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A new approach of a mobility analysis in mechanism design

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Abstract. *Since the end of the 19th century, researchers around the globe have proposed methods to evaluate mobility. Finding an efficient and universal method for determining mobility has been a relevant subject in mechanism science. This paper brings a literature review of such methods to investigate the current state of the art of mobility evaluation. As mechanism theory became more complex, the same happened in mobility analysis. In a mechanism, three types of mobilities are possible: actuated, passive and dangerous. It is a common mistake to evaluate the mobilities just considering the actuated mobilities. In fact, actuated mobilities are previously known, both for analysis and synthesis. In mechanism analysis, a simple count of the number of actuators shows the actuated mobility. In mechanism synthesis, the actuated mobility is the design input. However, passive and dangerous mobilities play a important role in mechanism design, and the engineers can not neglect them. Passive mobilities can occur intentionally to promote self-alignment, while dangerous mobilities can the mechanism to fail. They are possible threats to the users and environmental safety. This paper proposes a new approach for mobility evaluation considering the effect of passive and dangerous mobilities using: screw theory, Davies's Method, and redundant constraints knowledge. The redundant constraints are responsible for residual stresses, assembling problems, premature wear in mechanisms, among other issues. The proposed approach highlights the presence and effects of all types of mobility. This paper uses some case studies to discuss a new point of view in mechanism mobility analysis that could lead to new synthesis methods in the future.*

Keywords: *Mobility, Analysis, Method, Mechanisms*

1. INTRODUCTION

The studies regarding mechanisms can be either synthesis or analysis. Synthesis is the method of obtaining the mechanism geometry from a given set of design specifications (Martins and Murai, 2019). Thus, the mechanism mobility and the redundant constraints are considered design inputs. On the other hand, analysis is the process of obtaining the mechanism behavior through its geometry. Then, the mechanism mobility and the number of redundant constraints are the output of the analysis process. Fig. 1 shows the characteristics for synthesis and analysis.

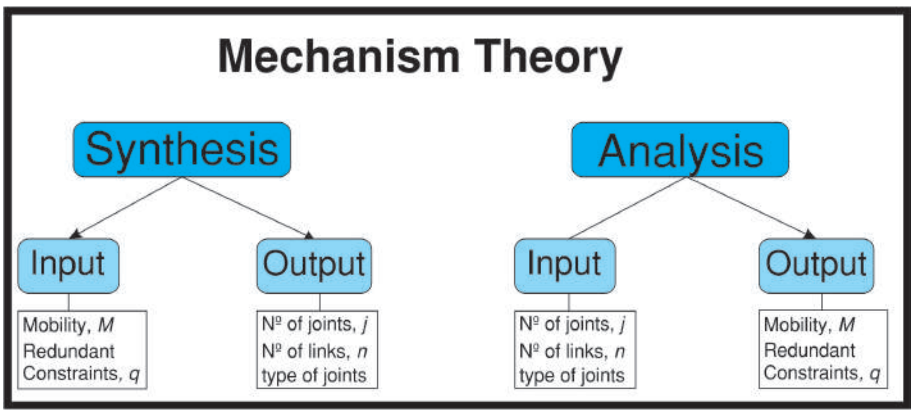


Figure 1. Synthesis and Analysis of mechanisms.

The mobility of mechanisms is a research topic since the second half of the XIX century. It is the main structural parameter and one of the most fundamental concepts in kinematics and dynamics of mechanisms and robots (Gogu (2005)).

Moreover, the redundant constraints are those whose elimination does not change the mobility of the mechanism (Reshetov (1979)). The elimination of redundant constraints does not change how the mechanism works (Barreto (2020)). Mobility and redundant constraints are related to each other. Therefore, it is necessary to consider both characteristics together either for synthesis or analysis tasks. Several researchers used the matroid theory, screw theory, Davies' method, and optimization algorithms to develop self-aligning mechanisms, which are linkages free of redundant constraints (Carboni (2015)), (Artmann (2019)), (Barreto (2020)).

This present work intends to use the knowledge of those prior researches and propose a different approach for mechanism analysis using the practicality of Reshethov's table (Reshetov (1979)) with the accuracy of Davies' Method (Davies (1983)). In Reshetov's method, the designer uses a table to fill the freedoms of each joint and then derive the mobilities and redundant constraints. Davies' method uses screw and graph theory to create matrices representing the kinematics and statics of mechanisms. The current research uses the motion matrix from Davies' method to create Reshetov's table. This new method is mathematically accurate, practical, and didactic.

