COB-2021-0028
STRATIFIED TWO-PHASE FLOW MODEL FOR EFFERVESCENT ATOMIZATION

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Abstract. This work consists in the development of a stratified two-phase flow model to calculate the physical variables of fluids in the exit orifice of the effervescent atomizer. The physical variables of fluids determined by the model are the gas volume fraction, dimensionless two-phase pressure, dimensionless gas temperature, dimensionless two-phase density, liquid velocity, gas velocity, speed of sound and the liquid thickness. From the results determined by the model, it was observed that the geometric variables of the atomizer have very little influence. Because the model allows reasonable results, it is considered that it can be useful in further research on primary and secondary atomization, and thus better estimate the characteristic parameters of the spray. Additionally, a dimensionless model was developed for the prediction of the mean diameter of the drops (ID_{32}) which is characterized by having $R^2 = 0.734$ and $RMSE = 3.165$, which is why it is considered to be a good dimensionless model for the prediction.

Keywords: Stratified two-phase flow model, effervescent atomization, spray

1. INTRODUCTION

Atomization is a process that consists of the reduction of the ratio of volume and total surface area of a liquid using a device called the atomizer. Atomization breaks the liquid flow in filaments or liquid portions to generate a cloud of drops, also known as spray, which are better mixed with the oxidizer, that can vaporize with higher velocity for having a higher surface area (Zhou et al. 2010, Nasr et al. 2002). To represent the diameter of the spray droplets, a mean diameter is used, being for example, the Sauter mean diameter (SMD) the most common for applications of combustion. Sprays have several applications such as painting, nebulization, particle deposition, drying, cooling, cleaning, and combustion (Lefebvre and McDonell, 2017, Williams, 1990). In combustion, there are mainly three advantages for atomization to obtain a small diameter mean of drops, which are the followings: (1) better heat transfer due to a better homogeneous two-phase mixture (Sirignano, 2010, Sazhin, 2014), (2) reduction of combustion time (Ashgriz, 2011, Basu et al., 2018), and (3) reduction of the concentrations for emissions of gases of greenhouse effect (Lefebvre and McDonell, 2017).

There are several types of atomizers, some of them can be found in the works of Lefebvre and McDonell (2017) and Nasr et al. (2002). However, the operation of most atomizers is based on the transformation of the pressure in kinetic energy (pressure atomizers) and transfer of the movement quantity (twin-fluid atomizers such as air-assisted atomizers, airblast atomizers and effervescent atomizers). Twin-fluid atomizers are those which use a gaseous flow for injection in the liquid flow to produce a reduction of the transversal surface area of the liquid in the exit orifice of the atomizer, and consequently, reduce the mean diameter of drops.

The atomization in an effervescent regimen consists of the injection of bubbles inside the liquid flow (Lefebvre et al., 1988). Functioning of this atomizer consisted in the use of a cylindrical perforated tube (called aerator) for injection of

the gaseous flow in the liquid flow. Advantages of these effervescent atomizers are basically the following (Jedelsky and Jicha, 2013): (1) use of small injection pressure of the liquid, (2) use of small mass ratio of the gas and liquid flow rates (GLR), (3) obtention of a small Sauter mean diameter for the drops, and (4) stable atomization for large values of GLR.

Effervescent atomizers can be classified in three types according to the injection of the gaseous flow in the liquid flow, these atomizers were classified such as: (1) type A (outside-in), (2) type B (inside-out), and (3) type C (parallel flows). In Figure 1 was presented the effervescent atomizer of type A. From the literature revision done regarding effervescent atomizers it was observed that the spray produced depends on three types of independent variables: (1) geometric variables, (2) physical variables, and (3) operational variables.
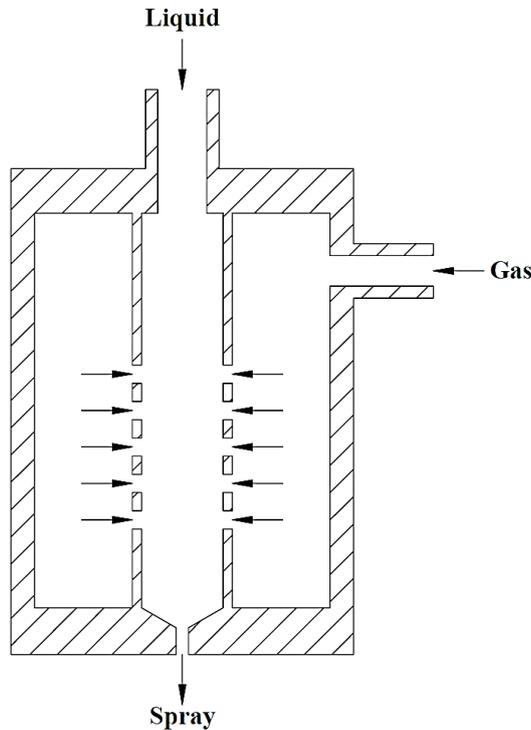

Figure 1. Effervescent atomizer of type A.

