k_r L'$, yields, from Eq. 3, a expression as

$$TL = 10log\left[1 + \left(\frac{\frac{\sqrt{\frac{VS}{L'}}}{2A_p}}{\frac{f}{f_r} - \frac{f_r}{f}}\right)^2\right] \tag{7}$$

Where f and f_r the emissive frequency and resonance frequency of the resonator, respectively.

However, experimental observations deviate from theoretical values due to the simplifications involved in reducing distributed, multi-dimensional phenomena to lumped parameters. In addition, the wave motion neglected by the classical approach can have a significant effect, once the neck or volume length reaches $5 \sim 10\%$ of the wavelength (Panton and Miller (1975)). As a method to approximate the results to real cases, the mass loading on the single neck without flange can be considered by correcting the neck length to an effective length as proposed in (Blackstock (2001))

$$L' = L' + 0.6r_n + \frac{8}{3\pi}r_n \tag{8}$$

Where r_n is the neck radius.

3.3 Equivalent impedance

Acoustic impedance is a magnitude that quantifies the resistance to propagation of sound waves in the system. It is indispensable when you want to describe and control the acoustic behavior of ducts and cylinders, for example.

Considering that there is no internal flow, i.e. mach = 0.0, the acoustic impedance in the resonator cavity, Z_c, and the acoustic impedance in the main duct, Z_t, can be calculated, respectively, by (Vér and Beranek (2005))

$$Z_c = j\frac{c}{kV_c} \tag{9}$$

$$Z_t = \frac{ck^2}{\pi} + j\frac{ckL'}{S_n} \tag{10}$$

Where k is the wave number, V_c is the volume of the resonator cavity in m³, S_n is the cross sectional area of the resonator neck in m² and L' the effective neck length of the resonator measured by Eq. (8).

3.3.1 Equivalent impedance for parallel combination

The equivalent impedance, Z_{eq}, of a parallel combinations of resonators, figure 2, is given by (Seo and Kim (2005))

$$Z_{eq} = \frac{Z_t Z_c}{n \cdot Z_t + Z_c} \tag{11}$$

Where Z_c the impedance of the resonator cavity, Z_t is the impedance of the main duct and n is the number of resonators in parallel.
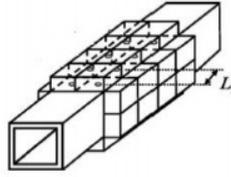
Figure 2. Parallel and series combinations of resonators.

3.3.2 Equivalent impedance for serial combination

Considering a pair of resonators in series, spaced by a distance **L** measured from the symmetry axis of the devices, as illustrated in figure 3 in which an electrical analogy of series resonators is demonstrated, the equivalent resonance of the resonator arrangement can be obtained as follows (Seo and Kim (2005)).
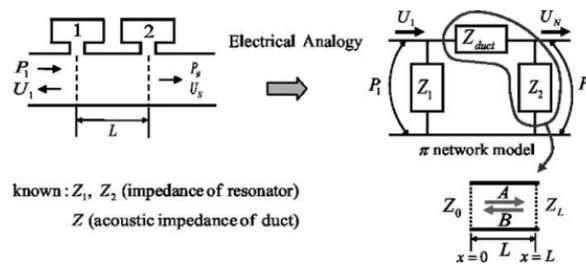

Figure 3. Electrical analogy of a simple model using two resonators, where P is the sound pressure and U is the volumetric speed. (Seo and Kim (2005))

First, considering only the resonator arranged at position **x = L**, its impedance is

$$Z_L = \frac{Z \cdot Z_2}{Z + Z_2} \tag{12}$$

Considering that the wave propagation has magnitudes A and B in the direction of the tube, the sound pressure can be represented by

$$P(x) = Ae^{-jk(x-L)} + Be^{jk(x-L)} \tag{13}$$

By this, the impedance at **x = 0** and at **x = L**, can be obtained as

$$Z_L = \frac{P(L)}{U(L)} = Z \cdot \frac{A + B}{A - B'} \tag{14}$$

$$Z_0 = \frac{P(0)}{U(0)} = Z \cdot \frac{Ae^{jkL} + Be^{-jkL}}{Ae^{jkL} + Be^{-jkL}} \tag{15}$$

