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EVALUATION OF SPECTRAL MODELS FOR DIFFERENT LEVELS OF TURBULENT INTENSITY IN A THREE-DIMENSIONAL DOMAIN

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Abstract. Turbulence-radiation interaction (TRI) is a complex phenomenon, where turbulent fluctuations affect the mean radiation field due to the highly non-linear nature of radiation, leading to what can be large errors in the predicted mean radiative quantities. The objective of the present paper is to study the effect of turbulent fluctuations on the accuracy of spectral models. The accuracy of the weighted-sum-of-gray-gases (WSGG) model are evaluated by comparison with a benchmark provided by line-by-line (LBL) integration. Turbulence is introduced in the analyses through a semi-empirical correction to the mean emission, while absorption-TRI is altogether neglected. The test cases consist of three-dimensional, non-coupled calculations, based on predefined temperature and species concentrations representative of realistic combustion scenarios for different levels of turbulence intensity. The participating medium is composed of typical combustion products, carbon dioxide and water vapor (the effect of soot is neglected). The results show that the presence of turbulent fluctuations substantially increases the radiative source term (S_r) and the radiative heat flux (q''_R). The average errors associated with the WSGG model are approximately 6% and 2% respectively. The present study contributes to the investigation of turbulence effects in reactive flows for a three-dimensional domain.

Keywords: turbulence-radiation-interaction, thermal radiation, spectral models, turbulent intensity, temperature fluctuations.

1. INTRODUCTION

The phenomenon of turbulence-radiation interaction (TRI) originates from the coupling between the effects of turbulence and radiation. Turbulent fluctuations play an essential role in representing the effects of turbulence, which are often recklessly neglected, causing errors between the values obtained computationally and the values obtained in practice (Gupta *et al.*, 2013; Coelho, 2007).

A major challenge in radiation exchange modeling is to capture how the radiative properties of a participating medium vary with the radiation spectrum. In order to cover different configurations, new spectral models are being developed. However, considering that turbulence affects different regions of the radiation spectrum in different ways (Hall and Vranos, 1994), as a consequence of this, the performance of these spectral models may change when evaluated in the presence or absence of turbulent fluctuations.

Turbulent fluctuations originate from the temporal decomposition of Reynolds, evaluating a decomposed term obtains mean values and fluctuating values, as an example the decomposition of the term related to temperature $T = \bar{T} + T'$. Average terms describe the main part of the term, representing the portion that can be predicted, while fluctuations represent the random part of the problem, often treated using statistical methods. (Wilcox, 2006).

Solving problems involving TRI requires the use of the radiative transfer equation (RTE), which establishes how the radiation intensity varies in a given path of a participating medium. For the solution, RTE for participating media needs spatial and spectral integrations, the spatial problem is solved through methods such as the finite volume method (FVM), while the spectral problem is solved through spectral models (Modest and Haworth, 2016). Spectral models resolve the different regions of the spectrum. The present work makes use of the line-by-line integration method (LBL) and the weighted-sum-of-grey-gases (WSGG) model. The former has the function of solving the line-by-line spectrum, while the latter replaces the hundreds of thousands of lines with a set of gray gases j (Cassol *et al.*, 2014; Dorigon *et al.*, 2013).

Recently Krishnamoorthy and collaborators developed a series of studies (Krishnamoorthy, 2012, 2010; Krishnamoorthy *et al.*, 2005a,b), which attempted to incorporate the effect of turbulent fluctuations in the evaluation of non-gray and gray gas radiation model. The effects of TRI were considered only in radiative emission, modeled using a simple approximation developed by Snegirev (2004). The results of these studies were evaluated in comparison with results obtained experimentally.

Differently from Krishnamoorthy's previous work, the present work aims to evaluate the accuracy of different WSGG models in the presence of different levels of turbulence intensity. As a form of validation, the method of line-by-line (LBL) integration was also simulated and used as a reference. The WSGG formulation of Dorigon *et al.* (2013) was simulated in a three-dimensional domain, according to the configurations used in the study of Fraga *et al.* (2019b).