The design of effervescent atomizers was improved from the study of the influence of geometric variables being the main goal of the reduction of the mean diameter of drops. Even with the advances in the design of effervescent atomizers it is very little known the influence of geometric variables, physical variables and operational variables regarding the two-phase flow in the exit orifice of the atomizer. A better physical comprehension of what happens in the exit orifice of the effervescent atomizer (or one twin-fluid atomizer in general) will allow know the influence of two-phase flow over the mean diameter of the drops, the cone angle or the penetration of the spray.

The current study proposes a stratified two-phase flow model to calculate some physical variables in the exit orifice of the effervescent atomizer. The physical variables determined in the exit orifice of the effervescent atomizer are the dimensionless two-phase pressure, liquid velocity, gas velocity and the liquid thickness. In this way, the model will determine whether the variables calculated at the exit orifice of the effervescent atomizer are significantly influenced by the geometric variables that define the atomizer design.

2. MATERIALS AND METHODS

The current section presents the stratified two-phase flow model in which the most important equations were shown. For the development of the model, the control volume method was used, which consists of applying the Reynolds Transport Theorem to a selected volume (continuous system). The Reynolds Transport Theorem is expressed in Eq. (1).

$$\iint_{CS} \eta \left(\rho \vec{V} \cdot \vec{n} \right) dA + \frac{\partial}{\partial t} \iiint_{CV} \eta \rho \, d\forall = \frac{DN}{Dt} \qquad (1)$$

Separately (in the application of the conservation of mass equation) and jointly (in the application of the energy conservation equation) in the liquid and gaseous flow, control volumes will be established for their analysis. All control

volumes are studied concerning for an inertial reference system. On the other hand, the injection pressure will be mentioned in this work as the storage pressure and will be expressed as the absolute pressure. In addition to simplify the naming of the absolute pressure in the storage tank (liquid or gas) it is simply mentioned as pressure.

2.1 Assumptions

In Figure 2, the annular two-phase flow nozzle is presented that is used to represent the two-phase flow inside of a twin-fluid atomizer with internal mixing, for example, effervescent atomizers. The entry orifice of the nozzle for the liquid flow and the gaseous flow is characterized by properties of the fluids corresponding to their respective storage tanks. The exit orifice of the nozzle for the two-phase flow is characterized by the properties of the fluids that will be investigated.
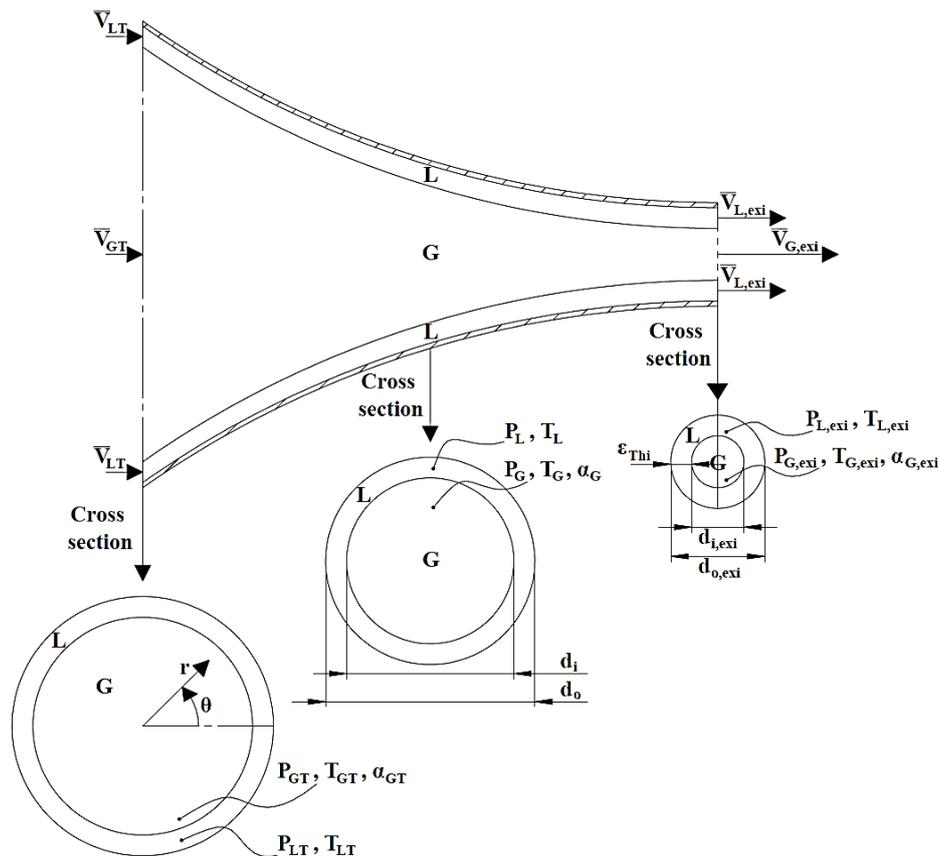
Figure 2. Annular two-phase flow.

