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COMPRESSIBLE DIRECT NUMERICAL SIMULATION OF TOLLMIEN-SCHLICHTING WAVES INTERACTING WITH A TWO-DIMENSIONAL ISOLATED ROUGHNESS

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Abstract.
Through Direct Numerical Simulations (DNS), this work investigates the boundary layer stability of two-dimensional Tollmien-Schlichting (TS) waves over a flat plate with a two-dimensional isolated roughness element immersed in a compressible laminar boundary layer flow. The rectangular roughness height was defined as 10% of the boundary layer displacement thickness at the position of the center of the roughness on a flat plate. Upstream from the roughness, there is a region capable of generating disturbances that travel downstream, interacting with the roughness. Different base flows for each Mach number were generated on the subsonic and transonic regimes, then sinusoidal two-dimensional TS waves were inserted with constant amplitudes and frequencies. Through hydrodynamic stability analysis, the roughness influence on the evolution of the TS waves shows greater amplitudes downstream, and increasing Mach numbers decrease the minimum critical Reynolds number.

Keywords: *Direct Numerical Simulation, Roughness, Tollmien-Schlichting waves, Boundary Layer Stability*

1. INTRODUCTION

In the aeronautical industry, structures similar to an isolated roughness are commonly present on aircraft surfaces, and can influence the drag coefficient if there is transition to the turbulent regime in the boundary layer. Even with a relatively small influence, preventing multiple sources like these is an interesting study that can be beneficial in saving fuel and operating costs.

The boundary layer transition is a process that can be initiated by instabilities. There is a large number of factors that influence on transition, several of which are interdependent. The study of each aspect individually promotes a better understanding of the complete mechanism and allows predictive modeling of phenomena, a great advantage in the aeronautical industry. In boundary layer flows, transition is often caused by primary instabilities and, as a consequence, the amplification of secondary instabilities. For small Mach numbers, the main primary instability source of these flows is the two-dimensional TS wave.

Through Direct Numerical Simulations (DNS), the objective of the current work is to develop a suitable two-dimensional mesh and investigate the effects of two-dimensional Tollmien-Schlichting (TS) waves over a flat plate with a two-dimensional isolated roughness element immersed in a compressible laminar boundary layer flow.

2. REVIEW

Early works investigated roughness elements and isolated imperfections on the surface of airfoils with zero pressure gradient or flat plates as factors that promoted transition from laminar to turbulent flow. Wind tunnel experiments from Fage (1943), Tani (1961, 1969) and those shown in the review by Dryden (1953) indicated that a smoother surface influenced in the conservation of stability in a laminar boundary layer. Therefore, to conserve laminar flow it was of great importance to establish the highest height h of the structures that could be tolerated without influencing the transition. For some combinations of roughness height-thickness ratio dependent, stream speed and location of roughness element, it was also determined that, for higher flow speeds, the distance between roughness and the transition point was gradually reduced.

On studies about two-dimensional roughnesses with disturbances, such as Klebanoff and Tidstrom (1972), Dovgal and Kozlov (1990), Morkovin (1990) and Wörner *et al.* (2003) , it was concluded that the flow region modified by the presence of the roughness is more sensitive to destabilizing influences. The degree of instability was dependent on the velocity profile and its interaction with the roughness geometry. The presence of waves with small oscillation amplitudes in the roughness region, around 1% of the velocity on the outer edge of the boundary layer U_0, already proved strong influence on transition.

In stability theory for compressible flows, the primary interest is in unstable rather than neutral waves (Mack, 1987). The maximum spatial amplification rate as a function of Mach for 2D waves indicates that the second and higher modes are most unstable as 2D waves, because they depend on the thickness of the relative supersonic region, but the first mode is most unstable as an oblique wave at all supersonic Mach numbers. For lower Mach numbers, an oblique wave can have an amplification rate several times larger than a 2D wave. For $M < 2.4$, an oblique first-mode wave is even more unstable than a 2D second-mode wave. For Dunn and Lin (1955) as the Mach Number increases, three-dimensional disturbances become significant under conditions that are less and less extreme, until finally, at a Mach Number between one and two, they begin to play the leading role in many cases of practical interest.

