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NUMERICAL STUDY OF THE DYNAMIC RESPONSE IN MULTI-DOFS STRUCTURES FROM THE SEISMIC EXCITATION

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Abstract. *Some engineering problems can be modeled mathematically using ordinary differential equations (ODEs), which allows evaluating the behavior of the physical phenomena of the problem. In this sense, the present study aims to compare the results from two solution methods, 4th-order Runge-Kutta Method and State-Space Method. The cases studied are structures with 1 and 2 degrees of freedom and a case with 3 degrees of freedom (DOFs) in a structure that receives excitation through a known earthquake. The Runge-Kutta Method consists of an explicit four-step method for the numerical solution of the set of ordinary differential equations, while the State-Space Method for a time-invariant linear system makes it possible to solve problems with multiple inputs and outputs in matrix form. In this way, the models will be discretized in a mass-spring-dampener system and then solved for both methods. With the responses of the two solutions, the Fast Fourier Transform (FFT) is applied to obtain the behavior of the structure in the frequency domain in order to verify the maximum amplitude of vibration. The calculations will be implemented in the Matlab software. The results of the numerical simulations are compared in terms of displacement and speed in the time and frequency domain.*

Keywords: *4th-order Runge-Kutta Method, State-Space Method, Fast Fourier Transform, dynamic analysis, seismic excitation*

1. INTRODUCTION

Among the various numerical techniques for first-order ODEs solutions, the Runge-Kutta methods can be highlighted (Gilat and Subramaniam, 2008) (Chapra and Canale, 2010). Butcher and Wanner (1996) in his article entitled "Runge-Kutta methods: some historical notes" presents the centenary history of the Runge-Kutta methods, approaching the first works and placing a little more attention on methods aimed at Hamiltonian systems. The first developers of the methods were Runge, Heun and Kutta. In 1895 Runge published on the numerical solution of differential equations. Later Heun in 1900 contributed to increasing the order of the method to 4. Kutta in 1901 published a version of the method which is known as the Runge-Kutta method. In 1925 Nyström corrected some of Kutta's fifth-order methods and also showed how to apply the Runge-Kutta Method to systems of second-order differential equations (Butcher, 2000) (Butcher, 1996).The different types of Runge-Kutta methods are classified according to their order, with the fourth-order or classic Runge-Kutta Method being the most commonly used (Gilat and Subramaniam, 2008) (Chapra and Canale, 2010) (Süli and Mayers, 2003).

Another way of solving ODEs is through the Space-State method, which consists of coupling the first-order differential equations with the state variables. The problem is solved in matrix form and is composed of input, output, and state variables (Rowell, 2002)(Ogata, 2017) (Chen, 1998). This emerged between the 1960s and 1970s in the field of control engineering in the representation of a type of differential equation represented by an equation of state in the time domain instead of using transfer functions (Williams II and Lawrence, 2007) (Commandeur *et al.*, 2011) (Fiiedland, 1986). One of the famous applications was the contributions to aerospace programs: Apollo and Polaris (Commandeur *et al.*, 2011) (Hutchinson, 1984) (Fiiedland, 1986). Physical systems can be represented in State-Space, as is the case of time-invariant linear systems. This system is a mathematical model that can represent dynamic systems, in which the output is a linear combination of the input signals being characterized by a set of first-order ordinary differential equations and which presents an independent variable, time (Rowell, 2002) (Phillips *et al.*, 2008) (Chen, 1998) (Williams II and Lawrence, 2007).

In civil engineering, with the construction of buildings that are slimmer and with ever-larger spans, structures become susceptible to vibrations and have a reduced capacity for energy dissipation. In addition, they are subject to external actions, whether from human activity or caused by nature, which can damage them and even compromise their functionality. Therefore, it is necessary to make a dynamic study of the actions involved and determine the vibrations in order to obtain control of these (Soriano, 2014). one of the useful applications of the State-Space model is structural control problems and it is more suitable for using certain computational schemes to solve equation of motion. (Datta, 2010).

The importance of structural analysis in events with dynamic loads is essential to ensure the integrity of buildings. Thus, the study seeks to analyze, using the 4th- order Runge-Kutta method and the State Space method, two systems, one with one degree of freedom and the other with two degrees of freedom, in order to compare the responses of the methods. Later, apply them to a structure with 3 degrees of freedom receiving an external dynamic load, represented by a known seism. Excitation is carried out on the first floor, as it is not a model with ground-structure coupling. The methods responses are in the time domain and present the structure behavior in terms of displacement and velocity. Furthermore, FFT is applied to identify the maximum vibration amplitudes.

2. NUMERICAL PROCEDURE

Attentive to the objective, analyze two methods in order to verify the behavior of a structure receiving direct seism excitation. Firsty, they will be discretized in a mass-spring-dampener system. In the sequence, the equations are presented using the 4th-order Runge-Kutta method for the cases of 1 and 2 DOFs and later the application involving the earthquake for 3 DOFs. The same procedure is performed using the State-Space method and finally, they are compared and applied to FFT.