2. THEORETICAL BASIS

2.1 Thermal Radiation in participating media

The variation of the radiative spectral intensity I_η along a path s is computed using the radiative transfer equation (RTE). For a non-scattering medium, this equation is described as (Modest, 2013)

$$\frac{dI_\eta}{ds} = -\kappa_\eta I_\eta + \kappa_\eta I_{b\eta}, \quad (1)$$

where $I_{b\eta}$ is the spectral blackbody radiation intensity or Planck function, κ_η is the spectral absorption coefficient, and η denotes the dependence on the radiation spectrum. In addition to the scattering phenomenon, neglected in this study, the RTE for participating media considers the radiative energy locally absorbed and emitted by the medium, respectively, as shown on the right-hand side of Eq. (1). The Eq. (1) has its spectrally integrated form as shown below

$$\frac{dI}{ds} = -\kappa_G I + \kappa_P I_b. \quad (2)$$

Analyzing Eq. (2) we have, I and I_b are the total local intensity and blackbody intensity (i.e., $I = \int_0^\infty I_b d\eta$ and $I_b = \int_0^\infty I_{b\eta} d\eta$), and κ_G and κ_P represent respectively the incident-mean and Planck-mean absorption coefficients.

2.2 Line-by-line (LBL) integration method

The line-by-line integration (LBL) method is considered to be the most accurate spectral model when compared to practical cases. If the dependence of the spectral absorption coefficient κ_η on the local thermodynamic state and on the radiation wavenumber is known, the total intensity is determined through the integral of the spectral intensity, resulting from each of the lines of the spectrum.

As it is considered the most accurate solution approach for the spectral part of the radiative transfer problem, the LBL integration method is often used as a benchmark for evaluations of other global spectral models. The LBL method uses a spectral database. The information related to the spectral absorption coefficient are taken from the HITEMP 2010 database (Rothman *et al.*, 2010). The absorption spectra of water vapor (H_2O) and carbon dioxide (CO_2) are extracted for 150 000 spectral intervals between $\eta = 0 \text{ cm}^{-1}$ and $\eta = 10^4 \text{ cm}^{-1}$, yielding a spectral resolution of 0.067 cm^{-1} .

The absorption spectra for H_2O are constructed in partial pressures of 0.01 atm, 0.1 atm, 0.2 atm and 0.4 atm, where the total pressure is always defined as 1.0 atm, this values are obtained through linear interpolations. For the absorption spectra referring to the CO_2 species, any partial pressure is obtained from the linear interpolation of the data produced in 0.1 atm (Howell *et al.*, 2016). The spectral data is generated for temperatures between 400 K and 2400 K, in intervals of 100 K, and the linear interpolation is again adopted for intermediate temperatures. More information on the procedure for obtaining the absorption lines H_2O and CO_2 is provided in Fraga *et al.* (2019a).

2.3 The weighted-sum-of-gray-gases (WSGG) model

Differently from the LBL integration method, the WSGG model replaces the absorption spectrum of a participating medium by a small set of J gray gases with constant absorption coefficient, these occupy non-contiguous spectral ranges and transparent windows with null absorption coefficient. Considering the sentence above, the integration of Eq. (1) along the spectral range corresponding to a single gray gas given by (Modest, 1991)

$$\frac{dI_j}{ds} = -\kappa_j I_j + \kappa_j a_j I_b, \quad (3)$$

where I_j represents the partial intensity of the gas j , κ_j is the absorption coefficient and a_j the emission weighting coefficient, which represents the fraction of the blackbody energy that falls within the spectral ranges corresponding to the gas j . After obtaining the radiative intensity for each gray gas in the Eq. (3) (and also for the transparent windows, to which the index $j = 0$ is assigned), the total radiative intensity is obtained through the sum of all partial intensities, given by

$$I = \sum_{j=0}^J I_j. \quad (4)$$

The dependence of the terms κ_j and a_j on the local thermodynamic state of the medium is determined by adjusting the total emittance considering some reference data. Various WSGG formulations, differing in how this dependence is expressed, have been developed so far, generally applicable to a specific group of applications, eg air-fuel combustion under atmospheric conditions, oxy-fuel combustion, soot flames.

In the present work, the formulation of Dorigon *et al.* (2013) is used. This formulation is applicable to the combustion of methane-air, where the ratio between the molar fractions is considered fixed.