Results in Lees (1947) and Lees and Reshotko (1962) indicate that, for the laminar boundary layer flow, the minimum critical Reynolds number decreases from its Mach number zero value, reaches a minimum somewhere around $M = 3$ and then increases again. In Criminale *et al.* (2018), it is shown that up to $M = 1.6$, the neutral stability curve is quite similar to the incompressible case, but at higher values of the Mach number the upper branch turns upward toward the inviscid limit. As pointed out by Mack in his calculations, inviscid disturbances begin to dominate at $M = 3$ and the stability characteristics are more like those of a free shear layer than of a low-speed zero-pressure gradient boundary layer.

3. METHODOLOGY

The numerical simulations were performed by a DNS for the compressible Navier-Stokes equations, developed by the Group of Aeroacoustics, Transition and Turbulence (GATT) of the Department of Aeronautical Engineering of the São Carlos School of Engineering, University of São Paulo (EESC-USP). The main works that present the development and validation of the DNS can be found in Bergamo (2014), Gaviria Martínez (2016), Mathias (2017), Mathias and Medeiros (2019).

In this work, the fourth order Runge-Kutta method is used for time marching. For the spatial derivatives, a sixth order compact spectral-like compact finite differences shown by Lele (1992) is used. The pre processing is done in MATLAB and the main processing is written in FORTRAN.

The governing equations were defined in a two-dimensional domain (x, y), and time (t), in terms of density (ρ), the two velocity components (u, v), and internal energy (e). The values presented here are non-dimensional, by the characteristic velocity at the outer edge of the boundary layer (U_0), the boundary layer displacement thickness at the roughness position (δ_r^*) and initial density (ρ_0).

For the boundary conditions, the inflow boundary is defined as an uniform flow at constant temperature and the pressure derivative is zero. In the outflow, pressure is kept constant and the second derivative is null for the other variables. The outer flow condition on the wall-normal direction sets the second derivative of all variables to zero.