This paper is structured as follows: Section 2 offers a review of mobility equations and criteria. Section 3 is dedicated to redundant constraints and Reshetov's Method. Section 4 presents an overview of Davies' Method. In Section 5, the proposed method is discussed with some examples and case studies. Section 6 is dedicated to conclusions and further works.

2. MOBILITY REVIEW

The mobility or degree of freedom (DoF) is the number of independent coordinates to define the configuration of a kinematic chain or mechanism, according to the IFToMM[1] definition (Ionescu *et al.* (2003)). There are several methods proposed in the literature for mobility calculation of mechanisms and, Gogu (2005) divides them into two types:

- Approaches based on setting up the kinematic constraint equations and their rank calculation for a given position;
- Formulas for a quick calculation without the need to develop the set of constraint equations.

The first approach is usually precise and mathematically correct, but the main disadvantage is that mobility is not quickly evaluated without setting up the kinematic model of the mechanism.The quick calculation formulas apply an explicit relation between structural parameters of the mechanism, such as the number of kinematic pairs and the number of rigid bodies. However, they usually fail for some complex mechanisms.

This section reviews relevant contributions to mobility formulas since the objective of this work is to propose a practical and accurate method to evaluate mobility and redundant constraints of mechanisms.

Chebychev (1869) was the first scientist to propose a mathematical formalization for the calculation of mechanism mobility. Chebychev's contribution (Eq. 1) is valid only to one degree of freedom planar mechanisms. Sylvester (1874) and Grübler (1883) also proposed similar equations to this problem.

$$M = 3n - 2(p_0 + p_n) \quad (1)$$

where:

- M is the mechanism mobility;
- n is the number of kinematic links;
- $2(p_0 + p_n)$ is the number of constraint equations imposed by the p_0 adjacent joints and p_n non-adjacent joints to the fixed base.

Somov (1887) proposed a criterion to analyze one degree of freedom mechanisms (Eq. 2). Somov's contribution works for both planar and spatial mechanisms.

$$m - q(b - 1) = 2 \quad (2)$$

where:

- m is the number of rigid bodies
- q is the number of loops
- b is the mobility number, ($b = 3$) for planar and ($b = 6$) for spatial mechanisms.

[1] IFTOMM stands for International Federation for the Promotion of Mechanism and Machine Science

Hochman (1890) made relevant contributions to the analysis of mechanisms. Hochman proposed a relation between the total number of elements m, the total number of joints p, and the number of independent closed loops q (Eq. 3). Hochman (1890) also proposed a general formulation for mechanism mobility, where C is the total number of constraints imposed by the joints (Eq. 4).

$$m + q - p = 1 \tag{3}$$

$$M = b(m - 1) - C \tag{4}$$

Several other authors developed formulas and criteria for the mobility evaluation based on Hochman (1890). Kutzbach (1929) proposed a mobility equation known as the Chebyshev-Gruebler-Kutzbach (CGK) criterion shown in Eq. 5. CGK Equation still fails to evaluate the mobility in more complex cases.

$$M = (n - 1)\lambda - \sum_{i=1}^{p}(c_i) \tag{5}$$

The contributions from Koenigs (1905), Dobrovolski (1949) Rössner (1961), Boden *et al.* (1962), Waldron (1966) are examples of works based on Hochman (1890). None of the previously cited authors considered the existence of redundant constraints in the mechanisms, which are important for a proper analysis.

Malishev (1923), on the other hand, proposed a formulation considering the redundant constraints (Eq 6). The author considers that the analysis of any mechanism should be done for the spatial case, since every machine will be assembled in space.

$$M = 6(n - 1) - \sum_{i=1}^{p}(ip_i) + q \tag{6}$$

where:

- M is the mechanism mobility;
- n is the number of rigid bodies;
- p_i is the number of restrictions imposed by i^{th} joint;
- q is the number of redundant constraints.

Other authors also considered the importance of redundant constrains such as Ozol (1964), Huang *et al.* (2009) and Reshetov (1979). Huang *et al.* (2009) proposed some methods to evaluate the redundant constraints and called the Eq. 6 as "modified Gruebler-Kutzback criterion".

Mobility and redundant constraints are connected to each other. Even though they are explicit in Eq. 6, both parameters cannot be determined by a simple mechanism inspection. Next section will discuss the method developed by Reshetov (1979) to analyze the mobilities and redundant constraints of a mechanism.

