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TRANSIENT RESPONSE OF SOIL-FOUNDATION SYSTEMS BY UPDATED STRUCTURAL MODAL QUANTITIES

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Abstract. *Soil behavior has a large influence on the dynamic response of the structural or mechanical systems. The dynamic modelling of unbounded domains, such as the soil, requires special techniques. In the last decades the Boundary Element Method (BEM) has established itself as a mature and efficient numerical tool to analyze response of unbounded domains. The response of systems interacting with the soil usually requires a sub-domain division, in which the soil is modelled by the BEM and the Structure by the FEM. In the present work, the stationary response of the structure is obtained by classical modal analysis procedures. This response, that may be obtained for an arbitrary number of modes, is coupled to the stationary response of the soil-foundation arrangement. This leads to modified frequency response functions (FRFs) of the structure, in which the soil-foundation influence is already taken into account. Based on these modified FRFs, new modal parameters of the coupled soil-foundation-structure system are extracted. These modal quantities are the base to determine the transient response of the coupled systems. This methodology is computationally more efficient than the interactive transient coupling of the soil and the structural system or the synthesis of the coupled system in the frequency domain followed by the application of the FFT algorithm to render the transient response.*

Keywords: *Dynamic Soil-Structure Interaction, Boundary Element Method, Transient Response, Modal Analysis*

1. INTRODUCTION

Numerical methods have become important tools for solving complex engineering problems, for example, in the case of dynamical systems with soil-structure coupling. The formulation of this type of system must include the effects of soil on the structure, and the use of the Finite Element Method (FEM) and the Boundary Element Method (BEM) combined has generated very satisfactory results, as seen in Tonyali and Ates (2018). The FEM is used to model the structure, since it is very suitable for heterogeneous and anisotropic materials, and BEM is used to describe the soil, as it presents the capacity for modelling infinite or semi-infinite domains. When using a proper auxiliary state or fundamental solution the BEM only requires the discretization of the domain boundary and it can also consider the geometric damping, i.e., the energy withdrawn from the system by outgoing and non-reflected waves.

Adolph (2001) did an analysis to obtain transient responses of two-dimensional structures coupled to a viscoelastic soil using an indirect version of the BEM. In that work Green's functions, in the frequency domain, are used to determine the stationary dynamic compliance matrix of a rigid foundation. Then, by applying the algorithm of the Fast Fourier Transform (FFT) it is possible to determine the transient responses of the soil domain.

In another study Louzada (2019a), extended the approach for determining transient responses of soil-structure systems, in which the soil response, obtained by BEM, is incorporated into the stationary response of the structure based on methodology proposed by Wu and Smith (1995). The structural response is obtained by superposition of modal quantities obtained in terms of structural displacements relative to the foundation movement (Louzada, 2019a). Also, by applying the inverse of the FFT in the frequency domain transfer function, the impulse response of the structure with soil influence is determined (Louzada *et al.*, 2019b). Thus, the general transient response is defined by the linear convolution of the impulse response function with the excitation force applied on the system.

Nowadays, there are still ongoing investigations aiming to determine the modal parameters of structural system interacting with the soil (Zangeneh *et al.*, 2021). In this work a methodology for modal analysis of soil-structure systems, based on a finite element layer model for two- and three-dimensional problems is presented, which allows the determination of the modal properties of the coupled system.

Based on the mentioned previous studies, the objective of this article is to determine transient response of soil-foundation-structures systems through direct Modal Superposition in the time domain. The modal parameters are extracted from the modified FRF curves, which include the influence of the soil. Three classical methods for extracting modal parameters from numerical or experimental FRFs are used in this study. The idea is to use the many stationary response of the soil-foundation systems already obtained by the research group of the authors to obtain a computationally efficient time domain response of the coupled soil-foundation-structures systems (Labaki *et al.*, 2021). The methodology can be extended to soil-foundation systems supported by piles. The only requirement is the stationary response of foundations arrangements, be it supported by piles, by layered soils or even embedded in the soil (Lima *et al.*, 2019). The proposed methodology has a lower computational cost than the methods used in previous studies to determine transient responses of a soil-structure system, which require a high processing capacity and large memory.

2. STATIONARY RESPONSE OF A SOIL-STRUCTURE SYSTEM

Consider the system of a structure with "n" degrees of freedom coupled to a viscoelastic half space and a rigid massless foundation, as seen in Figure 1. Presently only vertical displacement, $u_z(t)$ is analyzed. The system is divided into the structure and the soil subsystems, which are modeled by distinct methodologies and later coupled as formulated by Louzada (2019a).