For the development of the stratified two-phase flow model, some assumptions related to the physical variables of the fluids were established. For simplicity, the following paragraphs refer to the liquid-gas two-phase flow by simply two-phase flow. The two-phase flow was characterized by being in a steady state. In two-phase flow, it was assumed that there is no chemical reaction or phase change. A kinematic condition for velocity was assumed in Eq. (2), the normal vector $\vec{n}$ being relative to a given point on the control surface.

$$\vec{V} \times \vec{n} = 0 \tag{2}$$

The density of the liquid (ρ_L) is defined as constant in Eq. (3) for any pressure (P_L) and temperature (T_L) of the liquid from the storage tank to the exit orifice of the atomizer. In Eq. (3), the pressure and temperature in the liquid storage tank are represented by P_{LT} and T_{LT}, respectively. In addition to this equation, the pressure ($P_{L,k}$) and temperature ($T_{L,k}$) of the liquid in any exit orifice of the atomizer are equal in each case.

$$\rho_L \left(P_L, T_L \right) = \rho_L \ / \ \rho_L \text{ is a constant}$$

$$\forall \ P_L \in \left[P_{L,exi}, P_{LT} \right] = D_{P,L} \ \wedge \ \forall \ T_L \in \left[T_{L,exi}, T_{LT} \right] = D_{T,L} \ , \tag{3}$$

$$P_{L,k} = P_{L,exi} \quad , \quad T_{L,k} = T_{L,exi} \quad \wedge \quad h_{L,k} = h_{L,exi} \quad \forall \ k \in S \ / \ S = \left[1, e \right] \cap \mathbb{Z}^+$$

The atomization gas is defined as an ideal gas in Eq. (4). The thermodynamic transformation of the gas from the storage tank to the exit orifice of the atomizer is assumed to have a polytropic behavior, with "p" being the polytropic index. In Eq. (4), the pressure and temperature in the gas storage tank are represented by P_{GT} and T_{GT}, respectively. In addition to this equation, the pressure ($P_{G,k}$) and temperature ($T_{G,k}$) of the gas in any exit orifice of the atomizer are equal in each case.

$$P_G = R_G \rho_G T_G \quad , \quad P_G \propto \rho_G^p$$

$$\forall \ P_G \in \left[P_{G,exi}, P_{GT} \right] = D_{P,G} \quad , \quad \forall \ T_G \in \left[T_{G,exi}, T_{GT} \right] = D_{T,G} \ \wedge \ \forall \ p \in R \ , \tag{4}$$

$$P_{G,k} = P_{G,exi} \quad , \quad T_{G,k} = T_{G,exi} \quad , \quad \rho_{G,k} = \rho_{G,exi} \quad \wedge \quad h_{G,k} = h_{G,exi} \quad \forall \ k \in S$$

Four dimensionless variables referring to the pressure and temperature of the liquid and gas are established in Eq. (5).

$$\pi_{P,L} = \frac{P_L}{P_{LT}} \ / \ \pi_{P,L,exi} = \frac{P_{L,exi}}{P_{LT}} \quad , \quad \pi_{T,L} = \frac{T_L}{T_{LT}} \ / \ \pi_{T,L,exi} = \frac{T_{L,exi}}{T_{LT}} \quad ,$$

$$\pi_{P,G} = \frac{P_G}{P_{GT}} \ / \ \pi_{P,G,exi} = \frac{P_{G,exi}}{P_{GT}} \quad , \quad \pi_{T,G} = \frac{T_G}{T_{GT}} \ / \ \pi_{T,G,exi} = \frac{T_{G,exi}}{T_{GT}} \quad , \tag{5}$$

$$\forall \ P_L \in D_{P,L} \quad , \quad \forall \ T_L \in D_{T,L} \quad , \quad \forall \ P_G \in D_{P,G} \ \wedge \ \forall \ T_G \in D_{T,G}$$

The mass flow rates of the liquid ($\dot{m}_L$) and gas ($\dot{m}_G$) are defined in Eq. (6) as a function of their respective density and volumetric flow rate in the storage tank. The volumetric flow rates in the liquid and gas storage tanks are represented by $\dot{Q}_{LT}$ and $\dot{Q}_{GT}$, respectively. In this equation, the dimensionless parameter GLR is also defined as the ratio between the mass flow rate of the gas and the liquid. The dimensionless parameter GLR is a very important operating variable in twin-fluid atomization.

$$\dot{m}_L = \rho_L \dot{Q}_{LT} \quad , \quad \dot{Q}_{LT} = \iint_{S_{LT}} V_{LT} \, dA_L \quad ,$$

$$\dot{m}_G = \rho_{GT} \dot{Q}_{GT} \quad , \quad \dot{Q}_{GT} = \iint_{S_{GT}} V_{GT} \, dA_G \ \wedge \ GLR = \frac{\dot{m}_G}{\dot{m}_L} \tag{6}$$