2.4 Turbulence-radiation interaction (TRI)

The turbulence-radiation interaction (TRI) arises from the non-linear coupling between temperature fluctuations, medium composition and radiation intensity fluctuations (Coelho, 2007). The terms referring to fluctuations represent the effects of turbulence. These fluctuations arise through the temporal decomposition of Reynolds, where the terms are decomposed in average and fluctuating terms (Wilcox, 2006). Considering the average time of Eq. (1), we obtain the equation below, where the time-averaging operation is represented by the overbar.

$$\frac{d\overline{I_\eta}}{ds} = -\overline{\kappa_\eta I_\eta} + \overline{\kappa_\eta I_{b\eta}}, \quad (5)$$

The correlations that appear on the right-hand side of this equation ($\overline{\kappa_\eta I_\eta} \neq \overline{\kappa_\eta} \overline{I_\eta}$ and $\overline{\kappa_\eta I_{b\eta}} \neq \overline{\kappa_\eta} \overline{I_{b\eta}}$) often cannot be neglected when the turbulent fluctuations are present in calculations. The values of both $\overline{\kappa_\eta}$ and $\overline{I_{b\eta}}$, may differ significantly from the values evaluated in the mean temperature and mean medium composition. When combined explain the large errors that occur when the mean radiation field is solved.

In order to reduce the complexity of the problem while still partially capturing the TRI effects, this paper invokes the optically thin fine approximation (OTFA) to model the term $\overline{\kappa_\eta I_\eta}$. The use of this approximation simplifies the average absorption term by neglecting the cross-correlation between the absorption coefficient and the local intensity, resulting in $\overline{\kappa_\eta I_\eta} = \overline{\kappa_\eta} \overline{I_\eta}$. The validity for the use of OTFA was verified for a number of turbulent non-sooty flames (Coelho, 2007).

Based on Cox (1977), Snegirev (2004), proposes a modification in the RTE model, this model is adopted in the present study, where the emission term, originally introduced to approximate the average total emission and given as

$$\overline{\kappa_P I_b} = \overline{\kappa_P} I_b(\overline{T}) \left(1 + 6C_{TRI,1} \frac{\overline{T'^2}}{\overline{T}^2} + 4C_{TRI,2} \frac{\overline{T'^2}}{\overline{\kappa_P T}} \left. \frac{\partial \kappa_P}{\partial T} \right|_{\overline{T}} \right), \quad (6)$$

where the term $I_b(\overline{T})$ represents the total blackbody intensity evaluated at the local mean temperature, while $C_{TRI,1}$ ($= 2.5$) and $C_{TRI,2}$ are the constants of the model. As it was done in previous applications of Snegirev's model—as, for example, in the series of studies by Krishnamoorthy and collaborators (Krishnamoorthy, 2012, 2010; Krishnamoorthy *et al.*, 2005b,a)—the last term of Eq. (6) is neglected by setting $C_{TRI,2} = 0$. Considering the simplifications, the RTE in its mean spectral form is given by

$$\overline{\kappa_\eta I_{b\eta}} = \overline{\kappa_\eta} I_{b\eta}(\overline{T}) \underbrace{\left(1 + 6C_{TRI,1} \frac{\overline{T'^2}}{\overline{T}^2} \right)}_{\beta}, \quad (7)$$

where the Planck function evaluated at the mean temperature is represented by $I_{b\eta}(\overline{T})$. The β term represents an emission-TRI correction factor, which is represented by the terms in parentheses.

The final form of the time-averaged (spectral-based) RTE solved in this study is given as

$$\frac{d\overline{I_\eta}}{ds} = -\overline{\kappa_\eta} \overline{I_\eta} + \beta \overline{\kappa_\eta} I_{b\eta}(\overline{T}). \quad (8)$$

Considering the structure of the formulation of the WSGG models mentioned in the previous topics of this work, and considering the Eq. (8), the RTE time average for the WSGG model is similarly obtained by the time average Eq. (3):

$$\frac{d\overline{I_j}}{ds} = -\overline{\kappa_j} \overline{I_j} + \overline{\kappa_j} a_j \overline{I_b}. \quad (9)$$

Following similar modeling approaches as those adopted for Eq. (5), the above equation becomes

$$\frac{d\overline{I_j}}{ds} = -\overline{\kappa_j} \overline{I_j} + \beta \overline{\kappa_j} a_j(\overline{T}) I_b(\overline{T}), \quad (10)$$

where $a_j(\overline{T})$ is the weighting coefficient evaluated at the mean temperature. In the present calculations the terms $\overline{\kappa_j}$, and $\overline{\kappa_\eta}$ in Eq. (8), are evaluated at the mean temperature and mean medium composition.

