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SIMULATION OF FLOW IN SATURATED POROELASTIC MEDIA WITH ISOLATED VUGS

Monique Feitosa Dali
Frederico Carvalho Gomes
Marcio da Silveira Carvalho
Department of Mechanical Engineering, Pontifícia Universidade Católica do Rio de Janeiro, Rio de Janeiro, RJ 22451-900, Brazil
mfeitosa@lmmp.mec.puc-rio.br, fcgomes@lmmp.mec.puc-rio.br, msc@puc-rio.br

Abstract. *Modeling the interaction between fluids and porous structures subjected to mechanical forces is a complex problem. Because of its wide range of applications, this field has been attracted attention. Recently, substantial research effort has been directed to the study of poroelastic media in Brazil, due to the interest in determining the hydro-mechanical response of pre-salt carbonate reservoirs. Carbonates are heterogeneous formations that normally exhibit multiple spatial scales. Vuggy carbonates commonly present two different scales of pores, while even more scales can be found in fractured vuggy carbonates. Therefore, predictive modeling of flow in carbonates should take into account not only the transport processes on multi-porosity media but also the effects of geomechanical issues, such as subsidence, soil compaction and collapse of cavity regions, which can impact negatively the flow performance. We propose a finite element model for the coupled solution of flow and poromechanics in a vuggy deformable porous media, under quasi-static conditions. We construct an artificial porous medium model wherein the isolated macropores are explicitly marked regions with free flow and the microporous structure is replaced by averaged homogeneous properties. Furthermore, we apply the Stokes equation inside the vugs to model the free fluid transport, while the Biot model is used in the saturated poroelastic region. The latter is adopted to model the hydro-mechanical coupling effects of porous media based on a linear stress-strain relationship. The selected single-domain approach uses the Brinkman equation, which governs the flow interaction across both domains and vug-matrix interface, establishing a seamless transition between the two different flow regions. This flow scheme recovers Darcy's law on the fine-scale and the Stokes equation in the macropores through an appropriate choice of parameters. A two-dimensional model for the time-dependent displacement of a vuggy porous media is implemented in Python on the open-source FEniCS platform. The implementation of Biot's model is validated using two well-known poroelastic benchmark problems, the consolidation problem of Terzaghi and Mandel's problem. With the Biot-Brinkman system of equations, a load was applied to 2D porous medium samples containing circular vugs and filled with a fluid, in order to evaluate pressure, displacement and flow rate response. The impact of the circular inclusions on flow was assessed, and the macroscopic equivalent properties that are relevant to large-scale production predictions were calculated.*

Keywords: *Poroelasticity, Biot's model, Brinkman Equation, Finite Element, Vuggy porous media.*

1. INTRODUCTION

The simulation of the interaction of viscous fluid flow within deformable porous solid skeleton is very difficult. The coupled fluid-structure interaction is a challenging numerical problem. Flow and transport in poroelastic media occur in many applications, including groundwater hydrology, geomechanics, reservoir engineering, subsurface waste repositories and hemodynamics.

Carbonates are natural formations that present high heterogeneity. The variation in properties between sections of the reservoir is also due to the presence of cavities, fractures and vugs, where fluid preferentially flows (Choquette and Pray, 1970). According to Lucia (1999) and Yao and Huang (2017), vug is visible pores that are significantly larger than adjacent grains or crystals. Therefore, vugs can be considered a special case of double-porosity media because of this huge contrast between the microporous matrix and the average size of vugs (Lewandowska and Auriault, 2013) (Yao and Huang, 2017).

From a mathematical point of view, the challenge is the different scales of the problem between pore matrix and vugs. The coupling of porous flow and free-fluid flow uses basically the single domain and the two domain approach (Huang *et al.*, 2018). The first provides a model that continuously varies from a Darcy flow in the small pores to a Stokes flow inside the macropores (Popov *et al.*, 2009), with an appropriate choice of parameters. This use of the Stokes-Brinkman (Brinkman, 1949) equation avoids the necessity of an extra boundary condition at the porous-vug interface. He *et al.* (2015) highlighted that using a unified approach avoids some problems faced in coupled equations. The latter scheme employs Darcy and Stokes equations to describe flow at the different regions and couples with a specific boundary interfacial condition, the Beavers-Joseph-Saffman condition (Saffman, 1971).

Many papers Popov *et al.* (2009), Gulbransen *et al.* (2010), Qin *et al.* (2010), Krotkiewski *et al.* (2011), Oliveira and Carvalho (2014), He *et al.* (2015), Golfier *et al.* (2015), Guibert *et al.* (2016) and Yao and Huang (2017) employed Stokes-Brinkman equation to model single-phase steady-state flow as an effective alternative to obtain main flow characteristics.