3. REDUNDANT CONSTRAINTS

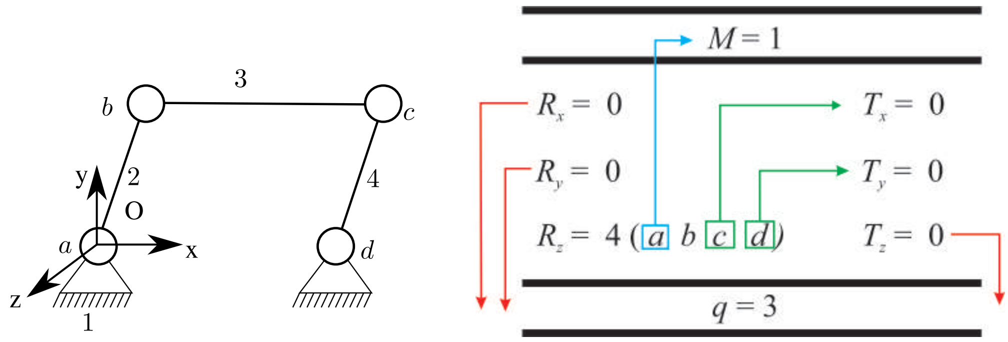
As stated beforehand, the redundant constraints are those whose elimination does not change the mobility of the mechanism (Reshetov, 1979). The elimination of redundant constraints, *i.e.* turning the original mechanism into a self-aligning one, does not change how the mechanism works (Barreto (2020)). Thus, by using self-aligning techniques, the designer can create a mechanism that is easier to manufacture and assemble. It is up to the designer to decide whether self-aligning is important to the mechanism or not (Barreto, 2020).

Reshetov (1979) proposed a method by which a table is used for completing the freedoms existing in the mechanism. From the table, the designer can derive mobilities, redundant constraints, and improve the mechanism. According to Reshetov's approach, if a loop contains all freedoms, no redundant constraints are present. Fig. 2 shows an example of Reshetov's table. The mechanism in Fig. 2(a) is a four-bar linkage with only revolute joints, allowing only rotations around the z-axis. These four rotations are placed in R_z in Reshetov's Table, in Fig. 2(b).

According to Reshetov (1979) rotations can generate translations, but not in their own axis. Thus, rotations from joints c and d can be placed at translations T_x and T_y. Without any further freedoms to relocate, it is not possible to obtain freedoms for R_x, R_y and T_z. The red arrows pointing downward determine the three redundant constraints present in the mechanism. The blue arrow pointing upward represent the one degree of freedom available in the four-bar linkage.

Reshetov's method is practical and relatively easy to use, but counterexamples exist for multi-loop mechanisms, as Carboni (2015) shows for the Tripteron parallel manipulator. For multi-loop linkages, it is necessary one table for each circuit. Since there will be joints belonging to more than one circuit, it is required attention to consider the mobilities appropriately. Reshetov (1979) developed some examples for multi-loop mechanisms, but the method it is not precise.

To overcome the drawbacks in Reshetov (1979) works, the next section explains the Davies' Method, a precise and computer efficient way to represent the kinematics and statics of mechanisms.



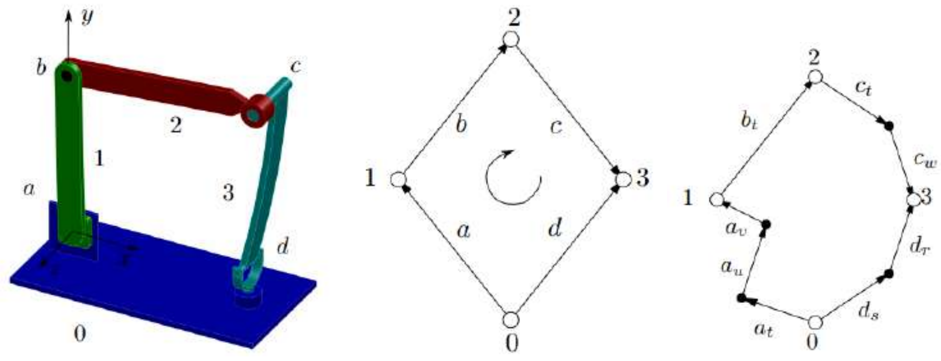
(a) Four-bar mechanism (b) Reshetov's Table

Figure 2. Reshetov's Table for the over-constrained four-bar mechanism

4. DAVIES' METHOD

Davies (1983) adapted Kirchhoff's circulation law and cutset law to multibody systems. This adaptation is based on the representation of a coupling network with n links and g joints by a graph, called coupling graph G_C, in which every link is represented by a node and every coupling between bodies by an edge (Davies, 1995), as shown in the example in Fig. 3. The coupling graph for the example is depicted in Fig. 3(b).