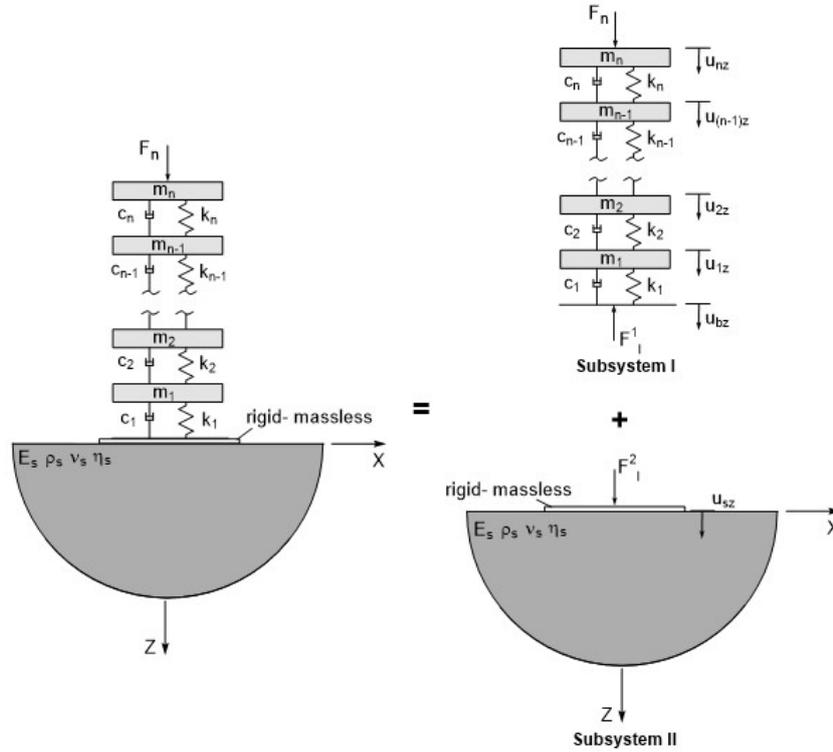


Figure 1. Decomposition of a soil-structure system.

Considering the relative displacement, $\{u_{relz}(t)\}$, of the structural degrees of freedom with respect to the base displacement, $\{u_{bz}(t)\}$, it is possible to determine the equations of motion of the subsystem I,

$$[M]\{\ddot{u}_{relz}(t)\} + [C]\{\dot{u}_{relz}(t)\} + [K]\{u_{relz}(t)\} = \{F(t)\} - [M]\{1\}\ddot{u}_{bz}(t), \quad (1)$$

where $[M]$, $[C]$, and $[K]$ are the mass, damping, and stiffness matrices, respectively, and $\{\ddot{u}_{relz}(t)\}$, $\{\dot{u}_{relz}(t)\}$, $\{u_{relz}(t)\}$, and $\{F(t)\}$ are the relative acceleration, velocity, displacement and input force vectors, respectively. After applying the Fourier Transform, and decoupling the equations of motion, using the matrix orthogonality conditions defined in Fu and He (2001), the structure response in terms of relative displacements is obtained in the frequency domain. In the sequence, the base response, $U_{bz}(\omega)$, is added and the total response of the structure is obtained, as seen in Eq. (2) (Louzada, 2019a):

$$\{U_z(\omega)\} = [\Phi][H(\omega)]([\Phi]^T\{F(\omega)\} + \omega^2[\Phi]^T[M]\{1\}U_{bz}(\omega)) + \{1\}U_{bz}(\omega), \quad (2)$$

where, $[H(\omega)]$ is the transfer function matrix of the system without the soil effect and $[\Phi]$ is the modal matrix. The dynamic response of the soil can be defined as the dynamic flexibility $S_z(\omega)$, the relating the soil-foundation response $U_{sz}(\omega)$ to the soil excitation vector $F_f^2(\omega)$ (Lima 2017):

$$U_{sz}(\omega) = S_z(\omega)F_f^2(\omega), \quad (3)$$

The coupling between the responses of the subsystems is carried out by kinematic compatibility and by the balance of forces at the soil-structure interface, resulting in the stationary response of the structure with soil influence,

$$\{U_z(\omega)\} = ([\Phi][H(\omega)]([\Phi]^T + \omega^2[\Gamma]^T\{H_s(\omega)\}) + \{1\}\{H_s(\omega)\}^T)\{F(\omega)\}, \quad (4)$$

where $\{H_s(\omega)\}$ is the vector with the stationary responses of the soil with influence of the structure and $[\Gamma]$ is the vector of generalized modal load coefficients defined by Louzada (2019a).

A great advantage of this formulation is the possibility of including the response of any soil-pile-foundation arrangement, in the frequency domain, by considering the appropriate soil-foundation impedance. In addition, this methodology allows the exclusion of any number of structural modes by removing the columns of the modal matrix, $[\Phi]$, and the rows and columns of the transfer function matrix, $[H(\omega)]$. An arbitrary number of higher structural modes may be disregarded in the formulation, resulting in a smaller system of equations. Figure 2a shows two FRFs. In the first case the structure is considered to lay on a fixed base. On the second, the structure is supported by a viscoelastic half-space. As can be seen, the soil influence on the foundation response is significant. In Figure 2a all structural vibration modes were considered. But as mentioned above, any arbitrary number of structural modes can be used to obtain the FRF with soil influence. This is shown in Figure 2b for the cases where only 1, 2 and 3 structural modes were considered to build the FRF.