The models gain complexity when the saturated double-porosity media are also deformable (Lewandowska and Auriault, 2013). Due to the complicated solution of the coupling between flow and solid deformation (Phillips and Wheeler, 2007) (Zhang *et al.*, 2015), traditionally reservoir simulators demand their effort on reservoir flow prediction, thereby the solid is simplified to a rigid matrix or approximated as constant rock compressibility (Mikelić *et al.*, 2014).

Many problems can occur as side effects of fluid-solid interaction that cannot be ignored such as volume reduction after air removal (compaction), the reduction of the total solid volume of a porous medium caused by the withdrawal of underground fluids (consolidation), and vertical compaction of unconsolidated systems (land subsidence) (Phillips and Wheeler, 2007) (Gai, 2004). The simplified porous media deformation could not be sufficient to give the appropriate rock response in naturally fractured reservoirs (Chen *et al.*, 1995). The decoupled flow simulation presents limitations and does not provide strain and stress of the solid (Mikelić *et al.*, 2014).

Computer simulations of drilling are used to predict fluid-flow induced deformation (Phillips and Wheeler, 2007). The usual procedure consists in apply loading on porous medium to evaluate the dynamic response of the fluid pressure and the elastic displacement of the solid. These initial parameters could be mechanical, thermal, chemical or biological conditions.

The quasi-static Biot's model is usually the equation presented to model the hydromechanical coupling. Analytical solutions are available for particular cases, normally simple geometries, boundary value problems and physical parameters (SILVA, 2018). The analytical solutions of Terzaghi's one-dimensional consolidation, Mandel's two-dimensional and Cryer's three-dimensional are a series of poroelastic problems used as the benchmark for numerical models.

There are typically three numerical techniques to model fluid-solid coupling problems: fully implicit, explicit coupling, and iterative coupling (Settari *et al.*, 2001). The fully implicit approach solves simultaneously a large system composed by all governing equations at each time step, which guarantees its robustness (Zhang *et al.*, 2015). The resulting of explicit coupling is less accurate and conditionally stable but experiences a lower computational cost (Brun *et al.*, 2020).

The iterative scheme solves the governing equation in a sequence and exchanges some shared variables among equations as correction terms to make sure that both solutions converge within tolerance at each time step (Mikelić *et al.*, 2014). This approach brings efficiency but may introduce splitting errors (Zhang *et al.*, 2015).

Badia *et al.* (2009), Ambartsumyan *et al.* (2014) and Ambartsumyan *et al.* (2019) employed numerical schemes to analyze poroelastic and fluid behavior implementing the Biot-Stokes coupling problem. Lewandowska and Auriault (2013) and Huang *et al.* (2018) took another path and developed an extension of the Biot model using the asymptotic expansion of the homogenization theory for elastic microporous media with disconnected vugs.

We present a poroelastic system in which the coupled processes are free fluid flow, porous flow and geomechanics. The problem under investigation is a single-phase flow within a deformable porous medium containing non-connected vugs. The Brinkman flow model is employed to model single-phase flow using the finite element method. Considering a fully-saturated, quasi-static regime, the mathematical model implemented is the quasi-static Biot's model to solve the fluid pressure and the elastic displacement of the solid matrix.

In this work, we adopt an iterative method for the coupled flow-structure problem because the implicit leads to a very large system. To overcome the splitting errors of iterative coupling, the fully coupled approach was also implemented to compare both solutions for well know poroelastic problems. Several numerical tests for the coupling porous-free flow problem, and the coupled flow and mechanics validate our proposed model.

The paper is organized in the following way. In Section 2, the mathematical formulation of the problem at Darcy-scale is presented. In Section 3, we present the monolithic and iterative algorithms. Section 4 provides the convergence and validity analysis. In Section 5, we present numerical experiments in deformable vuggy media. Finally, some conclusions remarks are presented in Section 6.

2. PROBLEM STATEMENT

We consider an elastic microporous medium with circular vugs. The entire region is saturated with viscous Newtonian and incompressible fluid, which is allowed to flow from the micropores to the vug. The scheme of the heterogeneous media is shown in Fig 1, where R represents the vug radius and k^*_{pm} the scalar permeability of the porous matrix. Here, Ω_{vug} is the region of the vug with free-flow area and Ω_{pm} is the deformable porous matrix governed by Biot's model for coupled Darcy and elastic equations. The boundaries of the geometry are divided into 4 segments corresponding to the left, top, right, and bottom of the domain.