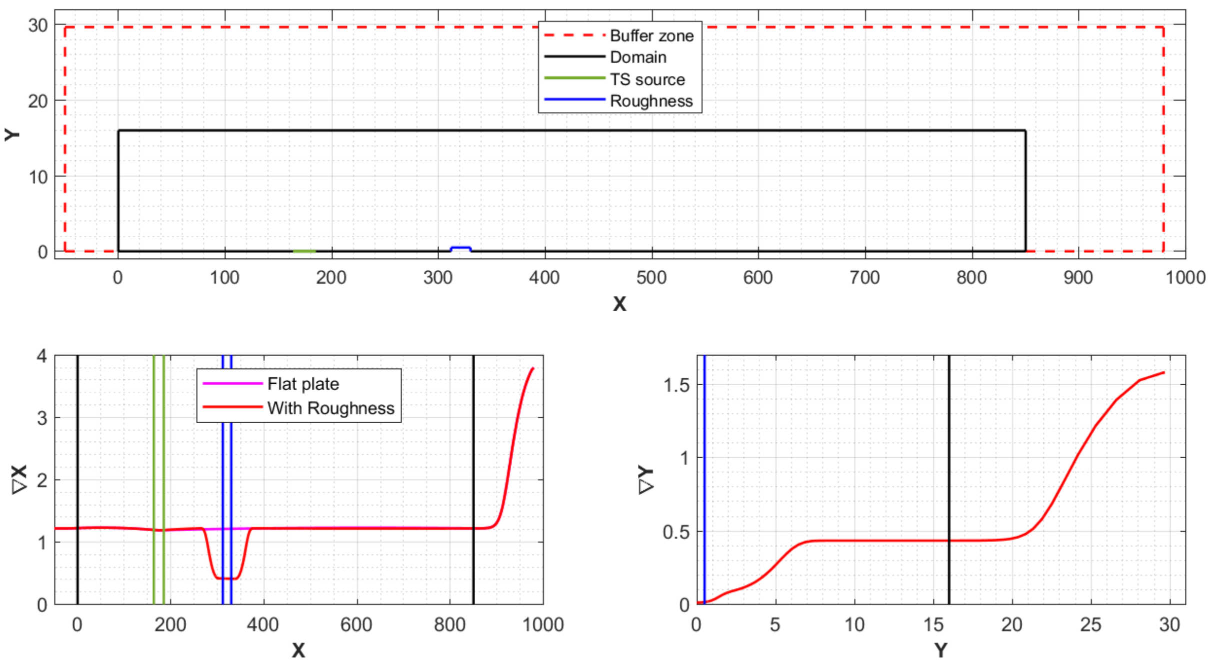

Figure 1. Illustration of the domain and mesh spacing (non-dimensional)

For the walls, including the roughness, there are no-slip and no-penetration conditions for velocity, the pressure gradient is zero in the normal direction and the temperature is fixed. From $X = -50$ to $X = 0$ there is a free-slip region in the wall, necessary to accommodate the flow before the boundary layer starts forming.

A rectangular roughness placed on a flat plate, according to Fig. 1. Upstream from the roughness, there is a region, represented by the green lines, capable of generating disturbances that travel downstream, interacting with the roughness.

Some flow parameters were taken from the experimental works by de Paula (2007) and de Paula *et al.* (2017), such as: Reynolds number in the position of the wave source $Re_{\delta^*_{TS}} = 700$, Reynolds number at the roughness position $Re_{\delta^*_r} = 950$, roughness diameter $d = 10\ mm$ and displacement thickness at the experimental roughness location $\delta^*_r = 0.55\ mm$. The roughness height was defined as 10% of the boundary layer displacement thickness at the position of the center of the roughness on a flat plate, that is, $0.1\delta^*_r$. The initial condition is a Blasius boundary layer at constant temperature and pressure. The characteristic Reynolds number is $Re = 950$.

The meshes are Cartesian and initially uniform, which can be stretched in certain regions, increasing the density of nodes as needed, as seen on Fig. 1. For the flows with the roughness element, the mesh is refined in the X direction around the roughness region. In the Y direction it is refined on the boundary layer region, and extra refined around the height of the roughness.

The buffer zone consists of the region from $X = 850$ to the end of the mesh, which is included to avoid problems in the simulation, such as reflections in the domain. In Y, there is a region with an intense refinement along the height of the roughness, from $Y = 0$ to $Y = 0.1$. The buffer zone starts at $Y = 16$ and ends at the last node.

4. RESULTS

The results will be presented for $M = 0.1$, $M = 0.3$, $M = 0.6$ and $M = 0.9$. For each Mach number, a different base flow was generated for the flat plate and for the plate with an isolated roughness, for comparison. Then, for each base flow, it was introduced a sinusoidal two-dimensional TS with constant amplitude and frequency.

4.1 Base Flow

The simulation of flat plate with roughness has a maximum relative error close to the order of 10^{-10}, an adequate value for a base flow simulation for the following analysis. According to Fig. 2, the roughness effects on the boundary layer are concentrated in their proximity. There is a velocity deficit in just near the roughness, resuming the flat plate values to a small longitudinal distance, as expected. The displacement and momentum thickness of the boundary layer were calculated using the equations in Schlichting and Gersten (2017).