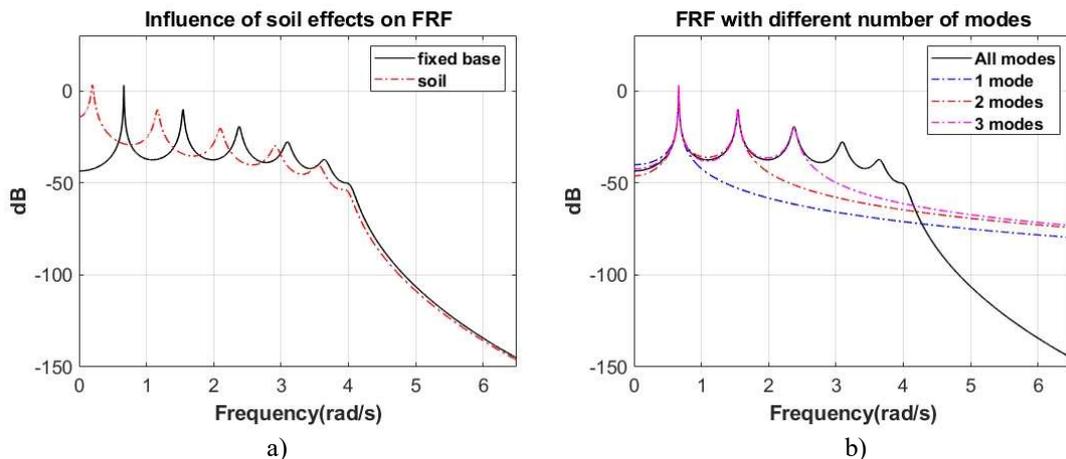


Figure 2. a) Comparison between FRF with fixed base vs FRF with soil response considered in the system; b) Representation of the FRF including soil-foundation influence and distinct number of structural modes.

3. MODAL SUPERPOSITION USING EXTRACTED MODAL QUANTITIES

To apply the modal superposition method, it is necessary to define the natural frequency, ω_n , the damping factor, ξ_n , and the modal matrix, $[\Phi]$, of the system analyzed. Thus, the following section presents some classic methods for extracting these modal parameters from the FRF. This may be applied to numerically synthesized or experimentally obtained FRFs.

According to Ewins (2000), the modal parameters extraction methods are divided into SDOF methods, in which only one resonance is analyzed at a time, i.e., each analysis only seeks to extract the properties of one of the system's modes. There are also MDOF methods, in which all or many modes are analyzed simultaneously. These methods are more powerful and generally have less user interference than SDOF methods, but present higher computational cost. In the next section, two SDOF methods and one MDOF method are presented.

3.1 Modal parameters extraction methods

The Circle Fit Method (CFM). This is a method that requires greater user interference, since each mode of showing in a FRF is analyzed individually (SDOF method) and the frequency range used in the analysis is manually determined. Based on the Nyquist format, i.e., in the complex plane, the FRF presents the formation of arcs of circles in regions close

to the natural frequency of the mode, as seen in Figure 3. Therefore, from trigonometric correlation between the modal circle and the equation of a curve FRF SDOF,

$$\tan(\theta) = \frac{1 - (\omega/\omega_n)^2}{2\xi_n}, \quad (5)$$

where θ is the angle between two points in modal circle and the natural frequency of a mode, ω_n , is defined by the pair of points that obtain the highest rate of angular variation in the modal circle (Ewins, 2000).

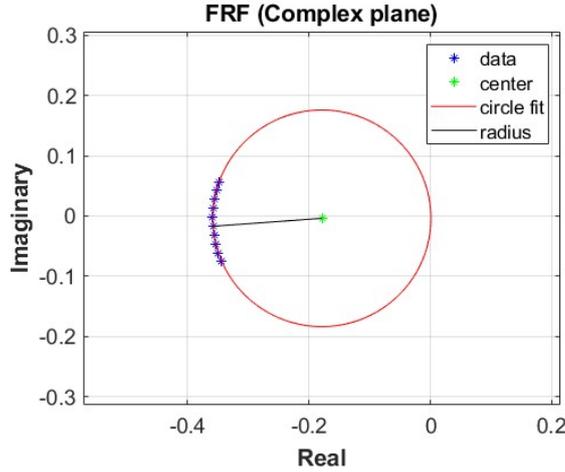


Figure 3. Modal circle of an FRF resonance.