The Brinkman equation, Eq. 1, is employed as a general interpolation equation that combines Darcy's law, which governs slow flow in porous media, and Stokes equation, in order to describe flow inside vug. The flow is governed by the

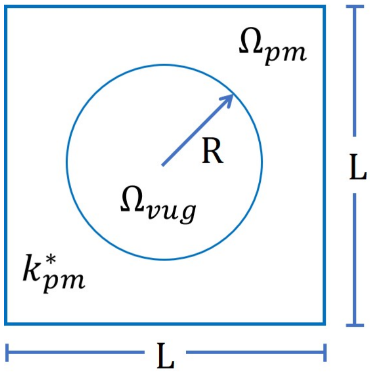


Figure 1: Schematic view of a 2D porous medium with a circular vug.

Brinkman equations on both regions, Ω_{vug} and Ω_{pm}. Apply Eq. 1 in the single-domain approach involves a penalization term to obtain a unique set of equations for the entire domain (Khadra *et al.*, 2000).

The purpose is to find the pressure p, the fluid velocity $\mathbf{w}$ and the displacement $\mathbf{u}$ such that

$$\nabla p = -\mu_f k^{-1}\mathbf{w} + \mu^* \nabla^2 \mathbf{w} \; in \; \Omega \tag{1}$$

$$\nabla \cdot \mathbf{w} = 0 \tag{2}$$

where p is the pressure, μ_f the fluid viscosity, μ^* a model parameter, $\mathbf{w}$ the velocity field and k the permeability.

The poroelastic area Ω_{pm} is governed by the quasi-static Biot system

$$-\nabla \cdot \sigma_{\mathbf{p}} \left(\mathbf{u}_{pm}, p_{pm}\right) = 0 \; in \; \Omega_{pm}, \tag{3}$$

$$\mu_f k^{-1}\mathbf{w} + \nabla p = 0 \; in \; \Omega_{pm}, \tag{4}$$

$$\frac{\partial}{\partial t}\left(s_0 p + \alpha \nabla \cdot \mathbf{u}\right) + \nabla \cdot \mathbf{w} = 0 \; in \; \Omega_{pm} \tag{5}$$

where α represents the Biot-Willis constant and s_0 is a storage coefficient.

Let $\sigma\left(\mathbf{u}, p\right)$ and $\sigma_{\mathbf{e}}\left(\mathbf{u}\right)$ be the poroelasticity stress tensor and elasticity, respectively:

$$\sigma_{\mathbf{p}}\left(\mathbf{u}, p\right) = \sigma_{\mathbf{e}}\left(\mathbf{u}\right) - \alpha p \mathbf{I}, \tag{6}$$

$$\sigma_{\mathbf{e}}\left(\mathbf{u}\right) = \lambda\left(\nabla \cdot \mathbf{w}\right)\mathbf{I} + 2\mu\epsilon\left(\mathbf{u}\right), \tag{7}$$

where $\mathbf{I}$ is the identity matrix and $\epsilon\left(\mathbf{u}\right)$ is the deformation rate tensor. Using the small strain theory:

$$\epsilon\left(\mathbf{u}\right) = \frac{1}{2}\left(\nabla \mathbf{u} + \nabla \mathbf{u}^T\right), \tag{8}$$

Considering the vugular system in Fig. 1, when the fluid particle is inside a void, we can assume in Eq. 1 that k is infinite, consequently, Darcy's term $\mu_f \mathbf{k^{-1}u}$ vanishes, which leads to Stokes equation. Whereas, if the particle is flowing within the porous media, we set μ^*=0, Darcy equation is recovered and the Biot's system is valid.

The assumptions considered for the model are single-phase, isothermal flow, constant fluid viscosity, homogeneous porous matrix, isotropic permeability, negligible gravitational and inertial effects. The fluid and the solid grains vary from incompressible to slightly compressible depending on the parameters of the problem. From Detournay and Cheng (1993) the following parameter correspondence is used for an ideal porous medium:

$$\frac{1}{M} = c_f \phi + c_s\left(\alpha - \phi\right), \tag{9}$$

where M is referred to the Biot modulus, this is also the inverse of a storage coefficient s_0. For simplified cases, incompressible fluid and solid constituents are assumed Biot's coefficient α = 1 and storage coefficient s_0 = 0.

3. NUMERICAL PROCEDURE

We based the discretization of this mathematical model on the finite element method is used as a numerical approach. We consider the weak formulation of Brinkman's flow, Eq. 1, and the continuity equation, Eq. 2 for an incompressible fluid in previous work Dali *et al.* (2019). The weak formulation of Biot's system is obtained by multiplying the equations by suitable test and trial functions, and further applying the usual concepts to turn a partial differential equation (PDE) into a variational problem (Langtangen and Logg, 2016). The backward Euler scheme was used for time discretization of Eq. 5. We refer to Zhang *et al.* (2015) for more details of these steps.