The volumetric fractions of the liquid and gas are defined in Eq. (7).

$$\alpha_L = \frac{\dot{Q}_L}{\dot{Q}_L + \dot{Q}_G} \ / \ \alpha_{LT} = \frac{\dot{Q}_{LT}}{\dot{Q}_{LT} + \dot{Q}_{GT}} \ \wedge \ \alpha_G = \frac{\dot{Q}_G}{\dot{Q}_L + \dot{Q}_G} \ / \ \alpha_{GT} = \frac{\dot{Q}_{GT}}{\dot{Q}_{LT} + \dot{Q}_{GT}} \tag{7}$$

$$\forall \ \dot{Q}_L \in R^+ \ \wedge \ \forall \ \dot{Q}_G \in R^+$$

The expression of α_{GT} as a function of the physical and operational variables of atomization is presented in Eq. (8).

$$\alpha_{GT} = \frac{R_G T_{GT} \rho_L GLR}{P_{GT} + R_G T_{GT} \rho_L GLR} \tag{8}$$

In Eq. (9), it is assumed that all the diameters of the exit orifices ($d_{o,k}$) and all the interface diameters ($d_{i,k}$) for the two-phase flow are the same in each case. In this equation it is also assumed that all the exit velocities of liquid ($V_{L,k}$) and gas ($V_{G,k}$) are equal in each case.

$$d_{o,k} = d_{o,exi} \quad , \quad d_{i,k} = d_{i,exi} \quad , \quad V_{L,k} = V_{L,exi} \quad \wedge \quad V_{G,k} = V_{G,exi} \quad \forall \, k \in S \tag{9}$$

The volumetric flow rates of liquid ($\dot{q}_{L,k}$) and gas ($\dot{q}_{G,k}$) in any exit orifice of the atomizer are defined in Eq. (10). In this equation, the exit total volumetric flow rates of liquid ($\dot{Q}_{L,exi}$) and gas ($\dot{Q}_{G,exi}$) are also defined.

$$\dot{q}_{L,k} = \iint_{S_{L,k}} V_{L,k} \, dA_L \quad , \quad \dot{q}_{G,k} = \iint_{S_{G,k}} V_{G,k} \, dA_G \quad ,$$

$$\dot{Q}_{L,exi} = \sum_{k=1}^{e} \dot{q}_{L,k} \quad \wedge \quad \dot{Q}_{G,exi} = \sum_{k=1}^{e} \dot{q}_{G,k} \quad \forall \, k \in S \tag{10}$$

In Eq. (11), it is assumed that all the exit volumetric flow rates of the liquid and gas are equal in each case. In this equation, it is also assumed that all the exit volumetric fractions of the liquid and gas are equal in each case.

$$\dot{q}_{L,k} = \dot{q}_{L,exi} \quad , \quad \dot{q}_{G,k} = \dot{q}_{G,exi} \quad ,$$

$$\alpha_{L,exi} = \frac{\dot{q}_{L,exi}}{\dot{q}_{L,exi} + \dot{q}_{G,exi}} \quad \wedge \quad \alpha_{G,exi} = \frac{\dot{q}_{G,exi}}{\dot{q}_{L,exi} + \dot{q}_{G,exi}} \quad \forall \, k \in S \tag{11}$$

2.2 Applying the mass conservation equation

The constancy of liquid volumetric flow rate

The application of the mass conservation equation ($\eta = 1$) of the liquid shown in Figure 2 is shown in Eq. (12).

$$\dot{Q}_{LT} = \dot{Q}_{L,exi} = \dot{Q}_L \tag{12}$$

Surface fraction of the gas in the exit orifice

To determine the surface fraction of the gas in the exit orifice, it was necessary to consider the mass flow rate of the liquid. After some algebraic manipulations was obtained the Eq. (13), in which the surface fraction of the gas is presented in the exit orifice.

$$\frac{A_{G,exi}}{A_{Exi}} = 1 - \frac{C_L}{V_{L,exi}} \tag{13}$$

Where:

$$C_L = \frac{\dot{m}_L}{e \rho_L A_{Exi}} \tag{14}$$

Dimensionless two-phase pressure

After some algebraic manipulations the Eq. (15) was obtained. The dimensionless relationship presented in this equation is an important relationship for a liquid-gas two-phase flow.

$$\pi_P = \left[\left(\frac{\alpha_{GT}}{1-\alpha_{GT}} \right) \left(\frac{1-\alpha_G}{\alpha_G} \right) \right]^p \qquad \forall \; \alpha_G \in \left[\alpha_{GT}, \alpha_{G,exi} \right] \tag{15}$$

Dimensionless gas temperature

Substituting $P_{T\text{-}P}$ and P_T as a function of the densities ρ_{GT} and ρ_G, the temperatures of the gas T_{GT} and T_G, and the gas constant R_G in the Eq. (15), Eq. (16) can be obtained.