Damping factors are determined from trigonometric correlations involving a point before, and after the resonance,

$$2\xi_n = \frac{\omega_a^2 - \omega_b^2}{2\omega_n^2(\tan(\theta_a/2) + \tan(\theta_b/2))}, \quad (6)$$

The modal constant, ${}_nA_{ij}$, used in the definition of modal forms, is defined by the relation,

$${}_nA_{ij} = 2\xi_n \omega_n^2 {}_nD_{ij}, \quad (7)$$

where ${}_nD_{ij}$ is the modal diameter, defined in the circle fit using data from the chosen frequency range, as in Figure 3.

The Inverse Method (IFM). As with the CFM, this method also considers a single mode per analysis. However, as seen in Ewins (2000), the method is based on the inverse of an FRF, which when plotted by the square of the frequency, behaves as a straight equation in the region close to the resonance of the mode, for both the real and the imaginary part, as seen in Figure 4.

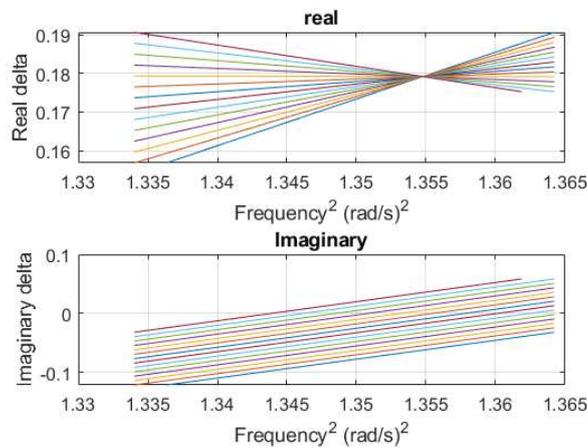


Figure 4. Set of lines obtained by the inverse of the FRF.

From the correlations established between the general equation of a line and the inverse equation of a FRF for several degrees of freedom, as seen in Eq. (8), the modal parameters are determined based on the set of angular and linear coefficients of the lines determined for each point in the analyzed frequency range, as stated in Ewins (2000).

$$\Delta(\omega) = \frac{(\omega^2 - \Omega^2)}{\alpha_{ij}(\omega) - \alpha_{ij}(\Omega)}, \quad (8)$$

where $\Delta(\omega)$ represents the inverse of the FRF, α_{ij} is the FRF analyzed and Ω is the fixed frequency.

The Rational Fraction Polynomial Method (RFPM). This is a more powerful method, which is based on high-order polynomials and all modes of an FRF are analyzed simultaneously (MDOF method). According to the formulation proposed in Richardson and Formenti (1982), the FRF can be represented as a rational fraction, as shown in Eq. (9).

$$H(\omega) = \frac{\sum_{k=0}^{2N-1} a_k(i\omega)^k}{\sum_{k=0}^{2N} b_k(i\omega)^k}, \quad (9)$$

where a_k e b_k are the indeterminate polynomial coefficients and N represents the number of FRF modes. Thus, from the minimization of the squared error criteria, the coefficients a_k and b_k are determined and it becomes possible to decompose Eq. (9) into partial fractions. Thus, the modal parameters are determined through the correlation between the poles and residues of the fractions obtained with the FRF equation.

Modal matrix. The matrix is obtained from the modal constants, as seen in the previous extraction methods, from the relation defined in Ewins (2000), as seen in Eq. (10).

$${}_n A_{ij} = \phi_{in} \phi_{jn}, \quad (10)$$

where ϕ is the component of the modal matrix and ${}_n A_{ij}$ is the modal constant of the n th mode of the component (FRF) ij of the receptivity matrix. Then, through the modal constants of the FRF jj , that is, the FRF measured at the excitation point itself, it is possible to determine the components of the mode matrix related to the j th degree of freedom, as noted in Eq. (11).

$$\phi_{jn} = \sqrt{{}_n A_{jj}}, \quad (11)$$

Therefore, Eq. (10) has only one unknown, which is the component of the modal forms relative to the other degrees of freedom, given that the modal constants of the remaining FRF curves are known.

3.2 Modal Superposition Method

The Modal Superposition Method is based on the superposition of the modal responses obtained from the system of equations of motion, and the Newmark integration was chosen to solve these equations, due to the stability of the method, as defined in Rao (2010). By the transformation to the modal coordinate of the equations of motion using the orthogonality conditions in the mass, stiffness and damping matrices, as seen in Fu and He (2001), the system of decoupled equations is obtained:

$$[I]\{\ddot{q}_z(t)\} + [2\xi_n \omega_n]\{\dot{q}_z(t)\} + [\omega_n^2]\{q_z(t)\} = [\Phi]^T \{F(t)\}, \quad (12)$$

where ξ_n and ω_n are the damping factor and the natural frequency to the n th system mode, respectively, $[\Phi]$ is the modal matrix and $\{q_z(t)\}$ is the vector of modal coordinates in z direction. Remember that modal parameters are determined using the extraction methods described in the previous section. Transient responses are obtained by using Newmark integration scheme on equations (12).