3. METHODOLOGY

The present work aims to test and analyze the accuracy of spectral models for different levels of turbulence intensity (TRI effect levels). The radiative transfer equation (RTE) was solved using the finite volume method (FVM), following the work sequence Gomes *et al.* (2020b,a) where studies were carried out for the line-of-sight (LOS) and one-dimensional (1D) domains. The present study represents a step beyond those mentioned, where the effects of TRI will be evaluated for a three-dimensional (3D) domain. The radiative source term (S_r) and the radiative heat flux (q''_R) for a position s were calculated from equations applied to the finite volume method, which is the method used in the realized solution through the open source software fluid dynamics simulator, for more information on the equation, see McGrattan *et al.* (2018).

The WSGG formulation of Dorigon *et al.* (2013), and the line-by-line integration method are simulated for five different levels of TRI effects, characterized by temperature fluctuations (or turbulent intensities, TI) of 0 %, representing the absence of TRI, and 20 %, 40 %, 60 % and 80 % representing the presence of TRI in the flow. To introduce turbulence effects into the radiative field, the temperature variance is calculated by artificially introducing a turbulent intensity (represented in the equation by the term TI and a temperature field (T), which is then inserted into the β term as shown below

$$T'^2 = ((TI)T)^2, \quad (11)$$

$$\beta = 1 + 6C_{TRI,1} \frac{T'^2}{T^2}, \quad (12)$$

This work is based on the Fraga *et al.* (2019b) study, for the simulations, the same domain and the same temperature profiles were used, as well as the profiles of the molar concentrations of the chemical species considered, water vapor (H_2O) and carbon dioxide (CO_2). The domain has the following dimensions 2 m, 2 m and 4 m for the x, y and z axes respectively, the origin of the x and y coordinates are at half the distance, while at the z coordinate, the origin is at the bottom base (at 0 m).

Fraga *et al.* (2019b), simulated six different cases for the three-dimensional domain. Of these six, only the first two were reproduced in this study. The temperature profiles used for the simulated cases are represented by

$$T(x, y, z) = \begin{cases} (T_0 - 800 \text{ K})(1 - 3r^2 + 2r^3) + 800 \text{ K}, & \text{if } r \leq 1 \text{ m} \\ 800 \text{ K}, & \text{if } r > 1 \text{ m} \end{cases} \quad (13)$$

where the T_0 term represents the temperature at the centerline, and the r term is the distance from the centerline of the domain. These two terms are calculated using the equations below

$$T_0(z) = \begin{cases} 400 \text{ K}(1 + \frac{3z}{0.375 \text{ m}}), & \text{if } z \leq 0.375 \text{ m} \\ 400 \text{ K}(4.5 - 2.5 \frac{z-0.375 \text{ m}}{3.625 \text{ m}}), & \text{if } z > 0.375 \text{ m} \end{cases}, \quad (14)$$

$$r = \sqrt{x^2 + y^2}, \quad (15)$$

For the molar concentrations of CO_2 and H_2O , a ratio equal to 2 was considered between the fractions of chemical species ($Y_w/Y_c = 2$). The profiles of molar concentrations used are shown according to shown below

$$Y_c(x, y, z) = 0.1, \quad (16)$$

$$Y_c(x, y, z) = \begin{cases} Y_{c,0}(1 - 3r^2 + 2r^3), & \text{if } r \leq 1 \text{ m} \\ 0, & \text{if } r > 1 \text{ m} \end{cases}, \quad (17)$$

where the term $Y_{c,0}$ is the molar fraction of carbon dioxide (CO_2) at the centerline of domain, this variable can be calculated using the Eq. (18)

$$Y_{c,0}(z) = \begin{cases} 0.1(\frac{z}{0.375 \text{ m}}), & \text{if } z \leq 0.375 \text{ m} \\ 0.1(1 - \frac{z-0.375 \text{ m}}{3.625 \text{ m}}), & \text{if } z > 0.375 \text{ m} \end{cases}, \quad (18)$$