Using the Eq. (14) e (15), the impedance Z_0, in **x = 0**, can be evaluated as

$$Z_0 = Z \cdot \frac{\left(\frac{Z_L}{Z_t}\right) + jtan(kL)}{1 + j\left(\frac{Z_L}{Z_t}\right)tan(kL)} \tag{16}$$

Finally, for the first resonator, the equivalent impedance Z_{eq} for a series combination can be measured by

$$Z_{eq} = \left(\frac{1}{Z_L} + \frac{1}{Z_0}\right)^{-1} \tag{17}$$

3.4 Transmission loss for resonator combination

The sound transmission for resonator combination can be obtained as follows. The ratio between the transmission and reflection pressure of waves in duct is given by (Kinsler *et al.* (2000))

$$\frac{Pt}{Pi} = \frac{Z_{eq}}{\frac{1}{2}\rho_0 cS + Z_{eq}} \tag{18}$$

Where $\mathbf{Z_{eq}}$ is the equivalent impedance of the system calculated by Eq. (17) or Eq. (11), ρ_0 the fluid density and $\mathbf{S}$ the cross sectional area of the main duct.

Also, the transmission loss can be calculated by (Serrano *et al.* (2014))

$$TL_{resonator} = 20\log\left(\frac{P_t}{P_i}\right) \tag{19}$$

So, for series or parallel combinations of resonators, it replaces the Eq. (18) in Eq. (19) to calculate the transmission loss of the system.

4. RESULTS AND DISCUSSION

The comparison between results for the primary resonance frequency obtained by the classical formulation, Eq. (6), and the one-dimensional acoustic theory, Eq. (4), for different l/d ratios of cavity are shown in the Table 1. For this, the l/d ratio of the neck for all cases is 0.2.

Table 1. Primary resonance frequency in (Hz) from analytical by Eq.(6) and by Eq. (4) for different l/d ratio of cavity.

l/d	fr, equation (6)	fr, equation (4)
1.0	766.5	708.9
2.5	766.5	604.9
5.0	766.5	475.4
10.0	766.5	336.7
15.0	766.5	194.4

There is a perceptible variation between the magnitudes calculated by the two formulations, which is accentuated the higher the l/d ratio of the cavity dimensions due to the simplifications involved in the distributed reduction, multidimensional phenomena for lumped parameters of the classical equation. (Davis Jr *et al.* (1954)).

Theoretical study of the transmission loss for five l/d resonator cavity size (with l/d ratio of the neck equal to 0.2) inserted in a circular duct of area equal to 1.385e-3, figure 4, shows that both the resonance frequency and the frequency band in which the resonator has some effectiveness is an inverse relationship to the l/d ratio.
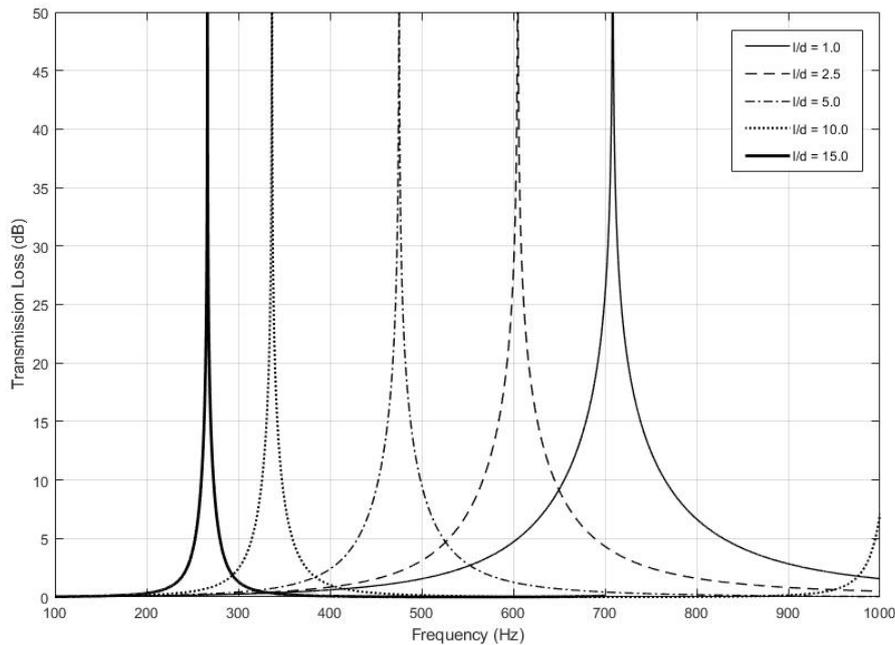

Figure 4. Transmission Loss for different l/d ratio of the resonator cavity.