2.1 Discrete model

In order to obtain the dynamic behavior through the 4th-order Runge-Kutta and State-Space Methods, three structures are modeled in the two-dimensional (2D) plane represented in the Cartesian plane by XY coordinates, as shown in Fig. 1. The Fig. 1(a) represents the model with one DOF, the Fig. 1(b) is already a structure with two DOFs and the Fig. 1(c) has three DOFs. The models discretized in a mass-spring-dampener system are presented in Fig. 2. The systems in Fig. 2(a) and Fig. 2(b) are excited harmonically by the variable force $f(t)$ and moves linearly in the x-axis direction. For the problem of case 1 the mass m_1 is fixed to a massive structure with a spring constant k_1 and a dampner with linear damping coefficient c_1. The second system (Fig. 2(b)) the masses are also fixed to a massive structure and force is applied to the mass m_1, in the same direction as the x-axis. The external harmonic force is given as $f(t) = F0\sin(\omega t)$, on what $F0$ is a constant. The case 3 (Fig. 2(c)) represents the application with 3 DOFs being dynamically excited by the acceleration of an seism ($\ddot{x}_g$) in mass m_1 and in the x direction. In this last model, as it is not coupled to the ground, that is, disregarding the effect of soil-structure interaction, the excitation is applied directly to the first floor of the structure. To facilitate the understanding of the 3 proposed problems, systems with 1, 2, and 3 DOFs will be called case 1, case 2, and case 3, respectively.

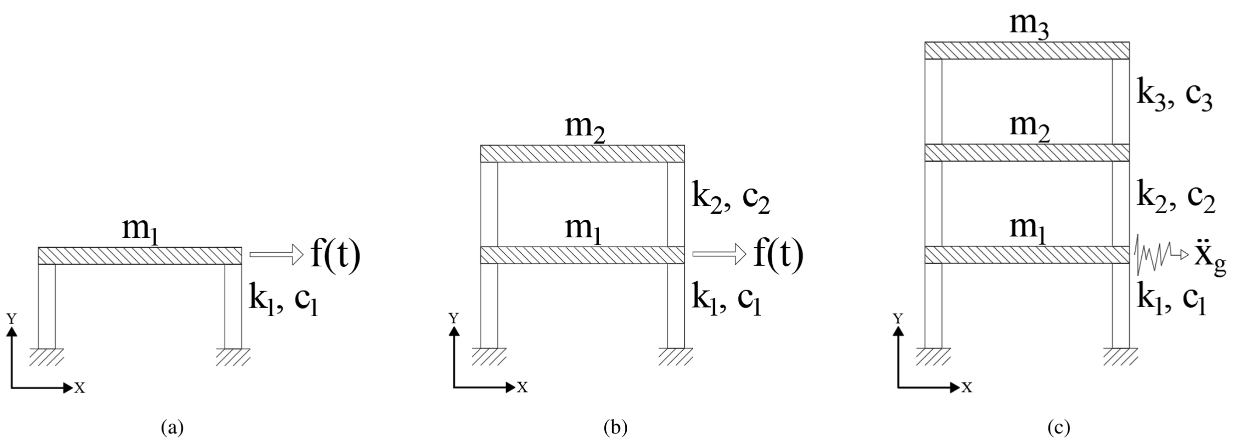

Figure 1. Models of structures in the plane. a) One-floor structure model - case 1; b) Two-floor structure model - case 2; c) Three-floor structure model - case 3.

The equations of motion representing the 3 cases are based on Newton's Second Law presented in canonical form. Cases 1 and 2 are given by the following equation

$$\mathbf{M}\,\ddot{\mathbf{x}}(t) + \mathbf{C}\,\dot{\mathbf{x}}(t) + \mathbf{K}\,\mathbf{x}(t) = \mathbf{f}(t). \tag{1}$$

The case 3 is represented for

$$\mathbf{M}\,\ddot{\mathbf{x}}(t) + \mathbf{C}\,\dot{\mathbf{x}}(t) + \mathbf{K}\,\mathbf{x}(t) = -\mathbf{M}\mathbf{j}\ddot{x}_g(t), \tag{2}$$