The simulated cases with the equations used for temperature profile and molar concentration of chemical species can be verified with the help of Table 1

4. RESULTS AND DISCUSSION

As already mentioned, two spectral models were simulated for a three-dimensional (3D) domain, the line-by-line (LBL) integration method and the weighted-sum-of-gray-gases (WSGG) formulation of Dorigon *et al.* (2013). To represent the

Table 1. Simulated cases (according to Fraga *et al.*, 2019b)

Case	1	2
Temperature profile	Eq. (13)	Eq. (13)
Molar concentration profile	Eq. (16)	Eq. (17)

Table 2. Differences between LBL and Dorigon *et al.* (2013) model (WSGG) for different levels of temperature fluctuation intensity

	Case 1		Case 2	
	$\Delta_{max}(\%)$	$\Delta_{avg}(\%)$	$\Delta_{max}(\%)$	$\Delta_{avg}(\%)$
Radiative heat source (0 m, 0 m, z):				
T.I. = 0 %	7.60	1.99	7.20	1.46
T.I. = 20 %	7.62	2.00	7.24	1.47
T.I. = 40 %	7.64	2.01	7.26	1.49
T.I. = 60 %	7.65	2.02	7.28	1.50
T.I. = 80 %	7.65	2.02	7.29	1.50
Radiative heat flux (-1 m, 0 m, z):				
T.I. = 0 %	9.11	5.78	3.99	1.86
T.I. = 20 %	9.26	5.94	4.11	1.94
T.I. = 40 %	9.40	6.07	4.21	2.01
T.I. = 60 %	9.45	6.13	4.25	2.04
T.I. = 80 %	9.48	6.15	4.27	2.06

effects of turbulence in the radiative field, turbulent fluctuations at different levels of intensity were used (0 %, 20 %, 40 %, 60 % and 80 %).

In order to analyze the behavior presented by the spectral models when subjected to the presence of different levels of turbulent fluctuations, the radiative source term (S_r) and the radiative heat flux (q''_R) were analyzed, the first was evaluated for the central axis of the domain along the z axis, while the second was evaluated at a position in the center of the wall along z (-1 m, 0 m, z). The results obtained are shown according to Table 2. In addition to the Table 2, the results are graphically displayed according to Figs. 1 and 2, which represent case 1 and case 2 respectively.

The results obtained demonstrate that the WSGG formulation of Dorigon *et al.* (2013) presented good accuracy when compared to the LBL method. The values referring to the radiative source term, for both case 1 and case 2, presented maximum error values lower than 7.65 %, and an average between the average errors of 2 % in case 1, and 1.48 % in case 2.

The values for the radiative heat flux in the wall in case 1 were shown to be higher than those for case 2. The first presented an average of 9.34 % for the maximum error means and 6 % for the average error, while the second presented values close to half of the first one, 4.16 % for the average of maximum errors and 1.98 % for the average errors.

Another point evaluated was the behavior of the variables analyzed in the presence of different levels of TRI. Through the tabulated results, it was found that for the radiative source term, the increase in the levels of turbulent fluctuations slightly increased the differences between the simulated percentages. The radiative heat flux for both cases showed a slightly greater increase, although also not very significant, when comparing the source term, these being not higher than 0.5 % when comparing the TRI levels 0 % and 80 %.

5. CONCLUSIONS

The present work performed the analysis of the effects of turbulence (TRI) on the accuracy of spectral models in a three-dimensional domain. Based on the results obtained for the WSGG formulation of Dorigon *et al.* (2013), the model presents good accuracy when compared to the line-by-line (LBL) integration method used as a benchmark, with error values between models less than 8 %.

Through the analysis of the radiative sources terms and heat fluxes, even with the increase in turbulent intensity, there was no significant change in the accuracy of the simulated WSGG model for the simulated cases, even though there was a