Replicating the analyses again, this time keeping the cavity dimensions with a fixed l/d ratio equal to 5.0 and changing only the l/d ratio of the neck dimensions with fixed volume, we have the same conclusions, where both the primary resonant frequency and the frequency band at which the device is effective relates inversely to the l/d ratio.
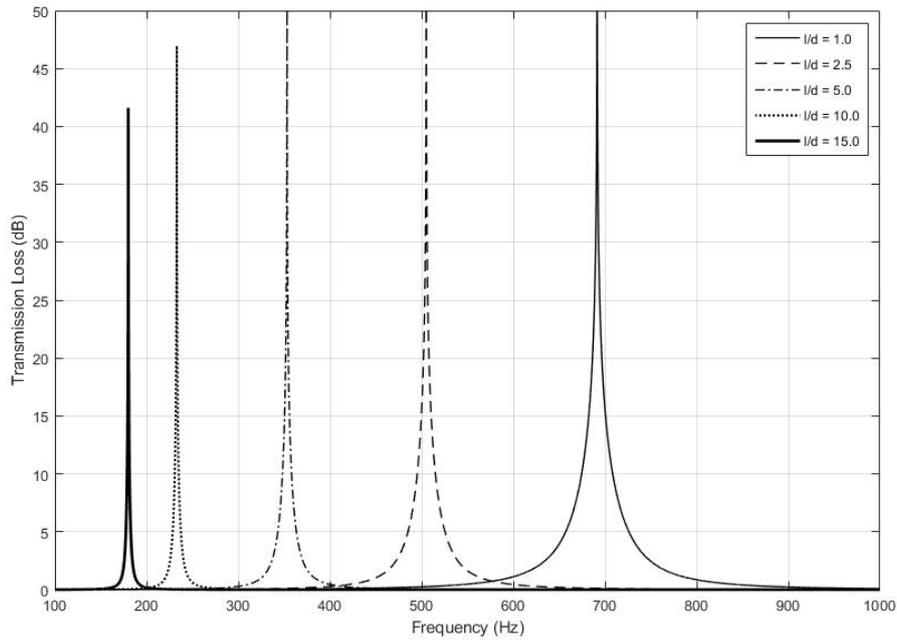
Figure 5. Transmission Loss for different l/d ratio of the resonad neck.

Figure 6, shows the analysis of the primary resonance frequencies calculated from the Eq. (4) for a fixed cavity volume and three different cross-section geometries varying only the length-to-diameter (l/d) ratio of the cavity or a l/d neck ratio equal to 0.2. A greater magnitude of the resonance frequency is realized when the l/d ratio approaches unity.
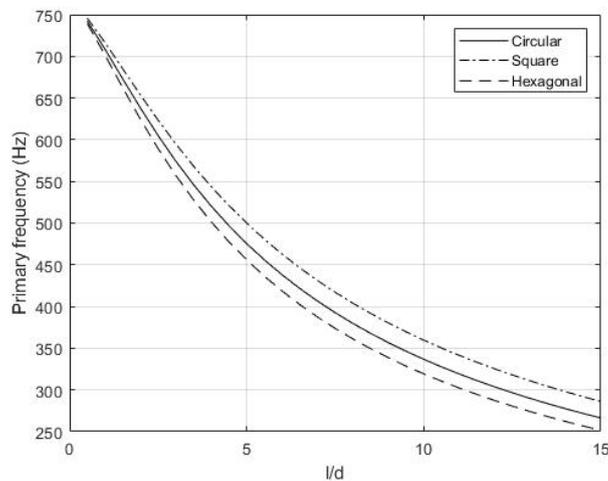

Figure 6. Primary resonance frequencies varying the l/d ratio for different cross sections of the resonator cavity.