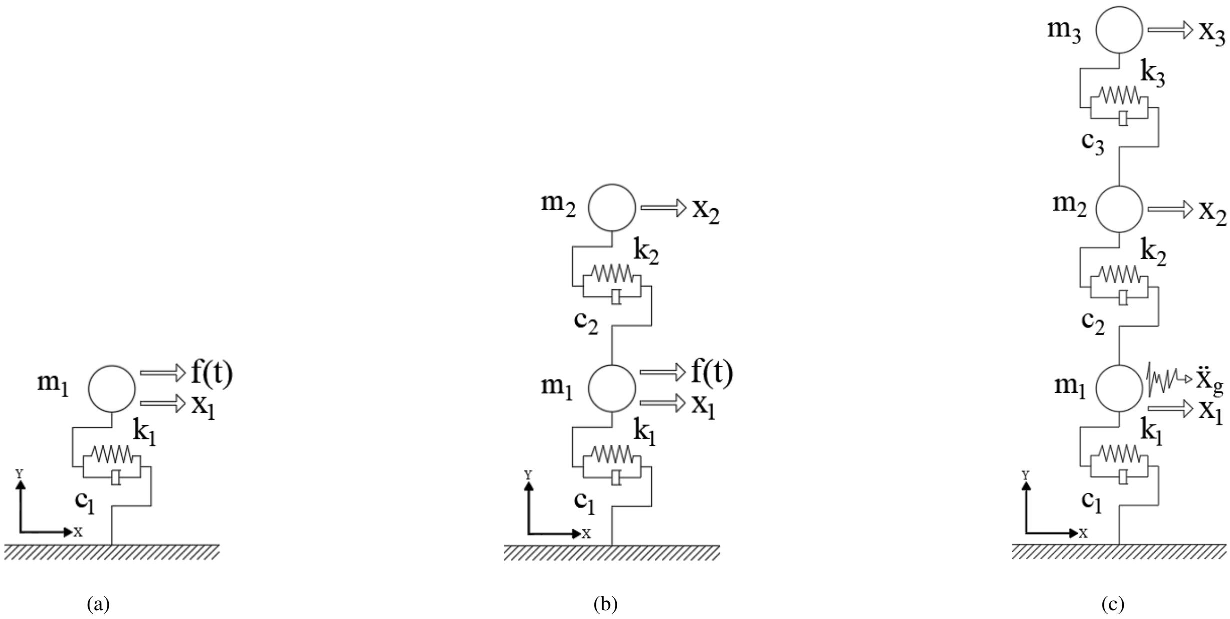

Figure 2. Discrete models. a) 1 DOF model - case 1; b) 2 DOFs model - case 2; c) 3 DOFs model - case 3.

in which $\mathbf{M}$, $\mathbf{K}$ and $\mathbf{C}$ are the mass, stiffness and damping matrices, respectively, presented below

$$\mathbf{M} = \begin{bmatrix} m_1 & & & \\ & \ddots & & \\ & & m_{n-1} & \\ & & & m_n \end{bmatrix}, \tag{3}$$

$$\mathbf{K} = \begin{bmatrix} k_1^2 + k_2^2 & -k_2^2 & & & & \\ -k_2^2 & k_2^2 + k_3^2 & -k_3^2 & & & \\ & -k_3^2 & \ddots & & & \\ & & & -k_{n-1}^2 & k_{n-1}^2 + k_n^2 & -k_n^2 \\ & & & & -k_n^2 & k_n^2 \end{bmatrix}, \tag{4}$$

$$\mathbf{C} = \begin{bmatrix} c_{11} & \dots & c_{1n} \\ \vdots & \ddots & \vdots \\ c_{n1} & \dots & c_{nn} \end{bmatrix}, \tag{5}$$

$\ddot{\mathbf{x}}$, $\dot{\mathbf{x}}$ and $\mathbf{x}$ are the acceleration, velocity and displacement vectors, respectively. $\mathbf{j}$ is the vector that defines the DOFs, $\mathbf{f}(t)$ is input force and $\ddot{x}_g(t)$ is the ground motion acceleration measured by a strong-motion seismograph during actual earthquake events. It is noteworthy that for each case it has its respective matrices and vectors, not being the same for the three models presented.

2.2 4th-order Runge-Kutta Method

The 4th-order Runge-Kutta Method (RK4) simulates Taylor series accuracy with n=4. In this case the local truncation error is O(h^5). The general equation of the Classical Method is

$$y_{i+1} = y_i + \underbrace{\frac{1}{6}(K_1 + 2K_2 + 2K_3 + K_4)\, h}_{\text{slope}}, \tag{6}$$

on what

$$K_1 = f(x_i, y_i), \tag{7}$$

$$K_2 = f(x_i + \frac{1}{2}h, y_i + \frac{1}{2}K_1 h), \tag{8}$$

$$K_3 = f(x_i + \frac{1}{2}h, y_i + \frac{1}{2}K_2 h), \tag{9}$$

$$K_4 = f(x_i + h, y_i + K_3 h), \tag{10}$$

in which h is the step size, K_1, K_2, K_3 and K_4 are the inclinations. The slope is a constant. The slope value in Eq. (6) is obtained by calculating the slope at various points within the subinterval. $f(x_i, y_i)$ is the differential equation evaluated in x_i and y_i, y_{i+1} is the approximation of RK4 determined by the current value y_i (Epperson, 2013) (Chapra and Canale, 2010) (Gilat and Subramaniam, 2008).

2.3 State-Space equations

A state-space representation for a linear time-invariant system has the general equation

$$\begin{array}{lll} \dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t) & \rightarrow & \textit{state equation} \\ \mathbf{y}(t) = \mathbf{C}\mathbf{x}(t) + \mathbf{D}\mathbf{u}(t) & \rightarrow & \textit{output equation} \end{array} \quad \rightarrow \quad \mathbf{x}(t_0) = \mathbf{x_0}, \tag{11}$$

in which $\mathbf{x(t)}$ is the n-dimensional state vector that contains internal system variables and $\dot{\mathbf{x}}(t)$ is the time-derivative on the left-hand side of Eq. (11). The vectors are represented, respectively, as follows

$$\mathbf{x}(t)^T = \left\{ \begin{array}{ccccc} x_1(t) & \dots & x_{n-1}(t) & \dots & x_n(t) \end{array} \right\}, \tag{12}$$

$$\dot{\mathbf{x}}(t)^T = \left\{ \begin{array}{ccccc} \dot{x}_1(t) & \dots & \dot{x}_{n-1}(t) & \dots & \dot{x}_n(t) \end{array} \right\}. \tag{13}$$