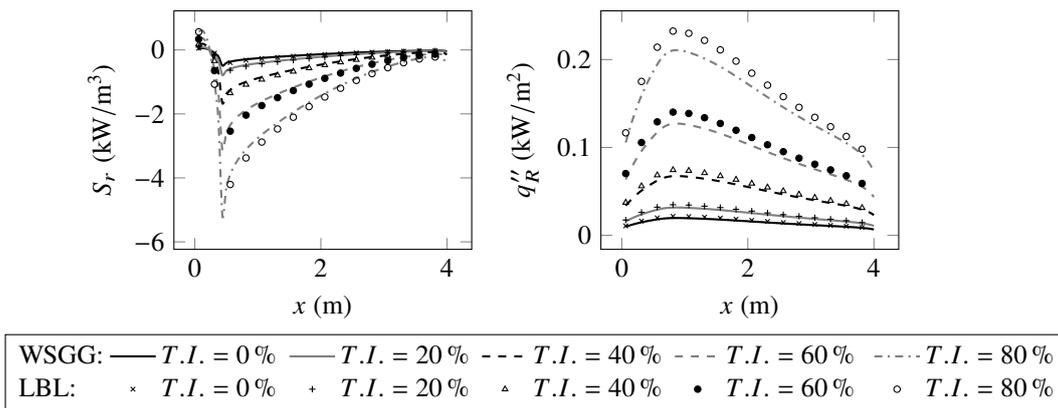


Figure 1. Case 1 - Different source term for models WSGG

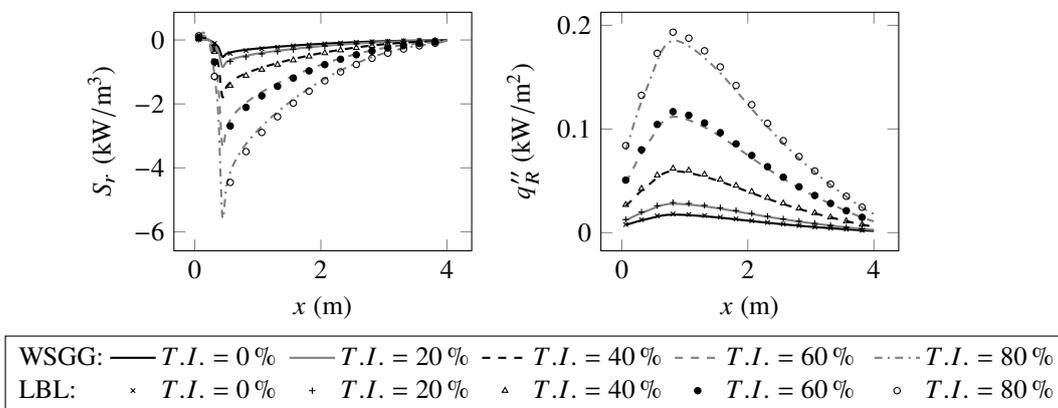


Figure 2. Case 2 - Different source term for models WSGG

subtle increase in error with the increase in the effects of the TRI.

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7. REFERENCES

- Cassol, F., Brittes, R., França, F.H. and Ezekoye, O.A., 2014. “Application of the weighted-sum-of-gray-gases model for media composed of arbitrary concentrations of H_2O , CO_2 and soot”. *International Journal of Heat and Mass Transfer*, Vol. 79, pp. 796–806. doi:10.1016/j.ijheatmasstransfer.2014.08.032.
- Coelho, P.J., 2007. “Numerical simulation of the interaction between turbulence and radiation in reactive flows”. *Progress in Energy and Combustion Science*, Vol. 33, No. 4, pp. 311–383. doi:10.1016/j.pecs.2006.11.002.
- Cox, G., 1977. “On radiant heat transfer from turbulent flames”. *Combustion Science and Technology*, Vol. 17, No. 1-2, pp. 75–78. doi:10.1080/00102209708946815.
- Dorigon, L.J., Duciak, G., Brittes, R., Cassol, F., Galarça, M. and França, F.H., 2013. “WSGG correlations based on HITEMP2010 for computation of thermal radiation in non-isothermal, non-homogeneous H_2O/CO_2 mixtures”. *International Journal of Heat and Mass Transfer*, Vol. 64, pp. 863–873. doi:10.1016/j.ijheatmasstransfer.2013.05.010.
- Fraga, G.C., Centeno, F.R., Petry, A.P., Coelho, P.J. and França, F.H.R., 2019a. “On the individual importance of temperature and concentration fluctuations in the turbulence-radiation interaction in pool fires”. *International Journal of Heat and Mass Transfer*, Vol. 136, pp. 1079–1089. doi:10.1016/j.ijheatmasstransfer.2019.03.054.
- Fraga, G.C., Zannoni, L., Centeno, F.R. and França, F.H.R.a., 2019b. “Evaluation of different gray gas formulations against line-by-line calculations in two- and three-dimensional configurations for participating media composed by CO_2 , H_2O and soot”. *Fire Safety Journal*.
- Gomes, B.d.L.Z., Fraga, G.C. and França, F.H.R., 2020a. “Analysis of thermal radiation models in their presence of different

levels of turbulent fluctuations”.