The developed code used the FEniCS package to facilitate the approximate solution of the partial differential equations. This platform can be used with Python or C++ interface. The code was implemented in Python code to numerically solve the hydromechanical coupling through the 2D domain.

3.1 Fully Coupling

The fully coupled scheme is employed in our model to solve the three governing equation simultaneously at the same reference time. At each time step n, the method tries to find $(p^n, \mathbf{w}^n, \mathbf{u}^n)$ given $\left(p^{n-1}, \mathbf{w}^{n-1}, \mathbf{u}^{n-1}\right)$. Although this method ensures stability and avoids convergence problems, it requires solving a large system (Brun *et al.*, 2020) presenting high computational cost. Therefore, this scheme was used to solve benchmark problems and validate the iterative scheme.

The triangular elements choose to solve the Biot system are elementwise constant space (DG0) for pressure p, the lowest order Raviart-Thomas(RT1) for the fluid velocity $\mathbf{w}$, and the quadratic Continuous Galerkin space (CG2) for solid displacement $\mathbf{u}$. As the DG0-RT1 pair of elements do not satisfy the stability conditions for the Stokes equation, the fully coupled method will only be used in the numerical solution of the Biot's system.

3.2 Iterative Coupling

In the iterative scheme, we decouple flow and mechanics into two problems which are solved sequentially at each iteration. At each iteration step i, the method try to find $\left(p^{n,i}, \mathbf{w}^{n,i}\right)$ given $\left(p^{n,i-1}, \mathbf{w}^{n,i-1}\right)$, and then, get $\left(\mathbf{u}^{n,i}\right)$ given $\left(p^{n,i}, \mathbf{w}^{n,i}, \mathbf{u}^{n,i-1}\right)$.

For this scheme, the Taylor-Hood elements(CG1-CG2) satisfies the conditions required for the flow problem and the polynomial Continuous Galerkin (CG2) is adopted for the solid displacement. The Taylor-Hood set of elements are standard and stable mixed elements for the solution of the Stokes equation. Therefore, the iterative method will be used in both implemented systems of equations, the Biot's model and the developed Biot-Brinkman model.

4. VERIFICATION AND VALIDITY

The model was verified through two problems found in the literature. First, we compare the flow model based on Brinkman equation to a channel-porous medium experiment presented by Yao and Huang (2017). Latter, the numerical algorithm was submitted to the one-dimensional Terzaghi's poroelastic problem and the two-dimensional Mandel's problem. Both problems posses analytical solutions to examine the numerical accuracy on fully coupled and iterative schemes.

4.1 Flow in porous media with macropores

The flow numerical accuracy is validated through velocity profiles in the free-flow domain obtained with LDA experiments by Yao and Huang (2017) and the boundary condition of Beavers and Joseph (1967). Attempt to numerically reproduce the same experiment, the simulation was performed using the same properties established in the former experiment and specified in Tab. 1. The porous medium used in the physical experiments is formed by compacted uniform rock particles.

Table 1: The experimental parameters from Yao and Huang (2017) used in the numerical procedure.

Parameter	**Value**
Pressure gradient $\nabla p(Pa/m)$	-0.33
Matrix permeability $k^*_{pm}(m^2)$	5 x 10^{-9}
Free flow thickness $h(m)$	0.02
Water viscosity $\mu_f(Pa.s)$	0.001
Matrix porosity ϕ	0.45

The computational scheme, as the experimental condition, consists of a single-phase flow forced by a pressure gradient to pass simultaneously through a porous medium and a free flow region. The domain is composed by a channel of width Δ = 0.02 m over a porous medium whose permeability is 5 x $10^{-9} m^2$. This setup is similar to Poiseuille flow in parallel plates. Since the flow at the channel is described by the Stokes flow, we need to assume a no-slip boundary condition on the upper boundary.