While $\mathbf{u}(t)$ is m-dimensional input vector that contains variables used to actuate the system and $\mathbf{y}(t)$ is p-dimensional output vector that contains the measurable quantities. The vectors are represented, respectively, as

$$\mathbf{u}(t)^T = \left\{ \begin{array}{ccccc} u_1(t) & \dots & u_{m-1}(t) & \dots & u_m(t) \end{array} \right\}, \tag{14}$$

$$\mathbf{y}(t)^T = \left\{ \begin{array}{ccccc} y_1(t) & \dots & y_{p-1}(t) & \dots & y_p(t) \end{array} \right\}. \tag{15}$$

$\mathbf{A_{n \times n}}$ is state matrix, $\mathbf{B}_{n \times m}$ is input matrix, $\mathbf{C}_{p \times n}$ is output matrix and $\mathbf{D}_{p \times m}$ is the feedforward matrix (Williams II and Lawrence, 2007)(Chen, 1998) (Hangos *et al.*, 2004).

2.4 Equations of motion in first order form

To solve the 3 systems, the RK4 and State-Space methods are implemented in MATLAB ® (2021) software. Firsty the equations Eq. (1) and Eq. (2) are transformed into a system of first-order ODEs. Case 1 is given by

$$\dot{x}_1 = x_2, \tag{16}$$

$$\dot{x}_2 = \frac{1}{m}\left[-cx_2 - kx_1 + f(t)\right], \tag{17}$$

case 2 is represented by the following set of equations

$$\dot{x}_1 = x_2, \tag{18}$$

$$\dot{x}_2 = \frac{1}{m_1}\left[-(c_1 + c_2)x_2 + c_2 x_4 - (k_1 + k_2)x_1 + k_2 x_3 + f(t)\right], \tag{19}$$

$$\dot{x}_3 = x_4, \tag{20}$$

$$\dot{x}_4 = \frac{1}{m_2}\left[c_2 x_2 - c_2 x_4 - k_2 x_3 + k_2 x_1\right], \tag{21}$$

and case 3 is composed of Eq. (22) to Eq. (27), noting that F is the external excitation force, in this case, represented by an earthquake.

$$\dot{x}_1 = x_2, \tag{22}$$

$$\dot{x}_2 = \frac{1}{m_1}\left[-c_{11}x_2 + c_{12}x_4 + c_{13}x_6 - (k_1 + k_2)x_1 + k_2 x_3 + F\right], \tag{23}$$

$$\dot{x}_3 = x_4, \tag{24}$$

$$\dot{x}_4 = \frac{1}{m_2}\left[c_{21}x_2 - c_{22}x_4 + c_{23}x_6 + k_2x_1 - (k_2+k_3)x_3 + k_3x_5\right], \tag{25}$$

$$\dot{x}_5 = x_6, \tag{26}$$

$$\dot{x}_6 = \frac{1}{m_3}\left[c_{31}x_2 + c_{32}x_4 - c_{33}x_6 + k_3x_3 - k_3x_5\right]. \tag{27}$$

Afterward, the three cases cases are solved using both methods.The numerical procedure for RK4 is shown in Fig. 3 it first defines the input parameters for each system, then the method consisting of four steps is applied and the results are plotted in terms of displacement and velocity in the time domain. The last process is applied to FFT to obtain in the frequency domain.

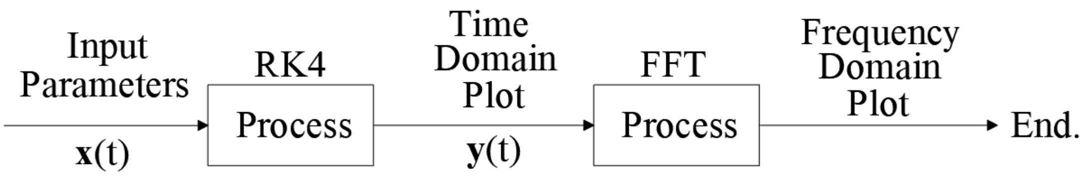

Figure 3. Schematic diagram of the 4th-order Runge-Kutta Method.

In State-Space, the initial characteristics of the RK4 method are used. In this system, the inputs $\mathbf{u}(t)$ pass through the input matrix $\mathbf{B}$ and later are added to the system represented by the matrix $\mathbf{A}$ and its state vectors having as the initial condition $\mathbf{x}(t_0) = \mathbf{x_0}$. Finally, the output equations, in which $\mathbf{C}$ is the output matrix that depends on the state variables and $\mathbf{y}(t)$ is the vector containing the responses in terms of displacement and velocity in the time domain. To obtain the response in the frequency domain, FFT is applied. The scheme of how the process was developed in the code is presented in Fig. 4.