$$\pi_{T,G} = \left[\left(\frac{\alpha_{GT}}{1-\alpha_{GT}} \right) \left(\frac{1-\alpha_G}{\alpha_G} \right) \right]^{p-1} \qquad \forall \; \alpha_G \in \left[\alpha_{GT}, \alpha_{G,exi} \right] \tag{16}$$

Fundamental kinematic relationship

To determine the fundamental kinematic relationship, it was necessary to consider the mass flow rate of the gas. After some algebraic manipulations the Eq. (17) was obtained.

$$\alpha_{GT} \left(1-\alpha_{G,exi} \right) V_{L,exi} V_{G,exi} - C_L \alpha_{GT} \left(1-\alpha_{G,exi} \right) V_{G,exi} - C_G \left(1-\alpha_{GT} \right) \alpha_{G,exi} V_{L,exi} = 0 \tag{17}$$

2.3 Application of the energy conservation equation

After a long algebraic development, an important energetic relationship presented in Eq. (18) was obtained.

$$\rho_L \left(1-\alpha_{GT} \right) \Delta E_L + \rho_{GT} \alpha_{GT} \Delta E_G = 0 \tag{18}$$

Where:

$$\Delta E_L = \frac{\beta_{L,exi} V_{L,exi}^2}{2} + \frac{P_T}{\rho_L} \left(\pi_{P,exi} - 1 \right) + \overline{c}_{v,L} T_{LT} \left(\pi_{T,L,exi} - 1 \right) \tag{19}$$

$$\Delta E_G = \frac{\beta_{G,exi} V_{G,exi}^2}{2} + \overline{c}_{p,G} T_{GT} \left(\pi_{T,G,exi} - 1 \right) \tag{20}$$

2.4 Stratified two-phase flow model

In this section, a stratified two-phase flow model will be defined for two-phase flow inside of an effervescent atomizer. A stratified two-phase flow model is one that considers the flow separately from two phases. When in the stratified two-phase flow model, it is assumed that there is no heat transfer between both phases, it is called the frozen stratified two-phase flow model. This assumption has an important consequence in the application of the energy conservation equation for a two-phase flow, from which originate that $\Delta E_L = 0$ and $\Delta E_G = 0$. This research proposes to develop a frozen stratified two-phase flow model.

Effervescent atomization is a process characterized essentially by isothermal thermodynamic transformations (Jedelsky et al. 2009). In a frozen stratified two-phase flow model it is possible to assume an isothermal transformation of the liquid by assuming $\pi_{T,L,exi} = 1$, but assuming an isothermal transformation of the gas implies that the polytropic index is equal to 1 ($\pi_{T,G,exi} = 1$) which it is not possible because this causes the gas exit velocity to be zero. For this reason, in this research, the polytropic index was evaluated from 1.10 to 1.40.

Equations (21) and (22) show the isolations of the liquid and gas exit velocities, respectively, obtained from the application of the energy conservation equation for the frozen stratified two-phase flow model.

$$V_{L,exi} = \sqrt{\frac{2P_T}{\beta_{L,exi} \rho_L} \left(1 - \pi_{P,exi} \right)} \tag{21}$$

$$V_{G,exi} = \sqrt{\frac{2\overline{c}_{p,G}T_{GT}}{\beta_{G,exi}}\left(1-\pi_{T,G,exi}\right)} \tag{22}$$

Finally, the velocities are shown in Eqs. (21) and (22) were replaced in the fundamental kinematic relationship presented in Eq. (18) to determine the $\alpha_{G,exi}$ value. Thus, the model determines in the exit orifice of the effervescent atomizer the variables such as the gas volume fraction, dimensionless two-phase pressure, dimensionless gas temperature, dimensionless two-phase density, liquid velocity, gas velocity, speed of sound and the liquid thickness.

3. RESULTS AND DISCUSSION

In this section, the results of the stratified two-phase flow model obtained from simulations for five operational conditions are presented. The operational conditions for the simulations were selected from the researches by Jedelsky et al. (2007) and Jedelsky et al. (2009). These researches used are complementary to each other, in the sense that some data that are not provided in one is provided in the other. In these researches, the influence of geometric variables of the effervescent atomizer on the ID_{32} of the spray drops is studied. In these researches, seventeen effervescent atomizers of type A are used. The atomizing fluids are light heating oil and air. The exit orifice diameter of all effervescent atomizers is 2.5 mm.