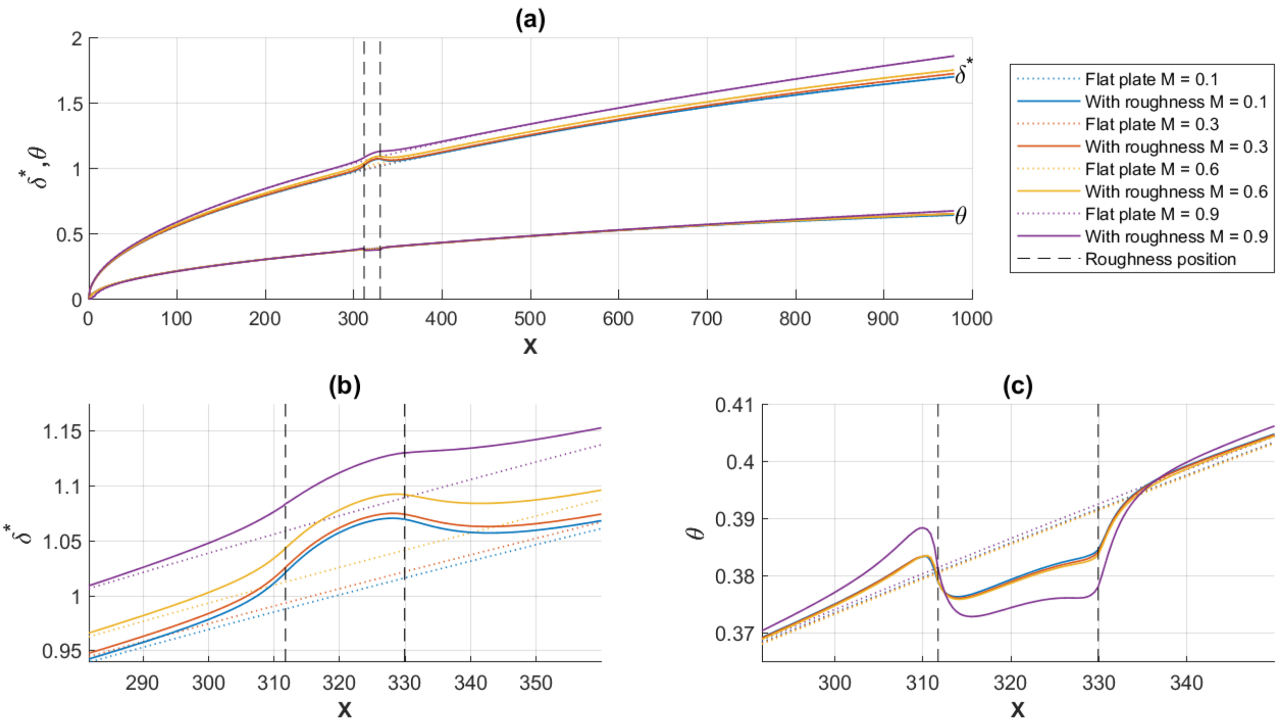

Figure 2. Displacement and momentum thickness on the boundary layer for the base flows

4.2 Base Flow with TS Wave

For each Mach number, a sinusoidal two-dimensional TS wave was generated with amplitude $A_{0-2D} = 0.75\%\, U_0$ at the roughness position, measured on the flat plate, and non-dimensional frequency $F = 90 \times 10^{-6}$. Fig. 3 compares the amplification curves for the flat plate and a flat plate with roughness, and Fig. 5 shows the mean flow distortion (MFD) for every flow with TS wave.