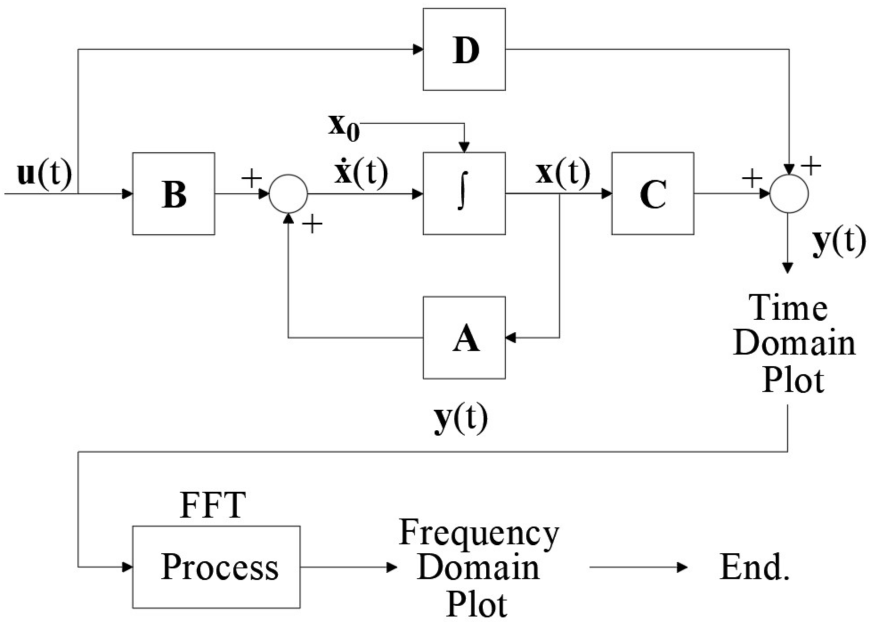

Figure 4. Schematic diagram of the State-Space Method.

The way the displacement and velocity variables are organized for the three cases is presented in Tab. 1.

The fast Fourier transform (FFT) is a computer algorithm that calculates the discrete Fourier transform (FDT). This plays a fundamental role, as it can be used to describe the relationship between the time domain and the frequency domain representation of discrete signals (Cochran *et al.*, 1967) (Brigham and Morrow, 1967) (Cooley *et al.*, 1969).

Table 1. Organization of the variables of the 4th-order Runge-Kutta and State-Space methods in terms of displacements and velocities.

Case 1		**Case 2**		**Case 3**	
SS	RK4	SS	RK4	SS	RK4
x_1	x_1	x_1	x_1	x_1	x_1
$\dot{x}_1$	$\dot{x}_1$	x_2	$\dot{x}_1$	x_2	$\dot{x}_1$
		$\dot{x}_1$	x_2	x_3	x_2
		$\dot{x}_2$	$\dot{x}_2$	$\dot{x}_1$	$\dot{x}_2$
				$\dot{x}_2$	x_3
				$\dot{x}_3$	$\dot{x}_3$

Note: x is displacement and $\dot{x}$ is velocity.

3. RESULTS

The present study proposes to dynamically analyze the application of a structure with 3 DOF receiving the excitation of an earthquake. For this, two methods were used: RK4 and State-Space. In order to verify if the methods were satisfactory to solve the application of case 3, two models were presented and compared with the literature.

Cases 1 and 2 were guided from research by Frankovský *et al.* (2013). In this study, they determined the mechanical vibration systems using MATLAB/Simulink. The results of Frankovský *et al.* (2013) through MATLAB with the 'ode45' function were compared with the results obtained using the State-Space method, as shown in Fig. 5, Fig. 6, Fig. 7 and Fig. 8, in which the first two figures refer to case 1 and consequently the last two to case 2, both presented in terms of displacement and velocity in the time domain. Furthermore, the caption "Runge-Kutta" refers to the numerical solution of Frankovský *et al.* (2013) and "State-Space" the solution proposed by the authors. The input parameters considered for both cases are found in Tab. 2 (Frankovský *et al.*, 2013).

Table 2. Input parameters.

	Case 1	**Case 2**
Mass [kg]	$m_1 = 1$	$m_1 = 75$ and $m_2 = 150$
Stiffness [N/m]	$k_1 = (2\pi)^2$	$k_1 = 500$ and $k_2 = 250$
Damping coefficient [Ns/m]	$c_1 = 0$	$c_1 = 10$ and $c_2 = 50$
Force [N]	$1\sin(\omega\ t)$	$100\sin(2t)$
ω [rad/s]	π	
t [s]	[0 20]	[0 10]

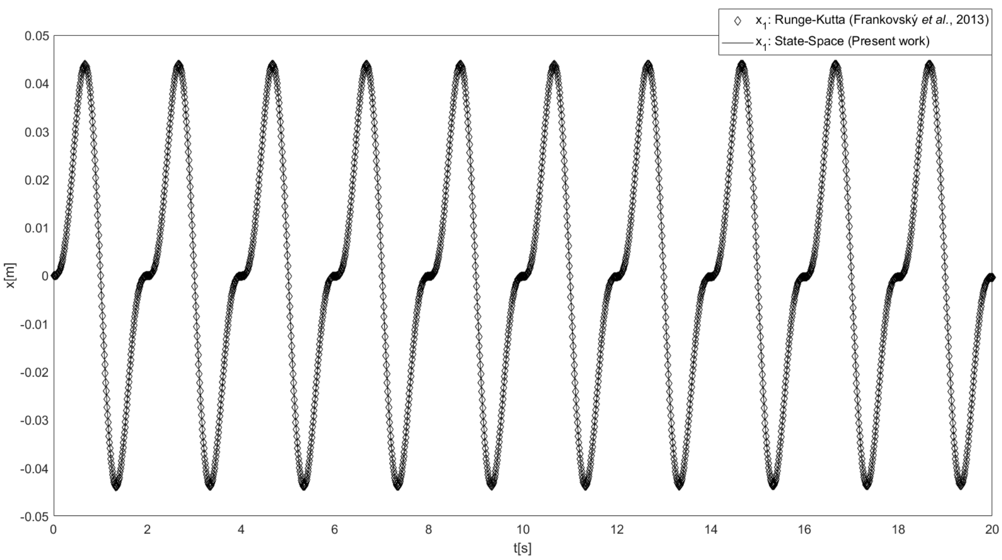

Figure 5. Comparison of the structure's dynamic response with respect to the displacement between the 4th-order Runge-Kutta and State-Space methods - case 1.