The physical variables estimated at the exit orifice of the effervescent atomizer are the volumetric fraction of gas ($\alpha_{G,exi}$), dimensionless two-phase pressure ($\pi_{P,exi}$), dimensionless gas temperature ($\pi_{T,G,exi}$), dimensionless two-phase density ($\pi_{\rho,exi}$), liquid velocity ($V_{L,exi}$), gas velocity ($V_{G,exi}$), two-phase speed of sound ($c_{T-P,exi}$), and liquid thickness (e_{Thi}). To determine the results of the physical variables, equation (17) is solved to calculate $\alpha_{G,exi}$ assuming the polytropic index from 1.10 to 1.40. From the substitution of the polytropic index from 1.15 to 1.40 it is possible to calculate $\alpha_{G,exi}$, but from the substitution of the polytropic index equal to 1.10 it is found only in a few cases that the value of equation (17) has a very small value close to zero.

From the results obtained from simulating five operating conditions in seventeen effervescent atomizers it is calculated that for each operating condition the standard deviation of the results is very small, the same happens for ID_{32}. For this reason, this section discusses the results of the physical variables of the phases in relation to the operating conditions, disregarding the analysis for each of the seventeen effervescent atomizers.

Figure 3 shows the exit velocity of liquid as a function of the polytropic index. In this figure it is observed that the behavior of the variable $V_{L,exi}$ is decreasing with respect to the increase of the polytropic index for all operating conditions. The range of the variable $V_{L,exi}$ is [7.976, 26.892 m/s]. For the pressure of 0.2 MPa and GLR = 5%, it is observed that the mean value of $V_{L,exi}$ is equal to 9.594 m/s. For the pressure of 0.4 MPa, the increase of GLR causes a decrease of the mean value of $V_{L,exi}$, which is equal to 15.501, 15.460 and 15.438 m/s for GLR equal to 2, 5 and 10%, respectively. For the pressure of 0.6 MPa and GLR = 5%, it is observed that $V_{L,exi}$ is higher compared to the pressures of 0.2 and 0.4 MPa, keeping the same GLR value. For this operating condition, the mean value of $V_{L,exi}$ is equal to 20.128 m/s.
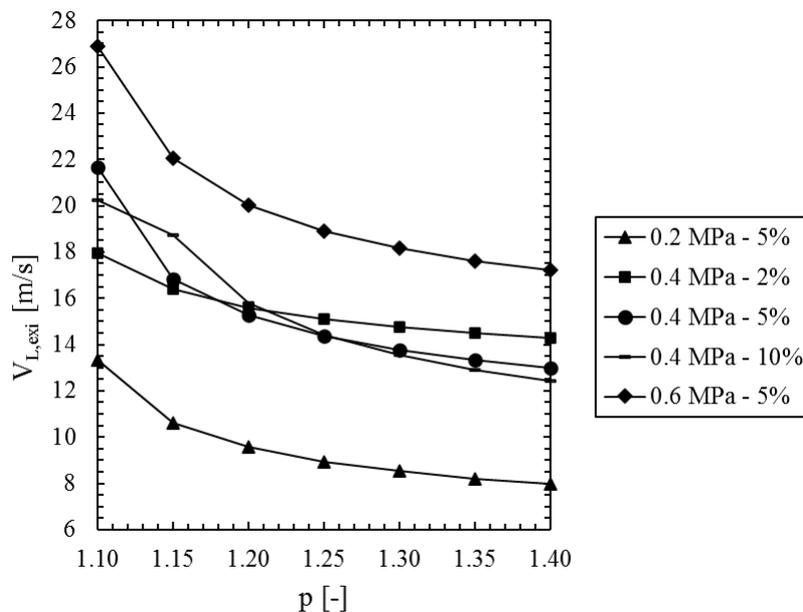

Figure 3. Exit velocity of liquid as a function of the polytropic index

For the analysis of the variable $V_{L,exi}$ as a function of pressure, operating conditions of 0.2, 0.4 and 0.6 MPa are selected, considering constant the GLR equal to 5%. For this analysis, it is essential to observe equation (21). From this equation, the inverse relationship between $V_{L,exi}$ and $\pi_{P,exi}$ can be seen, in the sense that if one of these variables increases, the other decreases. From this equation it can also be observed that the higher pressure, higher exit velocity of liquid, and consequently, higher mass flow rate of the liquid, which happens according to the selected experimental results.

For the analysis of the variable $V_{L,exi}$ as a function of GLR, operating conditions of 2, 5 and 10% are selected, considering constant pressure equal to 0.4 MPa. The trend of $V_{L,exi}$ as a function of the polytropic index (keeping the GLR value constant) is a constant decrease. The trend of $V_{L,exi}$ as a function of GLR (keeping the value of the polytropic index constant) is not very defined because in some cases it increases and in others it decreases. From Figure 3, it is observed that to obtain a higher value of $V_{L,exi}$, the pressure is more influential compared to the GLR.