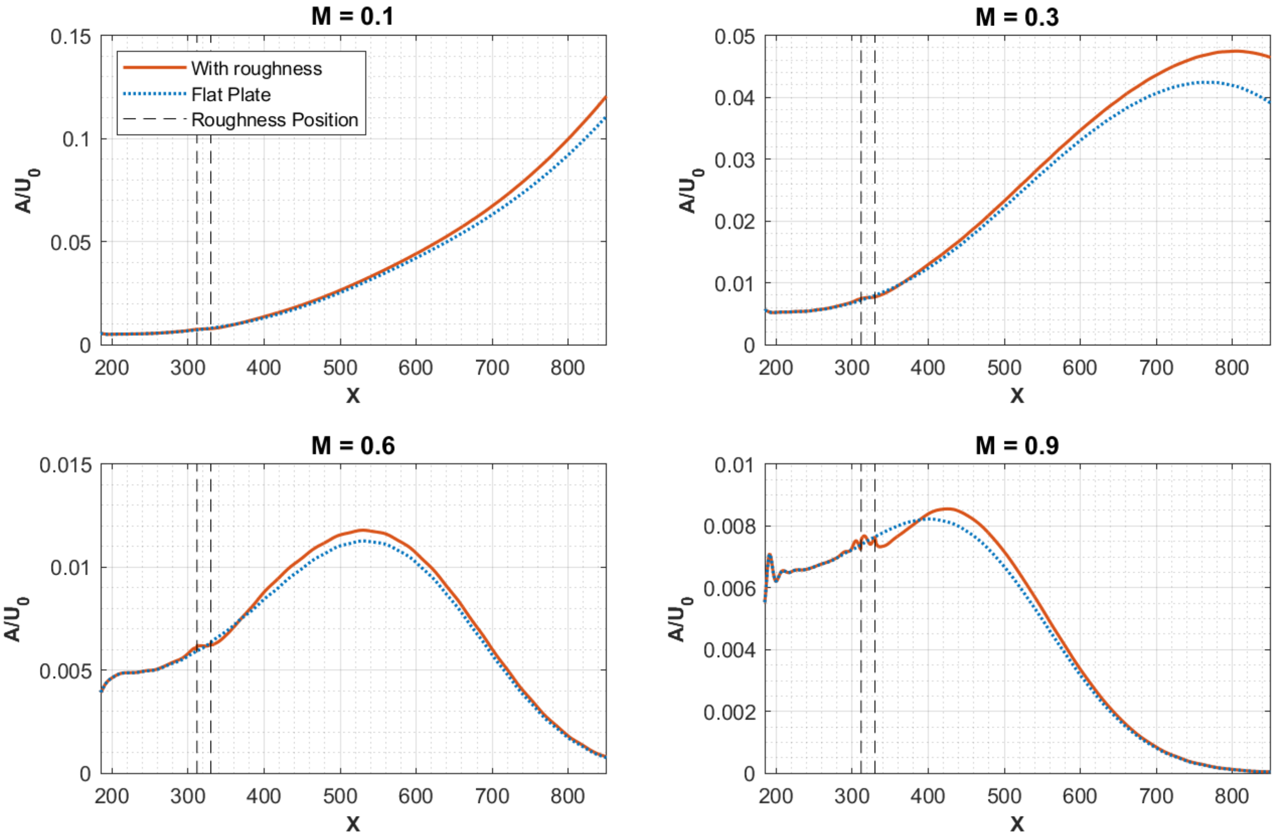

Figure 3. Amplitude development of a TS wave with $F = 90 \times 10^{-6}$

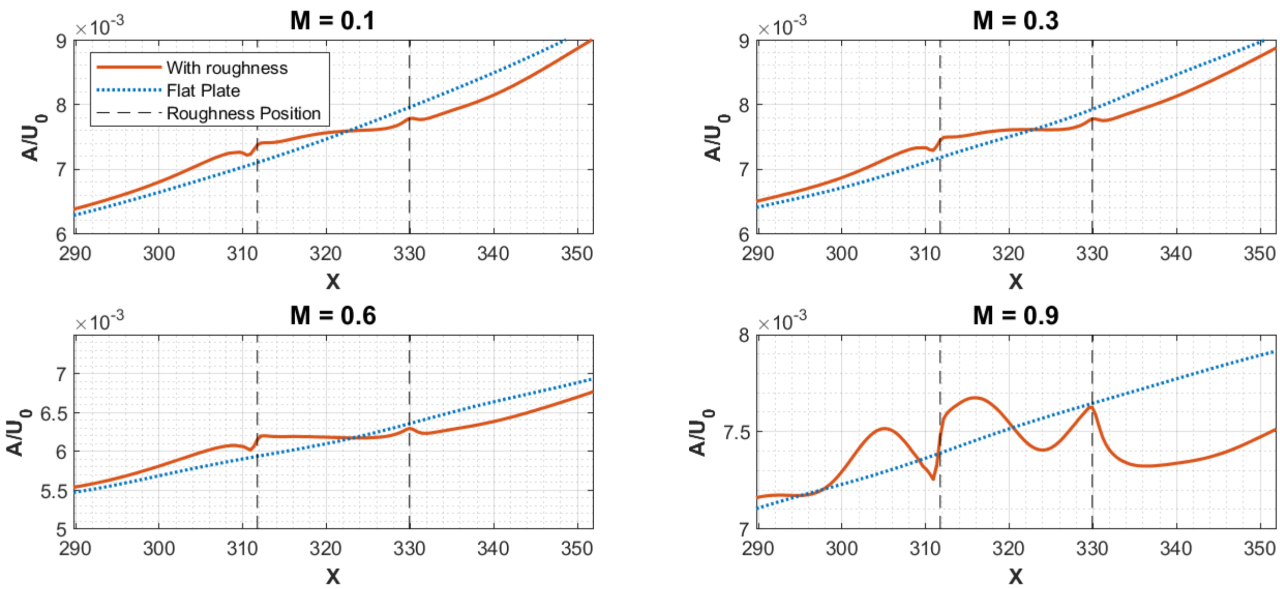

Figure 4. Amplitude development of a TS wave with $F = 90 \times 10^{-6}$, in detail

According to Wörner *et al.* (2003), at the rising edge of the roughness, TS wave amplitude decreases because of the thinner boundary layer. At the falling edge, the wave amplitude increases but does not surpasses the local amplitude for the flat plate, as seen on Fig. 4. The gap between wave amplitudes at the falling edge decreases as the Mach number grows.

This increase in amplification seems weaker than the stabilizing effect that occurs at the rising edge, showing that the global effect of a roughness at this location and with the chosen dimensions is not a destabilization of the boundary layer.

For $M = 0.1$ and $M = 0.3$, the maximum of the amplitude is not present on the current X domain, unlike for $M = 0.6$ and $M = 0.9$. This indicates a decrease on the minimum critical Reynolds number as the Mach number increases. For every Mach number included in this study, the amplitude development curve for the flat plate is generally lower than the amplitude for the plate with roughness downstream from the falling edge, as well as the maximum, when applicable.