The motion graph G_M is obtained from the coupling graph G_C. An edge in G_C that represents a direct coupling i of freedom f_i is replaced in G_M graph by f_i edges in series, representing a set of independent motions that together span the f_i system motion screws of coupling. So, the graph G_M contains F edges, where $F = \sum_1^g (f_i)$ is the gross DoF of coupling network. The motion graph for the example is shown in Fig. 3(c).



(a) Four-bar mechanism(Carboni (2015) (b) Coupling graph (c) Motion graph

Figure 3. Four bar mechanism (a), Coupling Graph (b) and Motion Graph (c)

Every motion is modeled as a geometric screw, requiring λ coordinates where λ is the minimum order of the screw system to which all motions belong ($1 \leq \lambda \geq 6$). Then, a motion screw $\$^m$ can be associated to each of the f_i freedom of joint i, *i.e.* to each of the edge of graph G_M. Hence, the adaptation of Kirchhoff's circulation law permits finding a set of independent instantaneous screws associated with the given kinematic chain. A screw is modelled as shows Eq. 7.

$$\$ = [S \quad S \; X \; S_0]^T \tag{7}$$

where

- $\$$ is the motion screw;
- S is the unitary vector representing the join axis;
- S_0 is a vector that indicates the position of a given joint;

As an example, the screw for the rotation movement allowed by joint b is:

$$\$_b = [0 \;\; 0 \;\; 1 \quad (0\,0\,1) \; X \; (0\,1\,0)]^T \tag{8}$$

$$\$_b = [0 \;\; 0 \;\; 1 \;\; 1 \;\; 0 \;\; 0]^T \tag{9}$$

The network unit motion matrix of the coupling network, M_D, contains one unit motion screw per column:

$$[M_D]_{\lambda,F} = \begin{bmatrix} \$^m_{a1} & \$^m_{ai} & \$^m_{b1} & \ldots & \$^m_F \end{bmatrix} \tag{10}$$

with $\$_i^m$ is the unit motion screw representing a single allowed motion of coupling i, *i.e* an edge of the motion graph G_M. For the example in Fig. 3(a) the unit motion matrix is given in Eq. 11.

$$M_D = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & -1 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \end{bmatrix}_{6\times 8} \tag{11}$$

According to Carboni (2015), it is possible to determine the number of redundant constraints with Eq. 12:

$$q = \lambda\nu - r \tag{12}$$

- q is the number of redundant constraints;
- λ is the screw system of the mechanism;
- ν is the number of closed loops;
- r rank of matrix M_D.

For the case presented in Fig. 3 the rank of the matrix M_D is 5. Thus, using Eq. 12, there is one redundant constraint in the mechanism. Even though the Davies' method is precise and mathematically correct, it is necessary a profound knowledge of mechanism theory in order to calculate to derive matrix M_D, evaluate its rank and then determine the number of redundant constraints.

The next section will discuss the properties of matrix M_D and then propose an practical approach in order to investigate the redundant constraints and mobilities.

The analysis provided in this section for the kinematics and single loop mechanisms can also be done for the statics and/or multi-loop mechanisms. More information about the Davies's method, screw theory and graph representation can be found at: Davies (1983), Davies (1995), Tsai (2000), Davidson *et al.* (2004), Carboni (2015).

5. PROPOSED APPROACH

This section will discuss some properties of the matrix M_D and then propose an approach for the redundant constraint and mobility analysis. First, a method for single loop mechanism will be discussed followed by an alternative for multi-loop linkages.

5.1 Single Loop Mechanisms

The discussions within this section will be exemplified by the mechanism depicted in Fig. 4. The proposed mechanism has the following characteristics:

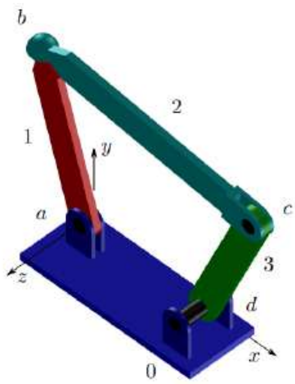
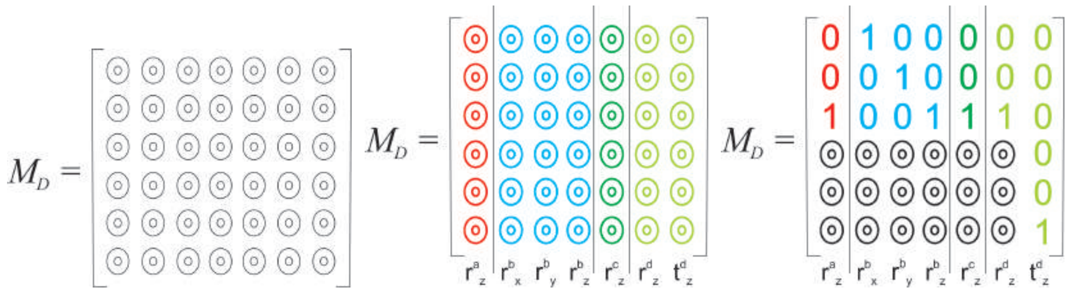


Figure 4. Four bar mechanism representation (Carboni (2015))

- the linkage is a four bar mechanism;
- joint a is a revolute joint, thus allowing one rotation r_z^a;
- joint b is a spherical joint, allowing three rotations r_x^b, r_y^b, r_z^b;
- joint c is a revolute joint, thus allowing one rotation r_z^c;
- joint d is a cylindrical joint, allowing one rotation r_z^d and a translation t_z^c.

In a single loop mechanisms, M_D is a $\lambda \times F$ matrix. Then, for the example of Fig. 4 it is necessary to create a 6×7 empty matrix, which is shown in Fig. 5(a). Since the columns represent the freedoms in the mechanism, the next step is to allocate the joints into the matrix. Fig.5(b) shows the M_D matrix with the freedoms allocated. Each joint is represented by a color and the number of colors for each joints depends on the number of freedoms they allow. For example, joint a is a revolute joint, therefore it takes just one column in matrix M_D. On the other hand, joint d is cylindrical and thus needs two columns in the matrix.

The next step is to fill the matrix with the motion screws for each joint freedom, as Fig. 5(c) shows. It is not necessary to calculate the whole motion screw as Eq. 7 suggests. For the present method, it is sufficient just to fill the joint axis into the matrix M_D. There is a region in matrix M_D has no defined elements, which is not necessary for the purpose of this analysis.



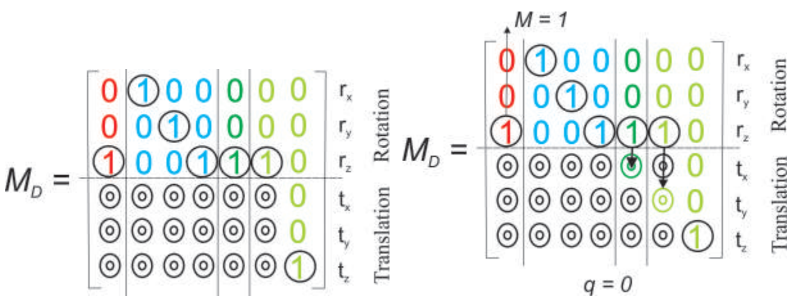
(a) Empty M_D matrix (b) M_D Matrix with freedoms allocated (c) M_D Matrix with screws

Figure 5. The creation of M_D matrix of the Four Bar Mechanism

With the matrix assembled, it is possible to start analyzing the mechanism. The M_D matrix is divided into rotation and translation areas, as shows Fig. 6(a). For the next step, it is necessary to identify the rotations and translations present in the mechanism. The mechanism depicted in Fig. 4 has six rotations and one translation, which are identified by a circle in Fig. 6(a).

According to Reshetov (1979), a single loop mechanism must contain all freedoms in its circuit. Moreover, the rotations can be transformed into translations, as presented in Reshetov's method. The mechanism already has one translation, t_z but the translations t_x and t_y are missing.

Also, the mechanism has one rotation r_x , one rotation r_y and four rotations r_z. Thus, two rotations r_z can be transformed into one translation t_x and one t_y, as shows Fig. 6(b). The remaining freedom from joint a (in red) is then, the mechanism mobility ($M = 1$). Since all freedoms are present in the loop, the mechanism does not have redundant constraints ($q = 0$).