4. NUMERICAL RESULTS

In this section, the influence of soil effects on the modal parameters and transient responses of a soil-structure system, as shown in the Figure 1, with “ n ” equal to six degrees of freedom is analyzed. The influence of the number of modes considered to obtain the transient responses is also investigated. The input data used in the simulations are shown in Table 1, and the model of a stiffness-proportional damping was used to define the damping. Also, the coefficients of the Newmark integration, β and γ , are set equal to 0.5 and 0.25, respectively, ensuring greater stability for the Newmark integration method (Louzada, 2019a).

Table 1. Input data.

| Mass | Stiffness | Damping |
|-----------------|-------------------|-------------------|
| $m_1 = 31.5$ kg | $k_1 = 151.5$ N/m | $c_1 = 1.57$ kg/s |
| $m_2 = 27.0$ kg | $k_2 = 129.8$ N/m | $c_2 = 1.35$ kg/s |
| $m_3 = 22.5$ kg | $k_3 = 108.2$ N/m | $c_3 = 1.12$ kg/s |
| $m_4 = 18.0$ kg | $k_4 = 86.6$ N/m | $c_4 = 0.90$ kg/s |
| $m_5 = 13.5$ kg | $k_5 = 64.9$ N/m | $c_5 = 0.67$ kg/s |
| $m_6 = 9.0$ kg | $k_6 = 43.3$ N/m | $c_6 = 0.45$ kg/s |

The properties of the viscoelastic homogeneous half space used in the analyzes are defined in Table 2.

Table 2. Half space properties.

| | |
|----------------------------------|--------------------------------|
| Soil density | $\rho_s = 1$ kg/m ³ |
| Modulus of elasticity | $E_s = 2.5$ Pa |
| Transverse modulus of elasticity | $G_s = 1.0$ Pa |
| Poisson's ratio | $\nu_s = 0.25$ |
| Damping ratio | $\eta = 0.01$ |

The excitation force, $F_6(t)$, is defined as a constant force applied for a short period of time on the structure, as in Figure 5.

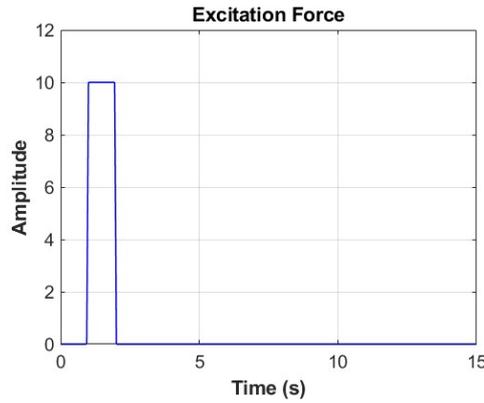


Figure 5. System excitation force.

4.1 Soil effects on modal parameters

For comparison purposes, the modal parameters of the structure without soil effects are determined, as can be seen in Table 3, by solving the eigenvalue problem defined in Fu and He (2001), in which the structure is on a fixed base.

Table 3. Modal parameters for fixed base structure.

| Natural Frequencies | Damping Factors | Modal Matrix |
|-------------------------|--------------------|--|
| $\omega_1 = 0.66$ rad/s | $\zeta_1 = 0.0034$ | $\begin{bmatrix} 0.029 & -0.064 & 0.089 & -0.095 & 0.082 & -0.057 \\ 0.059 & -0.102 & -0.070 & 0.015 & -0.088 & 0.102 \\ 0.089 & -0.087 & -0.051 & 0.111 & 0.001 & -0.120 \\ 0.116 & 0.014 & -0.127 & -0.046 & 0.109 & 0.108 \\ 0.138 & 0.092 & 0.030 & -0.133 & -0.149 & -0.076 \\ 0.152 & 0.183 & 0.169 & 0.134 & 0.084 & 0.032 \end{bmatrix}$ |
| $\omega_2 = 1.54$ rad/s | $\zeta_2 = 0.0080$ | |
| $\omega_3 = 2.38$ rad/s | $\zeta_3 = 0.0123$ | |
| $\omega_4 = 3.10$ rad/s | $\zeta_4 = 0.0160$ | |
| $\omega_5 = 3.65$ rad/s | $\zeta_5 = 0.0189$ | |
| $\omega_6 = 4.03$ rad/s | $\zeta_6 = 0.0209$ | |

Thus, through the methodology described in section 2, it is possible to determine the FRF curves with the inclusion of soil effects. A typical normalized flexibility, $S_z(\omega)$, as defined in Eq. (3), for a massless foundation resting on the homogeneous viscoelastic half space is given in Figure 6. The results were obtained by Labaki (2012).