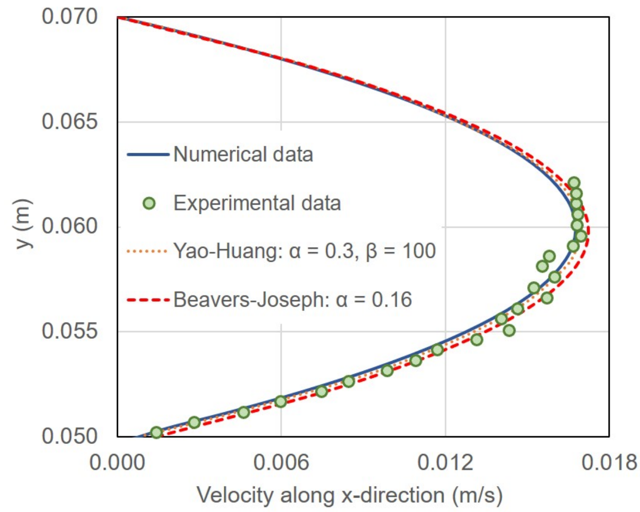


Figure 2: Comparison of velocity profile between numerical, theoretical and experimental values.

Figure 2 shows the numerical velocity profile obtained by our model, the semi-empirical Beavers and Joseph (1967) condition, experimental measurements and a theoretical interface condition proposed by Yao and Huang (2017). This comparison among the curves matches well LDA experiment values.

The velocity profile acquired by Yao and Huang (2017) experiment is similar to the aluminum sand of the Beavers and Joseph (1967) experiments. The computational data also shows a similar slip velocity at the interface of the channel and porous medium. It is important to note that this interface velocity captured is larger than Darcy velocity, in agreement with the literature. These comparisons provided comprehensive data to verify the model and evaluate whether the model can reproduce the Beavers-Joseph boundary condition on seepage flow.

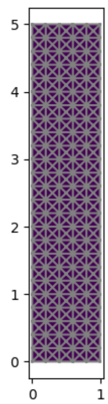


Figure 3: Mesh of Terzaghi's problem.

4.2 Terzaghi's One-Dimensional Consolidation

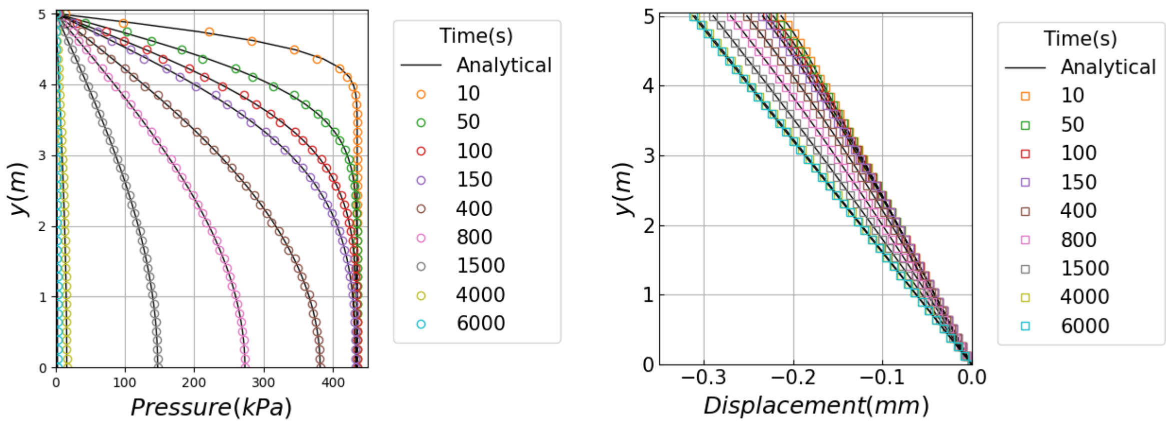
We first consider the test case presented by Tonelli *et al.* (2016) to verify the agreement with Terzaghi's analytical solutions. Figure 3 shows the fully saturated, homogeneous, porous vertical column with height H = 5 m and length L = 1 m.

In this 1D problem, a constant downward load F is imposed at the top of the beam, y = 5 m. This side is a free surface assumed to be fully drained, while the other surfaces are impermeable. The properties of the porous medium and the input parameters are listed in Tab. 2.

Table 2: Parameters for Terzaghi's problem.

Symbol	Parameter	Value
λ	First Lamé parameter	4.0x10^9 Pa
ν	Poisson's ratio	0.2
α	Biot's coefficient	0.777778
F	Overload	1.0x10^6 Pa
k^*_{pm}	Permeability	1.0x10^{-15} m^2
μ_f	Fluid viscosity	0.001 Pa.s
ϕ	Porosity	0.19
c_f	Fluid compressibility	3.030303x10^{-10} Pa^{-1}
c_s	Compressibility of the solid grains	2.777778x10^{-11} Pa^{-1}
Δt	Time step length	0.1 s