- Gomes, B.d.L.Z., Fraga, G.C. and França, F.H.R., 2020b. “Evaluation of the accuracy of thermal radiation models in the absence and presence of turbulent fluctuations”.
- Gupta, A., Haworth, D.C. and Modest, M.F., 2013. “Turbulence-radiation interactions in large-eddy simulations of luminous and nonluminous nonpremixed flames”. *Proceedings of the Combustion Institute*, Vol. 34, No. 1, pp. 1281–1288. ISSN 15407489. doi:10.1016/j.proci.2012.05.052.
- Hall, R.J. and Vranos, A., 1994. “Efficient calculations of gas radiation from turbulent flames”. *International Journal of Heat and Mass Transfer*, Vol. 37, No. 17, pp. 2745–2750. doi:10.1016/0017-9310(94)90391-3.
- Howell, J.R., Mengüç, M.P. and Siegel, R., 2016. *Thermal Radiation Heat Transfer*. CRC press, 6th edition.
- Krishnamoorthy, G., 2010. “A new weighted-sum-of-gray-gases model for CO₂-H₂O gas mixtures”. *International Communications in Heat and Mass Transfer*, Vol. 37, No. 9, pp. 1182–1186. doi:doi.org/10.1016/j.icheatmasstransfer.2010.07.007.
- Krishnamoorthy, G., 2012. “A comparison of angular discretization strategies for modeling radiative transfer in pool fire simulations”. *Heat Transfer Engineering*, Vol. 33, No. 12, pp. 1040–1051. doi:10.1080/01457632.2012.659626.
- Krishnamoorthy, G., Borodai, S., Rawat, R., Spinti, J. and Smith, P.J., 2005a. “Numerical modeling of radiative heat transfer in pool fire simulations”. In *Heat Transfer, Part A*. ASME, Vol. 2005, pp. 327–337. ISBN 0-7918-4221-5. doi:10.1115/IMECE2005-81095.
- Krishnamoorthy, G., Rawat, R. and Smith, P.J., 2005b. “Parallel computations of nongray radiative heat transfer”. *Numerical Heat Transfer: Part B: Fundamentals*, Vol. 48, No. 2, pp. 191–211. doi:10.1080/10407790590963631.
- McGrattan, K.B., Hostikka, S., McDermott, R., Floyd, J., Vanella, M. and Vanella, M., 2018. *Fire Dynamics Simulator User’s Guide*. National Institute of Standards and Technology, Building and Fire Research Laboratory.
- Modest, M.F., 1991. “The weighted-sum-of-gray-gases model for arbitrary solution methods in radiative transfer”. *Journal of Heat Transfer*, Vol. 113, No. 3, pp. 650–656. doi:10.1115/1.2910614.
- Modest, M.F., 2013. *Radiative Heat Transfer*. Academic Press.
- Modest, M.F. and Haworth, D.C., 2016. *Radiative Heat Transfer in Turbulent Combustion Systems: Theory and Applications*. Springer.
- Rothman, L., Gordon, I., Barber, R., Dothe, H., Gamache, R., Goldman, A., Perevalov, V., Tashkun, S. and Tennyson, J., 2010. “Hitemp, the high-temperature molecular spectroscopic database”. *Journal of Quantitative Spectroscopy and Radiative Transfer*, Vol. 111, No. 15, pp. 2139–2150. doi:10.1016/j.jqsrt.2010.05.001.
- Snegirev, A.Y., 2004. “Statistical modeling of thermal radiation transfer in buoyant turbulent diffusion flames”. *Combustion and Flame*, Vol. 136, No. 1, pp. 51–71. doi:10.1016/j.combustflame.2003.09.005.
- Wilcox, D.C., 2006. *Turbulence Modeling for CFD*. DCW Industries, 3rd edition.

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