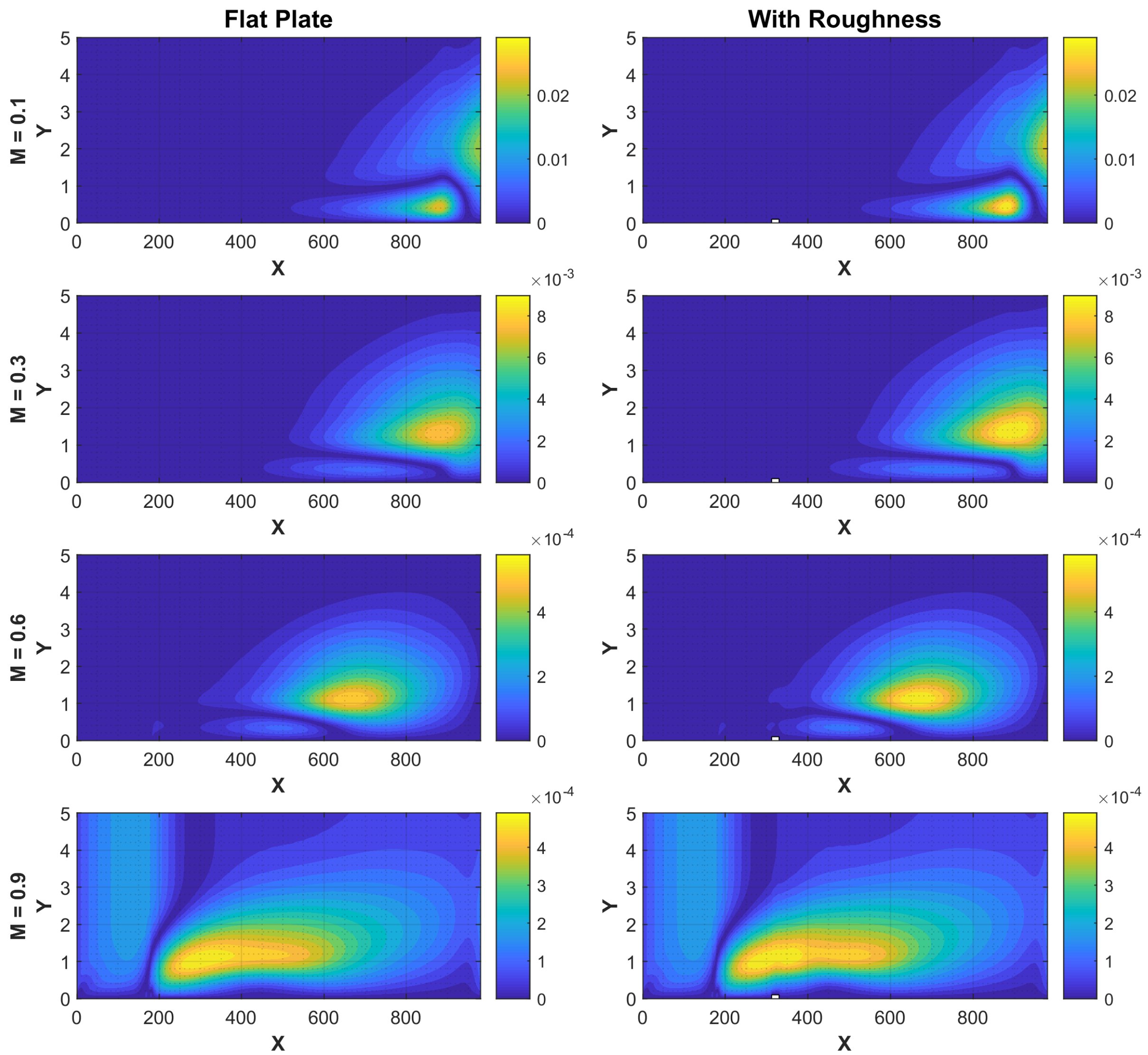

Figure 5. Mean flow distortion

For the available stream-wise domain, Fig. 5 displays weaker qualitative changes in amplitude between the flat plate and the plate with roughness for the smaller Mach numbers. As for their maximum values, smaller Mach numbers have greater variation, of approximately 21% on $M = 0.1$, 13% on $M = 0.3$, 9% on $M = 0.6$ and 0.02% on $M = 0.9$.

The mean flow distortion is the sole mode impacting the integral quantities of the boundary layer, and allows to compare the averaged transitional boundary layer and the laminar boundary layer, represented by the base flow (Appel, 2020). As the non-linear correction to the base flow, the MFD is the only mode impacting physical quantities such as the lift, drag and pressure coefficients, the displacement or the momentum thickness. $M = 0.9$ displays the largest MFD region over the roughness element, similar to Fig. 2, where is has the greatest shift among the others.

5. CONCLUSIONS

Through Direct Numerical Simulations (DNS), this work investigates the boundary layer stability of a sinusoidal two-dimensional Tollmien-Schlichting (TS) waves over a flat plate and a plate with a two-dimensional isolated roughness element, immersed in a compressible laminar boundary layer flow.

Results on this study presents the roughness' influence as a general increase on the amplitude of the TS wave downstream from its falling edge, and due to an increase on the Mach numbers, for $M = 0.1$, $M = 0.3$, $M = 0.6$ and $M = 0.9$, there is a decrease the minimum critical Reynolds number.

The amplification curves, as well as the span-wise and wall normal MFD distribution, are very different for each Mach, for a TS wave with the chosen frequency. For subsequent works, this could be a parameter determined individually. Besides that, the roughness height is another parameter vastly influential on the amplitude increase of the TS wave and boundary layer transition.