The same analysis is replicated keeping fixed, this time, the volume and dimensions of the cavity with a l/d ratio equal to 5.0, alternating only the length-diameter (l/d) ratio of the resonator neck for the three different proposed cavity cross section geometries, figure (7). Again, the magnitude of the primary resonant frequency increases inversely with the l/d ratio.
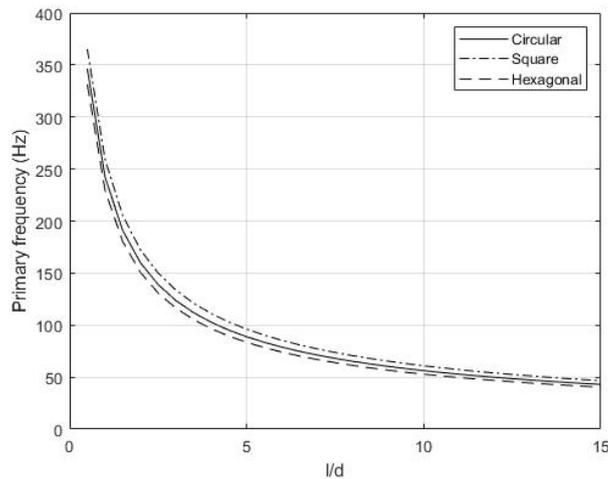
Figure 7. Primary resonance frequencies varying the l/d ratio for different cross sections of the resonator neck.

By the analysis of figure 6 and figure 7, it can be inferred that the use of a square cross-section cavity and neck is interesting when you want to achieve higher frequencies with a fixed volume.

The graphical analysis of the real and imaginary part of the normalized acoustic equivalent impedance ($\mathbf{Z_{eq}}/\mathbf{Z_t}$) in wave number equal to 15.0 for a combination of circular resonators in parallel obtained by Eq. (11), figure 8, shows an inverse relationship with the number of resonators and the l/d ratio of the neck dimensions for a l/d ratio of the cavity dimensions equal to 5.0, describing an asymptote starting at about four resonators for the cases analyzed.
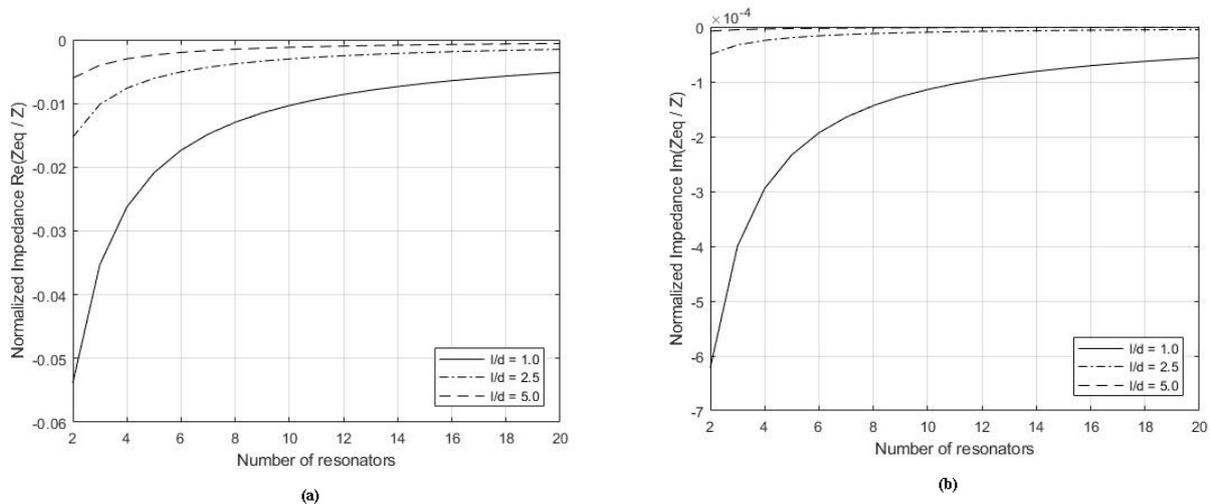

Figure 8. Normalized impedance varying the number of ressonators in parallel combinations for different l/d ratios. (a) real part and (b) imaginary part of impedance.