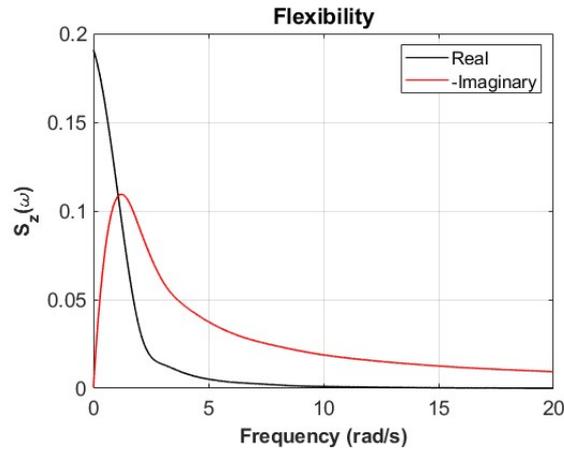


Figure 6. Typical normz dynamic flexibility of a massless foundation on a half space.

The modal parameters are determined by applying the methods, described in section 3.1, to the FRF curves modified by soil effects. The results are shown in Table 4.

Table 4. Modal parameters extracted with inclusion of soil effects.

| Method | Natural Frequencies | Damping Factors | Modal Matrix |
|--------|-------------------------|--------------------|--|
| CFM | $\omega_1 = 0.20$ rad/s | $\zeta_1 = 0.0956$ | $\begin{bmatrix} 0.087 & -0.093 & 0.085 & -0.074 & 0.061 & -0.064 \\ 0.090 & -0.065 & -0.006 & 0.076 & -0.121 & 0.171 \\ 0.092 & -0.011 & -0.106 & 0.093 & 0.042 & -0.204 \\ 0.093 & 0.061 & -0.112 & -0.090 & 0.102 & 0.157 \\ 0.094 & 0.134 & 0.016 & -0.118 & -0.164 & -0.082 \\ 0.095 & 0.186 & 0.183 & 0.148 & 0.091 & 0.027 \end{bmatrix}$ |
| | $\omega_2 = 1.16$ rad/s | $\zeta_2 = 0.0204$ | |
| | $\omega_3 = 2.10$ rad/s | $\zeta_3 = 0.0188$ | |
| | $\omega_4 = 2.92$ rad/s | $\zeta_4 = 0.0190$ | |
| | $\omega_5 = 3.56$ rad/s | $\zeta_5 = 0.0194$ | |
| | $\omega_6 = 3.99$ rad/s | $\zeta_6 = 0.0233$ | |
| IFM | $\omega_1 = 0.20$ rad/s | $\zeta_1 = 0.0971$ | $\begin{bmatrix} 0.087 & -0.092 & 0.086 & -0.075 & 0.066 & -0.091 \\ 0.090 & -0.065 & -0.006 & 0.077 & -0.130 & 0.240 \\ 0.092 & -0.012 & -0.106 & 0.095 & 0.042 & -0.276 \\ 0.093 & 0.061 & -0.113 & -0.089 & 0.111 & 0.196 \\ 0.095 & 0.133 & 0.015 & -0.120 & -0.170 & -0.090 \\ 0.095 & 0.185 & 0.183 & 0.146 & 0.086 & 0.021 \end{bmatrix}$ |
| | $\omega_2 = 1.16$ rad/s | $\zeta_2 = 0.0204$ | |
| | $\omega_3 = 2.10$ rad/s | $\zeta_3 = 0.0187$ | |
| | $\omega_4 = 2.92$ rad/s | $\zeta_4 = 0.0193$ | |
| | $\omega_5 = 3.56$ rad/s | $\zeta_5 = 0.0197$ | |
| | $\omega_6 = 4.00$ rad/s | $\zeta_6 = 0.0201$ | |
| RFPM | $\omega_1 = 0.20$ rad/s | $\zeta_1 = 0.0958$ | $\begin{bmatrix} 0.087 & -0.092 & 0.084 & -0.070 & 0.052 & -0.033 \\ 0.089 & -0.065 & -0.005 & 0.073 & -0.106 & 0.092 \\ 0.091 & -0.012 & -0.105 & 0.089 & 0.040 & -0.124 \\ 0.093 & 0.061 & -0.112 & -0.088 & 0.091 & 0.122 \\ 0.094 & 0.133 & 0.016 & -0.116 & -0.160 & -0.090 \\ 0.095 & 0.185 & 0.184 & 0.151 & 0.098 & 0.039 \end{bmatrix}$ |
| | $\omega_2 = 1.16$ rad/s | $\zeta_2 = 0.0204$ | |
| | $\omega_3 = 2.10$ rad/s | $\zeta_3 = 0.0185$ | |
| | $\omega_4 = 2.92$ rad/s | $\zeta_4 = 0.0193$ | |
| | $\omega_5 = 3.56$ rad/s | $\zeta_5 = 0.0203$ | |
| | $\omega_6 = 3.99$ rad/s | $\zeta_6 = 0.0213$ | |

First, there is a good approximation among the modal parameters extracted by the three methods. Comparing the parameters in Tables 3 and 4, it is possible to notice a decrease in the values of natural frequencies and an increase in the structure's damping factors due to the inclusion of soil in the system. Furthermore, it appears that there is a greater influence of the soil on the modal parameters for the low frequencies. It is also noted that the modal forms do not change with the inclusion of soil effects.