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7. REFERENCES

Appel, T., 2020. *Boundary Layer Instabilities Due to Surface Irregularities, a Harmonic Navier-Stokes Approach*. Ph.D. thesis, Imperial College London.

Bergamo, L.F., 2014. *Instabilidade hidrodinâmica linear do escoamento compressível em uma cavidade*. Master's thesis.

Criminale, W.O., Jackson, T.L. and Joslin, R.D., 2018. *Theory and Computation in Hydrodynamic Stability*. Cambridge University Press.

de Paula, I.B., Würz, W., Mendonça, M.T. and Medeiros, M.A.F., 2017. "*Interaction of instability waves and a three-dimensional roughness element in a boundary layer*". Journal of Fluid Mechanics, Vol. 824, pp. 624–660.

de Paula, I.B., 2007. *Influência de uma rugosidade tridimensional isolada na transição de uma camada limite sem gradiente de pressão*. Ph.D. thesis, Universidade de São Paulo.

Dovgal, A.V. and Kozlov, V.V., 1990. "*Hydrodynamic Instability and Receptivity of Small Scale Separation Regions*". Laminar-Turbulent Transition, pp. 523–531.

Dryden, H.L., 1953. "*Review of Published Data on the Effect of Roughness on Transition from Laminar to Turbulent Flow*". Journal of the Aeronautical Sciences, pp. 447–482.

Dunn, D.W. and Lin, C.C., 1955. "*On the stability of the laminar boundary layer in a compressible fluid*". Journal of the Aeronautical Sciences, Vol. 22, pp. 455–477.

Fage, A., 1943. "*The smallest size of spanwise surface corrugation which affects boundary-layer transition on an airfoil*". Aeronautical Research Council, R & M 2120.

Gaviria Martínez, G.A., 2016. *Towards natural transition in compressible boundary layers*. Ph.D. thesis, Universidade de São Paulo.

Klebanoff, P.S. and Tidstrom, K.D., 1972. "*Mechanism by Which a Two-Dimensional Roughness Element Induces Boundary-Layer Transition*". The Physics of Fluids, Vol. 15, No. 7, pp. 1173–1188.

Lees, L., 1947. "*The stability of the laminar boundary layer in a compressible fluid*". NACA TR-876.

Lees, L. and Reshotko, E., 1962. "*Stability of the compressible laminar boundary layer*". Journal of Fluid Mechanics, Vol. 12, p. 555–590.

Lele, S.K., 1992. "*Compact Finite Difference Schemes with Spectral-like Resolution*". Journal of Computational Physics, Vol. 106, pp. 16–42.

Mack, L., 1987. "*Review of compressible stability theory*". In *Stability of time dependent and spatially varying flows*. Springer.

Mathias, M.S., 2017. *Instability analysis of compressible flows over open cavities by a Jacobian-free numerical method*. Master's thesis, Universidade de São Paulo.

Mathias, M.S. and Medeiros, M.F., 2019. *Global instability analysis of a boundary layer flow over a small cavity*.

Morkovin, M.V., 1990. "*On Roughness-Induced Transition: Facts, Views, and Speculations*". pp. 281–295.

Schlichting, H. and Gersten, K., 2017. *Boundary-Layer Theory*. Springer, Berlin, Heidelberg.

Tani, I., 1961. "*Effect of Two-Dimensional and Isolated Roughness on Laminar Flow*". Boundary Layer and Flow Control, Vol. 2, pp. 637–656.

Tani, I., 1969. "*Boundary Layer Transition*". Annual Review of Fluid Mechanics, Vol. 1, pp. 169–196.

Wörner, A., Rist, U. and Wagner, S., 2003. "*Humps/Steps Influence on Stability Characteristics of Two-Dimensional Laminar Boundary Layer*". AIAA Journal, Vol. 41, No. 2, pp. 192–197.

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