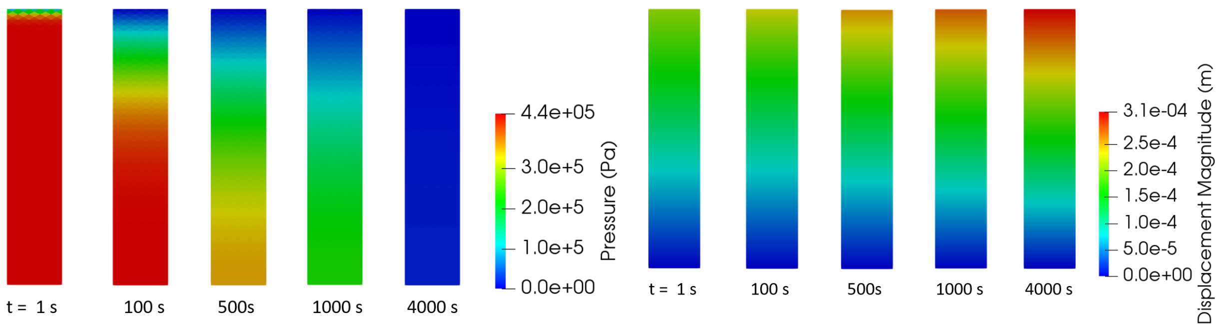
The good agreement between our numerical fully coupled result and the analytical solution are shown in Fig. 4. The analytical solutions for pressure and vertical displacement are taken from Haagenson (2020). Figure 5 shows the pressure and vertical displacement fields for selected times using the fully coupled scheme. Here is possible to check that the solution of pressure and displacement varies only along the y-direction.



(a) Pressure. (b) Displacement.

Figure 4: Comparison of the numerical and analytical solutions of Terzaghi's problem.

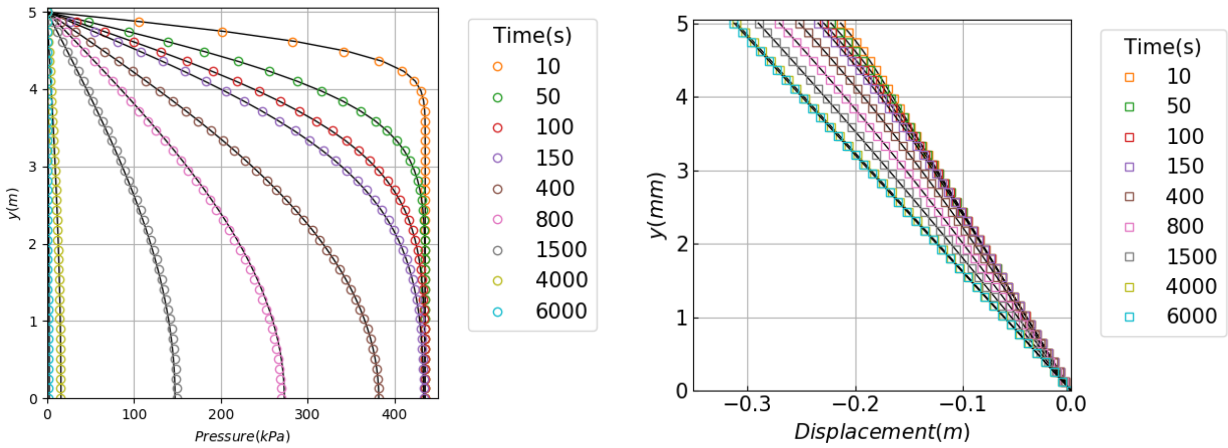
Analyzing Figs. 4 and 5 is possible to see the increase in water pressure by the load. Then, the water pumps, the velocity of the dissipation process depends on the permeability value and the pore pressure gradually decreases, the drained response, until the load be held by the solid part of the column.



(a) Pressure. (b) Displacement.

Figure 5: Comparison of the numerical and analytical solutions of Terzaghi's problem.

The second implementation of Terzaghi's problem applied the iterative approach. In this case, we used the convergence criteria for the iterative step based on the L_2 norm of the pressure and displacement relative error to the previous iterative value. The tolerance is set to $tol < 10^{-9}$ for the relative error. Figure 6 denotes that the iterative simulation achieved the expected one-dimensional behavior for the consolidation with a drained top boundary. Figures 4 and 6 demonstrate that both implemented strategies are correct.



(a) Pressure. (b) Displacement.

Figure 6: Comparison of the numerical and analytical solutions of Terzaghi's problem.