4.2 Soil effects on transient response

For numerical validation of the obtained results, a reference the transient response is used. This solution is determined by the convolution between the excitation force, $F_6(t)$, and the impulse response of the system, which is defined by applying the inverse of the Fourier Transform to the transfer function, i.e., on the same FRF curves used for modal parameters extraction.

Next, the Modal Superposition method, described in section 3.2, is used to determine the transient response of the coupled soil-structure system by using the modal parameters extracted from the three methods. Figure 7 shows a comparison between the transient reference responses and the responses obtained by the Modal Superposition method. It should be noted that there is a good agreement among the displayed responses. It should also be noticed that the modal

superposition method delivers transient responses at a much lower computational cost and avoids the use of the FFT algorithm.

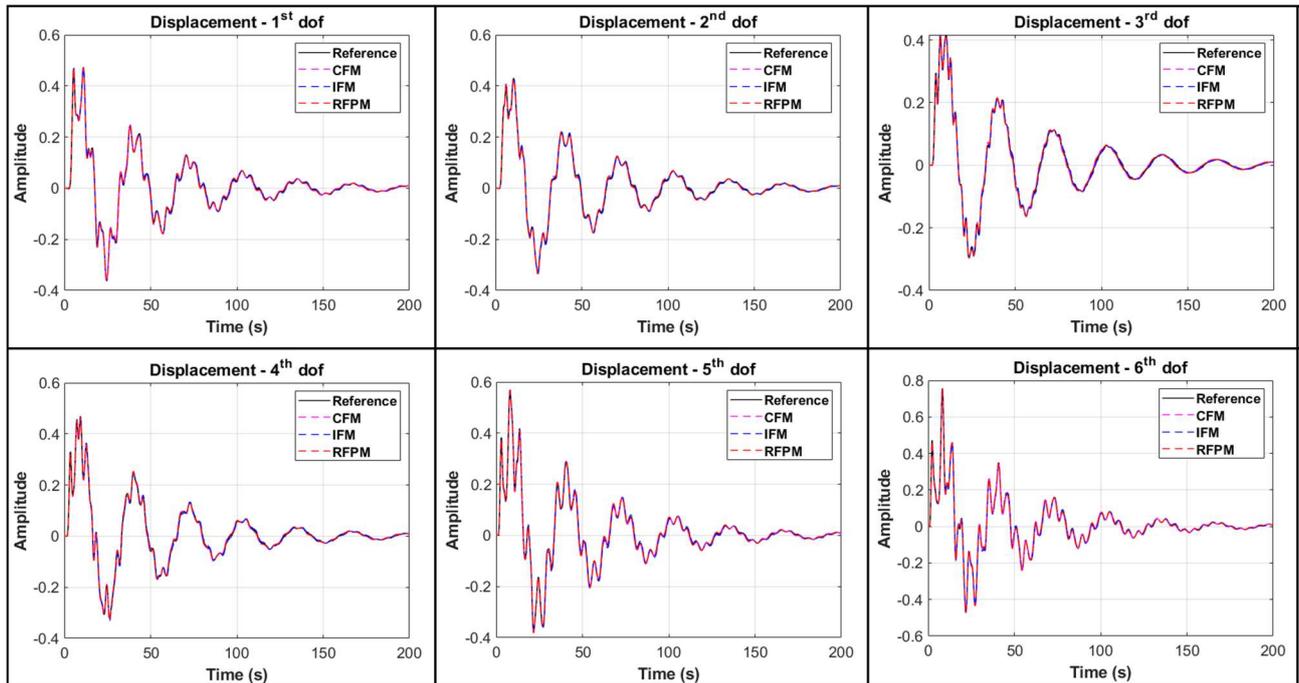


Figure 7. Transient responses for the studied extraction methods compared to the reference transient response.

The L2 norm is calculated between the transient responses obtained by the extraction methods and the response used as a reference. So, the L2 error measure obtained for each method is shown in Table 5. The better results of the RFPM extraction method is due to its greater robustness and the non-interference of the user in determining the modal parameters, which generates a possible new source of errors.

Table 5. L2 norm for the three extraction methods.

| Method | L2 norm |
|--------|----------|
| CFM | 2.29e-06 |
| IFM | 2.44e-06 |
| RFPM | 1.90e-06 |

4.3 Influence of the number of modes in the transient response

Generally, the analyzed soil-structure systems have many degrees of freedom, which generates high computational cost to obtain the transient response. Therefore, in this section the influence of using only some of the structural modes in the determination of transient response is analyzed. As mentioned in section 2 the current methodology allows the exclusion of FRF modes. The responses are defined, considering 1, 2 and 3 structural modes, by using the Modal Superposition method with the modal parameters extracted by the RFPM. In Figure 8, the transient responses with all modes included is used as a reference for comparison with the responses that include the first 3 modes of the modified FRFs.