(a) Matrix M_D: allocated screws (b) Matriz M_D: Mobility and Redundant Constraints

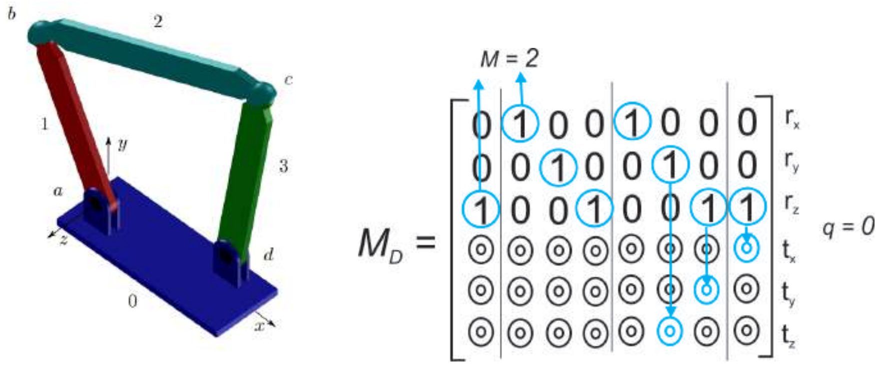
Figure 6. Analysis of the Four Bar Mechanism

The next example is given in Fig. 7. Another four bar mechanism will be analyzed wit the following characteristics:

- joint a and d are revolute joints, thus allowing rotations around z axis, r_z^a;
- joint b and c are spherical joints, allowing three rotations r_x^b, r_y^b, r_z^b.

Fig. 7(b) shows the M_D matrix for the mechanism in Fig. 7(a). Such mechanism has eight freedoms, which could be allocated in a way that no redundant constraint is present. However, there is an extra mobility in the mechanism. Apart from the expected rotation around z axis, this mechanism has a passive mobility around link 2.

A similar procedure for multi-loop mechanisms will be explained in the next section.



(a) Four bar mechanism with spherical joints (Carboni (2015).

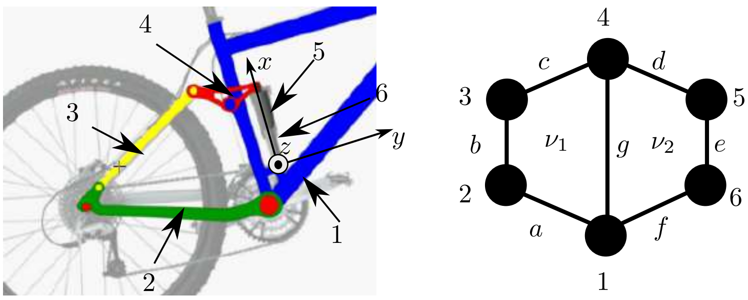
(b) Matrix M_D for the four bar mechanism

Figure 7. Analysis of Mobility and Redundant Constraints for a four bar mechanism

5.2 Multi-loop Mechanisms

This section will use a bicycle suspension as an example for a multi loop mechanism analysis, shown in Fig. 8. The mechanism has 6 links, 7 joints and 2 loops. The joints characteristics are:

- joint a, b, c, d, f and g, are revolute joints, thus allowing one rotation around z axis, r_z^a, r_z^b, r_z^c, r_z^d, r_z^f and r_z^g, respectively;
- joint e is a prismatic joint, allowing a linear displacement in x axis, t_x^e.



(a) Mechanism representation

(b) Graph

Figure 8. Suspension bike mechanism

In multi-loop mechanisms, M_D is a $\lambda\nu \times F$ matrix. For the example of Fig. 8(a) it is necessary to create a $12{\times}7$ empty matrix. Fig. 9(a) shows the empty matrix for the given example. The matrix is divided in two segments, accordingly to the circuits ν_1 and ν_2. Each segment has also the areas for rotation and translation freedoms.

For the next step is necessary to identify the joints belonging to the circuits. In Fig.8(b), it is clear that joint g intersects both circuits. In this case, it is necessary to choose in which circuit should joint g be considered. In Fig. 9(b), the green rectangles identify the joints contained in the circuits. Thus, joints a, b and c will be analyzed in circuit ν_1 and joints d. e, f and g will be analyzed in circuit ν_2 . The complete column of zeros represent that a certain joint does not belong to the circuit.