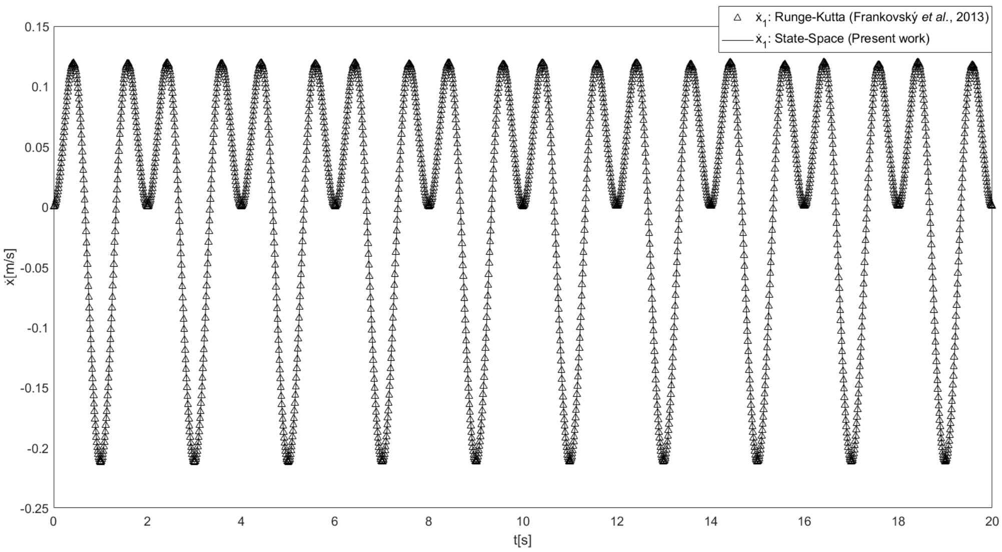

Figure 6. Comparison of the structure's dynamic response with velocity relation between the 4th-order Runge-Kutta and State-Space methods - case 1.

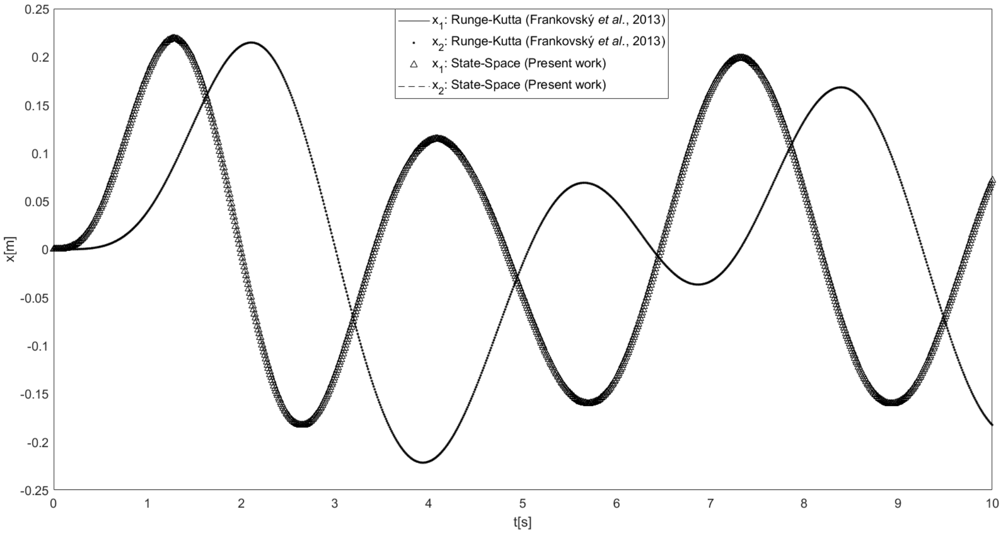

Figure 7. Comparison of the structure's dynamic response with respect to the displacement between the 4th-order Runge-Kutta and State-Space methods - case 2.

As can be seen in the figures for cases 1 and 2, the results obtained the same response, a fact identified in the plotting of the methods in which one response was over the other. Thus, it can be said that the State Space method proved to be adequate for solving dynamic systems.

The application of the RK4 and State-Space methods was used in a 3-story structural steel structure. The input parameters considered are: $m_1 = m_2 = m_3 = 0.1758$ kg; $k_1 = k_2 = k_3 = 182.0444$ N/m; height between floors (h) = 0.3 m; slab dimensions = 0.14 m × 0.2 m × $8{\times}10^{-4}$ m; pillar section = $8{\times}10^{-4}$ m × 0.012 m; Modulus of elasticity (E) = $2{\times}10^{11}$ Pa; steel density = 7850 kg/m^3. The damping matrix was obtained using the Caughey series. Finally, the excitation was the 1994 Northridge - LA earthquake shown in the Fig. 9. The data on the earthquake was obtained from the On-Line Ground-Motion Database NGA-West2 (Peer Ground Motion Database, 2013) tool.