4.3 Mandel's Two-Dimensional Problem

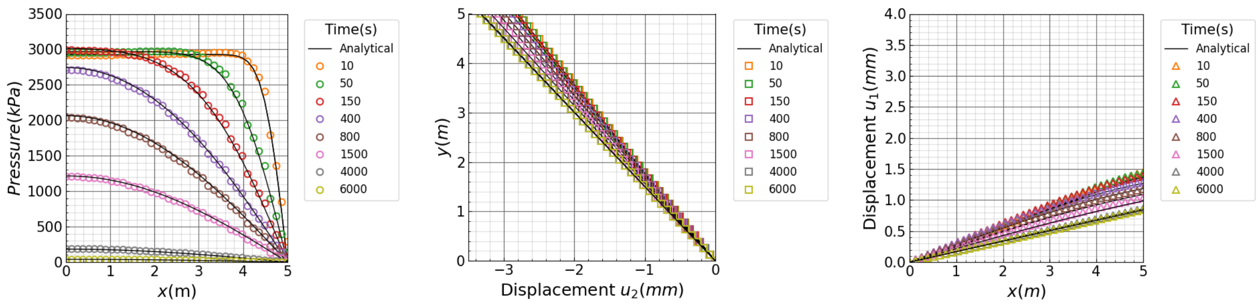
The proposed problem uses a two-dimensional porous medium with width 2a and height 2b, saturated with fluid. The section is subjected to a uniform and constant load F on the upper and lower edges through a rigid plate. The lateral boundaries are traction free and allowed to drain for instants of time t > 0. The geometry to be virtually constructed can be reduced to the upper right quadrant due to the symmetry of the problem in relation to the horizontal and vertical axis.

The Mandel-Cryer effect is the nonmonotonic fluid pressure variation with time in response to a constant loading. This critical phenomenon is a numerical challenge that can be replicated with a fully coupled poroelastic approach.

Since the plates are considered rigid, we would need to enforce that there is no shear strain on the top boundary by adding constrained equations to ensure that the vertical displacements along the top plate are all equal to an unknown constant value or further complicate the mathematical formulation by introducing Lagrange multipliers. On the other hand, the traction boundary condition can be easily applied through the weak form of the mechanical equilibrium.

Then, to verify the model behavior with compressible fluid and solid grains, we just apply the traction boundary condition. In this case, the rigid plate condition is not satisfied, however, the shear strain are naturally zero along the bottom boundary. Therefore, according to Haagenson (2020), the pressure solution along the bottom boundary can be compared to the analytical solution to verify the model's behavior. Nevertheless, we also present numerical solutions for the horizontal and vertical displacement.

Once again, we consider the test case presented by Tonelli *et al.* (2016) to verify the agreement with Mandel's analytical solutions. Poroelastic properties of the porous medium and other model parameters for the problem are almost the same as summarized in 2, the only difference is the loading, for this case the value is $F = 5\text{x}10^7$ N. A geometry with 5×5 m was evaluated.



(a) Pressure. (b) Displacement u_y. (c) Displacement u_x.

Figure 7: Comparison of the numerical and analytical solutions of Mandel's problem.

The numerical solutions for the Mandel's problem using the fully coupled method and the analytical solutions are presented in 7. Figure 7a demonstrates the comparison for pressure at eight different time steps. Even without the implementation of the rigid plate, the numerical solution for the pore pressure at bottom wall is very close to the analytical solutions.

Similar to Terzaghi's 1D problem, the pressure initially increases and then slowly dissipates as fluid continues to drain according to the permeability of the porous matrix. The instant loading leads to an undrained response, after an initial rise the fluid pressure increases far from the drained boundary before dissipating.

The graphs in Figs. 7b and 7c shows the comparison of solutions for displacements. The vertical displacement u_y of the upper surface was taken from $x = a/2$. The horizontal displacement u_x of the top boundary shown in Fig. 7b almost coincides with the analytical result.

5. COMPUTATIONAL RESULTS

In order to capture the structural influence on the flow, numerical simulations have been performed for a simplified unit cell Ω_{pm} involving a circular vug Ω_{vug} with macroporosity ϕ_m = 0.15. The two-dimensional analysis gives a qualitative description of the final shape of the macropore.

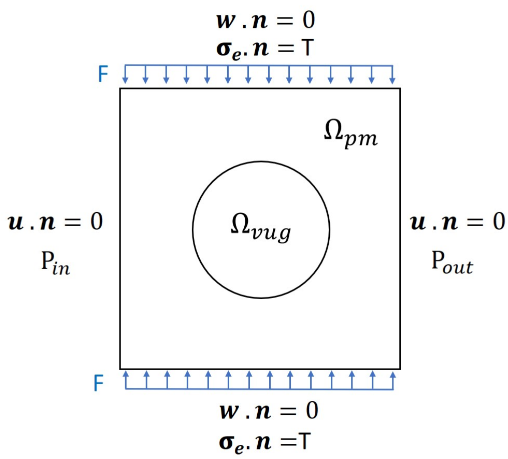


Figure 8: Schematic view of the porous medium with the boundary conditions.