Figure 4 shows the exit velocity of gas as a function of the polytropic index. In this figure it is observed that the behavior of the variable $V_{G,exi}$ is not stable with respect to the increase of the polytropic index, whereby there is no clear trend for all operating conditions. The range of the variable $V_{G,exi}$ is [145.322, 201.901 m/s]. For the pressure of 0.2 MPa and GLR = 5%, it is observed that the mean value of is equal to 151.751 m/s. For the pressure of 0.4 MPa, the increase of GLR causes a decrease of the mean value of $V_{G,exi}$, which is equal to 178.619, 176.215 and 175.596 m/s for GLR equal to 2, 5 and 10%, respectively. For the pressure of 0.6 MPa and GLR = 5%, it is observed that $V_{G,exi}$ is higher compared to the pressures of 0.2 and 0.4 MPa, keeping the same GLR value. For this operating condition, the mean value of $V_{G,exi}$ is equal to 189.599 m/s.
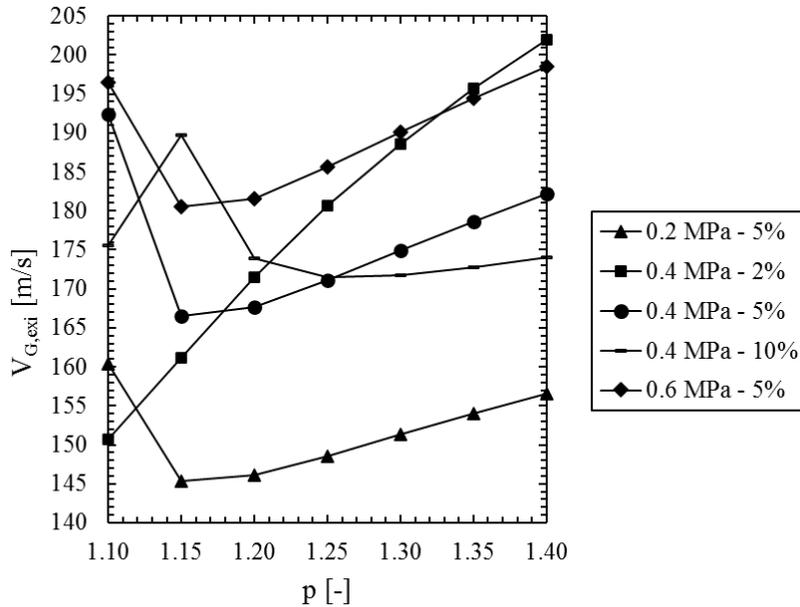

Figure 4. Exit velocity of gas as a function of the polytropic index

For the analysis of the variable $V_{G,exi}$ as a function of pressure and GLR, equation (22) must be observed. From this equation, the inverse relationship between $V_{G,exi}$ and $\pi_{T,G,exi}$ can be seen, in the sense that if one of these variables increases, the other decreases. In Figure 4 it is observed that there is no defined trend of $V_{G,exi}$ for all operating conditions except for the operating condition of pressure equal to 0.4 MPa and GLR = 2%.

From the determination of important variables in the exit orifice of the effervescent atomizer such as $V_{L,exi}$ and $V_{G,exi}$, a dimensionless model for the ID_{32} was developed. For the development of the dimensionless model, the same experimental data (Jedelsky et al. 2009) used for the application of the stratified flow model were used. In the dimensionless model it was established as an ID_{32} dependent variable and as independent variables the d_o, ρ_L, μ_L, γ, $V_{L,exi}$ and $V_{G,exi}$. The speeds $V_{L,exi}$ and $V_{G,exi}$ were those determined for the value of p = 1.15. The dimensionless model is presented in Eq. (23). In Eq. (23), important dimensionless numbers can be observed in the liquid-gas two-phase flow such as Reynolds Number (Re), Weber Number (We) and slip ratio (sr).

$$\frac{ID_{32}}{d_o} = 1.321 \, Re^{0.139} \, We^{-0.355} sr^{-0.519}$$

(23)

Where:

$$Re = \frac{\rho_L V_{L,exi} d_o}{\mu_L} \quad , \quad We = \frac{\rho_L V_{L,exi}^2 d_o}{\gamma} \quad \wedge \quad sr = \frac{V_{G,exi}}{V_{L,exi}} \tag{24}$$