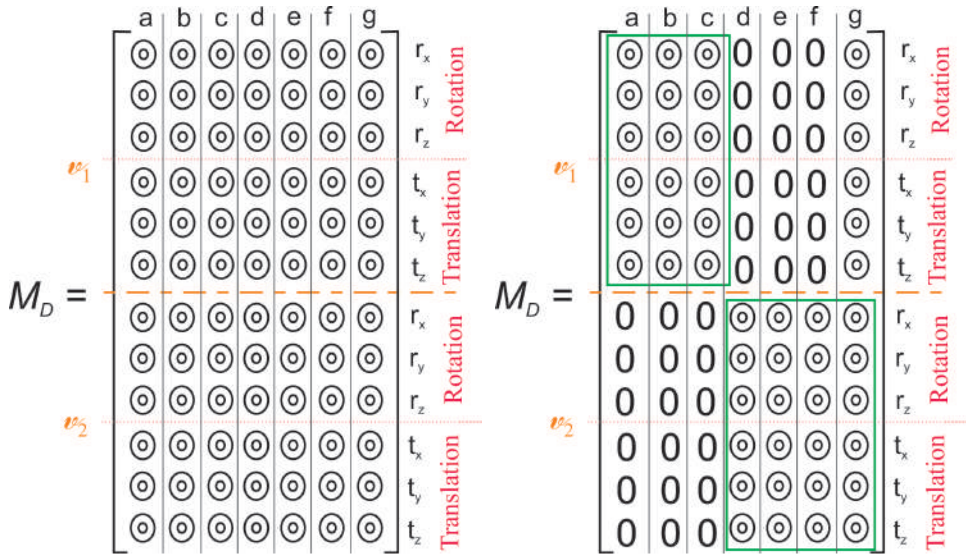
The next step is to fill the matrix with the motions screws, as shows Fig. 10(a). It is also important to identify the freedoms present in each screw, which is denoted by blue circles in 10(a). Then, mobility and redundant constraints can be analyzed. According to Reshetov (1979), all circuits must contain all freedoms six freedoms in order to the mechanism be considered self aligned. Otherwise, redundant constraints will be present.

Analyzing loop ν_1, two rotations r_z can be transformed into t_x and t_y translations respectively. Therefore, three redundant constraints are present in this loop and no available mobility, as shows Fig. 10(b).

In loop ν_2, one rotation r_z can be transformed in a translation t_y. The remaining freedom from joint g (in last column) is then, the mechanism mobility ($M = 1$). Hence, this loop has also three redundant constraints. The complete mechanism has one degree of freedom and six redundant constraints.

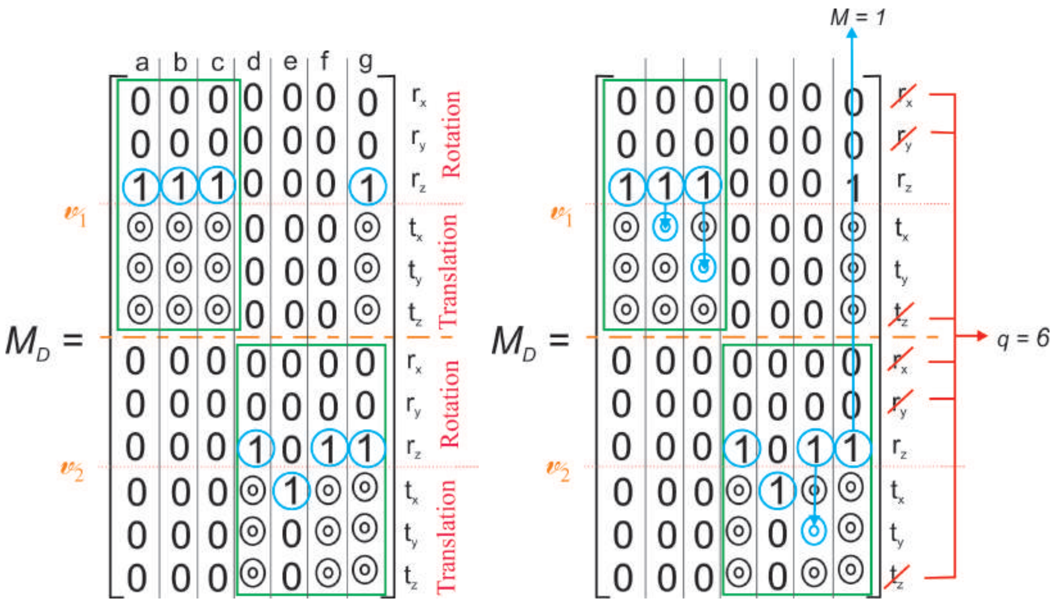
Another example to be studied is the Tripteron parallel mechanism presented by Kong and Gosselin (2001), presented in Fig. 11. This manipulator has a two-circuit kinematic chain with the following characteristics:

- joints a, e, and i, are prismatic joints, allowing displacements, t in t_x^a, t_y^e and t_z^i respectively;



(a) Empty M_D matrix (b) Joints separation in M_D matrix. analysis

Figure 9. M_D matrix for bicycle suspension mechanism



(a) M_D Matrix with motion screws (b) Mobility and Redundant Constraint Analysis for a Multi-loop mechanism

Figure 10. Mobility and Redundant Constraint Analysis for a bicycle suspension mechanism

- joints b, c and d are revolute joints around x axis;
- joints f, g and h are revolute joints around y axis;
- joints l, m and n are revolute joints around x axis.