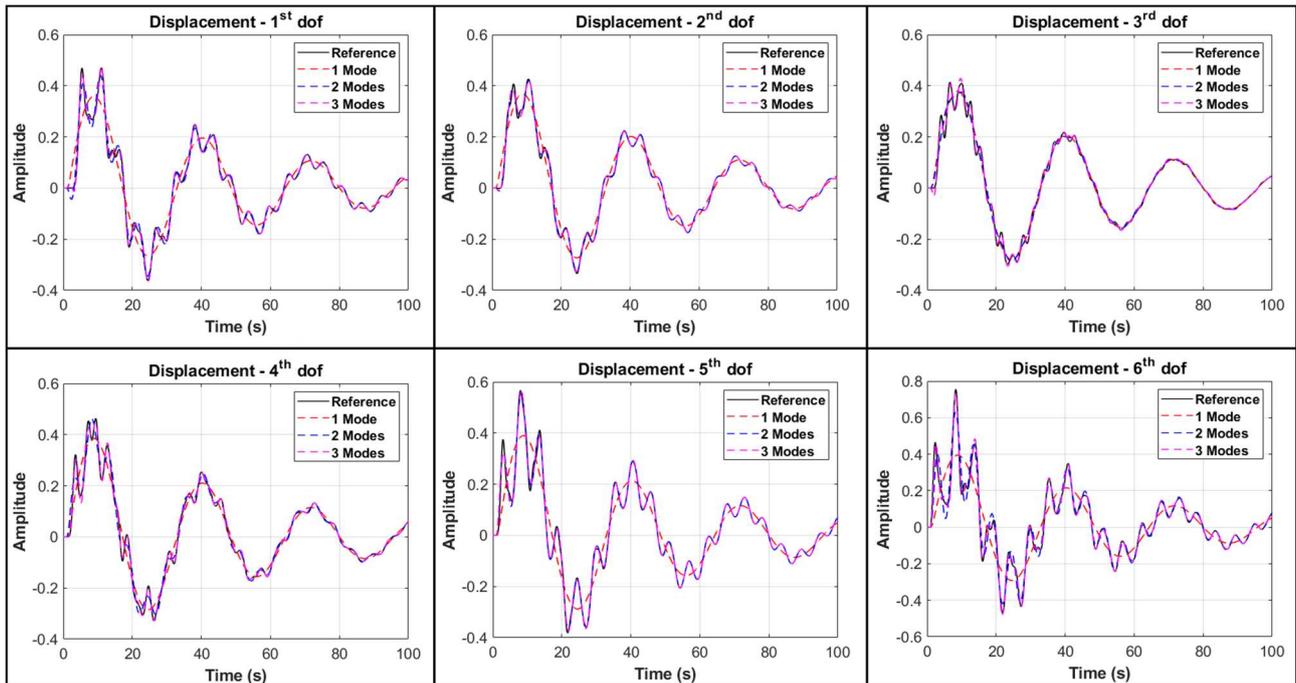


Figure 8. Transient responses for 1, 2 and 3 structural modes in the analysis compared to the case with all modes included.

In Figure 8, it can be seen that for the present analysis, a good convergence is already obtained for cases where only 2 and 3 structural modes are considered in the analysis. It also shows a large influence of the first FRF modes in the composition of the transient response of the system. In order to measure the approximation between the obtained responses, the L2 norm was calculated and is shown in Table 6. As expected, the best result was for the case with three modes in the analysis.

Table 6. L2 norm for the influence of the number of modes in the analysis.

| Number of modes | L2 norm |
|-----------------|----------|
| 1 | 2.01e-05 |
| 2 | 7.43e-06 |
| 3 | 3.14e-06 |

5. CONCLUSION

First, the extraction of modal parameters from modified FRFs by the inclusion of soil effects was satisfactory obtained for the three methods investigated. Furthermore, the validation of the modal superposition method is confirmed by the comparison of the transient responses rendered by this methodology in comparison with the reference transient response. This also reinforces the validation of the extraction methods, because the extracted modified modal parameters are used to obtain the transient responses.

The proposed methodology presented a good accuracy, satisfactory efficiency, and lower computational cost in relation to the methodologies generally used in obtaining transient responses. The possibility of considering only the first few structural modes in the analysis, may render this procedure very efficient for the transient analysis of large structural systems interacting with the soil. Furthermore, it is a very versatile methodology, due to the possibility of application to any soil-pile-foundation scheme when its response is known in the frequency domain. The methodology can also be used to synthesize transient responses of soil-foundation-structural systems, based on experimentally obtained modal quantities.

For the best of the authors' knowledge, this technique has not been applied to synthesize the transient response of coupled soil-foundation-structural system

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