The results of case 3 in terms of displacement by time and velocity by time are shown in Fig. 12 and Fig. 13, respectively, and in the domain of frequency are corresponding to Fig. 10 and Fig. 11. In view of the earthquake action, the greatest displacement amplitude and the greatest absolute value was on the third floor with 0.02626 m and the greatest absolute velocity amplitude was 0.348 m/s on the same floor.

The amplitudes maximum of vibrations is in the first mode and in the last floor at an amplitude of 0.003121 m with a

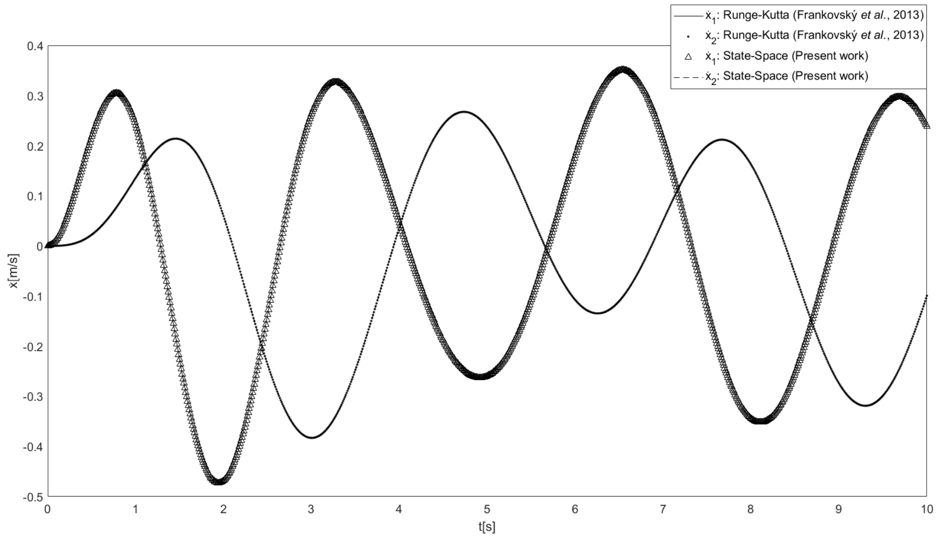

Figure 8. Comparison of the structure's dynamic response with velocity relation between the 4th-order Runge-Kutta and State-Space methods - case 2.

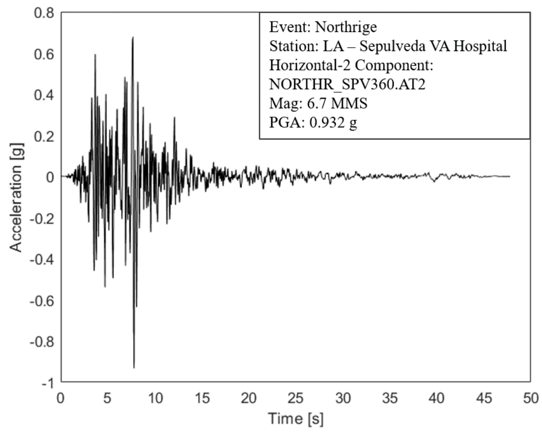

Figure 9. Earthquake Rising of Northridge - LA, in 1994. Source: Peer Ground Motion Database (2013)

frequency of 2.281 Hz considering the displacement and speed at an amplitude of 0.04472 m/s with a frequency of 2.281 Hz.

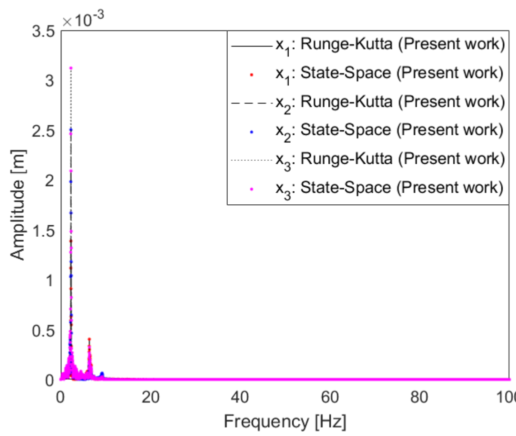

Figure 10. Response of the 3 floors of the structure in relation to displacement in the frequency domain - case 3.

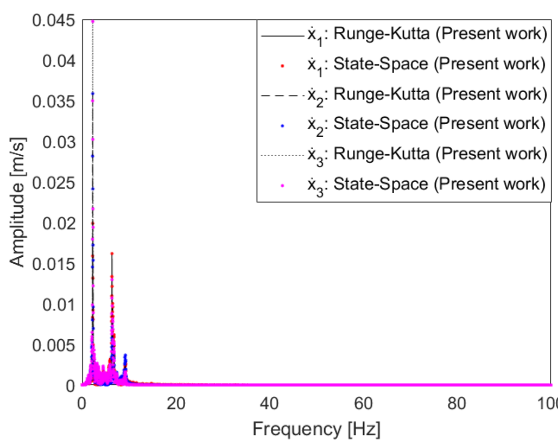

Figure 11. Response of the 3 floors of the structure in relation to velocity in the frequency domain - case 3.