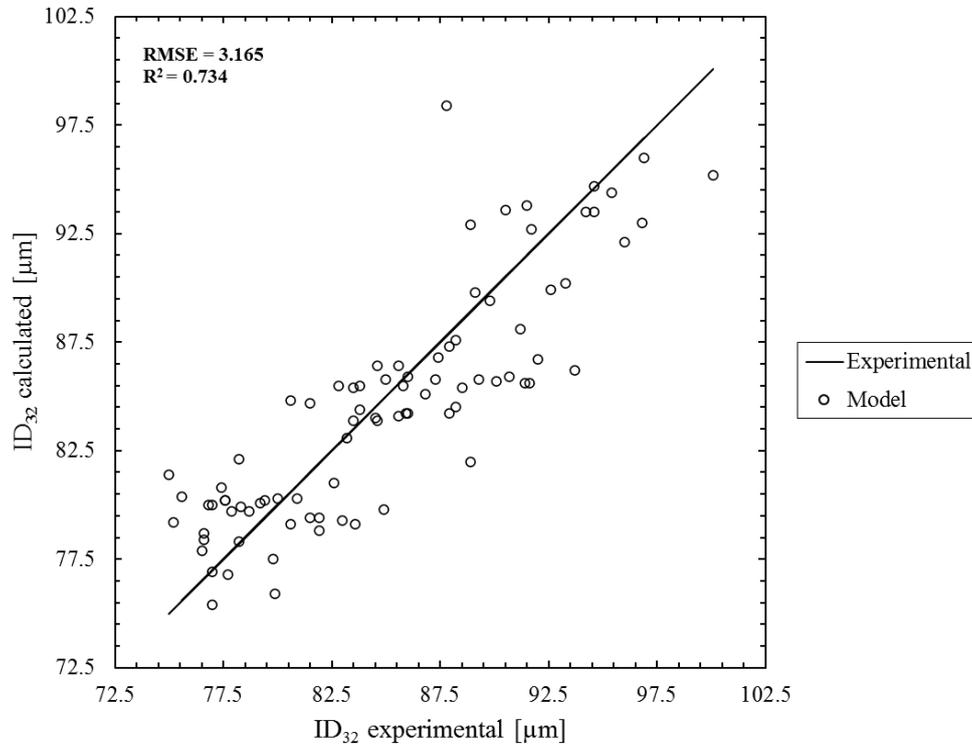The results of the Eq. (23) are compared with the experimental data in Figure 5.

Figure 5. Comparison between calculated and experimental ID_{32}

The dimensionless model is characterized by having $R^2 = 0.734$ and RMSE = 3.165. From the estimates of the dimensionless model, it was determined that the minimum, average and maximum relative error is 0.11, 2.92, and 12.07%, respectively. Additionally, from the estimates of the dimensionless model, it was observed that for 71 estimates the relative error is less than or equal to 5%, and for 14 estimates the relative error is greater than 5%. Due to the results obtained previously, it is considered that the proposed dimensionless model is good for the ID_{32} estimations.

4. CONCLUSIONS

The frozen stratified two-phase flow model determines many physical variables of interest in the exit orifice of the atomizer that helps to better physically understand the properties of the liquid and gas when atomization begins. The determination of these variables can help to develop future models for primary atomization (ligament division) and secondary atomization (droplet division and coalescence).

A very important advance for the stratified two-phase flow model would be the determination of the polytropic index of the gas in the exit orifice of the atomizer; this would help to determine more precisely the variables that can determine the model. On the other hand, the stratified two-phase flow model is very versatile for some future consideration of heat transfer between phases. Thus, it is also probable that the stratified two-phase flow model can be applicable in Y type atomizers. Additionally, a dimensionless model was developed for the prediction of the mean diameter of the drops (ID_{32}) which is characterized by having $R^2 = 0.734$ and RMSE = 3.165, which is why it is considered to be a good dimensionless model for the prediction.

5. ACKNOWLEDGEMENTS

6. REFERENCES

Zhou, Y., Zhang, M., Yu, J., Zhu, X., Peng, J., 2010. "Experimental investigation and model improvement on the atomization performance of single-hole Y-jet nozzle with high liquid flow rate". *Powder Technology*, Vol. 199, pp. 248–255.

Nasr, G.G., Yule, A.J., Bendig, L., 2002. Industrial Sprays and Atomization. First edition. Springer-Verlag London.

Lefebvre, A.H., McDonell, V.G., 2017. Atomization and Sprays. Second edition. CRC Press.

Williams, A., 1990. Combustion of liquid fuel sprays. First edition. Butterworth & Co.

Sirignano, W.A., 2010. Fluid dynamics and transport of droplets and sprays. Second edition. Cambridge University Press.

Sazhin, S., 2014. Droplets and Sprays. First edition. Springer-Verlag London.

Ashgriz, N., 2011. Handbook of Atomization and Sprays. First edition. Springer US.

Basu, S., Agarwal, A.K., Mukhopadhyay, A., Patel, C., 2018. Droplets and Sprays. First edition. Springer Singapore.

Lefebvre, A.H., Wang, X.F., Martin, C.A., 1988. Spray Characteristics of Aerated-Liquid Pressure Atomizers. *Journal of Propulsion and Power*, Vol. 4, pp. 293–298.

Jedelsky, J., Jicha, M., 2013. "Energy conversion during effervescent atomization". *Fuel,* Vol. 111, pp. 836–844.

Jedelsky, J., Jicha, M., Slama, J., Otahal, J., 2009. "Development of an Effervescent Atomizer for Industrial Burners". *Energy and Fuels*, Vol. 23, pp. 6121–6130.

Jedelsky J., Otahal J., Jicha M., 2007. "Effervescent atomizer: influence of the internal geometry on atomization performance". *Proc. 21th ILASS - Eur.* Meet., Mugla - Turkey: ILASS – Europe, pp. 1–6.

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