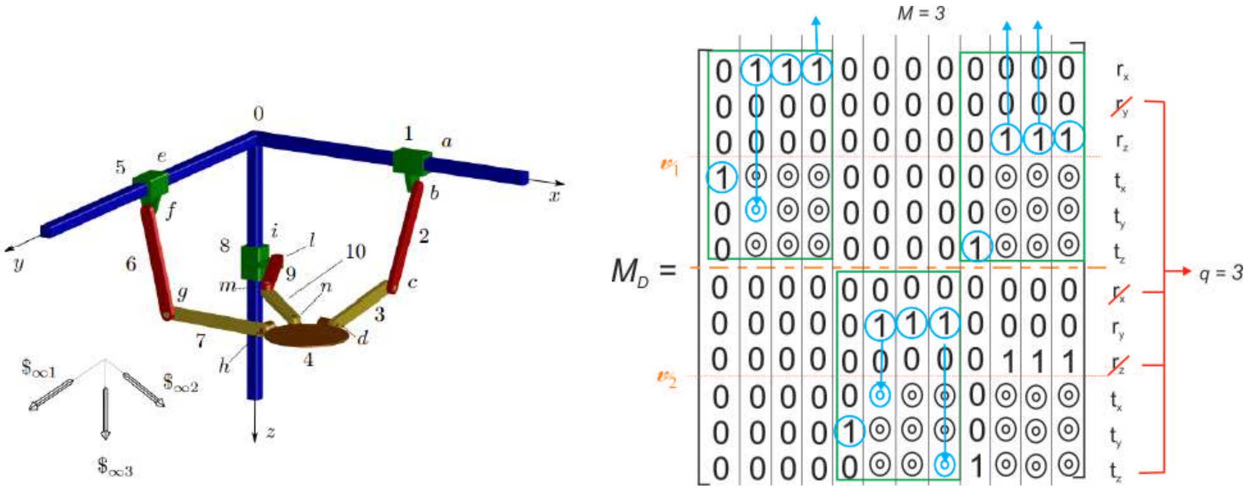
The M_D matrix in Fig. 11(b) is divided into two circuits. Joints a, b, c , d, i l, m and n are being considered in circuit ν_1. On the other hand, joints e, f, g and h are located solely on circuit ν_2

With the freedoms and screws properly allocated in the matrix M_D, it is possible to state that the mechanism has three redundant constraints and three mobilities which is in accordance with Gogu (2005) and Carboni (2015).

With the proposed approach it was possible to correctly determine the number of redundant constraints and the mobility of mechanisms.

6. CONCLUSIONS

This paper presented a brief review of some contributions on mobility calculation. Most of them had some limitations when applied to more complex mechanisms. Other approaches need the development of closure equations, which is



(a) Tripteron Manipulator (Carboni (2015).

(b) Matrix M_D for the Tripteron mechanism

Figure 11. Analysis of Mobility and Redundant Constraints for the Tripteron mechanism

complex, time consuming and dependent on the knowledge of several topics.

Being aware of the importance of redundant constraints, Reshetov (1979) developed a tool which could derive the redundant constraint and mobility of mechanism. However, this method presented some flaws in specific mechanism.

This work brought a novel approach for mobility and redundant constraint analysis using the accuracy of Davies' Method and the practicality of Reshetov's Method. The approach uses a simplification of a unit motion matrix network M_D which brings information about the freedoms and constraints for a given mechanism. Applying a technique similar to the Reshetov's Table, it was possible to determine the mobility and redundant constraints with accuracy for single and multi loop mechanisms.

For future works the approach will be implemented in a software which will facilitate the mechanism analysis. The designer will type some characteristics of the mechanism and the output will be the number of redundant constraints and mechanism mobility. Also, the research continues on developing methods to find passive and dangerous mobilities and to provide a tool useful for synthesis as well.

7. ACKNOWLEDGEMENTS

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