When t>0, the proper boundary conditions for the flow and solid problem are specified in Fig. 8. We assume no external forces or mass sources. Homogeneous Dirichlet displacement conditions were imposed on the left and right sides for zero normal poroelastic stress, while on the top and bottom, we imposed impermeable boundaries. For the solid part, the top and bottom surface are also exposed to a vertical load F. The natural condition of the pressure was imposed on the left and right sides.

Table 3: Physical parameters

Symbol	Parameter	Value
E	Young's modulus	5.0x10^7 Pa
ν	Poisson's ratio	0.2
α	Biot's coefficient	0.777778
F	Overload	1.0x10^6 Pa
k^*_{pm}	Permeability	1.0x10^{-11} m^2
μ_f	Fluid viscosity	0.001 Pa.s
ϕ	Porosity	0.27
ϕ_m	Macroporosity	0.15
s_0	Storage coefficient	6.06x10^{-11} Pa^{-1}
∇p	Pressure gradient	-6.9 kPa/m

Table 3 summarizes the dimensions of the geometry and its material properties used in the simulations. Figure 9 shows the solution fields for the pressure, flow, and displacement at the selected time t = 4.5 s.

The normalized permeability k_{Eq}/k_{pm} calculated using the flow rate on Darcy's equation is 1.34. The excess in equivalent permeability k_{Eq}>1 describes the gain in flow rate by the presence of the vug.

Furthermore, numerical calculations have been performed for different values of Young's modulus E and permeability of the porous matrix k^*_{pm} to verify the limitations of the model.

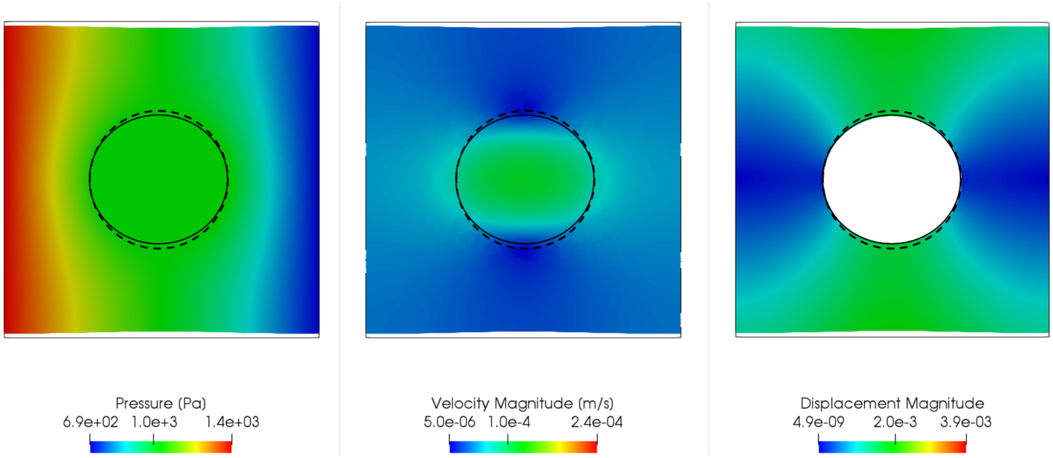


Figure 9: The solution of pressure, velocity, and displacement at time t = 4.5 s. The dashed line denotes the original geometry of the vug at t = 0.

As expected, the domain decreases more with a softer surrounded matrix. The micropores were compressed, then the flow rate decreases inside the porous matrix. With a high value for Young's modulus, the very hard material is slightly compressible, keeping almost the original domain size.

6. CONCLUSIONS

In this work, we have developed a fully coupled and iterative implementation of a hydromechanical coupling problem for linear poroelasticity using the finite element method. The flow simulation was validated through the Beavers and Joseph (1967) velocity equation and the experimental results of Yao and Huang (2017).

Solving the system of equations either fully coupled or iteratively yields compatible solutions. The numerical model was partially validated according to the Terzaghi's consolidation problem and Mandel's problem. The comparison of a fully coupled, iterative scheme and an analytical solution showed good agreement in Terzaghi's one-dimensional problem. The poroelastic model captured the Mandel-Cryer effect at the bottom boundary as the rigid plate condition was not implemented. The numerical model based on the Biot-Brinkman system of equations was successfully implemented using the iterative scheme and applied in a single-vug poroelastic medium.

7. ACKNOWLEDGEMENTS

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