The acoustic equivalent impedance of a pair of circular cross section resonators in series in circular duct is analytically evaluated by Eq. (17). In this, a fixed distance between the resonators (**L**) of 0.07 m and wave numer for the resonant frequencies of each resonator is considered. First, the neck dimensions are fixed and the impedance is evaluated for different l/d ratios of the cavity dimensions for a l/d ratio of neck equal to 0.2, figure 9(a). Next, the dimensions of the cavity are fixed and the impedance for different l/d ratios of the neck for a l/d ratio of cavity equal to 5.0 is evaluated, figure 9(b).
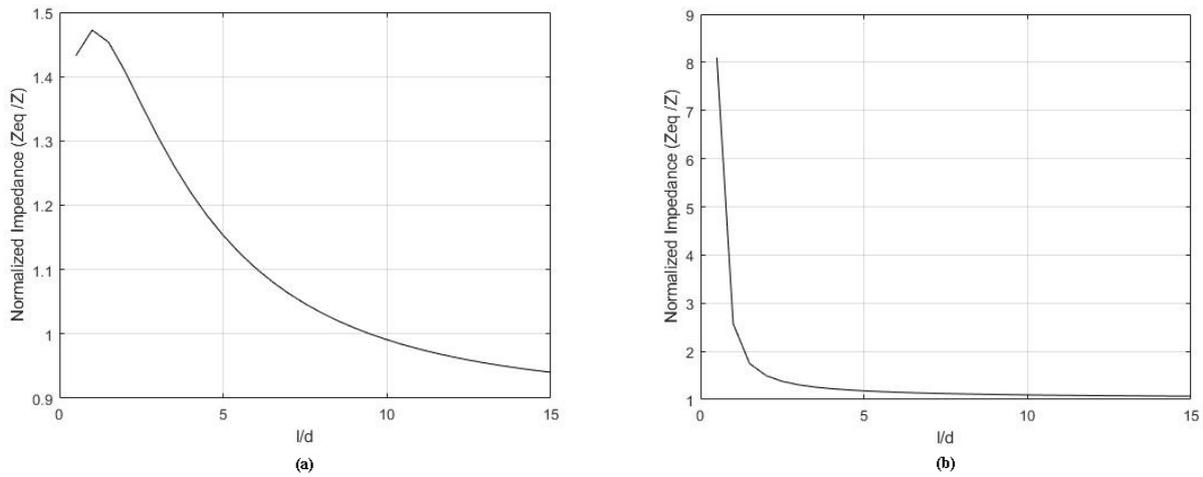
Figure 9. Normalized impedance for a series combination of resonators varying the l/d ratios of ressonator cavity (a) and of ressonator neck (b).

Finally, the transmission loss of the series combinations of resonators of the same geometry with l/d ratio of neck and cavity equal to 0.2 and 2.5, respectively, in 600 Hz of frequency is compared with the transmission loss of the single resonator, Table 2, where it is observed that the transmission loss for the arrangement of resonators becomes larger than the single resonator, going in agreement with the current literature.

Table 2. Comparison between the transmission loss of a parallel combination of resonators and the transmission loss of a single resonator in (dB) for a l/d ratio of cavity.

l/d	Single	2 resonators	3 resonators	4 resonators
1.0	36.6	42.4	79.3	116.1

5. CONCLUSION

In view of the above, one first notes the need to perform a precise study of the characteristic geometries of Helmholtz resonators when it is desired to insert them into acoustic cavities in the form of noise attenuation structures, since, the acoustic properties of a given device are only effective in a limited frequency band, and these are directly influenced by the dimensions of the resonator.

Therefore, using the classical formulation for calculating the resonance frequency and the transmission loss are not effective for optimal acoustic design, it is then preferable to use the formulations obtained by the planar propagation of sound waves, which present more accurate solutions to the experimental data, as shown in (Selamet *et al.* (1994)).

Regarding the graphical results obtained, can be seen that for the same geometry and volume of the cavity and neck, the l/d ratio has a great influence on the primary resonance frequency and the sound transmission loss, having an inverse relationship for these magnitude. Therefore, it is interesting to use a l/d ratio of resonator dimensions as close to unity as possible for the best use in acoustic noise mitigation projects. Graphical analysis also shows that resonators are more sensitive to changes in the neck than in the cavity

For parallel and series combinations of resonators, there is a direct relationship in the number of resonators and the impedance and acoustic transmission loss of the system converging from a finite number of resonators, so the amount of resonators inserted in a matrix configuration must be studied to define an optimal amount without wasting space and materials.

Future prospects include the improvement of analytical formulations related to resonator matrix combinations as part of the object of study of this research project in addition to the numerical study of the acoustic properties of the resonator and the relationship with variation of specific geometrical parameters.

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