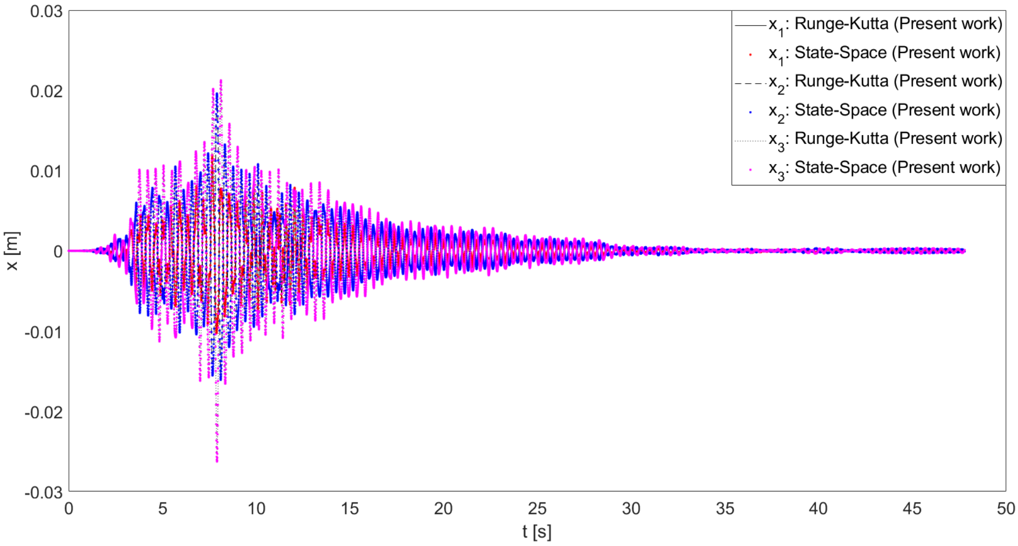

Figure 12. Comparison of the structure's dynamic response with respect to displacement between the rk4 and state-space methods - case 3.

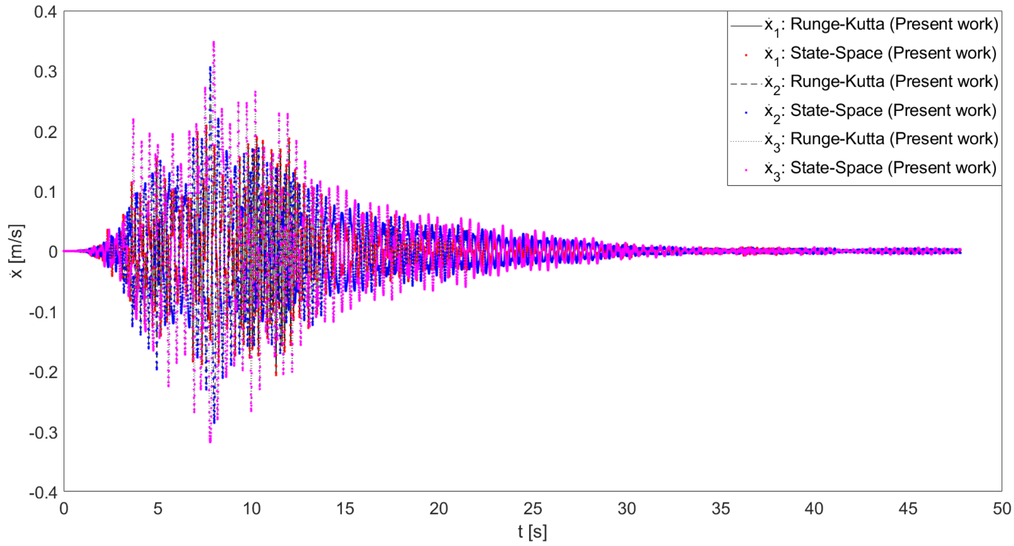

Figure 13. Comparison of the structure's dynamic response with velocity relation between the 4th-order Runge-Kutta and State-Space methods - case 3.

4. CONCLUSION

The study analyzed the dynamic behavior of three structures, which were represented by second-order equations of motion. To solve the problems, the equations were written in first-order ordinary differential equations and determined using the 4th-order Runge-Kutta and State-Space methods implemented in MATLAB ® (2021) software. The first two cases involving 1 and 2 DOFs were compared with the literature and it is possible to analyze that in both situations the results were similar. Meanwhile, the third case was an application with 3 DOFs. In this, he observed the behavior of the structure before a seismic action in which the displacements of the 3 floors of the structure were seen, as well as their respective velocities in the time domain. Furthermore, with the application of the FFT, the dynamic response of the structure in the frequency domain was obtained, making it possible to observe the maximum amplitude of vibration. The results of the latter case showed that the third floor had the greatest impact, as both in displacement and speed, the values were higher than the others.

According to the results presented in the study, I concluded that the procedures discussed to solve the 3 cases of dynamical systems involving first-order differential equations were satisfactory. This can be identified by analyzing the answers of the 3 proposed problems, and both solutions presented had their results plotted one over the other. Although the State-Space Method is more commonly used in control engineering, it proved to be efficient in the application of dynamic systems.

5. ACKNOWLEDGEMENTS

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7. RESPONSIBILITY NOTICE